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OBSERVER DESIGN FOR A CLASS OF NONLINEAR DESCRIPTOR SYSTEMS WITH MEASUREMENT NOISE

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In this paper, a robust H_∞ observer design method is proposed for a class of nonlinear descriptor systems subject to measurement noise and unknown disturbances. Specifically, a novel augmented system is constructed based on the output filtering method. Next, an observer resisting measurement noise is presented and the state estimation error system is derived. The observer parameters are parameterized by employing the generalized inverse technique for matrices. Finally, sufficient conditions ensuring the asymptotic stability of the state estimation error system with the H_∞ performance index are derived on the basis of the linear matrix inequalities (LMIs). Compared to the conventional robust H_∞ observers, simulation results demonstrate that the proposed method alleviates the adverse effects of measurement noise on the state estimation and results in smaller estimation errors.

Keywords: descriptor systems, H_∞ observer, linear matrix inequalities (LMIs), output filtering method

Classification: 93B36, 93C10

1. INTRODUCTION

Under actual operating conditions, disturbances and high-frequency measurement noise[3, 7, 14] are inevitable. Noise may be caused by sudden changes in the environment or component performance, such as nonlinear drifts and aging electronic components. When system outputs contain measurement noise, there may be product terms between the measurement noise and the observer gain, which can amplify the adverse effects of the measurement noise on state estimation. To mitigate their impact, the Kalman filter [13] is widely used and played a pivotal role in the Apollo moon landing program. However, its application is limited by the requirement that disturbance statistics be known a priori. Later, the H_∞ technique [30] was proposed, which only required disturbances/noise to be bounded. It aims to minimize the gain from disturbances/noise to the control objective, thereby enhancing the system's robustness to disturbances/noise.

Descriptor systems, also known as generalized state space systems, singular systems, or differential-algebraic systems, constitute a special class of systems that are described by both differential and algebraic equations. Many practical systems, such as biological

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systems [21], circuit systems [26], constrained mechanical systems [17], and energy systems [29], have been successfully modeled by descriptor systems. Xu and Lam were the first to introduce an H_∞ observer design method for linear regular descriptor systems [28]. Alma investigated an adaptive observer design method for a class of nonlinear descriptor systems, allowing the simultaneous estimation of both the system states and the unknown constant parameters [1]. In addition, numerous studies have addressed the design of state observers to cope with various challenges, such as uncertainties [12, 31], time delays [2, 10, 24], nonlinearities [18, 5, 23], unknown inputs [11, 25], prescribed-time estimation [15, 16], and interval estimation [19, 20].

However, few results have been presented for observer design of descriptor systems with measurement noise. Gupta et al. [9] used singular value decomposition (SVD) to divide the output of the system into two parts, and constructed a state observer using the noise-free output vector. Thereby the influence of noise was decoupled. Zhu et al. [32] utilized the generalized inverse technique to eliminate noise in the output equation. Tunga et al. [27] employed unknown input theory to decouple the influence of noise simultaneously presented in both the state equation and the output equation. However, all the above mentioned methods require the coefficient matrix of the system noise term to satisfy additional rank conditions. Thus, their application scope is limited.

In practical scenarios, noise terms may not always satisfy the aforementioned rank conditions. In order to solve this problem, we try to develop a general-purpose robust H_∞ observer for descriptor systems with measurement noise and unknown disturbances. Our contributions can be outlined as follows.

1. An augmented system is constructed based on the output filtering method, and an observer resisting measurement noise is presented.
2. The proposed method may yield smoother estimation errors with smaller magnitudes. Therefore, fluctuations in state estimation caused by measurement noise are mitigated.

The remaining structure of this article is organized as follows. Some preliminaries are given in Section 2. In Section 3, we give the main results, i. e., observer design based on the output filtering method. In Section 4, an example is used to illustrate the effectiveness of the proposed method. Finally, some conclusions are drawn in Section 5.

For a real symmetric matrix X, $X > 0 (X < 0)$ means X is a positive (negative) definite matrix. $\|\cdot\|$ denotes the induced matrix 2-norm. I is an identity matrix with appropriate dimensions. $\mathbb{R}^n$ denotes n-dimensional real Euclidean space. $\mathbb{C}$ is the set of complex numbers. A^+ represents the Moore-Penrose pseudoinverse of matrix A. $E^\perp$ stands for the orthogonal complement of E, satisfying the orthogonality condition $E^\perp E = 0$.

2. PRELIMINARIES

Consider a class of nonlinear descriptor systems described in the following form:

$$E\dot{x} = Ax + B_1 u + D_1 h(F_l x) + G_1 w, \tag{1a}$$

$$y = Cx + G_2 v, \tag{1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_u}$, $h(F_l x) \in \mathbb{R}^{n_h}$, and $y \in \mathbb{R}^{n_p}$ are the state vector, the input vector, a known nonlinear function, and the output vector, respectively. $w \in \mathbb{R}^{n_w}$ is the bounded external disturbance. $v \in \mathbb{R}^{n_v}$ is the measurement noise. Furthermore, $E \in \mathbb{R}^{n\times n}$ with $\operatorname{rank}(E) = r < n$, $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times n_u}$, $D_1 \in \mathbb{R}^{n\times n_h}$, $G_1 \in \mathbb{R}^{n\times n_w}$, $C \in \mathbb{R}^{n_p\times n}$, $G_2 \in \mathbb{R}^{n_p\times n_v}$, and $F_l \in \mathbb{R}^{n_h\times n}$ are known constant matrices.

Definition 2.1. (Dai [4]) For the system $E\dot{x} = Ax$, the pair (E, A) is said to be regular if there exists a scalar $s \in \mathbb{C}$ such that $\det(sE - A) \neq 0$, which ensures that the system $E\dot{x} = Ax$ has a unique solution. This system is said to be stable if all the roots of $\det(sE - A) = 0$ have negative real parts.

Assumption 2.2. (Gao and Ho [8]) For the system (1), the following impulse-free condition holds:

$$\operatorname{rank}\begin{bmatrix} E & 0 \\ A & E \end{bmatrix} = n + \operatorname{rank}(E).$$

Assumption 2.3. (Gao and Ho [8]) The triple (E, A, C) is finitely observable, that is for all $s \in \mathbb{C}$:

$$\operatorname{rank}\begin{bmatrix} sE - A \\ C \end{bmatrix} = n.$$

Assumption 2.4. The nonlinearity $h(F_l x)$ satisfies the Lipschitz condition. That is, for x_1, $x_2 \in \mathbb{R}^n$ and a Lipschitz constant $\lambda > 0$, it satisfies

$$\left\| h\left(F_l x_1\right) - h\left(F_l x_2\right) \right\| \leqslant \lambda \left\| F_l(x_1 - x_2) \right\|. \tag{2}$$

The following output filter transformation is introduced to convert the system measurement noise into input perturbation:

$$\begin{aligned} \dot{x}_f &= A_f x_f + B_f y, \\ y_f &= C_f x_f, \end{aligned} \tag{3}$$

where $x_f \in \mathbb{R}^f$ and $y_f \in \mathbb{R}^f$ are the state vector and the output vector of the filter, respectively. $A_f \in \mathbb{R}^{f\times f}$, $B_f \in \mathbb{R}^{f\times n_p}$, and $C_f \in \mathbb{R}^{f\times f}$ are some constant matrices to be selected.

Let $\bar{x}^T = \begin{bmatrix} x^T & x_f^T \end{bmatrix}^T$. An augmented system is constructed from (1) and (3), written in a compact form:

$$\bar{E}\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_1 u + \bar{D}_1 h(\bar{F}_l\bar{x}) + \bar{G}_1\bar{w}, \tag{4a}$$

$$\bar{y}^T = \begin{bmatrix} y^T & y_f^T \end{bmatrix}^T, \tag{4b}$$

$$y = C_1\bar{x} + \bar{G}_2\bar{w}, \tag{4c}$$

$$y_f = C_2\bar{x}, \tag{4d}$$

where $\bar{y}$ is the output vector of the augmented descriptor system, $\bar{w} = \begin{bmatrix} w \\ v \end{bmatrix}$, $\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}$, $\bar{A} = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix}$, $\bar{G}_1 = \begin{bmatrix} G_1 & 0 \\ 0 & B_f G_2 \end{bmatrix}$, $\bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$, $\bar{D}_1 = \begin{bmatrix} D_1 \\ 0 \end{bmatrix}$, $\bar{F}_l = \begin{bmatrix} F_l & 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} C & 0 \end{bmatrix}$, $\bar{G}_2 = \begin{bmatrix} 0 & G_2 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & C_f \end{bmatrix}$, $\bar{E}^{\perp} = \begin{bmatrix} E^{\perp} & 0 \end{bmatrix}$, and the order of system (4) is $n + f$.

Lemma 2.5. The triple $(\bar{E}, \bar{A}, C_2)$ is finitely observable provided that system (1) satisfies Assumption 2.3 and the matrices B_f, C_f have full column rank.

Proof. In this case, we have the following equality:

$$\operatorname{rank}\begin{bmatrix} s\bar{E}-\bar{A} \\ C_2 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} sE-A & 0 \\ -B_fC & sI-A_f \\ 0 & C_f \end{bmatrix} = n+f,$$

for all $s \in \mathbb{C}$. The proof is completed. □

Lemma 2.6. The augmented system (4) is impulse-free, and the following equation holds, if system (1) satisfies Assumption 2.2.

$$\operatorname{rank}\begin{bmatrix} \bar{E} \\ \bar{E}^{\perp}\bar{A} \end{bmatrix} = n+f. \tag{5}$$

Proof. Under Assumption 2.2, we have

$$\operatorname{rank}\begin{bmatrix} \bar{E} & 0 \\ \bar{A} & \bar{E} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} E & 0 & 0 & 0 \\ A & E & 0 & 0 \\ 0 & 0 & I & 0 \\ B_fC & 0 & A_f & I \end{bmatrix} = n+f+\operatorname{rank}(\bar{E}),$$

which means that the augmented system (4) is impulse-free. In addition,

$$\operatorname{rank}\begin{bmatrix} \bar{E} & 0 \\ \bar{A} & \bar{E} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} I & 0 \\ 0 & \bar{E}^{\perp} \\ 0 & \bar{E}\bar{E}^{+} \end{bmatrix}\begin{bmatrix} \bar{E} & 0 \\ \bar{A} & \bar{E} \end{bmatrix}\begin{bmatrix} I & 0 \\ -E^{+}A & I \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \bar{E} & 0 \\ \bar{E}^{\perp}\bar{A} & 0 \\ 0 & \bar{E} \end{bmatrix}.$$

The conclusion holds. The proof is completed. □

Lemma 2.7. For $W \in \mathbb{R}^{a\times b}$, $Z \in \mathbb{R}^{b\times c}$, and $U \in \mathbb{R}^{a\times c}$, if $\operatorname{rank}(Z) = c$, then the matrix equation $WZ = U$ has the following solution,

$$W = UZ^{+} + \Phi[I_b - ZZ^{+}],$$

where $\Phi \in \mathbb{R}^{a\times b}$ is an arbitrary matrix with proper dimensions.

3. MAIN RESULTS

In this section, a state observer is presented, and the stability of the state estimation error system is rigorously analyzed using Lyapunov-based methods. Thereafter, a parameterized design methodology for observer parameters is proposed. Ultimately, the explicit design methods of the observer are derived.

An observer for the augmented system (4) is designed as,

$$\dot{\xi} = N\xi + Ju + Fy + Hy_f + T\bar{D}_1 h(\bar{F}_l\hat{\bar{x}}), \tag{6a}$$

$$\hat{\bar{x}} = P\xi - R\bar{E}^{\perp}\bar{B}_1 u - R\bar{E}^{\perp}\bar{D}_1 h(\bar{F}_l\hat{\bar{x}}) + Ly_f, \tag{6b}$$

where $\xi \in \mathbb{R}^q$, and $\hat{\bar{x}} \in \mathbb{R}^{n+f}$ are the observer state vector and the estimate vector of $\bar{x}$, respectively. N, J, F, H, T, P, R, and L are unknown parameter matrices with appropriate dimensions.

Remark 3.1. Compared to the work in [5], an output filter (3) is introduced. Since the state x_f is available, we can select $C_f = I$. The matrices A_f and B_f can be designed by the trial and error method. Specifically, the matrix A_f is used to tune the filter's bandwidth. In order to reduce the impact of measurement noise on the output y_f, the elements of the matrix B_f can be selected small. In addition, the stricter rank conditions in [9, 27, 32] are removed.

3.1. Stability analysis

Multiplying both sides of (4a) by matrix $\bar{E}^{\perp}$ yields

$$-\bar{E}^{\perp}\bar{B}_1 u = \bar{E}^{\perp}\bar{A}x + \bar{E}^{\perp}\bar{D}_1 h(\bar{F}_l\bar{x}) + \bar{E}^{\perp}\bar{G}_1 w.$$

Define $e = \hat{\bar{x}} - \bar{x}, \varepsilon = \xi - T\bar{E}\bar{x}$, and $e_h = h(\bar{F}_l\hat{\bar{x}}) - h(\bar{F}_l\bar{x})$. The state estimation error system can be simplified as,

$$\dot{\varepsilon} = N\varepsilon + \hat{D}_1 e_h + \hat{G}_1\bar{w}, \tag{7a}$$

$$e = P\varepsilon + \hat{D}_2 e_h + \hat{G}_2\bar{w}, \tag{7b}$$

where $\hat{D}_1 = T\bar{D}_1,\ \hat{D}_2 = -R\bar{E}^{\perp}\bar{D}_1,\ \hat{G}_1 = F\bar{G}_2 - T\bar{G}_1,\ \hat{G}_2 = R\bar{E}^{\perp}\bar{G}_1$. The matrices N, J, F, H, T, P, R, and L are required to satisfy the following conditions:

$$J = T\bar{B}_1, \tag{8a}$$

$$T\bar{A} = NT\bar{E} + FC_1 + HC_2, \tag{8b}$$

$$I = PT\bar{E} + R\bar{E}^{\perp}\bar{A} + LC_2. \tag{8c}$$

The objective is to design the observer (6) such that the conditions (8) are satisfied and the state estimation error system (7) achieves asymptotic stability for $\bar{w} = 0$, and the following H_∞ performance index is guaranteed under $\bar{w} \neq 0$ [22],

$$\int_0^\infty \|e\|^2 \mathrm{d}t - \int_0^\infty \gamma^2 \|\bar{w}\|^2 \mathrm{d}t - x_0^T X x_0 < 0, \tag{9}$$

where x_0 is the value of the state at the initial time, and the matrix $X > 0$.

Sufficient conditions are established through the following theorem.

Theorem 3.2. For the augmented system (4) and the observer (6), if there exist a positive definite matrix X, and a matrix Λ such that the following LMIs hold:

$$\begin{bmatrix} N^TX+XN & X\hat{D}_1 & P^T & X\hat{G}_1 \\ \hat{D}_1^TX & -\Lambda & \hat{D}_2^T & 0 \\ P & \hat{D}_2 & -\Sigma^{-1} & \hat{G}_2 \\ \hat{G}_1^TX & 0 & \hat{G}_2^T & -\gamma^2 I \end{bmatrix} < 0, \Lambda < I, X > 0, \tag{10}$$

where $\Sigma = I_{n+f} + \lambda^2 \bar{F}_l^T \bar{F}_l$, then the state estimation error system (7) asymptotically converges to zero for $\bar{w} = 0$ and satisfies the H_∞ performance index (9) under $\bar{w} \neq 0$.

Proof. According to the Lipschitz condition (2), the following inequality can be obtained,

$$e_h^T e_h \leqslant \lambda^2 e^T \bar{F}_l^T \bar{F}_l e.$$

Choose the Lyapunov function as $V = \varepsilon^T X \varepsilon$. Thus, we have

$$\begin{aligned} \dot{V} =&(N\varepsilon + \hat{D}_1 e_h + \hat{G}_1 \bar{w})^T X\varepsilon + \varepsilon^T X(N\varepsilon + \hat{D}_1 e_h + \hat{G}_2 \bar{w}) \\ &+ e_h^T \Lambda e_h - e_h^T \Lambda e_h \\ <&(N\varepsilon + \hat{D}_1 e_h + \hat{G}_1 \bar{w})^T X\varepsilon + \varepsilon^T X(N\varepsilon + \hat{D}_1 e_h + \hat{G}_2 \bar{w}) \\ &+ \lambda^2 e^T \bar{F}_l^T \bar{F}_l e - e_h^T \Lambda e_h. \end{aligned}$$

In the case of $\bar{w} \neq 0$, let $\zeta^T = \begin{bmatrix} \varepsilon^T & e_h{}^T & \bar{w}^T \end{bmatrix}$, then

$$\begin{aligned} &\int_0^\infty \left(e^T e - \gamma^2 \bar{w}^T \bar{w} + \dot{V} \right) \mathrm{d}t \\ <&\int_0^\infty e^T (I + \lambda^2 \bar{F}_l^T \bar{F}_l) e - \gamma^2 \bar{w}^T \bar{w} + (N\varepsilon + \hat{D}_1 e_h + \hat{G}_1 \bar{w})^T X\varepsilon \\ &+ \varepsilon^T X(N\varepsilon + \hat{D}_1 e_h + \hat{G}_2 \bar{w}) - e_h^T \Lambda e_h, \\ <&\int_0^\infty \zeta^T \begin{bmatrix} P^T\Sigma P + N^TX + XN & X\hat{D}_1 + P^T\Sigma\hat{D}_2 & P^T\Sigma\hat{G}_2 + X\hat{G}_1 \\ \hat{D}_1^TX + \hat{D}_2^T\Sigma P & -\Lambda + \hat{D}_2^T\Sigma\hat{D}_2 & 0 \\ \hat{G}_2^T\Sigma P + \hat{G}_1^TX & 0 & \hat{G}_2^T\Sigma\hat{G}_2 - \gamma^2 I \end{bmatrix} \zeta \mathrm{d}t < 0. \end{aligned}$$

By Schur complement, it has

$$\begin{aligned} &\begin{bmatrix} P^T\Sigma P + N^TX + XN & X\hat{D}_1 + P^T\Sigma\hat{D}_2 & P^T\Sigma\hat{G}_2 + X\hat{G}_1 \\ \hat{D}_1^TX + \hat{D}_2^T\Sigma P & -\Lambda + \hat{D}_2^T\Sigma\hat{D}_2 & 0 \\ \hat{G}_2^T\Sigma P + \hat{G}_1^TX & 0 & \hat{G}_2^T\Sigma\hat{G}_2 - \gamma^2 I \end{bmatrix} < 0, \\ \Leftrightarrow &\begin{bmatrix} N^TX+XN & X\hat{D}_1 & P^T & X\hat{G}_1 \\ \hat{D}_1^TX & -\Lambda & \hat{D}_2^T & 0 \\ P & \hat{D}_2 & -\Sigma^{-1} & \hat{G}_2 \\ \hat{G}_1^TX & 0 & \hat{G}_2^T & -\gamma^2 I \end{bmatrix} < 0, \end{aligned}$$

which guarantees the H_∞ performance index (9) is satisfied.

In the case of $\bar{w} = 0$, $\dot{V} < 0$ is equivalent to

$$\begin{bmatrix} N^T X + XN & X\hat{D}_1 & P^T \\ \hat{D}_1^T X & -\Lambda & \hat{D}_2^T \\ P & \hat{D}_2 & \Sigma^{-1} \end{bmatrix} < 0,$$

which can be guaranteed by the theorem 3.2. The proof is completed. □

3.2. Observer parameter parameterization

In this part, we parameterize the matrices N, J, F, H, T, P, R, and L satisfying the conditions in (8). From (5), rank $\begin{bmatrix} \bar{E} \\ \bar{E}^\perp \bar{A} \end{bmatrix} = n + f$. Therefore,

$$\text{rank} \begin{bmatrix} \bar{E} \\ \bar{E}^\perp \bar{A} \\ C_2 \\ U \end{bmatrix} = n + f,$$

where $U \in \mathbb{R}^{q \times (n+f)}$ is a known matrix and will be determined later. Then there exist matrices $\tilde{T}, K$ satisfying that

$$\tilde{T}\bar{E} + K \begin{bmatrix} \bar{E}^\perp \bar{A} \\ C_2 \end{bmatrix} = U. \tag{11}$$

Let $\Phi = 0$. From Lemma 2.7, we can obtain that,

$$\tilde{T} = U \begin{bmatrix} \bar{E} \\ \bar{E}^\perp \bar{A} \\ C_2 \end{bmatrix}^+ \begin{bmatrix} I \\ 0 \end{bmatrix}. \tag{12a}$$

From (11), it follows that

$$\text{rank} \begin{bmatrix} \tilde{T}\bar{E} \\ \bar{E}^\perp \bar{A} \\ C_2 \end{bmatrix} = \text{rank} \begin{bmatrix} I & -K \\ 0 & I \end{bmatrix} \begin{bmatrix} U \\ \bar{E}^\perp \bar{A} \\ C_2 \end{bmatrix} = \text{rank} \begin{bmatrix} U \\ \bar{E}^\perp \bar{A} \\ C_2 \end{bmatrix}.$$

On the other hand, the conditions (8b) and (8c) can be rewritten as,

$$\begin{bmatrix} N & \Psi & F & H \end{bmatrix} \begin{bmatrix} \tilde{T}\bar{E} \\ \bar{E}^\perp \bar{A} \\ C_1 \\ C_2 \end{bmatrix} = \tilde{T}\bar{A}, \tag{13a}$$

$$\begin{bmatrix} P & R & L \end{bmatrix} \begin{bmatrix} \tilde{T}\bar{E} \\ \bar{E}^\perp \bar{A} \\ C_2 \end{bmatrix} = I \tag{13b}$$

where $\tilde{T} = T + \Psi\bar{E}^{\perp}$, Ψ is an unknown matrix with appropriate dimensions. Based on Lemma 2.7, the condition for its solvability is

$$\operatorname{rank}\begin{bmatrix} \tilde{T}\bar{E} \\ \bar{E}^{\perp}\bar{A} \\ C_2 \end{bmatrix} = n + f. \tag{14}$$

Thus, under the condition of (14), $U \in \mathbb{R}^{q\times(n+f)}$ is needed to be selected such that

$$\operatorname{rank}\begin{bmatrix} U \\ \bar{E}^{\perp}\bar{A} \\ C_2 \end{bmatrix} = n + f. \tag{15}$$

Define $\mathcal{U}_1 = \begin{bmatrix} \tilde{T}\bar{E} \\ \bar{E}^{\perp}\bar{A} \\ C_1 \\ C_2 \end{bmatrix}$, $\mathcal{U}_2 = \begin{bmatrix} \tilde{T}\bar{E} \\ \bar{E}^{\perp}\bar{A} \\ C_2 \end{bmatrix}$. The solution of (13) is given by,

$$P = P_1 + \Phi_1 P_2, R = R_1 + \Phi_1 R_2, L = L_1 + \Phi_1 L_2, \tag{16a}$$

$$N = N_1 + \Phi_2 N_2, \Psi = \Psi_1 + \Phi_2 \Psi_2, F = F_1 + \Phi_2 F_2, \tag{16b}$$

$$H = H_1 + \Phi_2 H_2, \tag{16c}$$

where

$$\begin{aligned}
P_1 &= \mathcal{U}_2^{+}\begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T, P_2 = \left(I - \mathcal{U}_2\mathcal{U}_2^{+}\right)\begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T, \\
R_1 &= \mathcal{U}_2^{+}\begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T, R_2 = \left(I - \mathcal{U}_2\mathcal{U}_2^{+}\right)\begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T, \\
L_1 &= \mathcal{U}_2^{+}\begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T, L_2 = \left(I - \mathcal{U}_2\mathcal{U}_2^{+}\right)\begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T, \\
N_1 &= \mathcal{U}_1^{+}\begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T, N_2 = \left(I - \mathcal{U}_1\mathcal{U}_1^{+}\right)\begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T, \\
\Psi_1 &= \mathcal{U}_1^{+}\begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T, \Psi_2 = \left(I - \mathcal{U}_1\mathcal{U}_1^{+}\right)\begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T, \\
F_1 &= \mathcal{U}_1^{+}\begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}^T, F_2 = \left(I - \mathcal{U}_1\mathcal{U}_1^{+}\right)\begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}^T, \\
H_1 &= \mathcal{U}_1^{+}\begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T, H_2 = \left(I - \mathcal{U}_1\mathcal{U}_1^{+}\right)\begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T.
\end{aligned}$$

3.3. Observer parameter synthesis via LMIs

In this part, a parameterized solution for the observer gains is first proposed, and then an algorithm is given for the H_∞ observer (6).

Theorem 3.3. For any given matrix U satisfying (15), if there exist positive definite matrices X, Λ, matrix Φ_1, and matrix Y satisfying the following matrix inequalities

$$\begin{bmatrix} M_1 & M_2 & M_3 & M_4 \\ M_2{}^T & -\Lambda & M_5 & 0 \\ M_3{}^T & M_5^T & -\Sigma^{-1} & M_6 \\ M_4{}^T & 0 & M_6{}^T & -\gamma^2 I \end{bmatrix} < 0,\ \Lambda < I,\ X > 0, \tag{17}$$

where

$$M_1 = N_1^T X + XN_1 + YN_2 + N_2^T Y^T, M_2 = X\tilde{T}\bar{D}_1 - X\Psi_1\bar{E}^{\perp}\bar{D}_1 - Y\Psi_2\bar{E}^{\perp}\bar{D}_1,$$
$$M_3 = P_1^T + P_2^T\Phi_1^T, M_4 = XF_1\bar{G}_2 + YF_2\bar{G}_2 - X\tilde{T}\bar{G}_1 + X\Psi_1\bar{E}^{\perp}\bar{G}_1 + Y\Psi_2\bar{E}^{\perp}\bar{G}_1,$$
$$M_5 = -\bar{D}_1^T\bar{E}^{\perp T}R_1^T - \bar{D}_1^T\bar{E}^{\perp T}R_2^T\Phi_1^T, M_6 = R_1\bar{E}^{\perp}\bar{G}_1 + \Phi_1 R_2\bar{E}^{\perp}\bar{G}_1, \Phi_2 = X^{-1}Y,$$

then the state estimation error system (7) asymptotically converges to zero for $\bar{w} = 0$ and satisfies the H_∞ performance index (9) under $\bar{w} \neq 0$.

Proof. The proof is completed by substituting (12) and (16) into Theorem 3.2. □

The optimal index γ can be obtained by solving the following optimization problem:

$$\begin{aligned} &\min \quad \gamma \\ &\text{s.t. } (17) \end{aligned} \tag{18}$$

The following algorithm summarizes the procedure for designing the observer (6).

Algorithm 1 Procedure of the observer (6) design.

1: Select the filter parameter matrices A_f, B_f, and C_f;
2: Determine the order q of the observer;
3: Suppose that matrix $U \in \mathbb{R}^{q\times(n+f)}$ is a binary matrix with elements of 0 or 1 and iterate through all possible values;
4: Select all possible U matrices that satisfy (15);
5: Determine $\tilde{T}$ via (12) and solve the optimization problem (18). Calculate the H_∞ index γ and the observer parameters corresponding to each matrix U;
6: The design parameters of the observer (6) are selected from the solutions obtained by step 5 with the optimal H_∞ index γ.

Remark 3.4. Distinct from the approach presented in [5], the proposed observer utilizes a filtered output y_f, which introduces an auxiliary parameter H into (8b). This provides some degrees of freedom when seeking a solution for equation (13a). Additionally, by eliminating the term y in (6b), the proposed scheme successfully isolates the state estimates from the direct feedthrough of measurement noise, leading to improved estimation robustness.

4. NUMERICAL EXAMPLE

In this section, a numerical example is given to illustrate the effectiveness of the proposed observer design approach. Consider the following system in [5], which is in the form of the system (1), described by

$$E=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},\ A=\begin{bmatrix}-1&1&0&0\\1&-1&0&1\\0&-1&-1&0\\0&0&0&1\end{bmatrix},\ B_1=\begin{bmatrix}1&0\\0&0\\-1&0\\1&0\end{bmatrix},\ G_1=\begin{bmatrix}1&2\\1&-1\\1&0\\1&1\end{bmatrix},$$

$$D_1=\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\ C=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},\ G_2=\begin{bmatrix}0&1\\1&0\end{bmatrix},\ F_l=\begin{bmatrix}0&1&0&0\end{bmatrix}. \tag{19}$$

The initial conditions are $x(0) = [0.2, 0.1, 0.3, 0]^T$. Let $h(F_l x) = \sin(x_2)$, $w^T = \begin{bmatrix}w_1^T & w_2^T\end{bmatrix}$, $v^T = \begin{bmatrix}v_1^T & v_2^T\end{bmatrix}$,$w_1 = 0.25\sin(0.6t + 0.5\pi)$, $w_2 = 0.2\sin(0.3t + 0.6\pi)$. Set the Gaussian noise: $v_1 \sim \mathcal{N}\left(0, 0.1^2\right)$ and $v_2 \sim \mathcal{N}\left(0, 0.15^2\right)$. The order of the observer q is designed as 3. x_i, $\hat{x}_i$, and $e_i = \hat{x}_i - x_i$ (where $i = 1, 2, 3, 4$) denote the states of the system (19), the estimated states, and the estimation errors, respectively.

Set $A_f = \begin{bmatrix}-1&0\\0&-1\end{bmatrix}$, $B_f = C_f = \begin{bmatrix}1&0\\0&1\end{bmatrix}$, and solve the optimal problem (18). The matrices $U, T, N, F, H, J, P, R, L$, and optimal index γ_{min} are given as,

$$U=\begin{bmatrix}0&0&1&0&0&0\\0&1&1&0&0&1\\1&0&0&0&1&0\end{bmatrix},\ T=\begin{bmatrix}0&0&1&0&0&0\\0&1&1&-1&0&0.5\\1&0&0&0&0.5&0\end{bmatrix},$$

$$J=\begin{bmatrix}-1&0\\-2&0\\1&0\end{bmatrix},\ F=\begin{bmatrix}-11.0317&16.6818\\12.2662&-13.5857\\12.6466&-13.7846\end{bmatrix},\ H=\begin{bmatrix}-5.5158&0.5\\5.6331&0.5\\6.0733&-0.5\end{bmatrix},$$

$$P=\begin{bmatrix}0&0&1\\-1&1&0\\1&0&0\\0&0&0\\0&0&0\\0&0&0\end{bmatrix},\ L=\begin{bmatrix}-0.5&0\\0&-0.5\\0&0\\0&0\\1&0\\0&1\end{bmatrix},\ N=\begin{bmatrix}-16.6818&-1&11.0317\\15.0857&-2&-11.2662\\12.7846&1&-13.1466\end{bmatrix},$$

$$R=\begin{bmatrix}0&0&0&1&0&0\end{bmatrix}^T,\ \gamma_{min}=2.5965.$$

Figures 1, 2, 3, and 4 present the estimation errors of the system (19) by using the method in [5] and the observer (6). These figures show that when the output is contaminated with measurement noise, the method presented in Theorem 12 [5] leads to significant oscillations in the observation error, whereas our method can effectively suppress such oscillations. Additionally, the estimation errors e_1, e_2, e_3, and e_4 obtained by using the method in [5] are bounded within the intervals $[-0.4214, 0.4521]$, $[-0.6276, 0.6662]$, $[-0.4215, 0.3857]$, $[-0.5695, 0.6656]$, whereas our proposed method confines these errors to the intervals $[-0.1970, 0.1771]$, $[-0.4238, 0.4834]$, $[-0.1552, 0.1420]$, $[-0.3288, 0.4418]$. In summary, the developed scheme achieves superior smoothness and reduced error deviations.

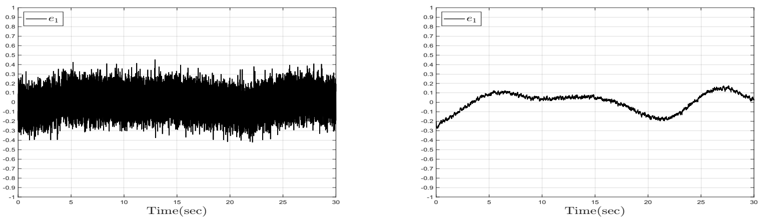

Fig. 1. The estimation error trajectories of e_1 obtained by the method in [5] (left) and the observer (6) (right).

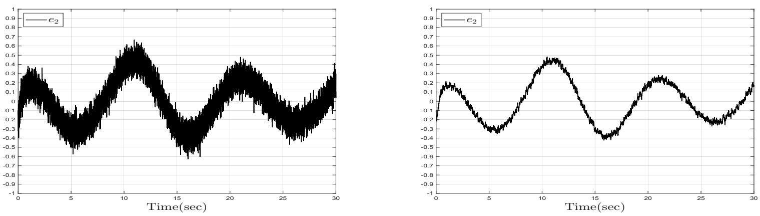

Fig. 2. The estimation error trajectories of e_2 obtained by the method in [5] (left) and the observer (6) (right).

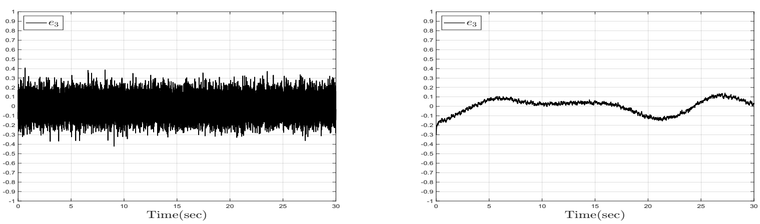

Fig. 3. The estimation error trajectories of e_3 obtained by the method in [5] (left) and the observer (6) (right).

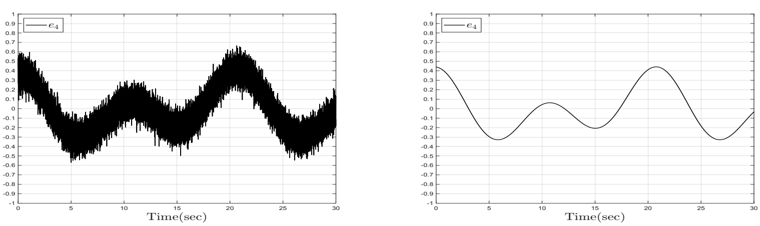

Fig. 4. The estimation error trajectories of e_4 obtained by the method in [5] (left) and the observer ((6) (right).

5. CONCLUSION

In this study, we proposed a novel nonlinear observer design methodology for a class of Lipschitz nonlinear descriptor systems with measurement noise. The developed method was applicable to both full-order and reduced-order observer architectures. Through the application of output filtering techniques, the influence of measurement noise was mitigated to a considerable extent. The observer synthesis problem was formulated through Lyapunov stability analysis, leveraging generalized matrix inversion theory and LMI techniques. Numerical simulations verified that the proposed observer effectively suppressed estimation oscillations caused by measurement noise and attained superior performance with smaller error deviations compared to the conventional approach. Future work will study more efficient observer designs to address high-frequency measurement noise.

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