STABILIZABILITY OF MULTI-AGENT SYSTEMS OVER FINITE FIELDS VIA FULLY ACTUATED SYSTEM APPROACHES

Yunsi Yang, Jun-e Feng and Lei Jia

The problem of stabilizability of high-order fully actuated (HOFA) multi-agent systems over finite fields is considered in this paper. The necessary and sufficient conditions for the stabilizability of HOFA multi-agent systems are presented, which indicates the stabilizability is closely related to the interaction topology among agents. Using the full-actuation property of HOFA models, a stabilization control protocol with neighbor interaction is given for HOFA multi-agent systems. Additionally, when the multi-agent system is stabilizable, the time for the system to reach a stable state can be determined through the control protocol. Finally, the results are employed to solve the formation control problem, and some sufficient and/or necessary conditions are proposed. Numerical examples are presented to demonstrate the effectiveness of the proposed results.

Keywords: finite fields, fully actuated system approach, stabilizability, multi-agent systems

Classification: 12E20, 93D99, 93A16

1. INTRODUCTION

With increasing informatization, multi-agent systems have garnered wide-spread popularity and applicability in different fields [1, 11, 27]. From robotics and autonomous vehicles to social networks and swarm intelligence, multi-agent systems have demonstrated their potential in solving complex problems and achieving collective behavior [12, 16, 25]. The concept of agents is inspired by numerous collective behaviors observed in nature, emphasizing collaboration among multiple systems to achieve complex objectives. However, the realization of ultimate goals relies on the fundamental properties of systems, including controllability, consensus, and stabilizability. Controllability, introduced by Kalman [18, 19], is a crucial notion in the control field. Tanner [42] extended this concept to first-order multi-agent systems. Subsequent researchers explored the controllability of multi-agent systems with different topologies [17, 42, 44]. A fundamental control problem is to make some agents reach an agreement under a given control protocol, which is known as the consensus problem [36, 37]. It entails achieving a consensus in

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the network state within a specified time frame, considering given constraints and information [30]. Scholars have extensively investigated the consensus problem under various conditions, such as time delay and switching topology [33, 39]. Notably, research on consensus has found applications in diverse practical domains, including robotics, drones, and collaborative robotic arm operations, among others.

Recently, the concept of stabilizability has been proposed for multi-agent systems, which is also the most basic and crucial problem [13, 14, 20]. As an intrinsic characteristics of systems, stabilizability represents the capacity of each agent to utilize information from itself and its neighbors as feedback such that the entire group of agents to arrive stable state. Kim [20] proposed decentralized stabilizability for systems with dynamics characterized by single integrators with a fixed network topology. Guan [14] analyzed the decentralized stabilizability of linear model under consensus protocol, and considered the stabilizability with fixed and switching topologies. Based on complete NSi graph partition and quotient graph $\mathcal{G}/\pi$, Sun [40] focused on the stabilizability of systems under structural unbalanced topology. They explored the relationship between stabilizability of uncontrollable multi-agent systems and network topology. Guan [15] extended the stabilizability problem to discrete-time linear multi-agent system, proposing several graph-theoretic conditions for both fixed and switching topologies. However, due to potential variations in the state dimensions among agents, it is referred to as a heterogeneous system. For the stabilizability of the heterogeneous system, Franceschelli [13] also gave some conclusions.

However, the issues mentioned earlier regarding the controllability, consensus, and stabilizability problems predominantly investigated within the framework of real number fields. Such investigations assumed infinite memory and communication resources, which may not be feasible in practice to achieve the desired objectives. Recognizing the constraints posed by limited communication bandwidth and storage capacity, researchers have increasingly turned their attention towards studying various problems related to multi-agent systems over finite fields [46]. Within these studies, states of systems are constrained to finite fields, and the process of state evolution includes operations based on modular arithmetic. Besides, systems over finite fields will possess some special properties and structures [35, 43]. In 2013, the authors [38] applied the concepts of structural controllability and observability to finite field leader-follower multi-agent systems. Subsequent researchers [28] investigated the structural controllability of finite field generalized linear systems with switching topologies, expanding on the findings of [38] to higher dimensions. Meanwhile, in 2014, Pasqualetti [34] developed some sufficient conditions for network consensus in finite fields using theories from graph theory and linear algebra. And then Li [21, 24] extended the results of [34] on finite fields consensus to networks with time-delays and switching topology. Meng [31] proposed the concept of synchronization, and gave some sufficient conditions for achieving synchronization of finite field networks. Xu [45] addressed the leader-follower consensus problem for finite field high-dimensionality dynamical systems, requiring that the weighted adjacency matrix of the system is a directed acyclic graph. In addition to above study, Li [22, 23] proposed a novel strategy to solve the problems of controllability and consensus of finite field dynamics based on semi-tensor product of matrices.

Some research methodologies require finite field systems to possess a specific topology structure or satisfy certain algebraic constraints for controllability and consensus. Other studies employ semi-tensor product, yielding effective conclusions. However, when dealing with high-dimensional systems or finite fields with a large number of dimensions, it may result in significant computational complexity. Regarding the stabilization and stabilizability of finite field networks [47], there is hardly any relevant research [46]. It is worth noting that some control methods for linear systems over the general real field cannot be directly applied to the analysis and control of dynamics over finite fields [32, 35]. Since the system state can only take values in a finite field, systems over finite fields are discrete. However, unlike general discrete linear systems, the stability of these systems cannot be determined by examining whether the eigenvalues of the system constant matrix lie on the unit circle in the complex plane. Controller parameter design methods based on matrix eigenvalues are difficult to apply to finite fields [47]. This limitation may stem from the fact that finite fields are not algebraically closed, implying that not all polynomials with coefficients in finite fields have roots within those fields [29]. Additionally, constructing a Lyapunov function for systems over finite fields is also a complex task [35]. Therefore, for finite field multi-agent systems, the aforementioned methods are not suitable for the control protocol design in this paper. Consequently, it is necessary to take into account the unique characteristics of finite fields and introduce new methods to address the control problem.

The fully actuated system (FAS) approach [4] has recently found wide application in complex systems and has shown considerable practical effectiveness, especially in cases where conventional algebraic methods fail to apply. Duan [5, 6] presented the FAS approach, illustrating the correlation of full-actuation with controllability. Indeed, both controllable linear and nonlinear unactuated systems can be transformed into high-order fully actuated (HOFA) models. Furthermore, a class of practical systems, which can be described by physical laws in the real world, can be directly modeled as HOFA systems, such as Lagrange Equations, Newton Laws, and so on [5]. After obtaining the HOFA systems, the full-actuation feature facilitates the design of the control protocol, enabling the elimination of dynamic characteristics in the original system and the creation of a new autonomous system with considerable degrees of freedom. In subsequent studies [7, 8, 9, 10], Duan specifically elaborated on the principles and implementation process of the FAS approach, particularly providing methods for handling various types of nonlinear systems. Due to the inherent advantages of HOFA models in system analysis and control, along with their practical relevance to real-world systems, the introduction of HOFA models and the FAS approach into multi-agent systems becomes a highly natural idea. However, research in this area is still lacking, especially when it comes to multi-agent systems within finite fields, which remains largely unexplored. Liu [26] addressed the coordinative control problem of a class of high-order fully actuated nonlinear multi-agent systems, introducing an HOFA predictive coordination method to mitigate for communication delays. Zhang [48, 49, 50] introduced a discrete HOFA model in multi-agent system field, and developed a predictive control strategy to achieve coordination objectives. The aforementioned researchers have studied fully actuated multi-agent systems, but the relevant studies were all conducted over the general real number field rather than finite fields. This paper introduces the concept of FAS into multi-agent systems within finite fields. It not only considers HOFA multi-agent systems but also transforms multi-agent systems with different dynamics into HOFA models via the FAS approach. This enables the resolution of stabilizability problems in such systems.

The main contributions and novelties are summarized as follows: First, the HOFA multi-agent system is defined over finite fields rather than general real number fields [26, 48]. Some necessary and sufficient conditions of stabilizability for HOFA multi-agent systems, along with corresponding stabilization protocol designs, are derived. A stabilization control method is obtained for linear multi-agent systems by FAS approach. Furthermore, utilizing the properties of HOFA systems and finite fields, it is possible to provide the time for the system to reach a stable state when designing stabilization control protocols. Additionally, results regarding stabilizability are used to tackle the formation control problem. Consequently, the time required by agents to achieve their objectives can also be determined through the design of control protocols. Compared with the existing works [14, 20], the HOFA multi-agent system is a completely new model. The stabilizability analysis over finite fields is different from discrete-time multi-agent systems over real number fields [15].

The rest of this paper is outlined as follows. In Section 2, some preliminary knowledge and definitions are given. Section 3 devotes to investigate the stabilizability of HOFA multi-agent systems over finite fields. In Section 4, the results are employed to solve the formation control problem. Numerical examples are presented in Section 5. Section 6 gets a conclusion.

2. PRELIMINARIES

2.1. Notations

A finite field is denoted as $\mathbb{F}_p$, with p being a prime number. The notation $\mathbb{F}_p^n$ represents the space of n-dimensional vector over $\mathbb{F}_p$, while $\mathbb{F}_p^{m\times n}$ denotes the space of $m \times n$ dimensional matrices over the same field, where the entries of vectors and matrices are in $\mathbb{F}_p$. The symbol I_n denotes the identity matrix, and $\varnothing$ represents the null set. Additionally, $det(A)$ denotes the determinant of the matrix A, while A^{-1} represents the inverse of matrix A.

For $K_i \in \mathbb{F}_p^{n\times n}$, $A_i \in \mathbb{F}_p^{n\times n}, i = 1, 2, \dots, n$, there is

$$\hat{K} = \begin{bmatrix} K_1 & \cdots & K_{n-1} & K_n \\ I_n & & & \\ & \ddots & & \\ & & I_n & 0 \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} A_1 & \cdots & A_{n-1} & A_n \\ I_n & & & \\ & \ddots & & \\ & & I_n & 0 \end{bmatrix}.$$

2.2. Finite fields and graph theory

According to the relevant definitions in abstract algebra, it can be understood that an abelian group is a set capable of addition and subtraction operations, whereas a ring extends this capability to include multiplication. A field, on the other hand, is a more powerful algebraic structure. It extends a ring by having commutative multiplication and ensuring that every nonzero element has a multiplicative inverse. Simply put, a field can be considered a commutative division ring. Fields are typically represented by the symbol $\mathbb{F}$. In this paper, the elements in the field are considered finite, consisting of a prime number p of elements. Then, $\mathbb{F}_p$ represents a finite field, and more properties about finite fields can be found in [34, 46].

Finally, we review some fundamental concepts in graph theory. A directed graph $\mathcal{G} = (\mathcal{V}, \varepsilon)$ consists of a set of vertices $\mathcal{V}$ and a set of edges $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$. An edge $(v, u) \in \varepsilon$ is directed, u is the parent node, and v is the child node receiving transmissions from v. For a vertex $v \in \mathcal{V}$, its neighbor set is defined as $\mathcal{N}_i = \{u \in \mathcal{V} : (v, u) \in \varepsilon\}$. The adjacency matrix of $\mathcal{G}$ is denoted as $\mathcal{A} = (a_{ij}) \in \mathbb{F}_p^{n\times n}$: if $u \in \mathcal{N}_i$, $a_{ij} \neq 0$, and $a_{ij} = 0$ otherwise. The in-degree of $v \in \mathcal{V}$ is $|\mathcal{N}_i| = \delta_i = \sum_{j=1}^{n} a_{ij}$, and the degree matrix is $\Delta = diag\{\delta_1, \delta_2, \ldots, \delta_n\}$. A path in $\mathcal{G}$ is a sequence of nodes $v_1, \ldots, v_{k+1}$ such that $(v_i, v_{i+1}) \in \varepsilon$ for all $i = 1, \ldots, k$. If any pair of nodes in a directed graph can be linked by a directed path, the graph is considered strongly connected. A cycle is defined as a path where the start and end nodes are identical. A directed graph without cycles is called a directed acyclic graph (DAG).

2.3. Fully actuated system approaches over finite fields

The fully actuated systems are a widely encountered class of systems in the natural world, and they can also be derived from common linear or nonlinear systems. Consider a linear system over $\mathbb{F}_p$:

$$x(t+1) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbb{F}_p^n$ is the state, $u(t) \in \mathbb{F}_p^r$ is the control input, both $A \in \mathbb{F}^{n\times n}$ and $B \in \mathbb{F}^{n\times r}$ are constant matrices.

It can be proved Theorem 3.4 of [10] holds over finite fields [47], that is, system (1) under Assumption 1, there is following lemma.

Lemma 1. System (1) over $\mathbb{F}_p$ is controllable if and only if it can be converted equivalently into a step forward HOFA model or a step backward HOFA model.

From the aforementioned lemma, system (1) can be converted into an HOFA model if it is controllable. Though two kinds of HOFA models are both FAS models, there still exist certain differences in aspects such as model transformation and control inputs [10, 47]. In order to facilitate the implementation of control for system (1), it is transformed into a step backward HOFA system, which can be written in the following form:

$$z(t+1) = \sum_{i=1}^{\mu} A_i z(t-i+1) + \hat{B}u(t), \tag{2}$$

where vector $z(t) \in \mathbb{F}_p^r$, input matrix $\hat{B}$ is nonsingular, and $\mu = \max\{\mu_i, i = 1, 2, \ldots, r\}$ is the largest controllability index of system (1).

Remark 1. By Lemma 1 proposed in this paper and Theorem 3.3 of [47], there exists a state transformation $y = Tx$, such that the system is transformed into the controllability canonical form:

$$y(t+1) = \hat{A}y(t) + \hat{B}u(t). \tag{3}$$

For some $y_j(t)$ in $y(t)$, there exists y_i(t) such that the following equation satisfies

$$y_j(t) = y_i(t-s+1), \tag{4}$$

where $i, j \in \{1, \dots, n\}, s = 1, 2, \dots, \mu_i$, and $\mu_i, i = 1, \dots, r$ are the controllability indices of system (1). Using equation (4), some state variables can be represented using the other r state variables. Then system (3) can be converted into system (2). This also implies that when the closed-loop system of HOFA system (2) is asymptotically stable, the closed-loop system of system (3) is also asymptotically stable.

It is shown in Remark 1 that state $y(t)$ is obtained from $x(t)$ by a non-singular state transformation. Similar to Lemma 6 in [40], it can be proved that the controllable subspace of system (3) and the eigenvalues of the dynamic matrix A are the same as system (1). Hence, system (1) and (3) are equivalent, then there is the following lemma.

Lemma 2. The stabilizability of system (1) is invariant under state transformation $y = Tx$.

Clearly, when system (3) achieves stabilization, after the corresponding inverse transformation, linear system (1) also achieves stabilization. Building on the preceding discussion, HOFA systems can be transformed from controllable linear systems. Once control input is implemented in the HOFA system, the initial linear system will also achieve the control objectives.

3. STABILIZABILITY OF MULTI-AGENT SYSTEMS OVER FINITE FIELDS

In the previous study of finite field networks, controllability and consensus of one-dimensional multi-agent systems both have garnered some attention, where each individual agent was represented as a one-dimensional entity. However, real-world systems often exhibit higher complexity, demanding the depiction of dynamics with agents possessing higher-level capabilities and intricate behaviors. Such advanced agents, with their sophisticated abilities and complex behaviors, interact to form multi-dimensional multi-agent systems.

3.1. Stabilizability analysis

Consider a group of multi-agent systems with general linear dynamics over $\mathbb{F}_p$, consisting of N agents. The dynamics of each agent is described by

$$x_i(t+1) = Ax_i(t) + Bu_i(t), i = 1, \dots, N, \tag{5}$$

where $x_i(t) \in \mathbb{F}_p^n$ is the state of agent i, $u_i(t) \in \mathbb{F}_p^r$ is the control input of agent i, A and B are identical to those in system (1). The interaction topology among the above N agents can be described by a graph $\mathcal{G}$, and the adjacency matrix of $\mathcal{G}$ is $\mathcal{A}$.

Assumption 1. The pair (A,B) is controllable.

Under Assumption 1, by Lemma 1, system (5) can be converted into an HOFA system:

$$z_i(t+1) = \sum_{s=1}^{\mu} A_s z_i(t-s+1) + \hat{B} u_i(t), i = 1, \ldots, N, \tag{6}$$

where vector $z_i(t) \in \mathbb{F}_p^r$, the input matrix $\hat{B}$ is nonsingular, and $\mu = \max\{\mu_i, i = 1, 2, \ldots, r\}$ is the largest controllability index of system (5).

In fact, through appropriate state transformation and variable elimination, HOFA systems can be derived not only from controllable linear systems, but also from strict-feedback nonlinear systems, feedback-linearizable nonlinear systems, and other related classes of systems. Furthermore, there exist an abundance of HOFA systems in the physical world, such as Eulerian law, Newtonian law, etc. These systems can be directly modeled as HOFA models. Therefore, the research concerning HOFA multi-agent systems (6) can not only solve the stabilizability of finite field linear multi-agent system (5), but also provide a control method for various systems represented by this model, which has great practical significance. The research in [48, 49, 50] addressed the predictive control problem of HOFA multi-agent systems. However, its model differs from system (6). In comparison to system (6), if the time index of the model in [48, 49, 50] shifts $n-1$ steps, it can be considered that there are time-delays in the control input. However, regarding those models, the control protocol considered in this paper are also effective.

For HOFA multi-agent system (6) over $\mathbb{F}_p$, a control protocol is presented as

$$\begin{cases} u_i(t) = \hat{B}^{-1}(u_{i1}(t) + u_{i2}(t)) \\ u_{i1}(t) = K_0 \sum\limits_{j \in \mathcal{N}_i} a_{ij}(z_i(t) - z_j(t)) \\ u_{i2}(t) = -\sum\limits_{s=1}^{\mu} A_s \bar{d}_i z_i(t-s+1) + \sum\limits_{s=1}^{\mu} K_s d_i z_i(t-s+1), \end{cases} \tag{7}$$

where $u_{i1}(t) \in \mathbb{F}_p^r$ is the cooperative control based on relative states between neighboring agents, and $u_{i2}(t) \in \mathbb{F}_p^r$ is the external control based on self-state feedback. Besides, $K_i \in \mathbb{F}_p^{r \times r}, i = 0, \ldots, \mu$ are the feedback gain matrices to be designed, and d_i indicates if agent i is impacted by an external control. Agent i acquires data provided by an external control if $d_i = 1$, otherwise $d_i = 0$, and $\bar{d}_i = 1 - d_i$. For the subsequent analysis, define $\Gamma_1 = \{i | i \in \mathcal{N}, d_i = 1\}$ and $\Gamma_2 = \{i | i \in \mathcal{N}, d_i = 0\}$, where $\mathcal{N} = \Gamma_1 \bigcup \Gamma_2 = \{1, 2, \ldots, N\}$, and $\Gamma_1 \bigcap \Gamma_2 = \varnothing$.

System (6) under protocol (7) can be written as

$$\begin{aligned} z_i(t+1) = &\sum_{s=1}^{\mu} d_i K_s z_i(t-s+1) + \sum_{s=1}^{\mu} \bar{d}_i A_s z_i(t-s+1) \\ &+ K_0 \sum_{j \in \mathcal{N}_i} a_{ij}(z_i(t) - z_j(t)). \end{aligned} \tag{8}$$

Let $y_i(t) = [z_i^{\mathrm{T}}(t), z_i^{\mathrm{T}}(t-1), \ldots, z_i^{\mathrm{T}}(t-\mu+1)]^{\mathrm{T}} \in \mathbb{F}_p^{\mu r}$, then there is the following equivalent form:

$$y_i(t+1) = \hat{K} d_i y_i(t) + \hat{A} \bar{d}_i y_i(t) + \bar{K} \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) - y_j(t)), \tag{9}$$

where

$$\bar{K} = diag\{K_0, 0, \ldots, 0\}.$$

In addition, denote $Y(t) = [y_1^{\mathrm{T}}(t), y_2^{\mathrm{T}}(t), \ldots, y_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{F}_p^{N\mu r}$, so

$$Y(t+1) = (D \otimes \hat{K} + \bar{D} \otimes \hat{A} - L \otimes \bar{K})Y(t), \tag{10}$$

where $D = diag\{d_1, d_2, \ldots, d_N\}$, $\bar{D} = I_N - D$, $L = \Delta - \mathcal{A}$, and $\Delta = diag\{\delta_1, \delta_2, \ldots, \delta_N\}$, $\delta_i = \sum_{j=1}^{N} a_{ij}$. Δ can be defined as the degree matrix in $\mathbb{F}_p$, which is different from the degree matrix in real number field. In the expression $\sum_{j=1}^{N} a_{ij}$, the addition follows modular arithmetic. L can be defined as the Laplacian matrix in $\mathbb{F}_p$.

Definition 1. For multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$, the stabilizability problem of multi-agent system (6) is said to be solvable, if there exists a series of feedback gain matrices $K_i, i = 0, 1, \ldots, \mu$ such that closed-loop system (10) is asymptotically stable.

From the above conclusion, closed-loop system (10) is asymptotically stable if and only if $D \otimes \hat{K} + \bar{D} \otimes \hat{A} - L \otimes \bar{K}$ is nilpotent in $\mathbb{F}_p$.

It can be seen if the matrix $K_0 = 0$, which means there is no information interaction among these neighbors. The stabilizability problem of system (6) can be converted to the stabilizability problem of several sub-systems (2). If there exists external control in each sub-systems, according to full-actuation property of each HOFA system in (6), closed-loop system (10) can be stable. Then there is the following assumption.

Assumption 2. Matrix $K_0 \neq 0$.

The topology graph $\mathcal{G}$ among the N agents is assumed to satisfy the following assumption, which was proposed in some studies of multi-agent systems over real number field [2, 3] and finite fields [45].

Assumption 3. Graph $\mathcal{G}$ is a DAG.

If Assumptions 3 holds, then there exists nonsingular matrix P such that $P\mathcal{A}P^{-1}$ is strict upper-triangular. Let $\bar{\mathcal{A}} = P\mathcal{A}P^{-1}$ and $\bar{\Delta} = P\Delta P^{-1} = diag\{\bar{\delta}_1, \bar{\delta}_2, \ldots \bar{\delta}_N\}$,

$$\begin{aligned} &(P \otimes I_{n\mu})(D \otimes \hat{K} + \bar{D} \otimes \hat{A} - L \otimes \bar{K})(P \otimes I_{n\mu})^{-1} \\ =&(D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K}), \end{aligned} \tag{11}$$

where $\bar{L} = PLP^{-1} = \bar{\Delta} - \bar{\mathcal{A}}$, $\bar{\mathcal{A}}$ is strictly upper-triangular, and $\bar{\Delta}$ is diagonal. Besides, D and $\bar{D}$ represent whether these agents receive information from the external control. Let $\bar{Y}(t) = (P \otimes I_{n\mu})Y(t)$, where $\bar{Y}(t) = [\bar{y}_1^{\mathrm{T}}(t), \bar{y}_2^{\mathrm{T}}(t), \ldots, \bar{y}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$. Without loss of generality, closed-loop system (10) can be rewritten as follows:

$$\bar{y}(t+1) = \Psi \bar{y}(t), \tag{12}$$

where

$$\Psi = \begin{bmatrix} \hat{K} - \bar{\delta}_1 \bar{K} & * & \dots & * \\ 0 & \hat{A} - \bar{\delta}_2 \bar{K} & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{K} - \bar{\delta}_N \bar{K} \end{bmatrix}.$$

Theorem 1. Under Assumptions 2-3, there exists a series of feedback gain matrices $K_s, s = 0, 1, \dots, \mu$ such that HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$ is stabilizable if and only if $\hat{K}_i = \hat{K} - \bar{\delta}_i \bar{K}, i \in \Gamma_1$ and $\hat{A}_i = \hat{A} - \bar{\delta}_i \bar{K}, i \in \Gamma_2$ are nilpotent. The upper bound of settling time for system (12) is $T = \sum_{i=1}^{N} \tau_i$, where $\tau_i = \min\{\max\{\hat{k}_1, \dots, \hat{k}_{N+1-i}\}, \max\{\hat{k}_i, \dots, \hat{k}_N\}\}$, $\hat{k}_i, i = 1, \dots, N$ are the nilpotency indices of $\hat{K}_i$ and $\hat{A}_i$.

Proof. $(D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K})$ is nilpotent if and only if its characteristic polynomial $det(\lambda I_{N\mu r} - (D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K})) = \lambda^{N\mu r}$. According to the form of closed-loop system (12), $(D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K})$ is block upper-triangular, $det(\lambda I_{N\mu r} - (D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K})) = det(\lambda I_{\mu r} - \hat{K} - \bar{\delta}_1 \bar{K}) det(\lambda I_{\mu r} - \hat{A} - \bar{\delta}_2 \bar{K}) \dots det(\lambda I_{\mu r} - \hat{K} - \bar{\delta}_N \bar{K})$. Then $det(\lambda I_{N\mu r} - (D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K})) = \lambda^{N\mu r}$ if and only if $det(\lambda I_{\mu r} - \hat{K} - \bar{\delta}_i \bar{K}) = \lambda^{\mu r}, i \in \Gamma_1$ and $det(\lambda I_{\mu r} - \hat{A} - \bar{\delta}_i \bar{K}) = \lambda^{\mu r}, i \in \Gamma_2$, that is, $\hat{K}_i = \hat{K} - \bar{\delta}_i \bar{K}, i \in \Gamma_1$ and $\hat{A}_i = \hat{A} - \bar{\delta}_i \bar{K}, i \in \Gamma_2$ are nilpotent. According to Lemma 4.3 of [45], for any $t \geq T$, $D \otimes \hat{K} + \bar{D} \otimes \hat{A} - \bar{L} \otimes \bar{K} = 0_{N\mu r}$, so the upper bound of settling time for (12) is T. $\square$

Remark 2. In this paper, the settling time refers to the duration within which the state of the multi-agent system reaches a fixed point from any initial state. This concept is essential in various fields and is widely applied in numerous control problems [41].

Theorem 2. Under Assumptions 2-3, there exists a series of feedback gain matrices $K_s, s = 0, 1, \dots, \mu$ such that HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$ is stabilizable if and only if one of the following statements holds

1. $\bar{\delta}_i \equiv c \in \mathbb{F}_p, i = 1, \dots, N$,
2. $\bar{\delta}_i \equiv c_1 \in \mathbb{F}_p, i \in \Gamma_1$ and $\bar{\delta}_i \equiv c_2 \in \mathbb{F}_p, i \in \Gamma_2$ such that $\hat{A}_i, i \in \Gamma_2$ are nilpotent.

Proof. By Theorem 1, under Assumptions 2-3, HOFA multi-agent system (6) is stabilizable if and only if $\hat{K}_i = \hat{K} - \delta_i \bar{K}, i \in \Gamma_1$ and $\hat{A}_i = \hat{A} - \delta_i \bar{K}, i \in \Gamma_2$ are nilpotent.

$$\hat{A}_i = \hat{A} - \bar{\delta}_i \bar{K} = \begin{bmatrix} A_1 - \bar{\delta}_i K_0 & A_2 & \dots & A_\mu \\ I & & & \\ & & \ddots & \\ & & I & 0 \end{bmatrix}, \tag{13}$$

$$\hat{K}_i = \hat{K} - \bar{\delta}_i \bar{K} = \begin{bmatrix} K_1 - \bar{\delta}_i K_0 & K_2 & \dots & K_\mu \\ I & & & \\ & & \ddots & \\ & & I & 0 \end{bmatrix}, \tag{14}$$

where $\bar{\delta}_i$ are elements on the diagonal of the matrix $\bar{L}$. The following three scenarios are discussed:

1. If there exists several $\bar{\delta}_i \equiv c_2 \in \mathbb{F}_p$ and a matrix $\bar{K}$ such that $\hat{A}_i = \hat{A} - \bar{\delta}_i\bar{K}$ is nilpotent ($\hat{A}_i = \hat{A}$ are nilpotent if $c_2 = 0$), these nodes are asymptotically stable without external control. Then Γ_2 represents numbers of these nodes, Γ_1 represents other nodes. For $i \in \Gamma_1$, we can find a series of feedback gain matrices $K_s, s = 0, 1, \ldots, \mu$ such that $\hat{K}_i = \hat{K} - \bar{\delta}_i\bar{K}$ are nilpotent if and only if $\delta_i \equiv c_1 \in \mathbb{F}_p$. In this case, in-degrees of all nodes in graph $\mathcal{G}$ have two different values: c_1, and c_2 ($c_2 = 0$ if $\hat{A}$ are nilpotent).

2. If $\hat{A}$ is not nilpotent, and there does not exist $\bar{\delta}_i \equiv c_2 \in \mathbb{F}_p$ such that $\hat{A}_i = \hat{A} - \bar{\delta}_i\bar{K}$ is nilpotent, Γ_2 is a empty set, which represents each nodes in graph $\mathcal{G}$ (or each agents) all have external control. Then $D = I, \hat{K}_i = \hat{K} - \bar{\delta}_i\bar{K}, i = 1, \ldots, N$ are nilpotent if and only if $\bar{\delta}_i \equiv c \in \mathbb{F}_p, i = 1, \ldots, N$. In this case, in-degrees of all nodes in graph $\mathcal{G}$ have the same in-degree c.

3. If in-degrees of nodes in graph $\mathcal{G}$ have more than two different values, $\hat{K}_i = \hat{K} - \bar{\delta}_i\bar{K}, i \in \Gamma_1$ and $\hat{A}_i = \hat{A} - \bar{\delta}_i\bar{K}, i \in \Gamma_2$ can not all be nilpotent. The nodes under external control and the nodes without external control should each have the same in-degree. Because there does not exist a matrix $\bar{K}$ and two different in-degrees c_1 and c_2, such that $\hat{A}_i = \hat{A} - \delta_i\bar{K}$ and $\hat{A}_i = \hat{A} - \delta_i\bar{K}$ (or $\hat{K}_i = \hat{K} - \delta_i\bar{K}$ and $\hat{K}_i = \hat{K} - \delta_i\bar{K}$) are all nilpotent. Unless the matrix $\bar{K} = 0$, which contradicts Assumption 2.

Based on the specific analysis of the three cases, the results can be summarized as the theorem above. □

Remark 3. According to the aforementioned theorem, the stabilizability of HOFA multi-agent system (6) actually depends on topology graph $\mathcal{G}$. For a DAG, there can be situations where certain nodes have equal in-degrees, allowing these agents to achieve stabilization without external control. However, for other agents, all of them need to be under external control, and their corresponding nodes in the graph should also have the same in-degree. If there are no such agents present which is stabilizable solely through interactions between neighbors and their own dynamic evolution, then in such a case, it is required that all nodes in the communication graph of the system have equal in-degrees, and all agents must be under external control for the stabilizability of HOFA multi-agent system (6).

Remark 4. Graph $\mathcal{G}$ is assumed to be a DAG. The agents, which has no parents nodes in the DAG, can be regarded as leaders, and all other agents can be regarded as followers. Without loss of generality, there exists a transformation such that the adjacency matrix of graph $\mathcal{G}$ becomes strictly upper triangular. In this transformed matrix, the preceding nodes are followers, while the succeeding nodes are leaders. The indices of agents are defined that an agent i for $i \in \{1, 2, \ldots, N_f\}$ is a follower, and an agent i for $i \in \{N_f + 1, N_f + 2, \ldots, N\}$ is a leader. Note that leaders of a multi-agent system only transmit information to their neighbors but do not accept information from other agents, so they all have zero in-degree. Therefore, if the in-degrees of nodes in a DAG are all equal, they can only be equal to zero. In this case, if $\hat{A}_i = \hat{A}$ for $i = 1, \ldots, N$, and they are all nilpotent, then the multi-agent system is stabilizable without external control. If they are not nilpotent, then all the agents are subject to external control in order for the system is stabilizable. According to Theorem 2, if the in-degrees of followers are not all zero, the in-degrees of other followers must be equal to a constant $c \neq 0 \in \mathbb{F}_p$. In this case, either $\hat{A}$ or $\hat{A}_i = \hat{A} - c\bar{K}$ is nilpotent. Then for the multi-agent system to achieve stabilization, either all leaders (and some followers) or followers with a non-zero in-degree are subject to external control.

Based on the above analysis, Theorem 2 can be rewritten as follows:

Theorem 3. Under Assumptions 2-3, there exists a series of feedback gain matrices $K_s, s = 0, 1, \ldots, \mu$ such that HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$ is stabilizable if and only if one of the following statements holds

1. $\bar{\delta}_i \equiv 0, i \in \{1, 2, \ldots, N_f\}$,
2. $\bar{\delta}_i \equiv 0, i \in \Gamma_1$ and $\bar{\delta}_i \equiv c \in \mathbb{F}_p, i \in \Gamma_2$ or $\bar{\delta}_i \equiv c \in \mathbb{F}_p, i \in \Gamma_1$ and $\bar{\delta}_i \equiv 0 \in \mathbb{F}_p, i \in \Gamma_2$ such that $\hat{A}_i, i \in \Gamma_2$ are nilpotent.

The following conclusion can be used for basic assessment of the stabilizability of system (6).

Corollary 1. Under Assumptions 2-3, there does not exist a series of feedback gain matrices $K_s, s = 0, 1, \ldots, \mu$ such that HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$ is stabilizable if the in-degrees of nodes in graph $\mathcal{G}$ take on more than two different values.

If an HOFA multi-agent system is derived from a linear multi-agent system, during the transformation process, the dimension of the system decreases. However, according to Remark 1, it can be proven that if the HOFA multi-agent system is stabilizable, then the multi-agent system obtained after dimension expansion also is stabilizable. By equation (4), if the HOFA multi-agent system reaches a steady state, the dimensionally expanded multi-agent system is stable after iterating $\mu - 1$ steps. Similar to Lemma 2, the stabilizability of system (5) is invariant under state transformation. When the dimensionally expanded multi-agent system reaches a fixed point or stable state under a given control protocol, the original linear multi-agent system also reaches a stable state. The following theorem demonstrates the above result.

Theorem 4. If HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$ is stabilizable, then multi-agent system (5) is stabilizable, and the upper bound of settling time for the closed-loop system of system (5) is $T + \mu - 1$, where T is defined in Theorem 1, $\mu = \max\{\mu_i, i = 1, 2, \ldots, r\}$ is the maximum controllability index of system (5).

3.2. Stabilization protocol design

In Theorem 1, a necessary and sufficient condition for the stabilizability of the HOFA multi-agent system is provided. Theorem 2 reveals that the stabilizability of the HOFA multi-agent system is closely related to the in-degrees of the nodes in graph $\mathcal{G}$. Theorem 3 provides a more specific equivalence condition for stabilizability under the assumption of a DAG. When the graph satisfies corresponding conditions, the stabilization problem is transformed into finding a suitable sequence of matrices that render system (10) asymptotically stable. To achieve stabilization of the HOFA multi-agent system, it may be necessary for each agent to rely on both the interaction with its neighbors and external control. With considerations such as resource conservation, the following stabilization control procedure has been designed in order to reduce external control as much as possible.

1. A controllable linear multi-agent system can be transformed into an HOFA multi-agent system. Alternatively, for a certain class of systems existing in practice, they can be directly modeled as HOFA multi-agent systems over $\mathbb{F}_p$.

2. For topology graph $\mathcal{G}$ (DAG) of the HOFA multi-agent system, the following four scenarios are discussed:

   - If the in-degrees of nodes in graph $\mathcal{G}$ take on more than two distinct values, the system is not stabilizable.
   - If the in-degrees of all nodes are zero, that is, $\bar{\delta}_i \equiv 0$, and $\hat{A}_i = \hat{A}$ is nilpotent, then the system can be stable without external control.
   - If the in-degrees of all nodes are zero, that is, $\bar{\delta}_i \equiv 0$, but $\hat{A}_i = \hat{A}$ is not nilpotent, then each agent needs to be under external control. This allows the stabilization control problem of the HOFA multi-agent system over $\mathbb{F}_p$ to be transformed into a parameter design problem of a series of matrices K_s, $s = 0, 1, \ldots, \mu$.
   - If the in-degrees of nodes take on two values 0 and c from $\mathbb{F}_p$, such that either $\hat{A}_i = \hat{A}$ or $\hat{A}_i = \hat{A} - c\bar{K}$ is nilpotent, then by designing appropriate a series of matrices K_s, $s = 0, 1, \ldots, \mu$, the system is stabilizable.

3. The FAS parameter design method for the feedback gain matrices is as follows:

   - First, based on different scenarios, choose a matrix K_0 such that $\hat{A}_i$ is nilpotent, or choose K_0 arbitrarily.
   - Then, determine other coefficient matrices in the control protocol, denoted as K_s, $s = 1, \ldots, \mu$. For the given matrix $J \in \mathbb{F}_p^{\mu r \times \mu r}$, which is composed of a series of nilpotent Jordan blocks, with the largest Jordan block having dimension $\hat{k}$, coefficient matrix $\hat{K}_i$ and arbitrarily nonsingular matrix $Q \in \mathbb{F}_p^{\mu r \times \mu r}$ satisfying

$$\hat{K}_i = \hat{K} - \bar{\delta}_i \bar{K} = QJQ^{-1}, \bar{\delta}_i \equiv c \in \mathbb{F}_p. \tag{15}$$

   The coefficient matrices K_s, $s = 1, \ldots, \mu$ are given by

$$\begin{bmatrix} K_1 - cK_0 & K_2 & \ldots & K_\mu \end{bmatrix} = ZJ^\mu Q^{-1}(Z, J), \tag{16}$$

and

$$Q = Q(Z, J) = \begin{bmatrix} ZJ^{\mu-1} \\ \vdots \\ ZJ \\ Z \end{bmatrix}, \tag{17}$$

where $Z \in \mathbb{F}_p^{r\times\mu r}$ satisfies

$$detQ(Z, J) \neq 0. \tag{18}$$

- Finally, coefficient matrices $K_s, s = 1, \ldots, \mu$ in control protocol (7) are

$$\begin{aligned} &\begin{bmatrix} K_1 & K_2 & \ldots & K_\mu \end{bmatrix} \\ =&ZJ^\mu V^{-1}(Z, J) - \begin{bmatrix} cK_0 & 0 & \ldots & 0 \end{bmatrix}. \end{aligned} \tag{19}$$

4. The design of control protocol u is accomplished, which can solve the stabilization problem of system (6).

5. Based on Theorem 4, system (5) is also stabilizable, and Theorem 4 provides the upper bound of settling time for the system.

Remark 5. Based on the above stabilization control procedure and subject to certain assumptions, it is possible to achieve stabilization of an HOFA multi-agent system, while also determining the upper bound of settling time for the closed-loop system. As mentioned earlier, HOFA multi-agent systems can be directly modeled based on real-world systems. Therefore, for this type of system, the stabilization problem has already been resolved. Moreover, HOFA multi-agent systems can also be derived from certain systems, such as linear multi-agent system (5). It has been proven that the stabilizability of a multi-agent system remains invariant under state transformations. When an HOFA multi-agent system is stabilizable, the corresponding original system is also stabilizable. The stabilization problem for a linear multi-agent system has been resolved, and the settling time can be obtained naturally. Therefore, the proposed stabilization control procedure in this section not only addresses the stabilization problem of HOFA multi-agent systems directly but also provides the FAS approach for linear multi-agent systems. By leveraging the feature of HOFA models, it facilitates the control protocol design for stabilization problem.

4. APPLICATION TO FORMATION CONTROL

The set-point formation control problem of multi-agent systems with different dynamics has been considered in recent research into stabilizability of multi-agent systems, such as single integrators under fixed topology, linear dynamics under fixed and switching topology, and discrete time linear dynamics under fixed and switching topology [14, 15, 20]. The stabilizability results from the previous section can be applied to this problem.

A set point, denoted as $h_0 \in \mathbb{F}_p^r$, along with $H = [h_1^{\mathrm{T}}, \ldots, h_1^{\mathrm{T}}, \ldots, h_N^{\mathrm{T}}, \ldots, h_N^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{F}_p^{N\mu r}$, characterizes a formation structure of a multi-agent system. h_i represents the formation vector associated with each agent i, and h_0 is utilized for moving the entire formation to a target position. Therefore, variable $h_i - h_j$ can represent the relative position between agent i and agent j.

Then, consider the following control protocol:

$$\begin{cases} u_i(t) = \hat{B}^{-1}(u_{i1}(t) + u_{i2}(t)) \\ u_{i1}(t) = K_0 \sum\limits_{j\in\mathcal{N}_i} a_{ij}((z_i(t) - h_i) - (z_j(t) - h_j)) \\ u_{i2}(t) = -\sum\limits_{s=1}^{\mu} A_s \bar{d}_i (z_i(t-s+1) - h_i - h_0) \\ \qquad\quad + \sum\limits_{s=1}^{\mu} K_s d_i (z_i(t-s+1) - h_i - h_0). \end{cases} \tag{20}$$

Definition 2. For HOFA multi-agent system (6) with control protocol (20) over $\mathbb{F}_p$, the set-point formation control problem is said to be solvable, if there exists a series of feedback gain matrices $K_i, i = 0, 1, \dots, \mu$ such that $\lim_{t\to\infty} \parallel z_i(t) - h_i - h_0 \parallel = 0$.

Let $\widetilde{z}_i(t) = z_i(t) - h_i - h_0$, system (6) under protocol (20) can be written as

$$\begin{aligned} \widetilde{z}_i(t+1) = & \sum_{s=1}^{\mu} d_i K_s \widetilde{z}_i(t-s+1) + \sum_{s=1}^{\mu} \bar{d}_i A_s \widetilde{z}_i(t-s+1) \\ & + K_0 \sum_{j\in\mathcal{N}_i} a_{ij}(\widetilde{z}_i(t) - \widetilde{z}_j(t)). \end{aligned} \tag{21}$$

Let $\widetilde{y}_i(t) = [\widetilde{z}_i^{\mathrm{T}}(t), \widetilde{z}_i^{\mathrm{T}}(t-1), \dots, \widetilde{z}_i^{\mathrm{T}}(t-\mu+1)]^{\mathrm{T}} \in \mathbb{F}_p^{\mu r}$, $\widetilde{Y}(t) = [\widetilde{y}_1^{\mathrm{T}}(t), \widetilde{y}_2^{\mathrm{T}}(t), \dots, \widetilde{y}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{F}_p^{N\mu r}$, then there is the following equivalent form:

$$\widetilde{Y}(t+1) = (D \otimes \hat{K} + \bar{D} \otimes \hat{A} - L \otimes \bar{K})\widetilde{Y}(t). \tag{22}$$

The stability analysis of $\widetilde{Y}(t)$ can be referred to the approach used for the closed-loop system (10) in this paper.

Corollary 2. Under Assumptions 2-3, there exists a series of feedback gain matrices $K_s, s = 0, 1, \dots, \mu$ such that the set-point formation control problem of multi-agent system (6) over $\mathbb{F}_p$ is solvable if and only if $\hat{K}_i = \hat{K} - \bar{\delta}_i \bar{K}, i \in \Gamma_1$ and $\hat{A}_i = \hat{A} - \bar{\delta}_i \bar{K}, i \in \Gamma_2$ are nilpotent. The upper bound of time for formation control is $T = \sum_{i=1}^{N} \tau_i$, where $\tau_i = \min\{\max\{\hat{k}_1, \dots, \hat{k}_{N+1-i}\}, \max\{\hat{k}_i, \dots, \hat{k}_N\}\}$, $\hat{k}_i, i = 1, \dots, N$ are the nilpotency indices of $\hat{K}_i$ and $\hat{A}_i$.

Corollary 3. Under Assumptions 2-3, for HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$, there exists a series of feedback gain matrices $K_s, s = 0, 1, \dots, \mu$ such that the set-point formation control problem is solvable if and only if one of the following statements holds

1. $\bar{\delta}_i \equiv 0, i \in \{1, 2, \dots, N_f\}$,
2. $\bar{\delta}_i \equiv 0, i \in \Gamma_1$ and $\bar{\delta}_i \equiv c \in \mathbb{F}_p, i \in \Gamma_2$ or $\bar{\delta}_i \equiv c \in \mathbb{F}_p, i \in \Gamma_1$ and $\bar{\delta}_i \equiv 0 \in \mathbb{F}_p, i \in \Gamma_2$ such that $\hat{A}_i, i \in \Gamma_2$ are nilpotent.

The following conclusion can be utilized for a basic assessment of the solvability of the formation control problem.

Corollary 4. Under Assumptions 2-3, consider HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_p$, there does not exist a series of feedback gain matrices $K_s, s = 0, 1, \ldots, \mu$ such that the set-point formation control problem is solvable if the in-degrees of nodes in graph $\mathcal{G}$ take on more than two different values.

5. NUMERICAL EXAMPLES

The practical applicability of the obtained results is validated through the following two examples. The first demonstrates the efficacy of results for the stabilizability of HOFA multi-agent systems.

Example 1. Consider HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_3$ with $z_i = \begin{bmatrix} z_{i1} & z_{i2} \end{bmatrix}^{\mathrm{T}}$ and $\mu = 2$, which consists of 4 agents. The communication topology is described in Figure 1, where blue circles $(1, 2)$ represent followers, red circles $(3, 4)$ represent leaders. In the following cases, the communication topology and coefficient matrices of the system vary, and the initial state is chosen randomly.

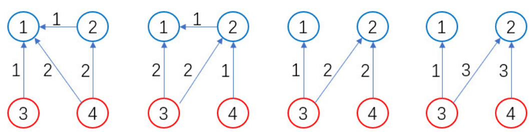

Fig. 1. The interaction topology among agents, with the sequence of graphs as $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ (from left to right).

Case 1

The communication topology is $\mathcal{G}_1$,

$$A_1 = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}.$$

The in-degrees for leaders are 0, while the in-degrees for followers are 1 (where 2+2=1 (mod 3)) and 2, so in-degrees of nodes in graph $\mathcal{G}_1$ take on three distinct values. $\hat{A}$ is not nilpotent, according to Corollary 3, even if each agent is under external control $(d_i = 1, i = 1, \ldots, 4)$, HOFA multi-agent system (6) is not stabilizable.

Case 2

The communication topology is $\mathcal{G}_2$,

$$A_1 = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The in-degree of each agent is 0 (where 2+1=0 (mod 3)), and $\hat{A}$ is nilpotent. By Theorem 3, HOFA multi-agent system (6) is stabilizable without external control, the upper bound of settling time is T=16. Let $K_0 = I_2$,

$$\begin{bmatrix} z_1(0)^{\mathrm{T}} & z_1(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_2(0)^{\mathrm{T}} & z_2(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_3(0)^{\mathrm{T}} & z_3(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 1 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_4(0)^{\mathrm{T}} & z_4(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}^{\mathrm{T}},$$

as shown in Figure 2, the settling time $T = 8 < 16$.

Case 3

The communication topology is $\mathcal{G}_2$,

$$A_1 = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}.$$

The in-degree of each agent is 0 (where 2+1=0 (mod 3)), but $\hat{A}$ is not nilpotent. By Theorem 3, HOFA multi-agent system (6) is stabilizable if each agent is under external control. Let parameter matrices of FAS design method are

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

so coefficient matrices of control protocol are

$$\hat{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; i = 1, \ldots, 4, \; \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix},$$

and the the upper bound of settling time is T=16.

Let $K_0 = I_2$,

$$\begin{bmatrix} z_1(0)^{\mathrm{T}} & z_1(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 0 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_2(0)^{\mathrm{T}} & z_2(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_3(0)^{\mathrm{T}} & z_3(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 1 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_4(0)^{\mathrm{T}} & z_4(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}^{\mathrm{T}},$$

as shown in Figure 2, the settling time $T = 9 < 16$.

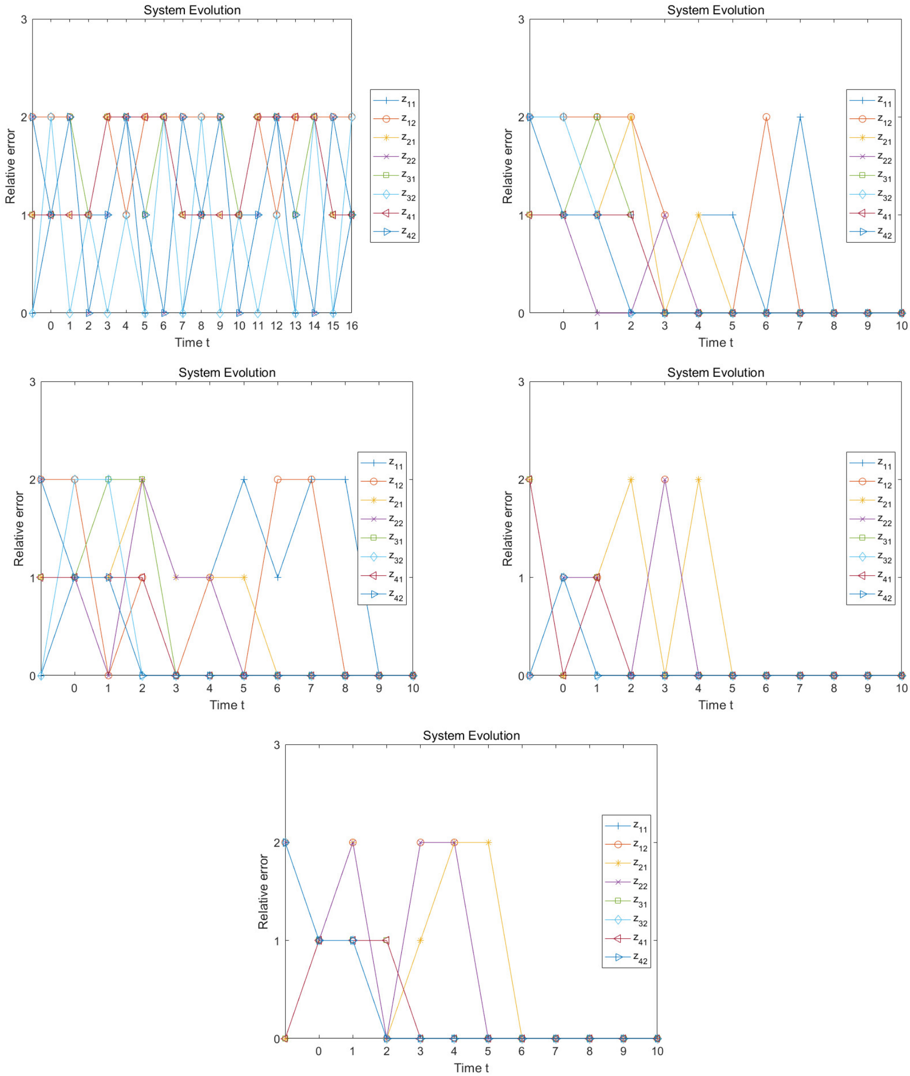

Fig. 2. The state trajectories of all agents in case 1-case 5.

Case 4

The communication topology is $\mathcal{G}_3$,

$$A_1 = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The in-degree of each leader is 0, and $\hat{A}_i = \hat{A}, i = 3, 4$ is nilpotent. The in-degree of each follower is 1 (where 2+2=1 (mod 3)), and $\hat{A}_i = \hat{A} - \bar{K}, i = 1, 2$ is not nilpotent.

By Theorem 3, HOFA multi-agent system (6) is stabilizable if each follower is under external control. Let parameter matrices of FAS design method are

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

so coefficient matrices of control protocol are

$$\hat{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ i = 1, 2, \ \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix},$$

and the upper bound of settling time is T=16. Let $K_0 = I_2$, $\begin{bmatrix} z_i(0)^{\mathrm{T}} & z_i(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 2 & 0 \end{bmatrix}^{\mathrm{T}}, i = 1, 2, 3, 4,$, as shown in Figure 2, the settling time $T = 5 < 16$.

Case 5

The communication topology is $\mathcal{G}_3$,

$$A_1 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The in-degree of each leader is 0, and $\hat{A}_i = \hat{A}, i = 3, 4$ is not nilpotent. The in-degree of each follower is 1, and $\hat{A}_i = \hat{A} - \bar{K}, i = 1, 2$ can be nilpotent, where $K_0 = I_2$. By Theorem 3, HOFA multi-agent system (6) is stabilizable if each leader is under external control. Let parameter matrices of FAS design method are

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

so coefficient matrices of control protocol are

$$\hat{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ i = 3, 4, \ \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix},$$

and the upper bound of settling time is T=16. Let $\begin{bmatrix} z_i(0)^{\mathrm{T}} & z_i(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 0 & 2 \end{bmatrix}^{\mathrm{T}}$, $i = 1, 2, 3, 4$, as shown in Figure 2, the settling time $T = 6 < 16$.

The second example is to validate the results for formation control. In this example, a multi-agent system is guided to transition from any given formation shape to a new formation shape through a control protocol. To validate the effectiveness of the control protocol, two formation control objectives are established to achieve.

Example 2. Consider HOFA multi-agent system (6) with control protocol (7) over $\mathbb{F}_5$ with $z_i = \begin{bmatrix} z_{i1} & z_{i2} \end{bmatrix}^{\mathrm{T}}$ and $\mu = 2$, which consists of 4 agents. Each agent represents a drone car, and each drone car can be controlled by input signals to move arbitrarily in the forward, backward, left, and right directions on a fixed plane. We divide the plane where the car is located, with the goal of moving the drone car from its initial position to the target position in sequence under the given control, as shown in the Figure 3.

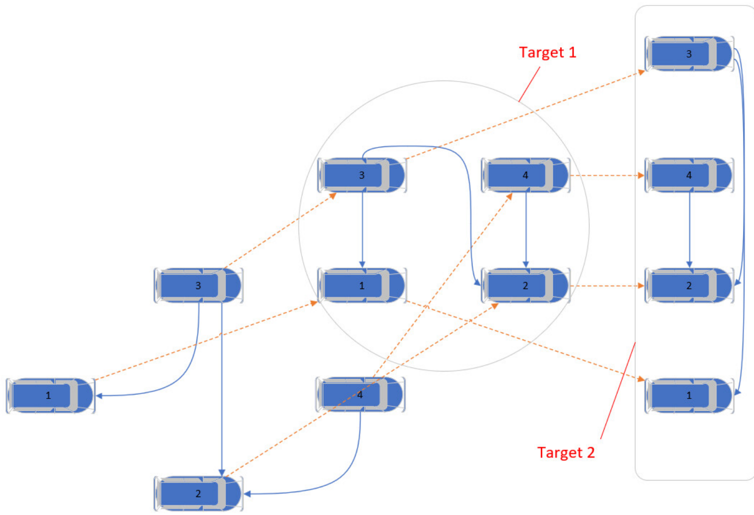

Fig. 3. Drone car formation targets.

Solid arrows represent the information transmission between drone cars, while dashed arrows indicate the trajectory of position changes for each drone car under the given control. The communication topology of HOFA multi-agent system (6) is described as $\mathcal{G}_4$ in Figure 1,

$$A_1 = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The in-degree of each leader is 0, and $\hat{A}_i = \hat{A}, i = 3, 4$ is not nilpotent. The in-degree of each follower is 1, and $\hat{A}_i = \hat{A} - \bar{K}, i = 1, 2$ can be nilpotent, where $K_0 = I_2$.

According to Definition 2, if $\lim_{t\to\infty} \| z_i(t) - h_i - h_0 \| = 0$, the set-point formation control problem is solved. The two formation objectives we have set are as follows. Let $h_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$, $H = \begin{bmatrix} h_1^{\mathrm{T}} & h_1^{\mathrm{T}} & h_2^{\mathrm{T}} & h_2^{\mathrm{T}} & h_3^{\mathrm{T}} & h_3^{\mathrm{T}} & h_4^{\mathrm{T}} & h_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, where $h_i = \begin{bmatrix} h_{i1} & h_{i2} \end{bmatrix}^{\mathrm{T}}$, for objective 1: $h_1 = \begin{bmatrix} 2 & 2 \end{bmatrix}^{\mathrm{T}}$, $h_2 = \begin{bmatrix} 3 & 2 \end{bmatrix}^{\mathrm{T}}$, $h_3 = \begin{bmatrix} 2 & 3 \end{bmatrix}^{\mathrm{T}}$, $h_4 = \begin{bmatrix} 3 & 3 \end{bmatrix}^{\mathrm{T}}$, and for objective 2: $h_1 = \begin{bmatrix} 4 & 1 \end{bmatrix}^{\mathrm{T}}$, $h_2 = \begin{bmatrix} 4 & 2 \end{bmatrix}^{\mathrm{T}}$, $h_3 = \begin{bmatrix} 4 & 4 \end{bmatrix}^{\mathrm{T}}$, $h_4 = \begin{bmatrix} 4 & 3 \end{bmatrix}^{\mathrm{T}}$. By Corollary 3, the set-point formation control problem is solvable if each leader is under external control. Let parameter matrices of FAS design method are

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

so coefficient matrices of control protocol are

$$\hat{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad i = 3, 4, \quad \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 4 & 1 & 0 & 1 \end{bmatrix},$$

and the upper bound of time for formation control is T=16. Let

$$\begin{bmatrix} z_1(0)^{\mathrm{T}} & z_1(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_2(0)^{\mathrm{T}} & z_2(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_3(0)^{\mathrm{T}} & z_3(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 1 & 2 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{bmatrix} z_4(0)^{\mathrm{T}} & z_4(-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 2 & 1 & 2 & 1 \end{bmatrix}^{\mathrm{T}}.$$

Figure 5 shows that the multi-agent system achieved formation control for two objectives: The first task for each agent is to transition from a larger square formation to a smaller square formation; The second task is to converge each agent into a straight line while maintaining a certain spacing. Figure 4 shows that the mission time for both formation control objectives is 6 steps, clearly less than the upper bound of time for formation control.

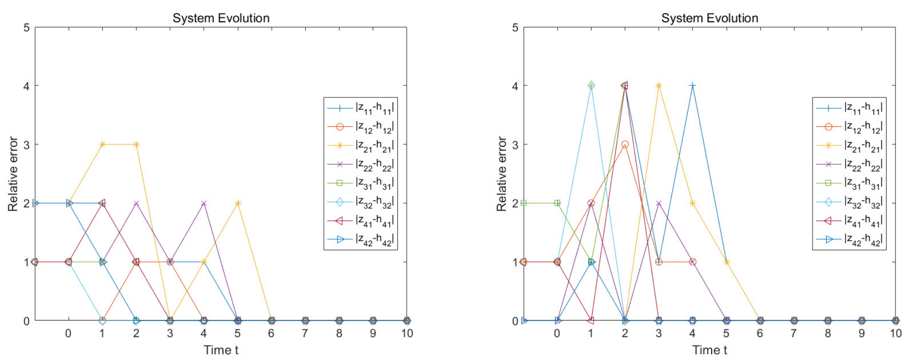

Fig. 4. The relative error between agent states and the desired formations.

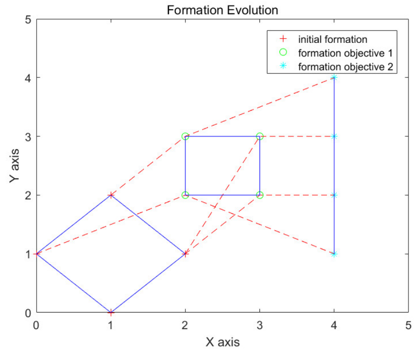

Fig. 5. The formation evolution trajectories of all agents.

6. CONCLUSION

In this paper, the stabilizability problem of HOFA multi-agent systems over finite fields was studied. Some necessary and/or sufficient conditions for the stabilizability of HOFA multi-agent systems have been proposed. It has been proved the stabilizability of HOFA multi-agent systems depends on the in-degrees of nodes in the interaction graph and coefficient matrices of systems. The general procedure of stabilization protocol design was presented, so that the desired coefficient matrices in control protocol was acquired, and the upper bound of settling time for the system was presented. Once an HOFA multi-agent system, derived from a linear multi-agent system, is stabilizable, the stabilizability problem of the linear multi-agent system can naturally be resolved through FAS approach. Finally, the results were employed to solve the formation control problem, and some sufficient and/or necessary conditions were developed. The problem of stabilizability and consensus in multi-agent systems with more general topological conditions and more complex dynamics remains to be explored.

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Yunsi Yang, Institute of Marine Science and Technology, Shandong University, Jinan 250100. P. R. China.
e-mail: yangyunsi@mail.sdu.edu.cn

Jun-e Feng, Corresponding author. School of Mathematics, Shandong University, Jinan 250100. P. R. China.
e-mail: fengjune@sdu.edu.cn

Lei Jia, Institute of Marine Science and Technology, Shandong University, Jinan 250100. P. R. China.
e-mail: jialei@sdu.edu.cn