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FIXED-TIME ADAPTIVE COMMAND-FILTER-BASED EVENT-TRIGGERED CONTROL OF CONSTRAINED SWITCHED NONLINEAR SYSTEMS WITH UNMODELED DYNAMICS

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In this paper, we investigate the problem of global output-feedback regulation for a class of switched nonlinear systems with unknown linear growth condition and uncertain output function. Based on the backstepping method, an adaptive output-feedback controller is designed to guarantee that the state of the switched nonlinear system can be globally regulated to the origin while maintaining global boundedness of the resulting closed-loop switched system under arbitrary switchings. A numerical example is given to demonstrate the effectiveness of the proposed control scheme.

Keywords: event-triggered control, command filter, unmodeled dynamics, function constraints, fixed-time stability

Classification: 93D21, 39A13

1. INTRODUCTION

As a category of important hybrid systems, the related control problems of switched system have attracted many scholars due to its extensive application in engineering [1, 2, 3]. In the research of switched systems, some methods, for instance, common Lyapunov function [4], multiple Lyapunov function [5] and average dwelling time [6] have been proposed. [7] designed finite-time controller of switched systems under different powers using common Lyapunov function method. In [8], a switched adaptive control method was developed combining multiple Lyapunov functions approach and parameter separation idea. [9] investigated stability for switched discrete-time systems under average dwell time. Above works consider stability and stabilization problems only for switched systems without unmodeled dynamics.

Meanwhile, state/output constraints have turned into a hot topic of control theory since its real applications, e.g., electromagnetic oscillators [10], electrostatic parallel plate micro-actuators [11] and so on. To overcome this issue, the barrier Lyapunov function (BLF) [12, 13, 14] and the nonlinear mapping [15] were constructed. In [16], a BLF-based adaptive control was considered for nonlinear systems under full state constraints.

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[17] investigated adaptive control of constrained nonlinear system with unknown control coefficients by Nussbaum gain technique. [18] further considered integral BLF-based adaptive control of nonlinear switched systems. In [19], adaptive stabilizer was designed for stochastic nonlinear constrained systems. [20] investigated an adaptive fuzzy tracking observer-based control approach for switched uncertain nonlinear constrained systems. Besides, [21] investigated adaptive neural control of nonlinear constrained systems utilizing the nonlinear mapping idea. To avoid repeated derivation of virtual stabilizers in recursive design procedure, command-filter based method [22] was borrowed. Subsequently, [23] proposed command-filter-based adaptive backstepping approach. In virtue of command filter, adaptive observer-based control of nonlinear systems was considered in [24]. On the basis of command filter, [25] constructed adaptive fuzzy stabilizer for nonlinear systems subject to unknown control gains. Noting that aforementioned thesis mainly concentrate on asymptotic properties when time attends to infinity.

To achieve better robustness and quicker response speed of system, [26] gave a criterion of finite-time stability for autonomous systems and the related results were achieved, such as [28]. Notice that the settling time functions depend on initial conditions in finite-time control results, which hinders their engineering applications since the desirable performance cannot be available without initial conditions. To this end, [29] proposed the concept of fixed-time stability, where the associated settling time functions are independent of initial conditions. Considering output/state constrains, in[30], an fixed-time adaptive controller was devised for output constrained multiple input multiple output (MIMO) systems . [31] researched fixed-time control of nonlinear systems subject to unmatched disturbances and output constraints. In [32], fixed-time controller was established for nonlinear switched systems with output limitations. On the basis of Levant differentiator [33], a new approach combining the backstepping technique and unbounded command-filter was further put forward for nonlinear systems in [34]. By an unbounded command filter, [35] designed adaptive finite-time stabilizer for quantized nonlinear systems. [36] studied predefined-time bipartite consensus tracking control for a class of constrained nonlinear multi-agent systems without unmodeled dynamics. Moreover, with the development of modern and intelligent control theories, control objectives and objects usually turn to be more and more complicated. For instance, unmodeled dynamics are appeared in real systems, which may result in the property degradation of stabilizers. As a consequence, it is necessary to study fixed-time adaptive control for switched nonlinear systems with output function constraints and unmodeled dynamics via bounded command filter.

To further save the resource of communication for nonlinear systems, event-triggered mechanism has attracted enormous attention, such as a static event-triggered method recommended in [37, 38]. [39] investigated adaptive neural network event-triggered formation fault-tolerant control issue for nonlinear multi-agent systems with intermittent actuator faults. Subsequently, a dynamic event-triggered strategy was expanded in [27, 40] and data-driven event-triggered algorithms were proposed in [41, 42]. To sum up, a significative question is proposed: how can we construct a command-filter-based adaptive fixed-time event-triggered stabilizer for uncertain constrained switched nonlinear systems subject to unmodeled dynamics by command filter and common barrier Lyapunov function (CBLF)? To reply this issue, a new fixed-time backstepping control scheme is proposed by joining bounded command filter into CBLF in this paper. The major advantages can be summarized below.

(i) The existing methods, such as asymptotic stability [27, 41] and finite-time stability [28, 42] are invalid to resolve the fixed-time control problem of universal nonlinear systems with unmodeled dynamics. Therefore, a new criterion of fixed-time stability in Assumption 3 is given and a novel dynamic signal in Lemma 1 is proposed by characterizing unmodeled impact for general dynamic systems.

(ii) Fuzzy logic systems (FLSs) and hyperbolic tangent function are simultaneously borrowed to handle complicated, unknown and continuous nonlinear function without the help of any linear/homogeneous growth condition. Meanwhile, the structure of the controller is simplified.

(iii) Distinct from [13, 14, 17], the CBLF of constrained switched nonlinear system is simplified based on command filter idea. Thus, a backstepping approach combining CBLF and bounded command filter is generalized such that only the reference signal y_d and its time derivative are bounded. Besides, the output constraints are the functions of both y_d and time t, which can be epitomized at Step 1 different from the constraints of only time t.

(iv) To reduce the control consumption, a new dynamic event-triggered stabilizer is devised to render that output can track the desired signal in fixed time and the output is kept within a constrained interval during operation.

Notations: $\mathbb{R}_+$ is the set of nonnegative real numbers, $\mathbb{R}^n$ is the set of n-dimensional real vectors.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider uncertain switched nonlinear systems:

$$\begin{cases} \dot{w} = \varrho_{\sigma(t)}(w,\zeta), \\ \dot{\zeta}_i = \zeta_{i+1} + \psi_{i,\sigma(t)}(\bar{\zeta}_i) + d_{i,\sigma(t)}(w,\zeta), \ i = 1,\dots,n-1, \\ \dot{\zeta}_n = u + \psi_{n,\sigma(t)}(\zeta) + d_{n,\sigma(t)}(w,\zeta), \\ y = \zeta_1 \end{cases} \tag{1}$$

where $\zeta = (\zeta_1,\dots,\zeta_n)^T \in \mathbb{R}^n$ is system state, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are control input and output, respectively. $\bar{\zeta}_i = [\zeta_1,\dots,\zeta_i]^T$ for $i = 1,\dots,n-1$. $\sigma(t)$ is called to be the switching signal and its values are taken in a limited set $\Pi = \{1,\dots,M\}$ and M is the quantity of subsystems. $\psi_{i,\pi}(\cdot)$ are unknown smooth functions and $\psi_{i,\pi}(0) = 0$. $w \in \mathbb{R}^m$ and $d_{i,\pi}(\cdot)$ represent unmodeled dynamics and external disturbances, severally. $\varrho_\pi(\cdot)$ and $d_{i,\pi}(\cdot)$ are unknown functions. $y_d(t)$ is the reference signal. $l(y_d,t) < y(t) < h(y_d,t)$ with $l(y_d,t)$ and $h(y_d,t)$ being differentiable functions of y_d and t. Moreover, we suppose that (1) has a unique solution in forward time for any initial condition except the origin.

The following assumption conditions are essential.

Assumption 2.1. The reference signal y_d and its first-order derivative $\dot{y}_d$ are available and bounded, i.e., $\underline{y}_0(t) \leq y_d(t) \leq \bar{y}_0(t)$ with $l(y_d, t) < \underline{y}_0(t) < \bar{y}_0(t) < h(y_d, t)$, and $|\dot{y}_d| \leq \bar{y}_1$ where $\bar{y}_1 > 0$ is a constant.

Assumption 2.2. For $i = 1, \ldots, n$ and $\pi \in \Pi$, the disturbances $d_{i,\pi}(w, \zeta)$ fulfil

$$|d_{i,\pi}(w, \zeta)| \leq \phi_{i,\pi,1}(\|\zeta\|) + \phi_{i,\pi,2}(\|w\|) \tag{2}$$

where the smooth functions $\phi_{i,\pi,1}(\|\zeta\|) > 0$ and $\phi_{i,\pi,2}(\|\omega\|) > 0$ are unknown.

Assumption 2.3. System $\dot{w} = \varrho_{\sigma(t)}(w, \zeta)$ are ISPS (input-to-state practically stable) when we can find $\mathcal{K}_\infty$ functions $\rho_1(\cdot), \rho_2(\cdot), \rho_3(\cdot)$, constants $\mu_1 > 0, \mu_2 > 0, \mu_3 \geq 0$, $\gamma = \frac{2n+o}{2n+1}$, where $n \in \mathbb{Z}^+$ and $o > 1$ is an odd integer and common ISPS function $V(w)$ ($\pi \in \Pi$) such that

$$\rho_1(\|w\|) \leq |V(w)| \leq \rho_2(\|w\|), \tag{3}$$

$$\frac{\partial V(w)}{\partial w} \varrho_\pi(w, \zeta) \leq -\mu_1 V(w) - \mu_2 V^\gamma(w) + \rho_3(|\zeta_1|) + \mu_3. \tag{4}$$

Remark 2.1. In Assumption 2.1, two points should be highlighted: (i) In this paper, the bound of y_d is the function of y_d and t simultaneously different from that is only the function of t in the existing works [13, 14], which can be seen in the later **Step 1**; (ii) Compared with [44], where y_d and ith order derivatives of y_d ($i = 1, \ldots, n$) must be bounded, only y_d and $\dot{y}_d$ need to be bounded in this paper. In Assumptions 2.2, the bound of disturbances is related to the functions of states and unmodeled dynamics. In contrast to [27], the term $-\mu_2 V^\gamma(w)$ with $\gamma > 1$ is introduced into Assumption 2.3. This gives a fixed-time stability criterion of the switched nonlinear system with unmodeled dynamics, which is also applied to non-switched case. Thus, Assumptions 2.1, 2.2 and 2.3 are the weaker conditions of nonlinear systems with unmodeled dynamics, and the proposed control scheme has more extensive applications, such as fixed-time tracking control for wheeled mobile robots.

The next definition and lemmas are vital for subsequent controller design and stability analysis.

Definition 2.1. (Polyakov [29]) The solution $\zeta(t, \zeta_0)$ of system $\dot{\zeta} = f(\zeta)$ is said to be practically fixed-time stable if for any positive constant c, there is a positive constant $T_{\max}$ independent of initial condition such that the settling-time function $T(\zeta_0)$ satisfies $\sup_{\zeta_0 \in \mathbb{R}^n} T(\zeta_0) \leq T_{\max}$, and $\|\zeta(t, \zeta_0)\| \leq c$ for all $t \geq T_{\max}$.

Lemma 2.1. When an ISPS function $V(w)$ gratifies (3) and (4), for $\forall \tilde{\mu}_1 \in (0, \mu_1)$, $\forall \tilde{\mu}_2 \in (0, \mu_2)$, any initial condition $w_0 = w(0)$, and any function $\tilde{\rho}_3(\zeta_1) \geq \rho_3(|\zeta_1|)$, there is a finite time $T_0 = T_0(\tilde{\mu}_1, \tilde{\mu}_2, r_0, w_0)$, a nonnegative function $B(t)$, $\forall t \geq 0$ and a signal given with

$$\dot{r} = -\tilde{\mu}_1 r - \tilde{\mu}_2 r^\gamma + \tilde{\rho}_3(\zeta_1(t)) + \mu_3, \; r(0) = r_0 > 0 \tag{5}$$

following that $B(t) = 0$ $(t \geq T_0)$

$$V(w(t)) \leq r(t) + B(t). \tag{6}$$

P r o o f. Consider the differential equation

$$\dot{x} = -\mu_1 x - \mu_2 x^\gamma,\ x(0) = x_0 > 0. \tag{7}$$

Multiplying by $(1-\gamma)x^{-\gamma}$ on both sides of (7) and taking $y = x^{1-\gamma}$, we have

$$\dot{y} + \mu_1(1-\gamma)y = \mu_2(\gamma - 1),\ y(0) = x_0^{1-\gamma}, \tag{8}$$

which is a first-order linear differential equation and its solution is

$$y(t) = (y(0) + \frac{\mu_2}{\mu_1})e^{\mu_1(\gamma-1)t} - \frac{\mu_2}{\mu_1}. \tag{9}$$

Substituting $y = x^{1-\gamma}$ and $y(0) = x_0^{1-\gamma}$ into (9) yields

$$x(t) = \big((x_0^{1-\gamma} + \frac{\mu_2}{\mu_1})e^{\mu_1(\gamma-1)t} - \frac{\mu_2}{\mu_1}\big)^{\frac{1}{1-\gamma}}. \tag{10}$$

With the aid of Gronwall's lemma, the combination of (4), (5) and (10) results in

$$\begin{aligned} V(w(t)) \le & r(t) + \big((V(w_0))^{1-\gamma} + \frac{\mu_2}{\mu_1})e^{\mu_1(\gamma-1)t} - \frac{\mu_2}{\mu_1}\big)^{\frac{1}{1-\gamma}} \\ & - \big(r_0^{1-\gamma} + \frac{\tilde{\mu}_2}{\tilde{\mu}_1})e^{\tilde{\mu}_1(\gamma-1)t} - \frac{\tilde{\mu}_2}{\tilde{\mu}_1}\big)^{\frac{1}{1-\gamma}}. \end{aligned} \tag{11}$$

Let

$$\begin{aligned} B(t) = & \max\{0, \\ & \big((V(w_0))^{1-\gamma} + \frac{\mu_2}{\mu_1})e^{\mu_1(\gamma-1)t} - \frac{\mu_2}{\mu_1}\big)^{\frac{1}{1-\gamma}} - \big(r_0^{1-\gamma} + \frac{\tilde{\mu}_2}{\tilde{\mu}_1})e^{\tilde{\mu}_1(\gamma-1)t} - \frac{\tilde{\mu}_2}{\tilde{\mu}_1}\big)^{\frac{1}{1-\gamma}}\} \end{aligned} \tag{12}$$

Due to $0 < \tilde{\mu}_1 < \mu_1$, $0 < \tilde{\mu}_2 < \mu_2$, $r_0 > 0$ and $\gamma > 1$, we can find a finite time $T_0 = T_0(\tilde{\mu}_1, \tilde{\mu}_2, r_0, w_0)$ to ensure that $B(t) = 0$, $\forall t \ge T_0$. By (11) and (12), (6) holds directly. □

Remark 2.2. A new dynamic signal in Lemma 1 is proposed by characterizing unmodeled impact for usual dynamic systems. Different from [27, 28], the term $-\tilde{\mu}_2 r^\gamma$ is introduced and it will lay the foundation of resolving practically fixed-time control issue of nonlinear systems with unmodeled dynamics. Without loss of generality, we choose $\tilde{\rho}_3(\zeta_1) = \zeta_1^2\rho_0(\zeta_1^2)$ with a smooth function $\rho_0(\cdot) \ge 0$.

Lemma 2.2. (Li et al. [27]) Provided that $f(\zeta)$ is a continuous function on a compact set Ω. Immediately a FLS $\theta^T\psi(\zeta)$ can be found to guarantee

$$\sup_{\zeta\in\Omega} |f(\zeta) - \theta^T\psi(\zeta)| \le \varepsilon,\ \forall\varepsilon > 0 \tag{13}$$

with $\zeta = [\zeta_1, \ldots, \zeta_n]^T$, $\theta = [\theta_1, \ldots, \theta_M]^T$ being weight vector, and $M > 1$ being quantity of fuzzy rules. $\psi(\zeta) = [\psi_1(\zeta), \ldots, \psi_M(\zeta)]^T$ and $\psi_m = \frac{\prod_{i=1}^n \mu_{F_i^m}(\zeta_i)}{\sum_{m=1}^M(\prod_{i=1}^n \mu_{F_i^m}(\zeta_i))}$ with $\mu_{F_i^m}(\zeta_i)$ being normally selected by a Gaussian-type function.

Lemma 2.3. (Polyakov [29]) For system $\dot{\zeta} = f(\zeta)$, a Lyapunov function $V(\zeta)$ guarantees $\dot{V}(\zeta) \leq -a_1 V^{\lambda}(\zeta) - a_2 V^{\eta}(\zeta) + \sigma$ with constants $a_1 > 0, a_2 > 0, \sigma > 0$, $0 < \lambda < 1$ and $\eta > 1$. Then, the system solution is practically fixed-time stable and and the residual set of the system solution is described as $\{\lim_{t\to T} \zeta | V(\zeta) \leq \min\{(\frac{\sigma}{a_1(1-\delta)})^{\frac{1}{\lambda}}, (\frac{\sigma}{a_2(1-\delta)})^{\frac{1}{\eta}}\}\}$. Moreover, the settling time T fulfils $T \leq T_{\max} := \frac{1}{(1-\lambda)\delta a_1} + \frac{1}{(\eta-1)\delta a_2}$ with $0 < \delta < 1$.

Lemma 2.4. (Yu et al. [34]) For system $\dot{\zeta} = f(\zeta)$, a Lyapunov function $V(\zeta)$ can render $\dot{V}(\zeta) \leq -c_1 V(\zeta) - c_2 V^m(\zeta)$, where constants $c_1 > 0$, $c_2 > 0$, $0 < m < 1$. Hence, the solution of $\dot{\zeta} = f(\zeta)$ is finite-time stable, where the settling time T fulfils $T \leq \frac{1}{c_1(1-m)} \ln(1 + \frac{c_1}{c_2} V^{1-m}(\zeta(0)))$.

Lemma 2.5. (Song and Li [32]) For $\nu_i \in \mathbb{R}$, $\left(\sum_{i=1}^{n} |\nu_i|\right)^a \leq \max\{n^{a-1}, 1\} \sum_{i=1}^{n} |\nu_i|^a$ holds with $a > 0$.

Lemma 2.6. (Xing et al. [37]) For $\xi \in \mathbb{R}$ and constant $k > 0$, we have

$$0 \leq |\xi| - \xi \tanh(\frac{\xi}{k}) \leq 0.2785k. \tag{14}$$

Lemma 2.7. (Li et al. [27]) Let $S_\nu = \{z|\ |z| < 0.2554\nu\}$. If $z \notin S_\nu$, then $1 - 16\tanh^2(z/\nu) \leq 0$.

Lemma 2.8. (Tee et al. [14]) For $|\zeta| < 1$, it follows $\log \frac{1}{1-\zeta^2} < \frac{\zeta^2}{1-\zeta^2}$.

Lemma 2.9. (Sun et al. [43]) Consider the differential equation: $\dot{\zeta} = -a\zeta - b\zeta^q + c\beta(t)$, $\zeta(0) \geq 0$ with the constants $a, b, c > 0$, $q > 1$ and the non-negative function $\beta(t)$. Then, it follows $\zeta(t) \geq 0$ for $\forall t \geq 0$.

Lemma 2.10. (Sun et al. [43]) For $x \geq y \geq 0$ and $p \geq 1$, then $y(x-y)^p \leq \frac{p}{p+1}(x^{p+1} - y^{p+1})$ holds.

In this paper, our goal is to devise a dynamic event-triggered fuzzy stabilizer such that the system output can track y_d in fixed time; all the states of the closed-loop systems are bounded; the function constraint on output signal is always maintained.

3. CONTROLLER DESIGN

In this section, a adaptive fixed-time controller is constructed by the backstepping method combining the command filter idea with the CBLF method. Its framework diagram is exhibited in Figure 1.

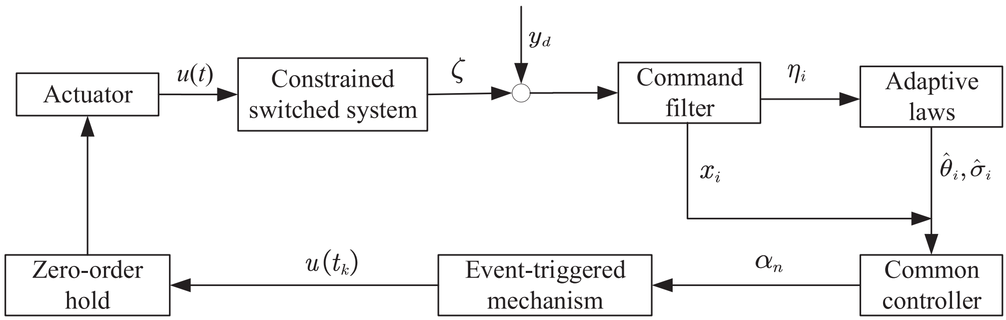

Fig. 1. The framework diagram of control scheme.

At first, we introduce coordinate changes:

$$x_1 = \zeta_1 - y_d,\ x_j = \zeta_j - \tilde{\alpha}_{j-1},\ j = 2, \ldots, n, \tag{15}$$

with $\tilde{\alpha}_{j-1}$ being command filter output. α_{j-1} is command filter input, which is described by

$$\begin{cases} \dot{\hbar}_{i1} & = \hbar_{i2} \\ \ell^2 \dot{\hbar}_{i2} & = -sat_{\varepsilon_b}\{sig(\ell \hbar_{i2})^{\beta}\} \\ & \quad -sat_{\varepsilon_b}\{sig(\phi_{\beta}(\hbar_{i1} - \alpha_i, \ell \hbar_{i2}))^{\frac{\beta}{2-\beta}}\} \end{cases} \tag{16}$$

with $0 < \beta < 1$, $\ell > 0$, $sig(\cdot)^p = sign(\cdot)|\cdot|^p$,

$$\phi_{\beta}(\hbar_{i1} - \alpha_i, \hbar_{i2}) = \hbar_{i1} - \alpha_i + \frac{sig(\ell \hbar_{i2})^{2-\beta}}{2-\beta},$$

$$sat_{\varepsilon_b}(\bullet) = \begin{cases} \bullet, & |\bullet| < \varepsilon_b, \\ \varepsilon_b sign(\bullet), & |\bullet| \geq \varepsilon_b, \end{cases} \tag{17}$$

where the signal α_i is a continuously differentiable, $\hbar_{i1} = \tilde{\alpha}_i$ with $\hbar_{i1}(0) = \alpha_i(0)$, and $\hbar_{i2}(0) = 0$. Therefore, constants $o_i > 0$ can be found to render that $|\hbar_{i1} - \alpha_i| \leq o_i$, $i = 1, \ldots, n-1$ by [27]. In addition, the compensating error signal η_i is depicted by

$$\eta_j = x_j - z_j, j = 1, \ldots, n, \tag{18}$$

with z_i being represented as

$$\begin{cases} \dot{z}_1 = -b_1 z_1 + z_2 + (\tilde{\alpha}_1 - \alpha_1) - \lambda_1 \text{sign}(z_1), \\ \dot{z}_i = -b_i z_i - z_{i-1} + z_{i+1} + (\tilde{\alpha}_i - \alpha_i) - \lambda_i \text{sign}(z_i), \\ \dot{z}_n = -b_n z_n - z_{n-1} - \lambda_n \text{sign}(z_n), \end{cases} \tag{19}$$

where $b_i > 0$ and $\lambda_i > 0$ are design constants.

By Lemma 2.2, FLSs are applied to handle unknown and complex functions:

$$F_{i,\pi}(s_i) = \theta_{i,\pi}^T \varphi_i(s_i) + \varepsilon_{i,\pi}(s_i),\ \pi \in \Pi,\ i = 1, \ldots, n \tag{20}$$

with the weight vector $\theta_{i,\pi}$ being unknown. Let $\theta_i = \max_{\pi\in\Pi}\{\|\theta_{i,\pi}\|^2\}$ and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, where $\hat{\theta}_i$ is estimation of θ_i, $\varepsilon_{i,\pi}(s_i)$ indicates approach error and $|\varepsilon_{i,\pi}(s_i)| \leq \bar{\varepsilon}_i$ with positive constant $\bar{\varepsilon}_i$ being unknown. $F_{i,\pi}$ and s_i are given in the sequel. Moreover, define $\sigma_i = \bar{\varepsilon}_i + \lambda_i$ and $\tilde{\sigma}_i = \sigma_i - \hat{\sigma}_i$ where $\hat{\sigma}_i$ is estimation of σ_i.

Next, a concrete fixed-time controller design process is given by a recursive technique.

Step 1: Choose common barrier function:

$$V_1 = \frac{q(\eta_1)}{2}\log\frac{L^2(y_d,t)}{L^2(y_d,t)-\eta_1^2} + \frac{1-q(\eta_1)}{2}\log\frac{H^2(y_d,t)}{H^2(y_d,t)-\eta_1^2} + \frac{1}{2m_1}\tilde{\theta}_i^2 + \frac{1}{2l_1}\tilde{\sigma}_1^2 + \frac{r}{l_0},$$

where $H(y_d,t) = h(y_d,t) - z_1 - y_d$, $L(y_d,t) = z_1 + y_d - l(y_d,t)$, $q(\eta_1) = \begin{cases} 1, \eta_1 \leq 0 \\ 0, \eta_1 > 0. \end{cases}$ $m_1,\ l_1,\ l_0$ are positive design constants. By a change of error coordinates

$$\begin{aligned} k_1(y_d,t) = \frac{\eta_1}{L(y_d,t)},\ k_2(y_d,t) = \frac{\eta_1}{H(y_d,t)}, \\ k(y_d,t) = q(\eta_1)k_1(y_d,t) + (1-q(\eta_1))k_2(y_d,t), \end{aligned} \tag{21}$$

V_1 can be simplified as $V_1 = \frac{1}{2}\log\frac{1}{1-k^2(y_d,t)} + \frac{1}{2m_1}\tilde{\theta}_1^2 + \frac{1}{2l_1}\tilde{\sigma}_1^2 + \frac{r}{l_0}$. Calculating the derivative of V_1, the use of (21) yields

$$\begin{aligned} \dot{V}_1 =& \frac{q(\eta_1)\eta_1}{L(y_d,t)^2-\eta_1^2}(\dot{\eta}_1 - \frac{\dot{L}(y_d,t)}{L(y_d,t)}\eta_1) + \frac{(1-q(\eta_1))\eta_1}{H(y_d,t)^2-\eta_1^2}(\dot{\eta}_1 - \frac{\dot{H}(y_d,t)}{H(y_d,t)}\eta_1) \\ &- \frac{1}{m_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 - \frac{1}{l_1}\tilde{\sigma}_1\dot{\hat{\sigma}}_1 + \frac{\dot{r}}{l_0} \\ =& v\eta_1\big(\eta_2 + \alpha_1 + \psi_{1,\pi}(\zeta) + d_{1,\pi}(w,\zeta) - \dot{y}_d + b_1 z_1 + \lambda_1\text{sign}(z_1) - q(\eta_1)\frac{\dot{L}(y_d,t)}{L(y_d,t)}\eta_1 \\ &- (1-q(\eta_1))\frac{\dot{H}(y_d,t)}{H(y_d,t)}\eta_1\big) - \frac{1}{l_1}\tilde{\sigma}_1\dot{\hat{\sigma}}_1 - \frac{1}{m_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0} + \frac{\tilde{\rho}_3(\zeta_1)}{l_0} + \frac{\mu_3}{l_0} \end{aligned} \tag{22}$$

with $\dot{L}(y_d,t) = \frac{\partial L(y_d,t)}{\partial t} + \frac{\partial L(y_d,t)}{\partial y_d}\dot{y}_d$, $\dot{H}(y_d,t) = \frac{\partial H(y_d,t)}{\partial t} + \frac{\partial H(y_d,t)}{\partial y_d}\dot{y}_d$, $v := q(\eta_1)/(L(y_d,t)^2 - \eta_1^2) + (1-q(\eta_1))/(H(y_d,t)^2-\eta_1^2)$.

By Assumption 2.2, it yields

$$v\eta_1 d_{1,\pi}(w,\zeta) \leq v|\eta_1|\phi_{1,\pi,1}(\|\zeta\|) + v|\eta_1|\phi_{1,\pi,2}(\|w\|). \tag{23}$$

Based on Lemma 2.6, one has

$$v|\eta_1|\phi_{1,\pi,1}(\|\zeta\|) \leq v\eta_1\phi_{1,\pi,1}(\|\zeta\|)\tanh\big(\frac{v\eta_1\phi_{1,\pi,1}(\|\zeta\|)}{\kappa_{11}}\big) + 0.2785\kappa_{11}, \tag{24}$$

with $\kappa_{11} > 0$ being a constant.

Applying Lemma 2.1, Assumption 2.3, the $\mathcal{K}_\infty$ characteristic of $\rho_1(\cdot)$ arrives at

$$\begin{aligned} v|\eta_1|\phi_{1,\pi,2}(\|w\|) \leq v|\eta_1|\phi_{1,\pi,2}(\rho_1^{-1}(r(t)+B(t))) \leq v|\eta_1|\phi_{1,\pi,2}(\rho_1^{-1}(2r(t))) \\ + v|\eta_1|\phi_{1,\pi,2}(\rho_1^{-1}(2B(t))) \end{aligned}$$

$$\begin{aligned}&\leq v\eta_1\phi_{1,\pi,2}(\rho_1^{-1}(2r(t)))\tanh\big(\frac{v\eta_1\phi_{1,\pi,2}(\rho_1^{-1}(2r(t)))}{\kappa_{12}}\big)+0.2785\kappa_{12}+v^2\eta_1^2\\&\quad+\frac{1}{4}\phi_{12}^2(\rho_1^{-1}(2B(t))),\end{aligned}\tag{25}$$

where $\phi_{1,2}(\rho_1^{-1}(2B(t)))=\max_{\pi\in\Pi}\{\phi_{1,\pi,2}(\rho_1^{-1}(2B(t)))\}$, and $\kappa_{12}>0$ is a constant.

Combining (23) – (25) and (22) gives

$$\begin{aligned}\dot V_1\leq& v\eta_1\big(\eta_2+\alpha_1+\psi_{1,\pi}(\zeta_1)+\bar d_{1,\pi}(\zeta,v\eta_1,r)+v\eta_1-q(\eta_1)\frac{\dot L(y_d,t)}{L(y_d,t)}\eta_1\\&-(1-q(\eta_1))\frac{\dot H(y_d,t)}{H(y_d,t)}\eta_1-\dot y_d+b_1z_1\\&+\lambda_1\mathrm{sign}(z_1)\big)-\frac{1}{m_1}\tilde\theta_1\dot{\hat\theta}_1-\frac{1}{l_1}\tilde\sigma_1\dot{\hat\sigma}_1-\frac{\tilde\mu_1r}{l_0}-\frac{\tilde\mu_2r^\gamma}{l_0}+\frac{\tilde\rho_3(\zeta_1)}{l_0}+\Lambda_1,\end{aligned}\tag{26}$$

where $\bar d_{1,\pi}(\zeta,v\eta_1,r)=\phi_{1,\pi,1}(\|\zeta\|)\tanh\big(\frac{v\eta_1\phi_{1,\pi,1}(\|\zeta\|)}{\kappa_{11}}\big)+\phi_{1,\pi,2}(\rho_1^{-1}(2r(t)))$ $\times\tanh\big(\frac{v\eta_1\phi_{1,\pi,2}(\rho_1^{-1}(2r(t)))}{\kappa_{12}}\big)$ and $\Lambda_1=0.2785\kappa_{11}+0.2785\kappa_{12}+\frac{\mu_3}{l_0}+\frac14\phi_{12}^2(\rho_1^{-1}(2B(t)))$.

Noting that the function $\frac{\tilde\rho_3(\zeta_1)}{v\eta_1l_0}$ is discontinuous at $\eta_1=0$, the function $\tanh^2(\frac{v\eta_1}{\nu})$ with constant $\nu>0$ is borrowed. Hence, we have

$$\begin{aligned}\dot V_1\leq& v\eta_1\big(\eta_2+\alpha_1+\psi_{1,\pi}(\zeta_1)+\bar d_{1,\pi}(\zeta,v\eta_1,r)+\frac{16}{v\eta_1}\tanh^2(\frac{v\eta_1}{\nu})\frac{\tilde\rho_3(\zeta)}{l_0}+v\eta_1\\&-q(\eta_1)\frac{\dot L(y_d,t)}{L(y_d,t)}\eta_1-(1-q(\eta_1))\frac{\dot H(y_d,t)}{H(y_d,t)}\eta_1-\dot y_d+b_1z_1\big)+v|\eta_1|\lambda_1-\frac{1}{m_1}\tilde\theta_1\dot{\hat\theta}_1\\&-\frac{1}{l_1}\tilde\sigma_1\dot{\hat\sigma}_1-\frac{\tilde\mu_1r}{l_0}-\frac{\tilde\mu_2r^\gamma}{l_0}+\big(1-16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde\rho_3(\zeta_1)}{l_0}+\Lambda_1.\end{aligned}\tag{27}$$

By (20), FLSs are applied to estimate uncertain switching functions $\psi_{1,\pi}(\zeta_1)$ and $\bar d_{1,\pi}(\zeta,v\eta_1,r)$, i.e., $F_{1,\pi}(s_1)=\psi_{1,\pi}(\zeta_1)+\bar d_{1,\pi}(\zeta,v\eta_1,r)=\theta_{1,\pi}^T\varphi_1(s_1)+\varepsilon_{1,\pi}(s_1)$ with $s_1=(\zeta,v\eta_1,r)$. By the definition of $\theta_1=\max_{\pi\in\Pi}\{\|\theta_{1,\pi}\|^2\}$ and Young's inequality, it leads to

$$v\eta_1\theta_{1,\pi}^T\varphi_1(s_1)\leq\frac14+v^2\eta_1^2\theta_1\varphi_1^T(s_1)\varphi_1(s_1).\tag{28}$$

This together with the definition $\sigma_1=\bar\varepsilon_1+\lambda_1$ and (27) yields

$$\begin{aligned}\dot V_1\leq& v\eta_1\big(\eta_2+\alpha_1+v\eta_1\tilde\theta_1\varphi_1^T(s_1)\varphi_1(s_1)+v\eta_1\hat\theta_1\varphi_1^T(s_1)\varphi_1(s_1)-\dot y_d+b_1x_1-b_1\eta_1\\&+\frac{16}{v\eta_1}\tanh^2(\frac{v\eta_1}{\nu})\frac{\tilde\rho_3(\zeta_1)}{l_0}+v\eta_1-q(\eta_1)\frac{\dot L(y_d,t)}{L(y_d,t)}\eta_1-(1-q(\eta_1))\frac{\dot H(y_d,t)}{H(y_d,t)}\eta_1\big)\\&+v|\eta_1|\sigma_1-\frac{1}{m_1}\tilde\theta_1\dot{\hat\theta}_1-\frac{1}{l_1}\tilde\sigma_1\dot{\hat\sigma}_1-\frac{\tilde\mu_1r}{l_0}-\frac{\tilde\mu_2r^\gamma}{l_0}+\big(1-16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde\rho_3(\zeta_1)}{l_0}\\&+\Lambda_1+\frac14.\end{aligned}\tag{29}$$

Next, the virtual stabilizer and adaptive laws are designed as

$$\begin{aligned}\alpha_1 =& - c_1 v^{p-1}\eta_1^{2p-1} - a_1 v^{\gamma-1}\eta_1^{2\gamma-1} - b_1 x_1 - (\bar{c}_1(t)+v)\eta_1 + \dot{y}_d\\ &- \frac{16}{v\eta_1}\tanh^2(\frac{v\eta_1}{\nu})\frac{\tilde{\rho}_3(\zeta_1)}{l_0} - v\eta_1\hat{\theta}_1\varphi_1^T(s_1)\varphi_1(s_1) - \hat{\sigma}_1\tanh(\frac{v\eta_1}{q_1}),\\ \dot{\hat{\theta}}_1 =& m_1 v^2\eta_1^2\varphi_1^T(s_1)\varphi_1(s_1) - \iota_1\hat{\theta}_1 - \gamma_1\hat{\theta}_1^{2\gamma-1},\ \hat{\theta}_1(0)\geq 0,\\ \dot{\hat{\sigma}}_1 =& l_1 v\eta_1\tanh(\frac{v\eta_1}{q_1}) - \tau_1\hat{\sigma}_1 - p_1\hat{\sigma}_1^{2\gamma-1},\ \hat{\sigma}_1(0)\geq 0,\end{aligned} \tag{30}$$

where $c_1, a_1, q_1, \iota_1, \tau_1, \gamma_1, p_1$ are positive constants, and $p = \frac{2n-1}{2n+1}$ with $n \in \mathbb{Z}$ and $n \geq 2$. Take $\bar{c}_1(t) = \max\{|\frac{\dot{L}(y_d,t)}{L(y_d,t)}|, |\frac{\dot{H}(y_d,t)}{H(y_d,t)}|\}$, which leads to $\bar{c}_1(t) + q(\eta_1)\frac{\dot{L}(y_d,t)}{L(y_d,t)} + (1 - q(\eta_1))\frac{\dot{H}(y_d,t)}{H(y_d,t)} \geq 0$. By Lemma 2.9, we have $\hat{\theta}_1 = \theta_1 - \tilde{\theta}_1 \geq 0$ and $\hat{\sigma}_1 = \sigma_1 - \tilde{\sigma}_1 \geq 0$. By Lemma 2.10, it attains

$$\begin{aligned}\tilde{\theta}_1\hat{\theta}_1^{2\gamma-1} &= \tilde{\theta}_1(\theta_1 - \tilde{\theta}_1)^{2\gamma-1} \leq \frac{2\gamma-1}{2\gamma}(\theta_1^{2\gamma} - \tilde{\theta}_1^{2\gamma}),\\ \tilde{\sigma}_1\hat{\sigma}_1^{2\gamma-1} &= \tilde{\sigma}_1(\sigma_1 - \tilde{\sigma}_1)^{2\gamma-1} \leq \frac{2\gamma-1}{2\gamma}(\sigma_1^{2\gamma} - \tilde{\sigma}_1^{2\gamma}).\end{aligned} \tag{31}$$

Notice the fact $\tilde{\theta}_1\theta_1 \leq \frac{1}{2}\tilde{\theta}_1^2 + \frac{1}{2}\theta_1^2$, $\tilde{\sigma}_1\sigma_1 \leq \frac{1}{2}\tilde{\sigma}_1^2 + \frac{1}{2}\sigma_1^2$ and substituting (30) and (31) into (29) yields

$$\begin{aligned}\dot{V}_1 \leq& - b_1 v\eta_1^2 - c_1 v^p\eta_1^{2p} - a_1 v^\gamma\eta_1^{2\gamma} + v\eta_1\eta_2 + v|\eta_1|\sigma_1 - \sigma_1 v\eta_1\tanh(\frac{v\eta_1}{q_1}) - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0}\\ &- \frac{\iota_1}{2m_1}\tilde{\theta}_1^2 - \frac{\tau_1}{2l_1}\tilde{\sigma}_1^2 - \frac{\gamma_1(2\gamma-1)}{2m_1\gamma}\tilde{\theta}_1^{2\gamma} - \frac{p_1(2\gamma-1)}{2l_1\gamma}\tilde{\sigma}_1^{2\gamma} + \big(1 - 16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}\\ &+ \frac{\iota_1}{2m_1}\theta_1^2 + \frac{\tau_1}{2l_1}\sigma_1^2 + \frac{\gamma_1(2\gamma-1)}{2m_1\gamma}\theta_1^{2\gamma} + \frac{p_1(2\gamma-1)}{2l_1\gamma}\sigma_1^{2\gamma} + \frac{1}{4} + \Lambda_1.\end{aligned} \tag{32}$$

From Lemma 2.6, it follows

$$v|\eta_1|\sigma_1 - \sigma_1 v\eta_1\tanh(\frac{v\eta_1}{q_1}) \leq 0.2785 q_1\sigma_1. \tag{33}$$

This gives rise to

$$\begin{aligned}\dot{V}_1 \leq& - b_1 v\eta_1^2 - c_1 v^p\eta_1^{2p} - a_1 v^\gamma\eta_1^{2\gamma} + v\eta_1\eta_2 - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0} - \frac{\iota_1}{2m_1}\tilde{\theta}_1^2 - \frac{\tau_1}{2l_1}\tilde{\sigma}_1^2\\ &- \frac{\gamma_1(2\gamma-1)}{2m_1\gamma}\tilde{\theta}_1^{2\gamma} - \frac{p_1(2\gamma-1)}{2l_1\gamma}\tilde{\sigma}_1^{2\gamma} + \big(1 - 16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0} + \bar{\Lambda}_1,\end{aligned} \tag{34}$$

where $\bar{\Lambda}_1 = \Lambda_1 + 0.2785 q_1\sigma_1 + \frac{\iota_1}{2m_1}\theta_1^2 + \frac{\tau_1}{2l_1}\sigma_1^2 + \frac{\gamma_1(2\gamma-1)}{2m_1\gamma}\theta_1^{2\gamma} + \frac{p_1(2\gamma-1)}{2l_1\gamma}\sigma_1^{2\gamma} + \frac{1}{4}$.

With (21) in mind, the application of Lemma 2.8 leads to

$$\dot{V}_1 \leq - b_1\frac{k^2}{1-k^2} - c_1\big(\frac{k^2}{1-k^2}\big)^p - a_1\big(\frac{k^2}{1-k^2}\big)^\gamma + v\eta_1\eta_2 - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0} - \frac{\iota_1}{2m_1}\tilde{\theta}_1^2$$

$$
\begin{aligned}
&-\frac{\tau_1}{2l_1}\tilde{\sigma}_1^2-\frac{\gamma_1(2\gamma-1)}{2m_1\gamma}\tilde{\theta}_1^{2\gamma}-\frac{p_1(2\gamma-1)}{2l_1\gamma}\tilde{\sigma}_1^{2\gamma}+\big(1-16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}+\bar{\Lambda}_1\\
\le&-b_1\log\frac{1}{1-k^2}-c_1\big(\log\frac{1}{1-k^2}\big)^p-a_1\big(\log\frac{1}{1-k^2}\big)^\gamma+v\eta_1\eta_2-\frac{\tilde{\mu}_1 r}{l_0}-\frac{\tilde{\mu}_2 r^\gamma}{l_0}\\
&-\frac{\iota_1}{2m_1}\tilde{\theta}_1^2-\frac{\tau_1}{2l_1}\tilde{\sigma}_1^2-\frac{\gamma_1(2\gamma-1)}{2m_1\gamma}\tilde{\theta}_1^{2\gamma}-\frac{p_1(2\gamma-1)}{2l_1\gamma}\tilde{\sigma}_1^{2\gamma}\\
&+\big(1-16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}+\bar{\Lambda}_1. \qquad (35)
\end{aligned}
$$

Step i ($2\le i\le n-1$): Choose $V_i=V_{i-1}+\frac{1}{2}\eta_i^2+\frac{1}{2m_i}\tilde{\theta}_i^2+\frac{1}{2l_i}\tilde{\sigma}_i^2$ with constants $m_i>0,\ l_i>0$.

In light of (1), (15), (18) and (19), one receives

$$
\begin{aligned}
\dot{V}_i\le&-b_1\log\frac{1}{1-k^2}-c_1\big(\log\frac{1}{1-k^2}\big)^p-a_1\big(\log\frac{1}{1-k^2}\big)^\gamma-\sum_{j=2}^{i-1}b_j\eta_j^2-\sum_{j=2}^{i-1}c_j\eta_j^{2p}\\
&-\sum_{j=2}^{i-1}a_j\eta_j^{2\gamma}+\eta_{i-1}\eta_i-\frac{\tilde{\mu}_1 r}{l_0}-\frac{\tilde{\mu}_2 r^\gamma}{l_0}-\sum_{j=1}^{i-1}\frac{\iota_j}{2m_j}\tilde{\theta}_j^2-\sum_{j=1}^{i-1}\frac{\tau_j}{2l_j}\tilde{\sigma}_j^2\\
&+\eta_i\big(\eta_{i+1}+\alpha_i+\psi_{i,\pi}(\bar{\zeta}_i)+d_{i,\pi}(w,\zeta)-\dot{\hat{\alpha}}_{i-1}+b_iz_i+z_{i-1}+\lambda_i\mathrm{sign}(z_i)\big)\\
&-\frac{1}{m_i}\tilde{\theta}_i\dot{\hat{\theta}}_i-\frac{1}{l_i}\tilde{\sigma}_i\dot{\hat{\sigma}}_i+\big(1-16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}+\sum_{j=1}^{i-1}\bar{\Lambda}_j\\
\le&-b_1\log\frac{1}{1-k^2}-c_1\big(\log\frac{1}{1-k^2}\big)^p-a_1\big(\log\frac{1}{1-k^2}\big)^\gamma-\sum_{j=2}^{i-1}b_j\eta_j^2-\sum_{j=2}^{i-1}c_j\eta_j^{2p}\\
&-\sum_{j=2}^{i-1}a_j\eta_j^{2\gamma}-\frac{\tilde{\mu}_1 r}{l_0}-\frac{\tilde{\mu}_2 r^\gamma}{l_0}-\sum_{j=1}^{i-1}\frac{\iota_j}{2m_j}\tilde{\theta}_j^2-\sum_{j=1}^{i-1}\frac{\tau_j}{2l_j}\tilde{\sigma}_j^2\\
&+\eta_i\big(\eta_{i+1}+\alpha_i+\psi_{i,\pi}(\bar{\zeta}_i)+\bar{d}_{i,\pi}(\zeta,\eta_i,r)+x_{i-1}+\eta_i-\dot{\hat{\alpha}}_{i-1}+b_iz_i\\
&+\lambda_i\mathrm{sign}(z_i)\big)-\frac{1}{m_i}\tilde{\theta}_i\dot{\hat{\theta}}_i-\frac{1}{l_i}\tilde{\sigma}_i\dot{\hat{\sigma}}_i+\big(1-16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}+\sum_{j=1}^{i-1}\bar{\Lambda}_j+\Lambda_i, \qquad (36)
\end{aligned}
$$

where $\bar{d}_{i,\pi}(\zeta,\eta_i,r)=\phi_{i,\pi,1}(\|\zeta\|)\tanh\big(\frac{\eta_i\phi_{i,\pi,1}(\|\zeta\|)}{\kappa_{i1}}\big)+\phi_{i,\pi,2}(\rho_1^{-1}(2r(t)))$ $\times\tanh\big(\frac{v\eta_i\phi_{i,\pi,2}(\rho_1^{-1}(2r(t)))}{\kappa_{i2}}\big)$ and $\Lambda_i=0.2785\kappa_{i1}+0.2785\kappa_{i2}+\frac{1}{4}\phi_{i2}^2(\rho_1^{-1}(2B(t)))$.

Resembling (20), we exploit the FLSs to approximate unknown switching functions $\psi_{i,\pi}(\bar{\zeta}_i)$ and $\bar{d}_{i,\pi}(\zeta,\eta_i,r)$, i.e., $F_{i,\pi}(s_i)=\psi_{i,\pi}(\bar{\zeta}_i)+\bar{d}_{i,\pi}(\zeta,\eta_i,r)=\theta_{i,\pi}^T\varphi_i(s_i)+\varepsilon_{i,\pi}(s_i)$ with $s_i=(\zeta,\eta_i,r)$. Based on the definition of $\theta_i=\max_{\pi\in\Pi}\{\|\theta_{i,\pi}\|^2\}$ and Young's inequality, we have

$$
\eta_i\theta_{i,\pi}^T\varphi_i(s_i)\le\frac{1}{4}+\eta_i^2\theta_i\varphi_i^T(s_i)\varphi_i(s_i). \qquad (37)
$$

Noting the definition $\sigma_i = \bar{\varepsilon}_i + \lambda_i$, (36) becomes

$$\begin{aligned}\dot{V}_i \leq &- b_1 \log \frac{1}{1-k^2} - c_1 \big(\log \frac{1}{1-k^2}\big)^p - a_1 \big(\log \frac{1}{1-k^2}\big)^\gamma - \sum_{j=2}^{i-1} b_j \eta_j^2 - \sum_{j=2}^{i-1} c_j \eta_j^{2p} \\ &- \sum_{j=2}^{i-1} a_j \eta_j^{2\gamma} - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0} - \sum_{j=1}^{i-1} \frac{\iota_j}{2m_j} \tilde{\theta}_j^2 - \sum_{j=1}^{i-1} \frac{\tau_j}{2l_j} \tilde{\sigma}_j^2 \\ &+ \eta_i \big(\eta_{i+1} + \alpha_i + \eta_i \tilde{\theta}_i \varphi_i^T(s_i)\varphi_i(s_i) + \eta_i \hat{\theta}_i \varphi_i^T(s_i)\varphi_i(s_i) + x_{i-1} + \eta_i - \dot{\hat{\alpha}}_{i-1} + b_i z_i\big) \\ &+ |\eta_i|\sigma_i - \frac{1}{m_i}\tilde{\theta}_i \dot{\hat{\theta}}_i - \frac{1}{l_i}\tilde{\sigma}_i \dot{\hat{\sigma}}_i + \big(1 - 16\tanh^2(\frac{\upsilon \eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0} + \sum_{j=1}^{i-1} \bar{\Lambda}_j + \Lambda_i + \frac{1}{4}. \quad (38)\end{aligned}$$

Subsequently, we construct virtual stabilizer and adaptive laws as

$$\begin{aligned}\alpha_i =& - c_i \eta_i^{2p-1} - a_i \eta_i^{2\gamma-1} - b_i x_i - x_{i-1} - \eta_i + \dot{\hat{\alpha}}_{i-1} - \eta_i \hat{\theta}_i \varphi_i^T(s_i)\varphi_i(s_i) - \hat{\sigma}_i \tanh(\frac{\eta_i}{q_i}), \\ \dot{\hat{\theta}}_i =& m_i \eta_i^2 \varphi_i^T(s_i)\varphi_i(s_i) - \iota_i \hat{\theta}_i - \gamma_i \hat{\theta}_i^{2\gamma-1}, \ \hat{\theta}_i(0) \geq 0, \\ \dot{\hat{\sigma}}_i =& l_i \eta_i \tanh(\frac{\eta_i}{q_i}) - \tau_i \hat{\sigma}_i - p_i \hat{\sigma}_i^{2\gamma-1}, \ \hat{\sigma}_i(0) \geq 0,\end{aligned} \quad (39)$$

where $c_i, a_i, q_i, \iota_i, \tau_i, \gamma_i, p_i$ are positive design constants.

Similar to (31), the application of Lemmas 2.9 and 2.10 gives

$$\tilde{\theta}_i \hat{\theta}_i^{2\gamma-1} \leq \frac{2\gamma-1}{2\gamma}(\theta_i^{2\gamma} - \tilde{\theta}_i^{2\gamma}), \quad \tilde{\sigma}_i \hat{\sigma}_i^{2\gamma-1} \leq \frac{2\gamma-1}{2\gamma}(\sigma_i^{2\gamma} - \tilde{\sigma}_i^{2\gamma}). \quad (40)$$

The use of Lemma 2.6 yields $|\eta_i|\sigma_i - \sigma_i \eta_i \tanh(\frac{\eta_i}{q_i}) \leq 0.2785\sigma_i q_i$, Then combination of (39) and (40) leads to

$$\begin{aligned}\dot{V}_i \leq &- b_1 \log \frac{1}{1-k^2} - c_1 \big(\log \frac{1}{1-k^2}\big)^p - a_1 \big(\log \frac{1}{1-k^2}\big)^\gamma - \sum_{j=2}^{i} b_j \eta_j^2 - \sum_{j=2}^{i} c_j \eta_j^{2p} \\ &- \sum_{j=2}^{i} a_j \eta_j^{2\gamma} - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0} - \sum_{j=1}^{i} \frac{\iota_j}{2m_j} \tilde{\theta}_j^2 - \sum_{j=1}^{i} \frac{\tau_j}{2l_j} \tilde{\sigma}_j^2 - \sum_{j=1}^{i} \frac{\gamma_j(2\gamma-1)}{2m_j\gamma} \tilde{\theta}_j^{2\gamma} \\ &- \sum_{j=1}^{i} \frac{p_j(2\gamma-1)}{2l_j\gamma} \tilde{\sigma}_j^{2\gamma} + \eta_i \eta_{i+1} + \big(1 - 16\tanh^2(\frac{\upsilon \eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0} + \sum_{j=1}^{i} \bar{\Lambda}_j, \quad (41)\end{aligned}$$

where $\bar{\Lambda}_i = \Lambda_i + 0.2785 q_i \sigma_i + \frac{\iota_i}{2m_i}\theta_i^2 + \frac{\tau_i}{2l_i}\sigma_i^2 + \frac{\gamma_i(2\gamma-1)}{2m_i\gamma}\theta_i^{2\gamma} + \frac{p_i(2\gamma-1)}{2l_i\gamma}\sigma_i^{2\gamma} + \frac{1}{4}$.

Step n: Choose the following common Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}\eta_n^2 + \frac{1}{2m_n}\tilde{\theta}_n^2 + \frac{1}{2l_n}\tilde{\sigma}_n^2 = \frac{1}{2}\log\frac{1}{1-k^2} + \sum_{i=2}^{n} \frac{1}{2}\eta_i^2$$

$$+\sum_{i=1}^{n}\frac{1}{2m_i}\tilde{\theta}_i^2+\sum_{i=1}^{n}\frac{1}{2l_i}\tilde{\sigma}_i^2+\frac{r}{l_0}, \tag{42}$$

where $m_n>0$ and $l_n>0$ are design parameters.

Similar to Step i, $F_{n,\pi}(s_n)=\psi_{n,\pi}(\bar{\zeta}_n)+\bar{d}_{n,\pi}(\zeta,\eta_n,r)=\theta_{n,\pi}^T\varphi_n(s_n)+\varepsilon_{i,\pi}(s_i)$ with $s_n=(\zeta,\eta_n,r)$, $\bar{d}_{n,\pi}(\zeta,\eta_n,r)=\phi_{n,\pi,1}(\|\zeta\|)\tanh\big(\frac{\eta_n\phi_{n,\pi,1}(\|\zeta\|)}{\kappa_{n1}}\big)+\phi_{n,\pi,2}(\rho_1^{-1}(2r(t)))$ $\times\tanh\big(\frac{\upsilon\eta_n\phi_{n,\pi,2}(\rho_1^{-1}(2r(t)))}{\kappa_{n2}}\big)$. The application of $\theta_n=\max_{\pi\in\Pi}\{\|\theta_{n,\pi}\|^2\}$ and Young's inequality yields $\eta_n\theta_{n,\pi}^T\varphi_n(s_n)\le\frac{1}{4}+\eta_n^2\theta_n\varphi_n^T(s_n)\varphi_n(s_n)$. Then, the employment of $\sigma_n=\bar{\varepsilon}_n+\lambda_n$ follows

$$\begin{aligned}\dot{V}_n\le&-b_1\log\frac{1}{1-k^2}-c_1\big(\log\frac{1}{1-k^2}\big)^p-\sum_{j=2}^{i}a_j\eta_j^{2\gamma}-\sum_{j=2}^{n-1}c_j\eta_j^2-\sum_{j=2}^{n-1}b_j\eta_j^{2p}-\sum_{j=2}^{n-1}a_j\eta_j^{2\gamma}\\&-\frac{\tilde{\mu}_1r}{l_0}-\frac{\tilde{\mu}_2r^\gamma}{l_0}-\sum_{j=1}^{n-1}\frac{\iota_j}{2m_j}\tilde{\theta}_j^2-\sum_{j=1}^{n-1}\frac{\tau_j}{2l_j}\tilde{\sigma}_j^2+\eta_n\big(u-\alpha_n+\alpha_n+\eta_n\tilde{\theta}_n\varphi_n^T(s_n)\varphi_n(s_n)\\&+\eta_n\hat{\theta}_n\varphi_n^T(s_n)\varphi_n(s_n)+x_{n-1}+\eta_n-\dot{\hat{\alpha}}_{n-1}+b_nz_n\big)+|\eta_n|\sigma_n-\frac{1}{m_n}\tilde{\theta}_n\dot{\hat{\theta}}_n-\frac{1}{l_n}\tilde{\sigma}_n\dot{\hat{\sigma}}_n\\&+\big(1-16\tanh^2(\frac{\upsilon\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}+\sum_{j=1}^{n-1}\bar{\Lambda}_j+\Lambda_n+\frac{1}{4},\end{aligned} \tag{43}$$

where $\Lambda_n=0.2785\kappa_{n1}+0.2785\kappa_{n2}+\frac{1}{4}\phi_{n2}^2(\rho_1^{-1}(2B(t)))$.

Therefore, we construct the virtual stabilizer and adaptive laws:

$$\begin{aligned}\alpha_n=&-c_n\eta_n^{2p-1}-a_n\eta_n^{2\gamma-1}-b_nx_n-x_{n-1}-\eta_n+\dot{\hat{\alpha}}_{n-1}\\&-\eta_n\hat{\theta}_n\varphi_n^T(s_n)\varphi_n(s_n)-\hat{\sigma}_n\tanh(\frac{\eta_n}{q_n}),\\\dot{\hat{\theta}}_n=&m_n\eta_n^2\varphi_n^T(s_n)\varphi_n(s_n)-\iota_n\hat{\theta}_n-\gamma_n\hat{\theta}_n^{2\gamma-1},\ \hat{\theta}_n(0)\ge0,\\\dot{\hat{\sigma}}_n=&l_n\eta_n\tanh(\frac{\eta_n}{q_n})-\tau_n\hat{\sigma}_n-p_n\hat{\sigma}_n^{2\gamma-1},\ \hat{\sigma}_n(0)\ge0,\end{aligned} \tag{44}$$

where $c_n,a_n,\iota_n,q_n,\tau_n,\gamma_n,p_n$ are positive parameters.

This together with (43) yields

$$\begin{aligned}\dot{V}_n\le&-b_1\log\frac{1}{1-k^2}-c_1\big(\log\frac{1}{1-k^2}\big)^p-a_1\big(\log\frac{1}{1-k^2}\big)^\gamma-\sum_{j=2}^{n}b_j\eta_j^2-\sum_{j=2}^{n}c_j\eta_j^{2p}\\&-\sum_{j=2}^{n}a_j\eta_j^{2\gamma}-\frac{\tilde{\mu}_1r}{l_0}-\frac{\tilde{\mu}_2r^\gamma}{l_0}-\sum_{j=1}^{n}\frac{\iota_j}{2m_j}\tilde{\theta}_j^2-\sum_{j=1}^{n}\frac{\tau_j}{2l_j}\tilde{\sigma}_j^2-\sum_{j=1}^{n}\frac{\gamma_j(2\gamma-1)}{2m_j\gamma}\tilde{\theta}_j^{2\gamma}\\&-\sum_{j=1}^{n}\frac{p_j(2\gamma-1)}{2l_j\gamma}\tilde{\sigma}_j^{2\gamma}+\eta_n\big(u-\alpha_n\big)+\big(1-16\tanh^2(\frac{\upsilon\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0}+\sum_{j=1}^{n}\bar{\Lambda}_j,\end{aligned} \tag{45}$$

where $\bar{\Lambda}_n=\Lambda_n+0.2785\sigma_nq_n+\frac{\iota_n}{2m_n}\theta_n^2+\frac{\tau_n}{2l_n}\sigma_n^2+\frac{\gamma_n(2\gamma-1)}{2m_n\gamma}\theta_n^{2\gamma}+\frac{p_n(2\gamma-1)}{2l_n\gamma}\sigma_n^{2\gamma}+\frac{1}{4}$.

initialize: Let $i = 1$, by choosing reference signal $y_d(t)$, command filter and BLF V_1, design controller α_1 and adaptive laws $\dot{\hat{\sigma}}_1$, $\dot{\hat{\theta}}_1$.
while $i \leq n$ **do**
Construct command filter and the BLF V_i and calculate its derivative $\dot{V}_i$.
Utilize FLSs to approximate unknown nonlinear function $\psi_{i,\pi}(\bar{\zeta}_i)$ and $\bar{d}_{i,\pi}(\zeta, \eta_i, r)$.
Design controller α_i and adaptive laws $\dot{\hat{\sigma}}_i$, $\dot{\hat{\theta}}_i$.
Construct dynamical event-triggered controller $u(t) = \vartheta(t_k),\ \forall t \in [t_k, t_{k+1})$.
end while
Output: $u(t)$.

Tab. 1. The control scheme algorithm.

In the following, to reduce the numbers of update for the stabilizer, a suitable dynamic event-triggered stabilizer is described by:

$$\vartheta(t) = -(1+\xi(t))\big(\bar{\epsilon}\tanh(\frac{\eta_n\bar{\epsilon}}{q_0}) + \alpha_n\tanh(\frac{\eta_n\alpha_n}{q_0})\big) \tag{46}$$

$$u(t) = \vartheta(t_k),\ \forall t \in [t_k, t_{k+1}), \tag{47}$$

$$t_{k+1} = \inf\{t > t_k \| e(t)| > \xi(t)|u(t)| + \epsilon(t)\}, \tag{48}$$

$$\dot{\xi} = -\varsigma_1|\alpha_n|\xi,\ 0 < \xi(0) < 1, \tag{49}$$

$$\dot{\epsilon} = -\varsigma_2|\alpha_n|\epsilon,\ \epsilon(0) > 0, \tag{50}$$

with q_0, ς_1 and ς_2 being positive design constants, $\bar{\epsilon} = \frac{\epsilon(t)}{1-\xi(t)}$, $e(t) = \vartheta(t) - u(t)$ is a measurement error, $\vartheta(t)$ represents a continuous stabilizer, t_k, $k \in \mathbb{Z}^+$ is the stabilizer update time, in other words , in case the event-triggered condition (48) is fulfilled, $u(t)$ is superseded with $u(t) = \vartheta(t_{k+1})$. If $t \in [t_k, t_{k+1})$, the controller $u(t)$ maintains at a constant value $\vartheta(t_k)$. In views of (48), we can find two variables $\varpi_1(t)$ and $\varpi_2(t)$ satisfying $|\varpi_1(t)| \leq 1$, $|\varpi_2(t)| \leq 1$, $\forall t \in [t_k, t_{k+1})$ to guarantee that

$$u(t) = \frac{\vartheta(t) - \varpi_2(t)\epsilon(t)}{1 + \varpi_1(t)\xi(t)}. \tag{51}$$

Remark 3.1. Compared with [48], $|\alpha_n|$ is introduced to dynamically adjust the convergent rate of $\xi(t)$ and $\epsilon(t)$. This reduces the conservativeness by the tunable control design parameters and further improves the control performance. Moreover, the Zeno behavior is ruled out.

For demystifying the process of controller design, a control scheme algorithm is provided as follows:

4. STABILITY ANALYSIS

In this section, Theorem 4.1 is provided below.

Theorem 4.1. Assumptions 2.1-2.3 hold for system (1). A dynamic event-triggered controller (46)−(50) is designed to render that: (1) all the signals of the closed-loop systems are bounded; (2) the function constraints on output is always kept during operation, that is, $l(y_d,t) < y(t) < h(y_d,t)$; (3) system output can follow the reference signal within fixed time; (4) Zeno behavior does not happen.

Proof. Substituting (51) into (45) leads to

$$\begin{aligned}\dot V_n \le & -b_1\log\frac{1}{1-k^2} - c_1\big(\log\frac{1}{1-k^2}\big)^p - a_1\big(\log\frac{1}{1-k^2}\big)^\gamma - \sum_{j=2}^n b_j\eta_j^2 - \sum_{j=2}^n c_j\eta_j^{2p}\\ & -\sum_{j=2}^n a_j\eta_j^{2\gamma} - \frac{\tilde\mu_1 r}{l_0} - \frac{\tilde\mu_2 r^\gamma}{l_0} - \sum_{j=1}^n\frac{\iota_j}{2m_j}\tilde\theta_j^2 - \sum_{j=1}^n\frac{\tau_j}{2l_j}\tilde\sigma_j^2 - \sum_{j=1}^n\frac{\gamma_j(2\gamma-1)}{2m_j\gamma}\tilde\theta_j^{2\gamma}\\ & -\sum_{j=1}^n\frac{p_j(2\gamma-1)}{2l_j\gamma}\tilde\sigma_j^{2\gamma} + \eta_n\big(\frac{\vartheta(t)-\varpi_2(t)\epsilon(t)}{1+\varpi_1(t)\xi(t)} - \alpha_n\big)\\ & + \big(1-16\tanh^2(\frac{\upsilon\eta_1}{\nu})\big)\frac{\tilde\rho_3(\zeta_1)}{l_0} + \sum_{j=1}^n\bar\Lambda_j. \end{aligned} \tag{52}$$

By Lemma 2.6, it follows $\eta_n\big(\frac{\vartheta(t)-\varpi_2(t)\epsilon(t)}{1+\varpi_1(t)\xi(t)} - \alpha_n\big) \le 0.557q_0$. This results in

$$\begin{aligned}\dot V_n \le & -b_1\log\frac{1}{1-k^2} - c_1\big(\log\frac{1}{1-k^2}\big)^p - a_1\big(\log\frac{1}{1-k^2}\big)^\gamma - \sum_{j=2}^n b_j\eta_j^2 - \sum_{j=2}^n c_j\eta_j^{2p}\\ & -\sum_{j=2}^n a_j\eta_j^{2\gamma} - \frac{\tilde\mu_1 r}{l_0} - \frac{\tilde\mu_2 r^\gamma}{l_0} - \sum_{j=1}^n\frac{\iota_j}{2m_j}\tilde\theta_j^2 - \sum_{j=1}^n\frac{\tau_j}{2l_j}\tilde\sigma_j^2 - \sum_{j=1}^n\frac{\gamma_j(2\gamma-1)}{2m_j\gamma}\tilde\theta_j^{2\gamma}\\ & -\sum_{j=1}^n\frac{p_j(2\gamma-1)}{2l_j\gamma}\tilde\sigma_j^{2\gamma} + \big(1-16\tanh^2(\frac{\upsilon\eta_1}{\nu})\big)\frac{\tilde\rho_3(\zeta_1)}{l_0} + \Lambda, \end{aligned} \tag{53}$$

with $\Lambda = 0.557q_0 + \sum_{j=1}^n\bar\Lambda_j$. Next, we certify that $z_1,\dots,z_n$ are bounded. Construct $V_z = \frac{1}{2}\sum_{i=1}^n z_i^2$, and its derivative is

$$\dot V_z \le -\sum_{i=1}^n b_i z_i^2 - \sum_{i=1}^n\lambda_i|z_i| + \sum_{i=1}^{n-1}|\tilde\alpha_i-\alpha_i||z_i| \le -\sum_{i=1}^n b_i z_i^2 - \sum_{i=1}^n(\lambda_i-o_i)|z_i|. \tag{54}$$

The last inequality holds owing to $|\tilde\alpha_i-\alpha_i| \le o_i$. By adjusting parameter λ_i, $\lambda_i - o_i \ge \bar\lambda_i > 0$ can be maintained. Hence, we have

$$\dot V_z \le -\sum_{i=1}^n b_i z_i^2 - \sum_{i=1}^n\bar\lambda_i|z_i| \le -\bar b V_z - \bar\lambda V_z^{\frac{1}{2}}, \tag{55}$$

where $\bar b = 2\min\{b_1,\dots,b_n\}$ and $\bar\lambda = \sqrt{2}\min\{\bar\lambda_1,\dots,\bar\lambda_n\}$. Based on Lemma 2.4, $z_1,\dots,z_n$ are bounded.

(1) Case 1: $\eta_1 \notin S_\nu$. The use of Lemma 2.7 yields $1-16\tanh^2(\frac{\upsilon\eta_1}{\nu}) \le 0$, and this together with (53) leads to

$$\begin{aligned}\dot{V}_n \le &- b_1 \log\frac{1}{1-k^2} - c_1\big(\log\frac{1}{1-k^2}\big)^p - a_1\big(\log\frac{1}{1-k^2}\big)^\gamma - \sum_{j=2}^n b_j\eta_j^2 - \sum_{j=2}^n c_j\eta_j^{2p}\\ &- \sum_{j=2}^n a_j\eta_j^{2\gamma} - \frac{\tilde{\mu}_1 r}{l_0} - \frac{\tilde{\mu}_2 r^\gamma}{l_0} - \sum_{j=1}^n \frac{\iota_j}{2m_j}\tilde{\theta}_j^2 - \sum_{j=1}^n \frac{\tau_j}{2l_j}\tilde{\sigma}_j^2\\ &- \sum_{j=1}^n \frac{\gamma_j(2\gamma-1)}{2m_j\gamma}\tilde{\theta}_j^{2\gamma} - \sum_{j=1}^n \frac{p_j(2\gamma-1)}{2l_j\gamma}\tilde{\sigma}_j^{2\gamma} + \Lambda. \end{aligned} \tag{56}$$

Applying $|m|^p|n|^q \le \frac{p}{p+q}c|m|^{p+q} + \frac{q}{p+q}c^{-\frac{p}{q}}|n|^{p+q}$ in [46], one has

$$1^{1-p}(\frac{\iota_j}{2m_j}\tilde{\theta}_j^2)^p \le (1-p)p^{\frac{p}{1-p}} + \frac{\iota_j}{2m_j}\tilde{\theta}_j^2, \tag{57}$$

$$1^{1-p}(\frac{\tau_j}{2l_j}\tilde{\sigma}_j^2)^p \le (1-p)p^{\frac{p}{1-p}} + \frac{\tau_j}{2l_j}\tilde{\sigma}_j^2, \tag{58}$$

$$(\frac{r}{l_0})^p = \big((\frac{1}{\tilde{\mu}_1})^{\frac{p}{1-p}}\big)^{1-p}(\frac{\tilde{\mu}_1 r}{l_0})^p \le (1-p)(\frac{p}{\tilde{\mu}_1})^{\frac{p}{1-p}} + \frac{\tilde{\mu}_1 r}{l_0}. \tag{59}$$

Substituting (57)–(59) into (56), the application of Lemma 2.5 results in

$$\begin{aligned}\dot{V}_n \le &- 2^p c_1\big(\frac{1}{2}\log\frac{1}{1-k^2}\big)^p - 2^p\sum_{j=2}^n c_j(\frac{1}{2}\eta_j^2)^p - (\frac{r}{l_0})^p - 2^\gamma a_1\big(\frac{1}{2}\log\frac{1}{1-k^2}\big)^\gamma\\ &- 2^\gamma\sum_{j=2}^n a_j(\frac{1}{2}\eta_j^2)^\gamma - \tilde{\mu}_2 l_0^{\gamma-1}(\frac{r}{l_0})^\gamma - \sum_{j=1}^n \iota_j^p(\frac{1}{2m_j}\tilde{\theta}_j^2)^p - \sum_{j=1}^n \tau_j^p(\frac{1}{2l_j}\tilde{\sigma}_j^2)^p\\ &- \sum_{j=1}^n \frac{\gamma_j(2\gamma-1)}{2m_j\gamma}\tilde{\theta}_j^{2\gamma} - \sum_{j=1}^n \frac{p_j(2\gamma-1)}{2l_j\gamma}\tilde{\sigma}_j^{2\gamma} + \Lambda + 2n(1-p)p^{\frac{p}{1-p}} + (1-p)(\frac{p}{\tilde{\mu}_1})^{\frac{p}{1-p}}\\ \le &- C(\frac{1}{2}\log\frac{1}{1-k^2} + \sum_{j=2}^n\frac{1}{2}\eta_j^2 + \frac{r}{l_0} + \sum_{j=1}^n\frac{1}{2m_j}\tilde{\theta}_j^2 + \sum_{j=1}^n\frac{1}{2l_j}\tilde{\sigma}_j^2)^p - D(\frac{1}{2}\log\frac{1}{1-k^2}\\ &+ \sum_{j=2}^n\frac{1}{2}\eta_j^2 + \frac{r}{l_0} + \sum_{j=1}^n\frac{1}{2m_j}\tilde{\theta}_j^2 + \sum_{j=1}^n\frac{1}{2l_j}\tilde{\sigma}_j^2)^\gamma + E = -CV_n^p - DV_n^\gamma + E, \end{aligned} \tag{60}$$

with $C = \min_{i=1,\dots,n}\{2^p b_i, \iota_i^p, \tau_i^p, 1\}$, $D = (2+3n)^{1-\gamma}\min\{\tilde{\mu}_2 l_0^{\gamma-1}, 2^\gamma a_i, \frac{\gamma_j(2\gamma-1)}{2m_j\gamma}, \frac{p_j(2\gamma-1)}{2l_j\gamma}\}(i=1,\dots,n)$ and $E = 2n(1-p)p^{\frac{p}{1-p}} + (1-p)(\frac{p}{\tilde{\mu}_1})^{\frac{p}{1-p}} + \Lambda$.

On the basis of (42) and (60), η_i, $\tilde{\theta}_i$, $\tilde{\sigma}_i$ are bounded for $i=1,\dots,n$. By $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\sigma}_i = \sigma_i - \hat{\sigma}_i$, it is known that $\hat{\theta}_i$ and $\hat{\sigma}_i$ are bounded. Together with the boundedness of z_i, (15) and (18), x_i and ζ_i are bounded in a recursive manner. Hence, boundedness of all signals are verified.

Case 2: $\eta_1 \in S_\nu$. This case implies $|\eta_1| \le 0.554\lambda_1$, that is, η_1 is bounded. Owing to the boundedness of z_1 and (18), x_1 is bounded. In light of (15), ζ_1 is bounded. Thus, there is a constant $H > 0$ such that $\big(1 - 16\tanh^2(\frac{v\eta_1}{\nu})\big)\frac{\tilde{\rho}_3(\zeta_1)}{l_0} \le H$. Similar to Case 1, we obtain $\dot{V}_n \le -CV_n^p - DV^\gamma + \bar{E}$ with $\bar{E} = E + H$. As a consequence, boundedness of the whole closed-loop system are recursively proved.

(2) From **(1)**, it is known that $\dot{V}_n \le -CV_n^p - DV_n^\gamma + E$ if$\eta_1 \notin S_\nu$, or $\dot{V}_n \le -CV_n^p - DV_n^\gamma + \bar{E}$ if $\eta_1 \in S_\nu$. Then, we consider $\dot{V}_n \le -CV_n^p - DV_n^\gamma + \bar{E}$ if $\eta_1 \in S_\nu$. From Lemma 2.3, it is known that the system solution is practically fixed-time stable, and the settling time T fulfils $T \le \frac{1}{(1-p)\lambda C} + \frac{1}{(\gamma-1)\lambda D}$ with $0 < \lambda < 1$. Moreover, one has $V_n \le \min\{(\frac{\bar{E}}{C(1-\lambda)})^{\frac{1}{p}}, (\frac{\bar{E}}{D(1-\lambda)})^{\frac{1}{\gamma}}\}$. This gives rise to $\frac{1}{2}\log\frac{1}{1-k(y_d,t)^2} \le \min\{(\frac{\bar{E}}{C(1-\lambda)})^{\frac{1}{p}}, (\frac{\bar{E}}{D(1-\lambda)})^{\frac{1}{\gamma}}\}$. Then, it follows

$$|k(y_d,t)| \le \sqrt{1 - e^{-2\min\{(\frac{\bar{E}}{C(1-\lambda)})^{\frac{1}{p}}, (\frac{\bar{E}}{D(1-\lambda)})^{\frac{1}{\gamma}}\}}} < 1, \tag{61}$$

which results in $-L(y_d,t) < \eta_1(t) < H(y_d,t)$. From $y(t) = y_d(t) + z_1(t) + \eta_1(t)$, it can be deduced that $y_d(t) + z_1(t) - L(y_d,t) < y(t) < y_d(t) + z_1(t) + H(y_d,t)$. By the forms of $L(y_d,t)$ and $H(y_d,t)$, we further obtain $l(y_d,t) < y(t) < h(y_d,t)$. For the same reason, the identical conclusion can be derived under the condition $\dot{V}_n \le -CV_n^p - DV_n^\gamma + E$ if $\eta_1 \notin S_\nu$. Therefore, it is omitted here.

(3) In views of **(2)**, the following error reaches to the region:

$$\begin{aligned} -L(y_d,t)\sqrt{1 - e^{-2\min\{(\frac{\bar{E}}{C(1-\lambda)})^{\frac{1}{p}}, (\frac{\bar{E}}{D(1-\lambda)})^{\frac{1}{\gamma}}\}}} &\le \zeta_1(t) - y_d(t) \\ &\le H(y_d,t)\sqrt{1 - e^{-2\min\{(\frac{\bar{E}}{C(1-\lambda)})^{\frac{1}{p}}, (\frac{\bar{E}}{D(1-\lambda)})^{\frac{1}{\gamma}}\}}}, \end{aligned}$$

when $t \ge \frac{1}{(1-p)\lambda C} + \frac{1}{(\gamma-1)\lambda D}$. Thus, output can approach the desired signal in fixed time.

(4) For $k \in \mathbb{Z}^+$, $t_{k+1} - t_k \ge t^* > 0$ should be verified. From (47), it follows $\dot{u}(t) = 0$, $\forall t \in [t_k, t_{k+1})$. From $e(t) = \vartheta(t) - u(t)$, it can be deduced

$$\frac{d|e(t)|}{dt} = \operatorname{sign}(e(t))\dot{e}(t) \le |\dot{\vartheta}(t)|, \ \forall t \in [t_k, t_{k+1}). \tag{62}$$

By (46), the existence of $\dot{\vartheta}(t)$ is ensured. Moreover, $\dot{\vartheta}(t)$ is bounded since it consists of bounded signals, that is, $|\dot{\vartheta}(t)| \le \bar{\vartheta}$, where $\bar{\vartheta}$ is a positive constant. By $e(t_k) = 0$ and $\lim_{t \to t_{k+1}} |e(t)| = \xi(t)|u(t)| + \epsilon(t)$, integral of (62) over $[t_k, t_{k+1})$ leads to $t_{k+1} - t_k \ge t^* \ge \frac{\xi(t)|u(t)|+\epsilon(t)}{\bar{\vartheta}} > 0$. As a consequence, Zeno phenomenon does not happen. □

Remark 4.1. Different from constraints, such as communication constraints and measurement quantization [35], actuator failures [37], and multiple packet dropouts under Markovian communication constraints [47], this paper employs the asymmetric BLF method to handle the asymmetric output constraints, which can be also used in symmetric output constraints. Moreover, we design the dynamic event-triggered controller to reduce the frequency of controller updates.

Remark 4.2. The computational complexity of our proposed scheme lies in introducing more design parameters, which can be used for achieving fixed-time stability. Following the previous control design and stability analysis, the larger parameters $c_i, b_i, a_i, l_0, \gamma, \iota_i, \tau_i, \gamma_i, p_i, \lambda_i$ and the smaller parameters $p, m_i, l_i, q_i, q_0, \ell$ can improve the tracking performance of the system. Since some parameters are derived by amplifying inequalities in backstepping manner, it will lead to conservativeness. Thus, some small design parameters can be tested in real applications.

5. ILLUSTRATIVE EXAMPLES

In this part, the proposed control scheme is validated by two examples.

Example 5.1. Consider the switched system with unmodeled dynamics:

$$\begin{cases} \dot{w} = \varrho_{\sigma(t)}(w, \zeta_1), \\ \dot{\zeta}_1 = \zeta_2 + \psi_{1,\sigma(t)}(\zeta_1) + d_{1,\sigma(t)}(w, \zeta), \\ \dot{\zeta}_2 = u + \psi_{2,\sigma(t)}(\zeta_1, \zeta_2) + d_{2,\sigma(t)}(w, \zeta), \\ y = \zeta_1 \end{cases} \tag{63}$$

where $\sigma(t) \in \{1, 2, 3\}$, $\varrho_1(w, \zeta_1) = \varrho_2(w, \zeta_1) = \varrho_3(w, \zeta_1) - w - w^{\frac{7}{3}} + 0.5\zeta_1^2 + 0.1$, $\psi_{1,1}(\zeta_1) = 0.2\zeta_1$, $d_{1,1}(w, \zeta) = w\zeta_2 \sin\zeta_1$, $\psi_{2,1}(\zeta_1, \zeta_2) = 0.1\zeta_2 \sin(\zeta_1)$, $d_{2,1}(w, \zeta) = 0.15w\zeta_2 \cos\zeta_1$, $\psi_{1,2}(\zeta_1) = 0.25\zeta_1$, $d_{1,2}(w, \zeta) = \zeta_2 \cos\zeta_1$, $\psi_{2,2}(\zeta_1, \zeta_2) = 0.1\zeta_2 \sin(\zeta_1)$, $d_{2,2}(w, \zeta) = 0.1w\zeta_2 \sin\zeta_1$, $\psi_{1,3}(\zeta_1) = 0.3\zeta_1$, $d_{1,3}(w, \zeta) = 0.2w\zeta_2 \sin\zeta_1$, $\psi_{2,3}(\zeta_1, \zeta_2) = 0.1\zeta_2 \sin(\zeta_3)$, $d_{2,3}(w, \zeta) = 0.1w\zeta_2 \cos\zeta_1$. The reference signal $y_d(t) = 0.3\sin(0.5t) + 0.6\sin(t)$. $y(t)$ and $y_d(t)$ are subject to function constraints $h(y_d, t) = 0.2 + 0.4e^{-t} + y_d(t)$ and $l(y_d, t) = -0.6 - e^{-2t} + y_d(t)$. Apparently, Assumption 2.1 remains true.

From the formulation of above disturbances, Assumption 2.2 is correct obviously. To confirm Assumption 2.3, one adopts $V(w) = w^2$ and its derivative is $\dot{V}(w) = -2w^2 - 2w^{\frac{10}{3}} + w\zeta_1^2 + 0.2w$. By Young's inequality, we give $\dot{V}(w) \leq -1.5w^2 - 2w^{\frac{10}{3}} + \zeta_1^4 + 0.04$. Thus, Assumption 2.3 holds. Selecting $\tilde{\mu}_1 = 1.4 \in (0, 1.5)$, $\tilde{\mu}_2 = 1.9 \in (0, 2)$, the application of Lemma 2.1 concludes that r is denoted by $\dot{r} = -1.4r - 1.9r^{\frac{5}{3}} + \zeta_1^4 + 0.04$, $r(0) = 0.3$. Select $\mu_{F_i^l}(x_i) = \exp[-\frac{1}{2}(x_i + l - 3)^2]$, $l = 1, \ldots, 5$, $i = 1, 2,$.

In the simulation, we choose the following initial states $w(0) = 1.6$, $(\zeta_1(0), \zeta_2(0)) = (0.3, 1.8)$, $(z_1(0), z_2(0)) = (0.2, 0.3)$, $(\hbar_{11}(0), \hbar_{12}(0)) = (\hbar_{21}(0), \hbar_{22}(0)) = (-1, -0.2)$, $(\hat{\theta}_1(0), \hat{\theta}_2(0)) = (-1, 2)$, $(\hat{\sigma}_1(0), \hat{\sigma}_2(0)) = (0.8, 0.2)$, $\xi(0) = 0.3$, $\epsilon(0) = 0.5$. The control design parameters are taken as $c_1 = 2$, $c_2 = 3$, $b_1 = 4$, $b_2 = 5$, $a_1 = 3$, $a_2 = 7$, $l_0 = 2$, $p = 0.7$, $\gamma = \frac{5}{3}$, $m_1 = 1$, $m_2 = 1$, $l_1 = 1$, $l_2 = 1$, $\iota_1 = 1$, $\iota_2 = 1$, $\gamma_1 = 1$, $\gamma_2 = 1$, $\tau_1 = 1, \tau_2 = 1$, $p_1 = 1$, $p_2 = 1$, $q_1 = 1$, $q_2 = 1, \lambda_1 = 0.0001$, $\lambda_2 = 0.0001$, $q_0 = 0.6$, $\varsigma_1 = 0.001$, $\varsigma_2 = 0.001$, $\ell = 0.12$.

With the help of the fixed-time adaptive fuzzy event-triggered control technique (46) – (50), the performance of output tracking and control input is exhibited in Figure 2 and Figure 7, respectively. Figures 3 – 6 indicates that all the signals of the closed-loop system are bounded. From Figure 8, Zeno behavior is ruled out. Figure 9 shows the switching signal. This indicates that the proposed control strategy is effective even if the initial condition is unknown.

In addition, to give quantitative criteria, we introduce $e_{RMS} = \left(\frac{1}{T}\int_0^T |y - y_d|^2\, dt\right)^{\frac{1}{2}}$, the rms value of the tracking error, where T denotes the total running time. For $T = 15$, by computation of Matlab, we derive $e_{RMS} = 0.0683$ by the method of [28], and $e_{RMS} = 0.0594$ by the method of the proposed method. This implies that the proposed control method has the better control performance.

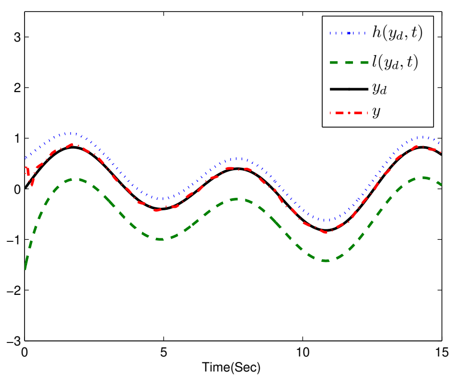

Fig. 2. The trajectories of y and y_d.

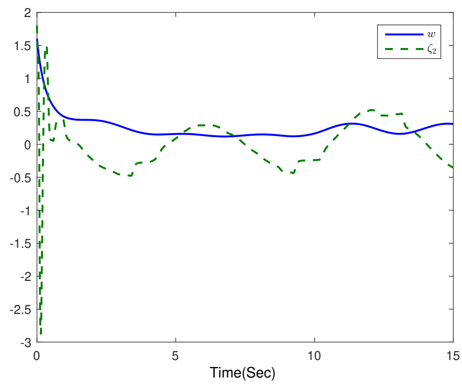

Fig. 3. The trajectories of w and ζ_2.

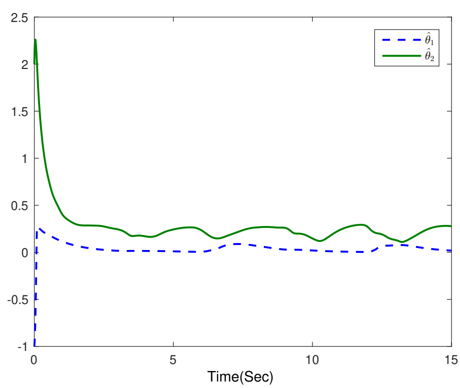

Fig. 4. The trajectories of adaptive laws $\hat{\theta}_1$ and $\hat{\theta}_2$.

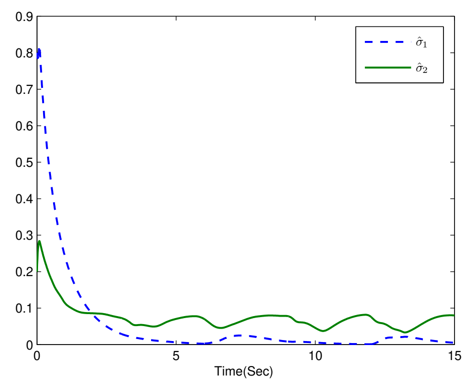

Fig. 5. The trajectories of adaptive laws $\hat{\sigma}_1$ and $\hat{\sigma}_2$.

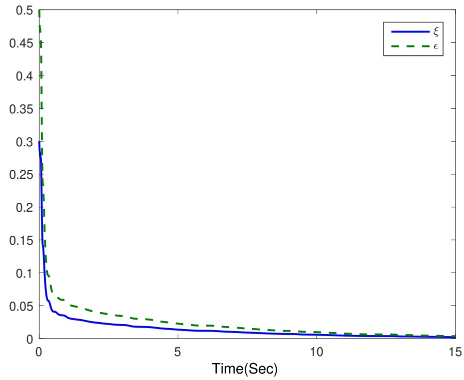

Fig. 6. The trajectories of ξ and ϵ.

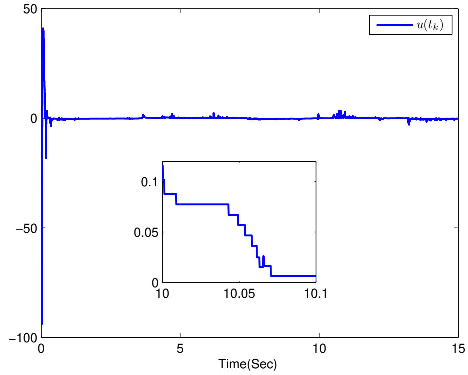

Fig. 7. Controller u.

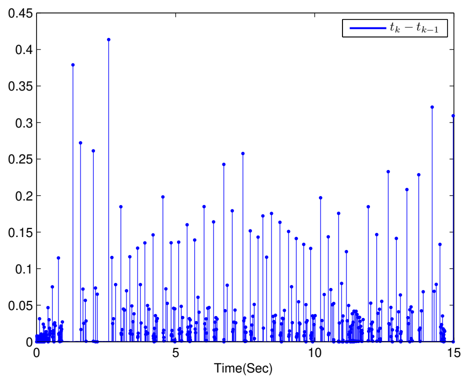

Fig. 8. Inter-execution time intervals.

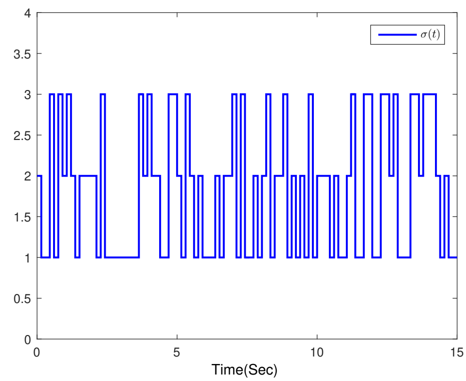

Fig. 9. Switching signal.

Example 5.2. As an application of the proposed control method, a switched RCL circuit [45] is considered. The two state variables ζ_1, ζ_2 are the charge in the capacitor q_C, and the flux in the inductance ϕ_L, respectively. The input u is the voltage. Then, the switched RCL circuit system is described by

$$\begin{cases} \dot{\zeta}_1 = \frac{1}{L}\zeta_2, \\ \dot{\zeta}_2 = u - \frac{1}{C_{\sigma(t)}}\zeta_1 - \frac{R}{L}\zeta_2, \\ y = \zeta_1 \end{cases} \tag{64}$$

Where $\sigma(t) \in \{1, 2\}$. Take $L = 0.9$, $R = 1$, $C_1 = 50$ and $C_2 = 100$. Select $\mu_{F_i^l}(x_i) = \exp[-\frac{1}{2}(x_i - 3 + l)^2]$, $i = 1, 2$, $l = 1, \ldots, 5$. The reference signal $y_d(t) = \sin(0.5t) + 0.5\sin(t)$. $l(y_d, t) = y_d(t) - 0.8e^{-t} - 0.3$ and $h(y_d, t) = y_d(t) + 1.2e^{-2t} + 0.18$.

In the simulation, we give the following initial states $(\zeta_1(0), \zeta_2(0)) = (0.3, -1.2)$, $(z_1(0), z_2(0)) = (0.2, 0.3)$, $(\hbar_{11}(0), \hbar_{12}(0)) = (\hbar_{21}(0), \hbar_{22}(0)) = (-1, -0.2)$, $(\hat{\theta}_1(0), \hat{\theta}_2(0)) =$

$(0.5, -1)$, $(\hat{\sigma}_1(0), \hat{\sigma}_2(0)) = (0.8, 0.2)$, $\xi(0) = 0.3$, $\epsilon(0) = 0.5$. we take the design parameters as $c_1 = 3$, $c_2 = 3$, $b_1 = 3$, $b_2 = 2$, $a_1 = 3$, $a_2 = 5$, $m_1 = 1$, $m_2 = 1$, $l_1 = 6$, $l_2 = 13$, $p = 0.95$, $\gamma = \frac{5}{3}$, $\iota_1 = 1$, $\iota_2 = 1$, $\gamma_1 = 1$, $\gamma_2 = 1$, $\tau_1 = 1$, $\tau_2 = 1$, $p_1 = 1$, $p_2 = 1$, $q_1 = 1$, $q_2 = 1, \lambda_1 = 0.01$ $\lambda_2 = 0.01$, $q_0 = 0.6$, $\varsigma_1 = 0.0001$, $\varsigma_2 = 0.0001$, $\ell = 0.22$.

By fixed-time adaptive event-triggered control scheme (46) − (50), Figure 10 indicates the better tracking performance. Figures 11 − 14 indicates that all the signals of the closed-loop system are bounded. Figure 15 shows the response of input. Figure 16 means that Zeno behavior does not happen.

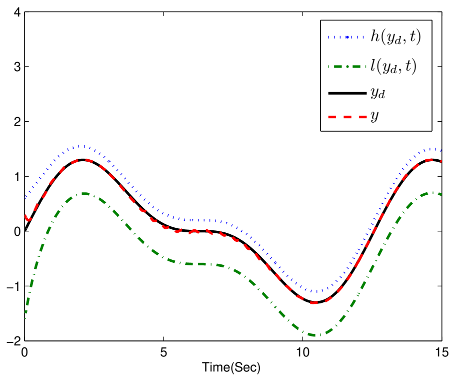

Fig. 10. The response of y and y_d.

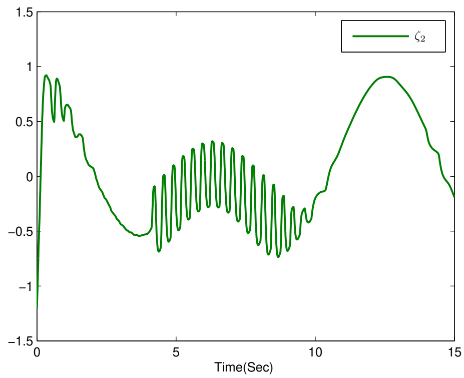

Fig. 11. The trajectories of ζ_2.

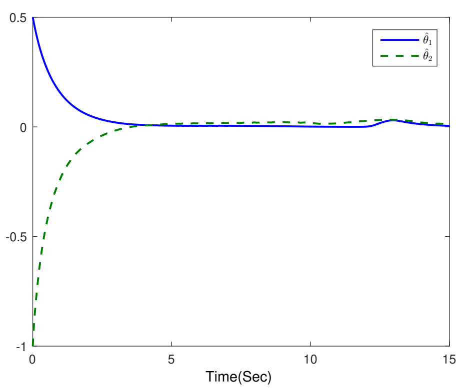

Fig. 12. The trajectories of adaptive laws $\hat{\theta}_1$ and $\hat{\theta}_2$.

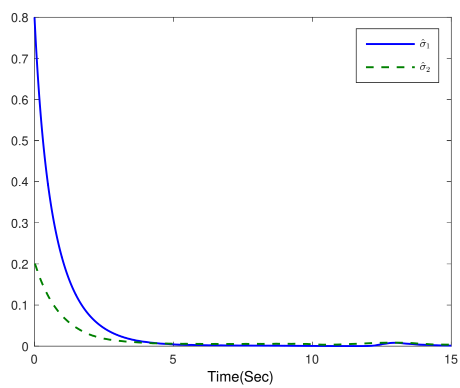

Fig. 13. The trajectories of adaptive laws $\hat{\sigma}_1$ and $\hat{\sigma}_2$.

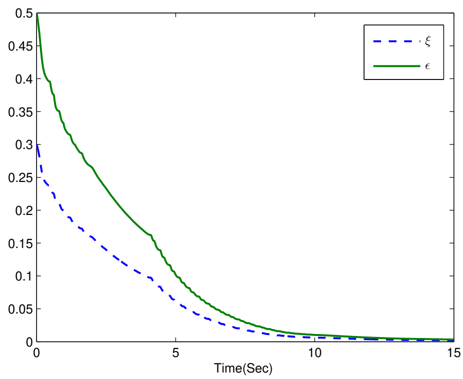

Fig. 14. The trajectories of ξ and ϵ.

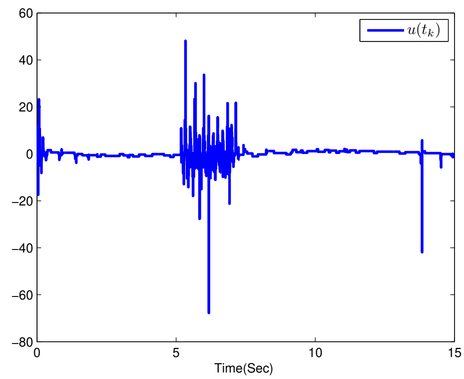

Fig. 15. Controller u.

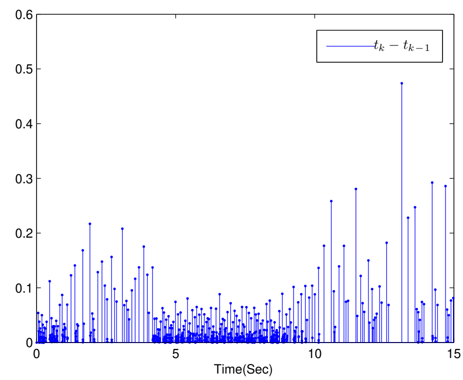

Fig. 16. Inter-execution time intervals.

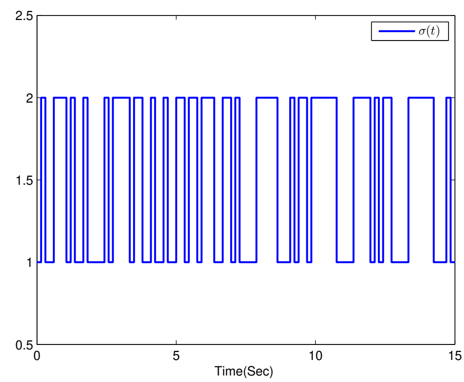

Fig. 17. Switching signal.

6. CONCLUSION

In this paper, adaptive fixed-time fuzzy control has been investigated for uncertain constrained switched nonlinear systems under unmodeled dynamics. For general uncertain nonlinear systems with unmodeled dynamics, the existing results, such as [27, 28] can not be applied to resolve its fixed-time stability problem. Hence, a novel criterion of fixed-time stability is provided and a corresponding dynamic signal is proposed by characterizing unmodeled impact. Owing to complex nonlinear functions caused by uncertain nonlinearities and external disturbances, fuzzy logic systems (FLSs) are applied to estimate them. Based on bounded command filter and CBLF, a common adaptive fuzzy event-triggered stabilizer is constructed to ensure fixed-time stability of the whole system under output-function constraints. Since stochastic disturbances widely exist in practical systems, fixed-time adaptive event-triggered control will be our future work for constrained switched stochastic nonlinear systems.

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