KYBERNETIKA — VOLUME *60* (2024), NUMBER 6, PAGES 754–778

EQUILIBRIUM ANALYSIS OF DISTRIBUTED AGGREGATIVE GAME WITH MISINFORMATION

Meng Yuan, Zhaoyang Cheng, and Te Ma

This paper considers a distributed aggregative game problem for a group of players with misinformation, where each player has a different perception of the game. Player's deception behavior is inevitable in this situation for reducing its own cost. We utilize hypergame to model the above problems and adopt ϵ-Nash equilibrium for hypergame to investigate whether players believe in their own cognition. Additionally, we propose a distributed deceptive algorithm for a player implementing deception and demonstrate the algorithm converges to ϵ-Nash equilibrium for hypergame. Further, we provide conditions for the deceptive player to enhance its profit and offer the optimal deceptive strategy at a given tolerance ϵ. Finally, we present the effectiveness of the algorithm through numerical experiments.

Keywords: distributed aggregative game, deceptive strategy, hypergame, ϵ-Nash equilibrium for hypergame

Classification: 91A10, 68W15, 68W40

1. INTRODUCTION

In recent years, aggregative games have received extensive attention, due to their widespread applications in fields such as the Cournot oligopoly model in the management of smart grids [2], the route selection of road network [21], and so on. The aggregative game of multi-agent systems is an important subclass of non-cooperative games, where the cost function of participants not only depends on their own actions but also the aggregative function of actions taken by all participants. Specifically, due to the expansion of communication network scale, communication burden, and the demand for individual privacy protection, distributed aggregative games are widely used to depict complex real-life scenarios, where players achieve their goals through communication networks and local data [15, 31].

Misinformation in games describes that players have different perceptions of the game, and commonly arises in numerous practical scenarios and the player's deception behavior is inevitably in these complex situations [1, 4, 30]. For example, [16] used evolutionary game theory to study the impact of spreading erroneous/misleading messages on the evolution of network collective cooperation. Based on this, [17] further analyzed the

DOI: 10.14736/kyb-2024-6-0754

effect of dynamic updates and elimination of erroneous information in collective cooperation under personalized strategies. Moreover, in order to analyze how a player with more information benefits from the misinformation situation, many deception methods have been deployed in the game model. Actually, deception typically stems from the manipulation and concealment of beliefs by other players. Combining cooperative game theory and reinforcement learning methods, the problem of false information dissemination in online social networks was investigated [29]. [1] designed a mechanism to combat misinformation attacks for each battlefield Internet of Battlefield Things (IoBT) node by finding the optimal probability of receiving given information. In non-cooperative stochastic games aimed at military action control, defenders increased their rewards by manipulating the information available to attackers [5]. Therefore, deception are one popular method for a player to improve its own utility when it has some knowledge of the misinformation situation.

However, whether players trust their own cognition is very important in the situation with players' deception behaviors and different observations [24]. Indeed, players' doubts about their own perceptions could disturb the equilibrium or potentially lead to the collapse of the game. For instance, players might recognize biased misperceptions and aim to uncover the truth [3]. Regrettably, existing research failed to adequately address the role of trusting their own cognition in distributed aggregative games with different players' cognition. In addition, in aggregative games with misinformation, it is also crucial for players to increase their own profits through actively engaging in deceptive behavior [22]. This is important for enhancing the competitiveness of players in non-cooperative game processes. For example, [9] pretends to attack successfully and provides false data to mislead the attacker and collect their information (such as IP addresses). In [22], defenders actively send deceptive signals to attackers; In [7], players relieve insider threat mitigation through deceptive behavior. Motivated by the above consideration, the capabilities of the player to have high utility and trust in their own cognition are important and rarely considered in existing literature.

Fortunately, hypergames provide an effective tool for analyzing whether players trust their own cognition. Roughly speaking, hypergames describe complex situations where players have different understandings by breaking down a game into multiple subjective games. Hyper Nash Equilibrium (HNE) [23] is a core concept in hypergames that represents the best response of each player in the subjective game. Considering the strict conditions for implementing HNE and referring to ϵ-NE [28], the definition of ϵ-NE is needed to be analyzed in many practical problems. ϵ-NE for hypergame allows players to have a certain degree of deviation from the subjective optimal strategy, which is measured by ϵ. When players reach ϵ-NE for hypergame, everyone not only trusts their cognition but also does not need to change their strategies with a tolerance of ϵ. A similar discussion on HNE and ϵ-NE has been applied to various situations, including resource allocation, military conflicts, and economics [12].

The following are the main contributions of the paper:

- We propose a distributed aggregative game problem with misinformation, where each player has a biased estimate of the cost function of other players, and only one player is aware of the existence of misinformation and intends to adopt deceptive strategies. We model the above problem as a hypergame and propose ϵ-NE for hypergame as the criterion.

- We design a distributed deceptive strategy to guide player's deception behavior and prove its convergence to the ϵ-NE for hypergame. Moreover, we give an upper bound of the tolerance metric ϵ, where the bound is related to the corresponding deceptive strategy.
- Furthermore, we provide the condition for a player to be willing to adopt the deception and get the optimal utility in the aggregative game with quadratic form. Speciffcally, we show the range of deceptive behavior that can reduce costs, and for a fixed tolerance ϵ, we present the optimal deception behavior from the perspective of the deceptive player.

Notations: $\mathbf{1}_N$ *denotes an* N*-dimensional column vector whose elements are all 1. The transpose of* Ψ *is denoted by* Ψ^T*, where* Ψ *is either a matrix or a vector. The notation* $[g_i]_{vec}$ *for* $i \in \{1, 2, \ldots, N\}$ *denotes a column vector whose ith element is* g_i*.*

2. PROBLEM FORMULATION

In this section, we first formulate our problem and then give some preliminary knowledge.

2.1. Problem statement

Consider a game with N players or agents, where the players are indexed by $\mathcal{V} = \{1, \ldots, N\}$. For each player $i \in \mathcal{V}$, player i has an action strategy x_i in local constraint set $\mathcal{X}_i \subseteq \mathbb{R}^n$. Denote $\mathcal{X} \triangleq \prod_{i=1}^N \mathcal{X}_i \subset \mathbb{R}^{nN}$, $\mathbf{x} \triangleq \left(x_1^T, \ldots, x_N^T\right)^T$ as the action profile for all players, and $\mathbf{x}_{-i} \triangleq \left(x_1^T, \ldots, x_{i-1}^T, x_{i+1}^T, \ldots, x_N^T\right)^T$ as the action profile for all players except player i. Player i has its own cost function $f_i(x_i, \mathbf{x}_{-i}) : \mathbb{R}^{nN} \to \mathbb{R}$. Furthermore, define the aggregate term of all players as follows:

$$\sigma(\mathbf{x}) \triangleq \frac{1}{N} \sum_{i=1}^N \varphi_i(x_i),$$

where $\phi_i : \mathbb{R}^n \to \mathbb{R}^m$ is a map for the local contribution to the aggregate term. Given $\mathbf{x}_{-i}$, the objective of player i is to solve the following optimization problem:

$$\min_{x_i \in \mathcal{X}_i} f_i\left(x_i, \mathbf{x}_{-i}\right), \tag{1}$$

where $f_i(x_i, \mathbf{x}_{-i}) = \tilde{f}_i(x_i, \sigma(\mathbf{x}))$.

The game in a normal form is defined as a triple $\Gamma = \{\mathcal{V}, X, f\}$. The Nash equilibrium is a core concept of Γ, defined as follows.

Definition 2.1. (Nash Equilibrium) An action profile $\mathbf{x}^* = \left(x_i^*, \mathbf{x}_{-i}^*\right) \in \mathcal{X}$ is a Nash equilibrium if for $i \in \mathcal{V}$, $x_i \in \mathcal{X}_i$,

$$f_i\left(x_i^*, \mathbf{x}_{-i}^*\right) \leq f_i\left(x_i, \mathbf{x}_{-i}^*\right),$$

where $\mathbf{x}_{-i} = [x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N]^T$.

The Nash equilibrium is an action profile on which no player can reduce its cost by unilaterally changing its own action, In other words, $\mathbf{x}^*$ is an NE if no player can decrease its cost by unilaterally deviating from its strategy x_i^*.

Due to the fact that NE may not exist or be difficult to calculate in real situations, the following definition is introduced.

Definition 2.2. (ϵ-Nash Equilibrium) A profile $\mathbf{x}^*$ is said to be an ϵ-Nash equilibrium of game (1) if

$$f_i\left(x_i^*, \mathbf{x}_{-i}^*\right) \leq f_i\left(x_i, \mathbf{x}_{-i}^*\right) + \epsilon, \forall i \in \mathcal{V}, \forall x_i \in \mathcal{X}_i$$

where the constant $\epsilon > 0$. Particularly, $\mathbf{x}^*$ is said to be an NE when $\epsilon = 0$.

Consider the aggregative game (1), where each player only knows its own cost function and can not have access to the aggregate term $\delta(\mathbf{x})$, while players may exchange information with neighbors through the network $\mathcal{G}$ to update their estimates of the aggregate value. The information exchange among the N players is described by a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{E} = \{(i,j) \mid i,j \in V\}$ denotes the edge set where $(i,j) \in \mathcal{E}$ when agent i and agent j have information exchanges. Define $N \times N$-dimensional matrix $W = [w_{ij}]$ as the adjacency matrix of the communication graph, where $w_{ij} > 0$ if $(i,j) \in \mathcal{E}$ and $w_{ij} = 0$, otherwise.

In fact, players may receive some information of other players' cost functions. For example, in non-cooperative games of various circumstances, players aim to obtain the private information of other players to enrich their information base, in order to generate better results for themselves, e. g., [27] has considered the existence of honest-but-curious adversaries in stochastic aggregative games, who follow all protocol steps correctly but collect all intermediate and input/output information in an attempt to learn sensitive information about other participating players. In the rest of this paper, we consider that players have a cognition of others' cost function before players take actions.

However, the cognition of cost functions among players may be different, due to external environmental interference or self-awareness. For instance, external interference in communication channels may result in imperfect observations in sensor systems [19], while individuals with bounded rationality may exhibit biased observations within the IoT [3]. Specially, the jth player may not know the actual cost function f_i of player i, but only receive a fictitious cost function with biased errors. We denote the cost function of player i under player j's cognition by f_{ij}, where $i,j \in \mathcal{V}$. Actually, $f_{ii} = f_i$, since the ith player knows its own real cost function.

Moreover, throughout the entire network, we suppose that only one player is aware of the observed differences. It is common in the actual agent interactions. For instance, [4] describes the leader realizes the fact that all followers are not aware of the observed differences in the single-leader-multiple-followers Stackelberg security game. Without loss of generality, we consider player r is aware of others' cost functions and their partial perceptions. In this situation, player r intends to adopt a deceptive strategy to receive as beneficial results as possible for itself.

2.2. Hypergame model

Note that hypergame theory is very advantageous in modeling complex situations, where players' cognition is observed by others, we use hypergame theory to model distributed aggregative games with misinformation. Hypergame theory delineates varying cognitive perspectives among players within strategic interactions occurring in contexts rife with misinformation. At its core, the hypergame theory seeks to break down intricate scenarios involving misinformation into several subjective games [12].

There are different levels for describing cognitive situations [26]. For convenience, we denote $\Gamma(f_1, \ldots, f_N)$ as a game where f_i represents the local cost function of player i. Thus, as mentioned before, for $j \neq r$, player j supposes that they are playing a first-level hypergame $H_j^I = \{G_j\}$, where $G_j = \Gamma(f_{1j}, \ldots, f_{Nj})$, since it does not notice that they mutually estimate the local cost function and have the different cognitions among them. For player r, the first level hypergame observed by player r is denoted by $H_r^I = \{G'_1, \ldots, G'_N\}$, where G'_j is the perception of player r towards the first level game of player j since it perceives misinformation from other players and receives partial information on their cognitions of cost functions. Then, this second level hypergame is denoted as $H^{II} = \{H_1^I, \ldots, H_N^I\}$.

In reality with various uncertainties, the conditions for meeting NE may be too stringent. Therefore, similar to the ϵ-NE proposed in definition 2.2, we relax the restrictions and introduce the following loose inequality, ϵ-Nash equilibrium for hypergame. A profile $\mathbf{x}^*$ is said to be a ϵ-Nash equilibrium for hypergame of game Γ if for all $i \in \mathcal{V}$,

$$f_{ji}\left(x_j^*, \mathbf{x}_{-j}^*\right) \leq f_{ji}\left(x_j, \mathbf{x}_{-j}^*\right) + \epsilon, \text{ for all } x_j \in \mathcal{X}_j, j \in \mathcal{V},$$

with a constant $\epsilon > 0$.

In fact, ϵ-NE for hypergame is such a strategy profile that near the best response strategy in everyone's subjective game. Moreover, due to the overly stringent conditions for implementing NE, we relaxed the conditions and proposed a definition for ϵ-NE for hypergame. ϵ-NE for hypergame allows players to have a certain degree of deviation from the subjective optimal strategy, which is measured by ϵ.

When the strategies of other players do not match their cognition, some players may doubt their observation of the game. According to [4], suspicions about their cognitions may lead to the updation of their observations, and even make the game model collapse.

Actually, the communication network $\mathcal{G}(\mathcal{V}, \mathcal{E})$ can be utilized by player r to spread its own deceptive strategy. Each player sends a message through the communication network. Player r is able to send a deceptive strategy to others through the network such as adding strategy $\theta \in \mathbb{R}^m$ when updating the local aggregate variable. For example, in sensor systems, false data injection (FDI) attacks modify data at the network communication layer to achieve network attacks [32].

The main task of this paper is to design a distributed deceptive strategy or algorithm for distributed aggregative games with misinformation and prove the proposed algorithm converges to ϵ-NE for hypergame, and provide an upper bound for ϵ and the condition for promoting the benefit of the deceptive player.

2.3. Assumptions

To establish main results, we impose some assumptions as follows.

The following assumptions were widely used for the strategy sets and the cost functions [11].

Assumption 2.3. For each player $i \in \mathcal{V}$,

(a) the strategy set $\mathcal{X}_i$ is convex and compact;

(b) the cost function $f_i(\cdot, \mathbf{x}_{-i})$ is convex over $\mathcal{X}_i$ for every $\mathbf{x}_{-i} \in \mathcal{X}_{-i}$;

(c) $f_i(\cdot, u)$ is continuously differentiable in x_i over $\mathcal{X}_i$ for every fixed $u \in \mathbb{R}^n$, and $f_i(x_i, u)$ is continuously differentiable in $u \in \mathbb{R}^n$ for any fixed $x_i \in \mathcal{X}_i$.

Assumption 2.3 is a standard assumption that guarantees the existence of a Nash equilibrium.

In order to explicitly show the aggregate term of the game, we define map $G_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n, i \in \mathcal{V}$ as

$$G_i(x_i, \eta_i) \triangleq \nabla_{x_i} f_i(\cdot, \mathbf{x}_{-i})\Big|_{\sigma(\mathbf{x})=\eta_i} = \left(\nabla_{x_i} \tilde{f}_i(\cdot, \sigma) + \frac{1}{N} \nabla_\sigma \tilde{f}_i(x_i, \cdot)^T \nabla \varphi_i\right)\Big|_{\sigma=\eta_i}.$$

Also, let $G(\mathbf{x}, \boldsymbol{\eta}) \triangleq \operatorname{col}(G_1(x_1, \eta_1), \ldots, G_N(x_N, \eta_N))$. Clearly, $G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x})) = F(\mathbf{x})$, where the pseudo-gradient map $F : \mathbb{R}^n \to \mathbb{R}^n$ is defined as

$$F(\mathbf{x}) \triangleq \operatorname{col}\{\nabla_{x_1} f_1(\cdot, \mathbf{x}_{-1}), \ldots, \nabla_{x_N} f_N(\cdot, \mathbf{x}_{-N})\}.$$

Assumption 2.4. $F(\mathbf{x})$ is k-Lipschitz continuous and μ-strongly monotone for some constants $k > 0$ and $\mu > 0$,

$$\|F(\mathbf{x}) - F(\mathbf{y})\| \leq k\|\mathbf{x} - \mathbf{y}\|,$$
$$(\mathbf{x} - \mathbf{y})^\top (F(\mathbf{x}) - F(\mathbf{y})) \geq \mu\|\mathbf{x} - \mathbf{y}\|^2, \quad \forall \mathbf{x}, \mathbf{y} \in \mathcal{X}.$$

Assumption 2.5. $G(\mathbf{x}, \boldsymbol{\eta})$ is k_1-Lipschitz continuous with respect to $\mathbf{x} \in \Omega$ and k_2-Lipschitz continuous with respect to $\boldsymbol{\eta}$ for some constants $k_1, k_2 > 0$,

$$\|G(\mathbf{x}, \cdot) - G(\mathbf{y}, \cdot)\| \leq k_1\|\mathbf{x} - \mathbf{y}\|,$$
$$\|G(\cdot, \boldsymbol{\eta}_1) - G(\cdot, \boldsymbol{\eta}_2)\| \leq k_2\|\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2\|.$$

Assumption 2.6. For any $i \in \mathcal{V}, \varphi_i$ is k_3-Lipschitz continuous on $\mathcal{X}_i$ for constant $k_3 > 0$,

$$\|\varphi_i(x) - \varphi_i(y)\| \leq k_3\|x - y\|, \quad \forall x, y \in \mathcal{X}_i.$$

Assumption 2.4-2.6 are needed to ensure the existence and uniqueness of the Nash equilibrium and also to facilitate algorithm design. Note that the strong monotonicity of the pseudo-gradient map F has been widely adopted in the literature such as [14].

Assumption 2.7. The communication graph $\mathcal{G}$ is undirected and connected. Moreover, The associated adjacent matrix of graph A satisfies

1) $a_{ij} > 0, \text{if } (i,j) \in \mathcal{E}$ and $a_{ij} > 0, \text{if } (i,j) \notin \mathcal{E}$;

2) $A = A^T, A\mathbf{1}_N = \mathbf{1}_N$.

The network model in Assumption 2.7 was widely used in distributed multi-agent systems (see, e. g., [18]). It is easy to achieve in a distributed setting.

Assumption 2.8. For every fixed $\mathbf{x} \in \mathcal{X}$, the difference between the cost function of each player i's own perception of other players and the actual cost function is bounded, i. e., there exists $\mathcal{B}_1, \mathcal{B}_2 \in \mathbb{R}^n$, such that

$$\mathcal{B}_1^\top x_i \leq f_{ij}(\mathbf{x}) - f_{ii}(\mathbf{x}) \leq \mathcal{B}_2^\top x_i, i, j \in \mathcal{V}.$$

Remark 2.9. Assumption 2.8 ensures that even if each player's cognition is different, the difference in their cognition of the same player's cost function is bounded by linear functions.

3. MAIN RESULTS

In this section, we develop our distributed projected deception algorithm, followed by the analysis of the proposed algorithm convergence to ϵ-NE for hypergame and a bound analysis of ϵ. Moreover, for aggregative games with quadratic functions, we also provide conditions that enhance the benefits of the deceptive player and completely avoid suspicion from other players. Additionally, for a fixed tolerance ϵ, we provide the optimal deceptive strategy from the perspective of deceptive player.

3.1. Distributed deceptive algorithm

For the distributed aggregative game, each player has different perceptions of the ongoing game. Suppose that only one player realizes the existence of this difference. The player aims to use this information gap to adopt deceptive strategies in order to produce as favorable results as possible for itself. We use the hypergame model, which focuses on the cognitive stability of players with misinformation, as it displays the conditions under which each player trusts their current cognition. Otherwise, once the player's expectations are inconsistent with the strategies of others, the player will doubt their own cognition, which may cause the game model to collapse. Motivated by it, we design a distributed projected deception algorithm and prove the proposed algorithm converges to ϵ-NE for hypergame. The detailed algorithm is given in Algorithm 1 as follows.

The updation for deceptive player r is:

$$i = r, \quad \begin{cases} \dot{x}_i = P_{\Omega_i}\left(x_i - \alpha G_i\left(x_i, \eta_i\right)\right) - x_i, & x_i(0) \in \mathcal{X}_i, \\ \dot{\delta}_i = \beta \sum_{j=1}^N a_{ij}\left(\eta_j - \eta_i\right) + \beta(a_{rr} - 1)\theta, & \delta_i(0) = \mathbf{0}_m, \\ \eta_i = \delta_i + \varphi_i\left(x_i\right). \end{cases}$$

The updation for others is:

$$i \neq r, \quad \begin{cases} \dot{x}_i = P_{\Omega_i}\left(x_i - \alpha G_i\left(x_i, \eta_i\right)\right) - x_i, & x_i(0) \in \mathcal{X}_i, \\ \dot{\delta}_i = \beta \sum_{j=1}^N a_{ij}\left(\eta_j - \eta_i\right) + \beta a_{ir}\theta, & \delta_i(0) = \mathbf{0}_m, \\ \eta_i = \delta_i + \varphi_i\left(x_i\right). \end{cases}$$

Firstly, each player initializes their decision values and aggregated estimates. In each iteration, each player updates their local decision values using the projected gradient method. Since each player i has their own perceived games, perturbation θ is used to mask the real aggregate estimate η_i^k. Furthermore, as player r becomes aware of the existence of deceptive information from others, it subtracts the deceptive value when updating the local estimate η_r^k.

In Algorithm 1, parameters α and β satisfy

$$0<\alpha<\frac{2\mu\beta\lambda_2-4k_2k_3}{k^2\beta\lambda_2+2\mu k_2k_3},\quad \beta>\frac{2k_2k_3}{\mu\lambda_2},$$

where λ_2 is the smallest positive eigenvalue of L (L is the Laplacian matrix of communication graph $\mathcal{G}$).

The compact form can be written as

$$\begin{cases}\dot{\mathbf{x}}=P_\Omega(\mathbf{x}-\alpha G(\mathbf{x},\boldsymbol{\eta}))-\mathbf{x}, & \mathbf{x}(0)\in\Omega,\\ \dot{\boldsymbol{\delta}}=-\beta\left(L\otimes I_m\boldsymbol{\eta}+\mathbf{W}\otimes I_m\theta\right), & \boldsymbol{\delta}(0)=\mathbf{0}_{mN},\\ \boldsymbol{\eta}=\boldsymbol{\delta}+\boldsymbol{\varphi}(\mathbf{x}),\end{cases}$$

where $\boldsymbol{\varphi}(\mathbf{x}) = \operatorname{col}\left(\varphi_1\left(x_1\right),\ldots,\varphi_N\left(x_N\right)\right)$ and $\boldsymbol{W} = [-a_{1r},\ldots,\sum_{i=1}^N a_{ir} - a_{rr},\ldots,-a_{Nr}]^\top$. Furthermore, we can rewrite the above form as

$$\begin{cases}\dot{\mathbf{x}}=P_\Omega(\mathbf{x}-\alpha G(\mathbf{x},\boldsymbol{\eta}))-\mathbf{x}, & \mathbf{x}(0)\in\Omega,\\ \dot{\boldsymbol{\eta}}=-\beta\left(L\otimes I_m\boldsymbol{\eta}+\mathbf{W}\otimes I_m\theta\right)+\frac{d}{dt}\boldsymbol{\varphi}(\mathbf{x}), & \boldsymbol{\delta}(0)=\boldsymbol{\varphi}(\mathbf{x}(\mathbf{0})).\end{cases}\tag{2}$$

Since the communication graph is doubly stochastic, $\mathbf{1}_N^\top L=\mathbf{0}_N^\top$ and $\mathbf{1}_N^\top\mathbf{W}=0$. Therefore,

$$\frac{1}{N}\sum_{i=1}^N\dot{\eta}_i=\frac{d}{dt}\sigma(\mathbf{x}).$$

As a result,

$$\mathcal{M}\triangleq\left\{\operatorname{col}(\mathbf{x},\boldsymbol{\eta})\in\Omega\times\mathbb{R}^N\;\middle|\;\frac{1}{N}\sum_{i=1}^N\eta_i=\sigma(\mathbf{x})\right\}$$

is an invariant set of (2).

Similar to reference [14], the analysis in this problem is limited to $\mathcal{M}$ rather than the entire space, and the Nash equilibrium point of this problem is unique and can be expressed as

$$\left[\begin{array}{c}\mathbf{x}\\ \boldsymbol{\eta}\end{array}\right]=\left[\begin{array}{c}\mathbf{x}^*\\ \boldsymbol{\eta}^*\end{array}\right]=\left[\begin{array}{c}\mathbf{x}^*\\ \mathbf{1}_N\otimes\sigma\left(\mathbf{x}^*\right)\end{array}\right].$$

3.2. Convergence analysis

In this subsection, the convergence of the proposed deceptive algorithm is analyzed.

As we model this distributed aggregative game with a second-level hypergame for the situation where players have cognitive errors, a quantitative analysis for the approximation of the hyper Nash equilibrium is given in the following.

Theorem 3.1. (Convergence to ϵ-NE for hypergame) Under Assumptions 2.3-2.7, the sequence $\mathbf{x}$ generated by Algorithm 1 converges to ϵ-NE for hypergame, with

$$\epsilon \leq 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta\| + 2\mathcal{B}M,$$

where $\rho = \frac{1}{\min\left\{\frac{2}{c_\theta}\left(\omega_1 - \frac{\xi_1}{2} - \frac{\xi_2}{2}\right), \frac{2}{3c_\theta}\left(\omega_2 - \frac{\xi_1}{2} - \frac{\xi_2}{2}\right)\right\}}$.

Proof.

Step 1: Denote $\mathbf{y} \triangleq \boldsymbol{\eta} - \mathbf{1}_N \otimes \sigma(\mathbf{x})$. Then, it follows from $L\mathbf{1}_N = \mathbf{0}_N$ and (2) that

$$\dot{\mathbf{x}} = P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{y} + \mathbf{1}_N \otimes \sigma(\mathbf{x}))) - \mathbf{x}, \tag{3}$$

$$\begin{aligned}
\dot{\mathbf{y}} &= \dot{\boldsymbol{\eta}} - \frac{d}{dt}\left(\mathbf{1}_N \otimes \sigma\left(\mathbf{x}\right)\right) \qquad (4)\\
&= -\beta\left(L \otimes I_m\boldsymbol{\eta} + \mathbf{W} \otimes I_m\theta\right) + \frac{d}{dt}\boldsymbol{\varphi}(\mathbf{x}) - \frac{d}{dt}\left(\mathbf{1}_N \otimes \sigma\left(\mathbf{x}\right)\right)\\
&= -\beta \otimes I_m L - \beta \otimes I_m\mathbf{W}\theta + \left(\nabla\boldsymbol{\varphi}(\mathbf{x}) - \mathbf{1}_N \otimes \nabla\sigma\left(\mathbf{x}\right)\right)^\top\left(P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \boldsymbol{\eta})) - \mathbf{x}\right).
\end{aligned}$$

Denote

$$\begin{aligned}
&H(\mathbf{x}) \triangleq \mathbf{x} - P_{\boldsymbol{\Omega}}(\mathbf{x} - \alpha F(\mathbf{x})),\\
&\widetilde{H}(\mathbf{x}, \mathbf{y}) \triangleq \mathbf{x} - P_{\boldsymbol{\Omega}}\left(\mathbf{x} - \alpha G\left(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y}\right)\right),\\
&\boldsymbol{\xi}(\mathbf{x}, \mathbf{y}) \triangleq \widetilde{H}(\mathbf{x}, \mathbf{y}) - H(\mathbf{x}).
\end{aligned}$$

Then,

$$\begin{aligned}
\|\boldsymbol{\xi}(\mathbf{x}, \mathbf{y})\| &= \|\widetilde{H}(\mathbf{x}, \mathbf{y}) - H(\mathbf{x})\|\\
&\leq \|P_{\boldsymbol{\Omega}}(\mathbf{x} - \alpha F(\mathbf{x})) - P_{\boldsymbol{\Omega}}\left(\mathbf{x} - \alpha G\left(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y}\right)\right)\|\\
&\leq \alpha\|F(\mathbf{x}) - G\left(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y}\right)\|\\
&\leq \alpha k_2\|\mathbf{y}\|.
\end{aligned}$$

Then,

$$\begin{aligned}
&(\mathbf{x} - \mathbf{y})^T(H(\mathbf{x}) - H(\mathbf{y}))\\
=&\|\mathbf{x} - \mathbf{y}\|^2 - (\mathbf{x} - \mathbf{y})^T \cdot (P_\Omega(\mathbf{x} - \alpha F(\mathbf{x})) - P_{\boldsymbol{\Omega}}(\mathbf{y} - \alpha F(\mathbf{y})))\\
\geq&\|\mathbf{x} - \mathbf{y}\|(\|\mathbf{x} - \mathbf{y}\| - \|P_\Omega(\mathbf{x} - \alpha F(\mathbf{x})) - P_{\boldsymbol{\Omega}}(\mathbf{y} - \alpha F(\mathbf{y}))\|)\\
\geq&\|\mathbf{x} - \mathbf{y}\|(\|\mathbf{x} - \mathbf{y}\| - \|\mathbf{x} - \alpha F(\mathbf{x}) - (\mathbf{y} - \alpha F(\mathbf{y}))\|,
\end{aligned}$$

and

$$\begin{aligned}
&\|\mathbf{x} - \mathbf{y}\| - \|\mathbf{x} - \alpha F(\mathbf{x}) - (\mathbf{y} - \alpha F(\mathbf{y}))\|\\
=&\frac{\|\mathbf{x} - \mathbf{y}\|^2 - \|\mathbf{x} - \alpha F(\mathbf{x}) - (\mathbf{y} - \alpha F(\mathbf{y}))\|^2}{\|\mathbf{x} - \mathbf{y}\| + \|\mathbf{x} - \alpha F(\mathbf{x}) - (\mathbf{y} - \alpha F(\mathbf{y}))\|}\\
\geq&\frac{2\alpha(\mathbf{x} - \mathbf{y})^T(F(\mathbf{x}) - F(\mathbf{y})) - \alpha^2\|F(\mathbf{x}) - F(\mathbf{y})\|^2}{(2 + \alpha \cdot k)\|\mathbf{x} - \mathbf{y}\|}\\
\geq&\frac{2\alpha \cdot \mu - \alpha^2 \cdot k^2}{2 + \alpha \cdot k}\|\mathbf{x} - \mathbf{y}\|.
\end{aligned}$$

In addition, there holds the identity $H(\mathbf{x}^*) = \mathbf{0}$, since $\mathbf{x}^*$ is the Nash equilibrium. It can be conclude that the map H is ω_1-strongly monotone.

Define Lyapunov candidate function as

$$V_1(\mathbf{x}) = \frac{1}{2}\|\mathbf{x} - \mathbf{x}^*\|^2. \tag{5}$$

Its time derivative along the trajectory of (3) is

$$\begin{aligned}
\dot{V}_1 &= -(\mathbf{x} - \mathbf{x}^*)^T \widetilde{H}(\mathbf{x}, \mathbf{y}) \\
&= -(\mathbf{x} - \mathbf{x}^*)^T (H(\mathbf{x}) + \boldsymbol{\xi}(\mathbf{x}, \mathbf{y})) \\
&= -(\mathbf{x} - \mathbf{x}^*)^T (H(\mathbf{x}) - H(\mathbf{x}^*)) - (\mathbf{x} - \mathbf{x}^*)^T \boldsymbol{\xi}(\mathbf{x}, \mathbf{y}) \\
&\le -(\mathbf{x} - \mathbf{x}^*)^T (H(\mathbf{x}) - H(\mathbf{x}^*)) + \|\mathbf{x} - \mathbf{x}^*\| \|\boldsymbol{\xi}(\mathbf{x}, \mathbf{y})\| \\
&\le -\frac{2\alpha \cdot \mu - \alpha^2 \cdot \kappa^2}{2 + \alpha \cdot \kappa} \|\mathbf{x} - \mathbf{x}^*\|^2 + \alpha k_2 \|\mathbf{x} - \mathbf{x}^*\| \|\mathbf{y}\| \\
&\le -\omega_1 \|\mathbf{x} - \mathbf{x}^*\|^2 + \xi_1 \|\mathbf{x} - \mathbf{x}^*\| \|\mathbf{y}\| \\
&\le -\omega_1 \|\mathbf{x} - \mathbf{x}^*\|^2 + \frac{\xi_1}{2} \|\mathbf{x} - \mathbf{x}^*\|^2 + \frac{\xi_1}{2} \|\mathbf{y}\|^2 \\
&= -\left(\omega_1 - \frac{\xi_1}{2}\right) \|\mathbf{x} - \mathbf{x}^*\|^2 - \left(-\frac{\xi_1}{2}\right) \|\mathbf{y}\|^2.
\end{aligned}$$

Next, we focus on dynamics (4). Let

$$\begin{aligned}
\mathbf{g}(\mathbf{x}, \mathbf{y}) &\triangleq \frac{d}{dt}(\boldsymbol{\varphi}(\mathbf{x}) - \mathbf{1}_N \otimes \sigma(\mathbf{x})) \\
&= (\nabla \boldsymbol{\varphi}(\mathbf{x}) - \mathbf{1}_N \otimes \nabla \sigma(\mathbf{x}))^T (P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y})) - \mathbf{x}),
\end{aligned}$$

where the time derivative $\dot{\mathbf{x}}$ is along the dynamics (3). Clearly, $\mathbf{1}_N^T \otimes I_m \mathbf{g}(\mathbf{x}, \mathbf{y}) = \mathbf{0}$. Also, since

$$\begin{aligned}
&\|P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y})) - \mathbf{x}^*\| \\
\le& \|P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y})) - P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x})))\| \\
&+ \|P_\Omega(\mathbf{x} - \alpha F(\mathbf{x})) - P_\Omega(\mathbf{x}^* - \alpha F(\mathbf{x}^*))\| \\
\le& \alpha k_2 \|\mathbf{y}\| + \|\mathbf{x} - \mathbf{x}^*\| + \alpha k \|\mathbf{x} - \mathbf{x}^*\|,
\end{aligned}$$

there holds

$$\begin{aligned}
\|\mathbf{g}(\mathbf{x}, \mathbf{y})\| &\le \|\nabla \boldsymbol{\varphi}(\mathbf{x}) - \mathbf{1}_N \otimes \nabla \sigma(\mathbf{x})\| \|P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y})) - \mathbf{x}\| \\
&\le k_3 \|P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \mathbf{1}_N \otimes \sigma(\mathbf{x}) + \mathbf{y})) - \mathbf{x}^*\| + 2\sqrt{N} k_3 \|\mathbf{x} - \mathbf{x}^*\| \\
&\le 2\sqrt{N} k_3 (2 + \alpha k) \|\mathbf{x} - \mathbf{x}^*\| + 2\sqrt{N} \alpha k_2 k_3 \|\mathbf{y}\|.
\end{aligned}$$

Define the following Lyapunov candidate function

$$V_2(\mathbf{y}) = \frac{1}{2}\|\mathbf{y}\|^2.$$

The time derivative of V_2 along the trajectory of (4) is

$$\begin{aligned}
\dot{V}_2 &= \mathbf{y}^\top \dot{\mathbf{y}} \\
&= \mathbf{y}^\top \left(-\beta L \boldsymbol{\eta} - \beta \mathbf{W} \theta + \frac{d}{dt} \left(\boldsymbol{\varphi}(\mathbf{x}) - \mathbf{1}_N \otimes \sigma(\mathbf{x}) \right) \right) \\
&= -\beta \mathbf{y}^T L \mathbf{y} + \mathbf{y}^T \mathbf{g}(\mathbf{x}, \mathbf{y}) - \beta \mathbf{y}^\top \mathbf{W} \theta \\
&\le -\beta \lambda_2 \|\mathbf{y}\|^2 + \mathbf{y}^T \mathbf{g}(\mathbf{x}, \mathbf{y}) - \beta \mathbf{y}^\top \mathbf{W} \theta \\
&\le -\beta \lambda_2 \|\mathbf{y}\|^2 + \|\mathbf{y}\| \|\mathbf{g}(\mathbf{x}, \mathbf{y})\| + \beta \|\mathbf{y}\| \|\mathbf{W}\| \|\theta\| \\
&\le -\beta \lambda_2 \|\mathbf{y}\|^2 + \|\mathbf{y}\| \left[2\sqrt{N} \kappa_3 (2 + \alpha\kappa) \|\mathbf{x} - \mathbf{x}^*\| + 2\sqrt{N} \alpha \kappa_2 \kappa_3 \|\mathbf{y}\| \right] + \beta \|\mathbf{W}\| \|\theta\| \|\mathbf{y}\| \\
&= -\left(\beta \lambda_2 - 2\sqrt{N} \alpha \kappa_2 \kappa_3 \right) \|\mathbf{y}\|^2 + 2\sqrt{N} \kappa_3 (2 + \alpha\kappa) \|\mathbf{x} - \mathbf{x}^*\| \|\mathbf{y}\| + \beta \|\mathbf{W}\| \|\theta\| \|\mathbf{y}\|,
\end{aligned}$$

where λ_2 is the smallest positive eigenvalue of L.

Denote $c_\theta = \beta \|\mathbf{W}\|$ and $\|\theta\| = k_\theta \sqrt{\|\mathbf{x} - \mathbf{x}^*\|^2 + \|\mathbf{y}\|^2}$, then $\|\theta\| \le k_\theta \left(\|\mathbf{x} - \mathbf{x}^*\| + \|\mathbf{y}\| \right)$. Then,

$$\begin{aligned}
\dot{V}_2 &\le -\left(\beta \lambda_2 - 2\sqrt{N} \alpha \kappa_2 \kappa_3 \right) \|\mathbf{y}\|^2 + 2\sqrt{N} \kappa_3 (2 + \alpha\kappa) \|\mathbf{x} - \mathbf{x}^*\| \|\mathbf{y}\| + c_\theta k_\theta \|\mathbf{y}\| \left(\|\mathbf{x} - \mathbf{x}^*\| + \|\mathbf{y}\| \right) \\
&\triangleq -\omega_2 \|\mathbf{y}\|^2 + \xi_2 \|\mathbf{x} - \mathbf{x}^*\| \|\mathbf{y}\| + c_\theta k_\theta \|\mathbf{x} - \mathbf{x}^*\| \|\mathbf{y}\| + c_\theta k_\theta \|\mathbf{y}\|^2 \\
&\le -\left(\omega_2 - c_\theta k_\theta \right) \|\mathbf{y}\|^2 + \frac{\xi_2}{2} \|\mathbf{x} - \mathbf{x}^*\|^2 + \frac{\xi_2}{2} \|\mathbf{y}\|^2 + \frac{c_\theta k_\theta}{2} \|\mathbf{x} - \mathbf{x}^*\|^2 + \frac{c_\theta k_\theta}{2} \|\mathbf{y}\|^2 \\
&= -\left(-\frac{\xi_2}{2} - \frac{c_\theta k_\theta}{2} \right) \|\mathbf{x} - \mathbf{x}^*\|^2 - \left(\omega_2 - \frac{\xi_2}{2} - \frac{3 c_\theta k_\theta}{2} \right) \|\mathbf{y}\|^2.
\end{aligned}$$

Take

$$V = V_1 + V_2.$$

Then,

$$\begin{aligned}
\dot{V} =& \dot{V}_1 + \dot{V}_2 \\
\le& -\left(\omega_1 - \frac{\xi_1}{2} \right) \|\mathbf{x} - \mathbf{x}^*\|^2 - \left(-\frac{\xi_1}{2} \right) \|\mathbf{y}\|^2 \\
& - \left(-\frac{\xi_2}{2} - \frac{c_\theta k_\theta}{2} \right) \|\mathbf{x} - \mathbf{x}^*\|^2 - \left(\omega_2 - c_\theta k_\theta - \frac{c_\theta k_\theta}{2} \right) \|\mathbf{y}\|^2 \\
\le& -\left(\omega_1 - \frac{\xi_1}{2} - \frac{\xi_2}{2} - \frac{c_\theta k_\theta}{2} \right) \|\mathbf{x} - \mathbf{x}^*\|^2 - \left(\omega_2 - \frac{\xi_1}{2} - \frac{\xi_2}{2} - \frac{3 c_\theta k_\theta}{2} \right) \|\mathbf{y}\|^2.
\end{aligned}$$

The following inequality is required

$$\begin{cases} \omega_1 - \frac{\xi_1}{2} - \frac{\xi_2}{2} - \frac{c_\theta k_\theta}{2} > 0, \\ \omega_2 - \frac{\xi_1}{2} - \frac{\xi_2}{2} - \frac{3 c_\theta k_\theta}{2} > 0. \end{cases}$$

Then, apply

$$k_\theta < \min\left\{\frac{2}{c_\theta}\left(\omega_1 - \frac{\xi_1}{2} - \frac{\xi_2}{2}\right), \frac{2}{3c_\theta}\left(\omega_2 - \frac{\xi_1}{2} - \frac{\xi_2}{2}\right)\right\}.$$

Let

$$\rho = \frac{1}{\min\left\{\frac{2}{c_\theta}\left(\omega_1 - \frac{\xi_1}{2} - \frac{\xi_2}{2}\right), \frac{2}{3c_\theta}\left(\omega_2 - \frac{\xi_1}{2} - \frac{\xi_2}{2}\right)\right\}}.$$

Also, there exists $0 < c_1 < \frac{1}{2} < c_2$, such that

$$c_1\left(\|\mathbf{x}-\mathbf{x}^*\|^2 + \|\mathbf{y}\|^2\right) \le V = \frac{1}{2}\|\mathbf{x}-\mathbf{x}^*\|^2 + \frac{1}{2}\|\mathbf{y}\|^2 \le c_2\left(\|\mathbf{x}-\mathbf{x}^*\|^2 + \|\mathbf{y}\|^2\right).$$

Then we obtain

$$\gamma = \rho\sqrt{\frac{c_2}{c_1}}.$$

Thus,

$$\|\mathbf{x}-\mathbf{x}^*\| \le \sqrt{\|\mathbf{x}-\mathbf{x}^*\|^2 + \|\mathbf{y}\|^2} \le \beta\left(\|\mathbf{x}_0-\mathbf{x}^*\|^2 + \|\mathbf{y}_0\|^2, t-t_0\right) + \gamma\|\theta\|.$$

Step 2: Assume that the mapping of the proposed Algorithm 1 reaches equilibrium $(x_i, \mathbf{x}_{-i})$. For $\forall x_i' \in \mathcal{X}_i, i \in [N]$, we have

$$\begin{aligned}
&f_{ij}(x_i, \mathbf{x}_{-i}) - f_{ij}(x_i', \mathbf{x}_{-i})\\
=&f_{ij}(x_i', \mathbf{x}_{-i}^*) - f_{ij}(x_i', \mathbf{x}_{-i})\\
&+ f_{ij}(x_i, \mathbf{x}_{-i}) - f_{ij}(x_i^*, \mathbf{x}_{-i}^*)\\
&+ f_{ij}(x_i^*, \mathbf{x}_{-i}^*) - f_{ij}(x_i', \mathbf{x}_{-i})\\
\triangleq& \text{ Term 1 } + \text{ Term 2 } + \text{ Term 3 }.
\end{aligned}$$

Denote $[x_i', \mathbf{x}_{-i}^*]_{\text{ vec }} = \left[x_1^*, \ldots, x_{i-1}^*, x_i', x_{i+1}^*, \ldots, x_N^*\right]^\top$,

$$\begin{aligned}
\text{Term 1 } =&f_{ij}(x_i', \mathbf{x}_{-i}^*) - f_{ij}(x_i', \mathbf{x}_{-i})\\
\le&\langle \nabla f_{ij}(x_i', \mathbf{x}_{-i}^*), [x_i', \mathbf{x}_{-i}^*]_{\text{ vec }} - [x_i', \mathbf{x}_{-i}]_{\text{ vec }}\rangle\\
\le&\|\nabla f_{ij}(x_i', \mathbf{x}_{-i}^*)\|\|\mathbf{x}^* - \mathbf{x}\|\\
=&\mathcal{C}\|\mathbf{x}^* - \mathbf{x}\|,
\end{aligned}$$

where the first inequality comes from the cost function f_{ij} is convex, the second inequality comes from Cauchy-Swartch inequality and the third inequality comes from the gradient of cost function is bounded.

$$\begin{aligned}
\text{Term 2 } =&f_{ij}(x_i, \mathbf{x}_{-i}) - f_{ij}(x_i^*, \mathbf{x}_{-i}^*)\\
\le&\langle \nabla f_{ij}(x_i, \mathbf{x}_{-i}), \mathbf{x} - \mathbf{x}^*\rangle\\
\le&\|\nabla f_{ij}(\mathbf{x})\|\|\mathbf{x} - \mathbf{x}^*\|\\
=&\mathcal{C}\|\mathbf{x} - \mathbf{x}^*\|,
\end{aligned}$$

where the first inequality comes from the cost function f_{ij} is convex, the second inequality comes from Cauchy-Swartch inequality, and the third inequality comes from the gradient of cost function f_{ij} is bounded.

$$
\begin{aligned}
f_{ij}(x_i^*, \mathbf{x}_{-i}^*) &\leq f_{ii}(x_i^*, \mathbf{x}_{-i}^*) + \mathcal{B}_2^\top x_i^*, \\
-f_{ij}(x_i', \mathbf{x}_{-i}^*) &\leq -f_{ii}(x_i', \mathbf{x}_{-i}^*) - \mathcal{B}_1^\top x_i'.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\text{Term 3 } =& f_{ij}(x_i^*, \mathbf{x}_{-i}^*) - f_{ij}(x_i', \mathbf{x}_{-i}^*) \\
\leq& f_{ii}(x_i^*, \mathbf{x}_{-i}^*) + \mathcal{B}_2^\top x_i^* - f_{ii}(x_i', \mathbf{x}_{-i}^*) - \mathcal{B}_1^\top x_i' \\
\leq& \mathcal{B}_2^\top x_i^* - \mathcal{B}_1^\top x_i' \\
\leq& \|\mathcal{B}_2^\top x_i^* - \mathcal{B}_1^\top x_i'\| \\
\leq& \|\mathcal{B}_2^\top x_i^*\| + \|\mathcal{B}_1^\top x_i'\| \\
\leq& 2\mathcal{B}M,
\end{aligned}
$$

where $M = \sum_{i=1}^N \max_{x_i \in \mathcal{X}_i} \|x_i\|$, $\mathcal{B} = \max\{\mathcal{B}_1, \mathcal{B}_2\}$ and the first inequality comes from Assumption 2.8. □

Theorem 1 demonstrates that the influence of deceptive behavior, where we provide an upper bound on ϵ, and show the convergence of the proposed distributed deceptive algorithm. Different from existing literature [11], in our work, we consider the existence of players with deceptive behavior and other players as little unaware of the existence of misinformation as possible. This leads to the instability of the network system, thereby affecting ϵ. From (3.1), as the level of deception by such players increases (as θ becomes higher), the network system becomes more difficult to reach the true hyper Nash equilibrium point, and consequently, the upper bound of ϵ also increases. Therefore, considering the tolerance of other players towards profit differences, player r can control its own deceptive strategy θ to make other players as little unaware of the existence of misinformation as possible.

Remark 3.2. In the absence of misinformation and deception, the algorithm can achieve exponential convergence to the optimal value, as in reference [14].

3.3. Deceptive analysis

In this section, we analyze the motivation behind players adopting deceptive strategies. Typically, players prioritize minimizing their cost functions. We outline how player r can reduce its costs by deceptive behavior, which is the condition under which the player is willing to engage in deceptive strategy.

Suppose that there are N players in a quadratic aggregative game [20], where each player i aims at minimizing a quadratic cost function $f_i(x_i, \mathbf{x}_{-i}) : \mathbb{R}^{Nn} \to \mathbb{R}$

$$
f_i(x_i, \mathbf{x}_{-i}) := x_i^T Q_i x_i + (C\sigma(x) + c_i)^T x_i,
$$

where $Q_i \in \mathbb{R}^{n\times n}, C_i \in \mathbb{R}^{n\times n}, c_i \in \mathbb{R}^n$ and $\sigma(x) := \frac{1}{N}\sum_{j=1}^N x_j$.

Note that the cost of each agent depends on the other players' strategies only via the interaction function $C_i\sigma(x) + c_i$. Assuming that player j's estimation of the parameter c_i in the cost function of any player i is biased, denoted as f_{ij}. Thus, the cost function of player i under the cognition of player j is

$$f_{ij}(x_i, \mathbf{x}_{-i}) := x_i^T Q_i x_i + (C\sigma(x) + c_{ij})^T x_i.$$

The deceptive player r will increase or decrease its own cost after adopting the deceptive strategy. Obviously, player r is willing to adopt a deceptive strategy when its own cost can be reduced.

The following theorem provides the conditions, under which the deceptive player is willing to adopt a deceptive strategy.

$$\gamma_i = \left(Q_i + Q_i^\top + \frac{C^\top}{N}\right)^{-1},$$

$$\mu = C\left[NE + \sum_{j=1}^{N}\left(Q_j + Q_j^\top + \frac{C^\top}{N}\right)^{-1} C\right]^{-1},$$

$$\tilde{\mu} = \left[E + \frac{C}{N}\sum_{j=1}^{N}\left(Q_j + Q_j^\top + \frac{C^\top}{N}\right)^{-1}\right]^{-1},$$

$$\lambda_i = \left[NC^{-1} + \sum_{j=1}^{N}\left(Q_j + Q_j^\top + \frac{C^\top}{N}\right)^{-1}\right] c_{ir}, \tilde{\eta} = \sum_{j=1}^{N}\left(Q_j + Q_j^\top + \frac{C^\top}{N}\right)^{-1} c_{jr},$$

$$A_1 = \left[(\gamma_r\tilde{\mu}CW_2)^\top Q_r + \frac{1}{N}\left(\sum_{i=1}^{N}\gamma_i\tilde{\mu}CW_2\right)^\top\right](\gamma_r\mu CW_2), \tag{6}$$

$$A_2 = \left[(\gamma_r\tilde{\mu}CW_2)^\top Q_r + \frac{1}{N}\left(\sum_{i=1}^{N}\gamma_i\tilde{\mu}CW_2\right)^\top\right](-\gamma_r\tilde{\mu}c_r), \tag{7}$$

$$A_3 = \left[(-\gamma_r\tilde{\mu}c_r)^\top Q_r - \frac{1}{N}\left(\sum_{i=1}^{N}\gamma_i\tilde{\mu}c_i\right)^\top C^\top + c_r^\top\right](\gamma_r\mu CW_2), \tag{8}$$

$$\begin{aligned} B = &\left[\gamma\mu(\lambda_r + \tilde{\eta}) + \frac{C}{N}\sum_{i=1}^{N}\gamma_i\mu(-\lambda_i + \tilde{\eta}) + c_r\right]\gamma_r\mu(-\lambda_r + \tilde{\eta}) \\ &- \left[(-\gamma_r\tilde{\mu}c_r)^\top Q_r - \frac{1}{N}\left(\sum_{i=1}^{N}\gamma_i\tilde{\mu}c_i\right)^\top C^\top + c_r^\top\right](-\gamma_r\tilde{\mu}c_r). \end{aligned} \tag{9}$$

Theorem 3.3. When $\theta^\top A_1\theta + \theta^\top A_2 + A_3\theta < B$,, the inequality

$$f_r(\hat{\mathbf{x}}) < f_r(\mathbf{x}_r^*),$$

is satisfied, where A_1, A_2, A_3 and B are defined as (6), (7), (8) and (9).

Proof. Consider Algorithm 1. When the player cost function is a quadratic function, for each agent the gradients of the first and second term of the cost function depend only on the local strategy x_i and on the aggregate quantity $\sigma(x)$, that is,

$$\begin{aligned}\nabla_{x_i} f_i\left(x_i, \mathbf{x}_{-i}\right) &= \left(Q_i + Q_i^\top + \frac{C^\top}{N}\right) x_i + C\sigma(x) + c_i \\ &=: F_i\left(x_i, \sigma(x)\right) \in \mathbb{R}^n.\end{aligned}$$

Therefore, the equilibrium point in the above map can be simplified as

$$\begin{aligned}\hat{x_i} &= \left(Q_i + Q_i^\top + \frac{C^\top}{N}\right)^{-1}\left[E + \frac{C}{N}\sum_{j=1}^{N}\left(Q_j + Q_j^\top + \frac{C^\top}{N}\right)^{-1}\right]^{-1}\left(CW_2\theta - c_i\right) \\ &= \gamma_i\tilde{\mu}\left(CW_2\theta - c_i\right).\end{aligned}$$

The deceptive player r believes that the Nash equilibrium is $\mathbf{x}_r^* = \{x_{1r}^*, \ldots, x_{Nr}^*\}$,

$$x_{ir}^* = \gamma_i\mu\left(-\lambda_i + \tilde{\eta}\right), \forall i \in [N].$$

Only by reducing its own cost can player r be willing to adopt deceptive strategies, that is to say, when

$$f_r(\hat{\mathbf{x}}) < f_r(\mathbf{x}_r^*), \tag{10}$$

is satisfied, player r is willing to adopt deceptive strategies. Simplifying inequality (10), we obtain

$$\theta^\top A_1\theta + \theta^\top A_2 + A_3\theta < B,$$

which is the condition for player r to be willing to adopt the deceptive strategy. □

Theorem 3.3 outlines how player r can increase profit through deceptive behavior and provides conditions for player r to be willing to engage in deceptive behavior. It can be concluded from the above theorem that the player r is willing to adopt a deceptive strategy θ which meets the above condition, as this can reduce the player's own costs. For instance, in CPS, while the hacker is probing the system, the network administrator might change the system's TCP/IP stack and obfuscate the services running on the por to ensure the confidence and security of users in the network.

Then we consider the optimal deceptive strategy of the deceiver under a fixed tolerance ϵ, that is, what deceptive strategy does the deceiver adopt to minimize its own costs. For a given tolerance ϵ, if the deceptive strategy θ is taken according to the following theorem, the deceptive player obtains the minimum costs.

Theorem 3.4. If $\epsilon \in (0, 2\rho\sqrt{\frac{c_2}{c_1}}\| \left[(S_3+S_2)+(S_3+S_2)^\top\right]^{-1}\left[(R_3S_2)^\top+(S_3R_2)\right]\| + 2\mathcal{B}M]$, then $\theta^* = -\left[(S_3+S_2)+(S_3+S_2)^\top\right]^{-1}\left[(R_3S_2)^\top+(S_3R_2)\right]$ is optimal for player r to implement deception.

Moreover, if $\epsilon \in [2\mathcal{B}M + \sqrt{\frac{c_2}{Nc_1}}\frac{1}{\lambda_{B'_{\min}}}\left[(R_3S_2)^\top+(S_3R_2)\right]\mathbf{1}_N, \infty)$, then the player r's optimal deception θ^* is the solution of the following equation:

$$\begin{aligned}
&\frac{\|\theta^*\|}{2\mathcal{M}\rho\sqrt{\frac{c_2}{c_1}}}\left[(R_3S_2)^\top+(S_3R_2)+[(S_3S_2)+(S_3S_2)^\top]\theta^*\right] \\
&=\left[R_3S_2\theta^*+(S_2R_2)^\top\theta^*+\theta^{*\top}[(S_3S_2)+(S_3S_2)^\top]\theta^*\right]\theta^*,
\end{aligned}$$

where $\mathcal{M} = \frac{1}{\left[2\rho\sqrt{\frac{c_2}{c_1}}\frac{1}{\|\theta^*\|}\right]\theta^{*\top}\theta^*}$ and $\lambda_{B'_{\min}}$ is the smallest eigenvalue of $[(S_3S_2)+(S_3S_2)^\top]$.

P r o o f. Consider the deception algorithm shown in Algorithm 1. For the case where the player cost function is a quadratic function, for each agent the gradients of the first and second term of the cost function depend only on the local strategy x_i and on the aggregate quantity $\sigma(x)$, that is,

$$\begin{aligned}
\nabla_{x_i} f_i(x_i, x_{-i}) &= \left(Q_i + Q_i^\top + \frac{C_i^\top}{N}\right)x_i + C_i\sigma(x) + c_i \\
&=: F_i(x_i, \sigma(x)) \in \mathbb{R}^n.
\end{aligned}$$

Assume that the sequence values after each gradient descent fall within the constraint set. Then, there is iterative formulas as follows, for $\forall i$

$$\begin{aligned}
\dot{\mathbf{x}} &= P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \boldsymbol{\eta})) - \mathbf{x}, \\
\dot{\mathbf{y}} &= -\beta L - \beta \mathbf{W}\theta + \left(\nabla\boldsymbol{\varphi}(\mathbf{x}) - \mathbf{1}_N \otimes \nabla\sigma(\mathbf{x})\right)^\top\left(P_\Omega(\mathbf{x} - \alpha G(\mathbf{x}, \boldsymbol{\eta})) - \mathbf{x}\right).
\end{aligned} \tag{11}$$

Then, the equilibrium point in the above map can be simplified as

$$\hat{x_i} = \gamma_i\tilde{\mu}\left(CW_2\theta - c_i\right).$$

Denote $\hat{x_r} = \gamma_r\tilde{\mu}\left(CW_2\theta - c_r\right) \triangleq R_2 + S_2\theta$. Therefore,

$$\begin{aligned}
\sum_{i=1}^N \hat{x}_i =& \sum_{i=1}^N \left[\alpha_i C_i \beta\left(-\gamma_i - \mu_i c_{ii} + \tilde{\eta}\right)\right] \\
&+ \left[\alpha_r C_r \beta(NE + 2\lambda) - \sum_{i=1}^N \alpha_i C_i \beta(E+\lambda)\right]\theta \\
\triangleq& R_1 + S_1\theta.
\end{aligned}$$

Then, the deceptive player's cost function can be calculated as follows

$$
\begin{aligned}
&f_r(\hat{x}_r, \hat{\mathbf{x}}_{-r}) \\
=&\left[\left(R_2^\top Q_r + \frac{1}{N}R_1^\top C_r^\top + C_r^\top\right) + \theta^\top\left(S_2^\top Q_r + \frac{1}{N}S_1^\top C_r^\top\right)\right] \\
&(R_2 + S_2\theta) \\
\triangleq&(R_3 + \theta^\top S_3)(R_2 + S_2\theta) \\
=&R_3R_2 + R_3S_2\theta + \theta^\top S_3R_2 + \theta^\top S_3S_2\theta \triangleq h(\theta).
\end{aligned}
$$

From Theorem 3.1,

$$\epsilon \le 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta\| + 2\mathcal{B}M.$$

Given a tolerance of ϵ, our goal is to seek the minimum cost function of the deception player, that is, to solve the following optimization problem with inequality constraint:

$$
\begin{aligned}
\min h(\theta) &= R_3R_2 + R_3S_2\theta + \theta^\top S_3R_2 + \theta^\top S_3S_2\theta, \\
\text{s.t. } \epsilon &\le 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta\| + 2\mathcal{B}M,
\end{aligned}
$$

The above optimization problem is equivalent to

$$
\begin{aligned}
&\min h(\theta) = R_3R_2 + R_3S_2\theta + \theta^\top S_3R_2 + \theta^\top S_3S_2\theta, \\
&\text{s.t. } g(\theta) = \epsilon - 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta\| - 2\mathcal{B}M \le 0.
\end{aligned}
$$

Next, KKT condition is applied to optimization problems with inequality constraints.

$$
\begin{array}{ll} \min & h(\theta) \\ \text{s.t.} & g(\theta) \le 0 \end{array}
\Rightarrow
\begin{array}{l} \text{KKT} \\ \text{condition} \end{array}
\begin{cases}
\nabla h(\theta^*) + \lambda \nabla g(\theta^*) = 0 \\
\lambda g(\theta^*) = 0 \\
\lambda \ge 0 \\
g(\theta^*) \le 0
\end{cases}
$$

$$
\begin{aligned}
\nabla h(\theta) &= (R_3S_2)^\top + (S_3R_2) + [(S_3S_2) + (S_3S_2)^\top]\theta, \\
\nabla g(\theta) &= -2\rho\sqrt{\frac{c_2}{c_1}}\frac{\theta}{\|\theta\|}.
\end{aligned}
$$

Case 1:
If $\lambda = 0$, then $\nabla h(\theta^*) = 0 \Rightarrow \theta^* = -\left[(S_3 + S_2) + (S_3 + S_2)^\top\right]^{-1}\left[(R_3S_2)^\top + (S_3R_2)\right]$, Substituting into $g(\theta^*) \le 0$, we have

$$g(\theta^*) = \epsilon - 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta^*\| - 2\mathcal{B}M \le 0,$$

i. e., $\epsilon \le 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta^*\| + 2\mathcal{B}M$.

Case 2:
If $\lambda \neq 0$, then $g(\theta^*) = 0$, i. e.,

$$g(\theta^*) = \epsilon - 2\rho\sqrt{\frac{c_2}{c_1}}\|\theta^*\| - 2\mathcal{B}M = 0.$$

Then we have

$$\|\theta^*\| = \sqrt{\frac{c_1}{c_2}}\frac{\epsilon - 2\mathcal{B}M}{2\rho}. \tag{12}$$

Substituting θ^* into $\nabla h(\theta^*) + \lambda \nabla g(\theta^*) = 0$, we have

$$(R_3S_2)^\top + (S_3R_2) + [(S_3S_2) + (S_3S_2)^\top]\theta^* + \lambda\left[-2\rho\sqrt{\frac{c_2}{c_1}}\frac{\theta^*}{\|\theta^*\|}\right] = 0.$$

Then,

$$(R_3S_2)^\top + (S_3R_2) + [(S_3S_2) + (S_3S_2)^\top]\theta^* = \lambda\left[2\rho\sqrt{\frac{c_2}{c_1}}\frac{1}{\|\theta^*\|}\right]\theta^*,$$

$$\theta^{*\top}(R_3S_2)^{*\top} + \theta^{*\top}(S_3R_2) + \theta^{*\top}[(S_3S_2) + (S_3S_2)^\top]\theta^* = \lambda\left[2\rho\sqrt{\frac{c_2}{c_1}}\frac{1}{\|\theta^*\|}\right]\theta^{*\top}\theta^*.$$

Denote $\mathcal{M} = \frac{1}{\left[2\rho\sqrt{\frac{c_2}{c_1}}\frac{1}{\|\theta^*\|}\right]\theta^{*\top}\theta^*}$, and then

$$\lambda = \mathcal{M}\left[R_3S_2\theta^* + (S_2R_2)^\top\theta^* + \theta^{*\top}[(S_3S_2) + (S_3S_2)^\top]\theta^*\right].$$

The optimal deception θ^* a is the solution of the following equation:

$$\begin{aligned}&\frac{\|\theta^*\|}{2\mathcal{M}\rho\sqrt{\frac{c_2}{c_1}}}\left[(R_3S_2)^\top + (S_3R_2) + [(S_3S_2) + (S_3S_2)^\top]\theta^*\right]\\ &= \left[R_3S_2\theta^* + (S_2R_2)^\top\theta^* + \theta^{*\top}[(S_3S_2) + (S_3S_2)^\top]\theta^*\right]\theta^*.\end{aligned} \tag{13}$$

Then, verify $\lambda > 0$ by combining equations (12) and (13). Therefore,

$$\epsilon \geq 2\mathcal{B}M + \sqrt{\frac{c_2}{Nc_1}}\frac{1}{\lambda_{B'_{\min}}}\left[(R_3S_2)^\top + (S_3R_2)\right]\mathbf{1}_N,$$

where $\lambda_{B'_{\min}}$ is the smallest eigenvalue of the symmetric matrix $[(S_3S_2) + (S_3S_2)^\top]$. □

From Theorem 3.4, it can be concluded that when the tolerance ϵ meets certain conditions, player r can achieve the lowest cost consumption by adopting the above-mentioned deceptive strategy.

4. NUMERICAL SIMULATION

In this section, we provide simulations to illustrate the convergence performance and the effectiveness of the deceptive strategy.

4.1. Convergence

For simulation, we first discretize the continuous-time algorithm as follows

$$\overline{\boldsymbol{x}}[k+1] = \overline{\boldsymbol{x}}[k] + h\frac{d}{dt}\boldsymbol{x}(t)\bigg|_{t=kh},$$
$$\overline{\boldsymbol{\eta}}[k+1] = \overline{\boldsymbol{\eta}}[k] + h\frac{d}{dt}\boldsymbol{\eta}(t)\bigg|_{t=kh},$$

where $h > 0$ is a fixed stepsize. Here, we consider the 5-player energy consumption game, where player i's objective function is given by

$$f_i(\mathbf{x}) = (x_i - \hat{x}_i)^2 + \left(0.04\sum_{i=1}^{5} x_i + 5\right) x_i,$$

with $\hat{x}_1 = 50, \hat{x}_2 = 55, \hat{x}_3 = 60, \hat{x}_4 = 65, \hat{x}_5 = 70$. The game has the unique pure-strategy Nash equilibrium at $\mathbf{x}^* = (41.5, 46.4, 51.3, 56.2, 61.1)$. In the following, fixed communication topologies and time-varying communication topologies will be considered.

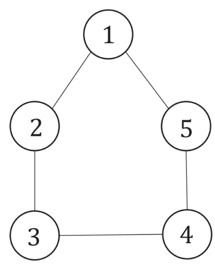

Fig. 1. The communication graph for the players.

In the simulation, the players are supposed to communicate via a cycle depicted in Figure 1. Correspondingly, the weight matrix is given as

$$W = \begin{bmatrix} 0.5 & 0.2 & 0 & 0 & 0.3 \\ 0.2 & 0.5 & 0.3 & 0 & 0 \\ 0 & 0.3 & 0.5 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0.3 & 0 & 0 & 0.3 & 0.4 \end{bmatrix},$$

Moreover, the proposed method is run for 50 times for the observation of the simulation results.

Figure 2 shows the players' squared equilibrium errors, i.e., $\|x_i^k - x_i^*\|^2$ for $i \in \{1, 2, \ldots, 5\}$ from which we see that the proposed method drives $\|x_i^k - x_i^*\|^2$ to a small neighborhood of zero even with a deceptive strategy θ.

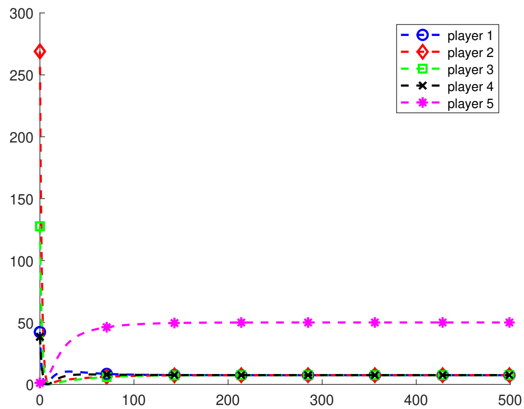

Fig. 2. Convergence to NE ($\theta = 100$).

Next, we implement Algorithm 1 with step-size α as in Theorem 1, and display the empirical result (maximize across sampling trajectories with the same initial points) in Figure 3. Besides, we compare our algorithm with the projected gradient method [11] and extra-gradient method [8] when applied to the considered aggregative game, while the network aggregate value is still estimated with the dynamical average tracking. It can be seen that the distributed aggregative game problem with misinformation proposed in this paper, compared to the existing non-deceptive algorithm, when selecting an appropriate deceptive strategy, our deceptive algorithm is more in line with the cognition of each player.

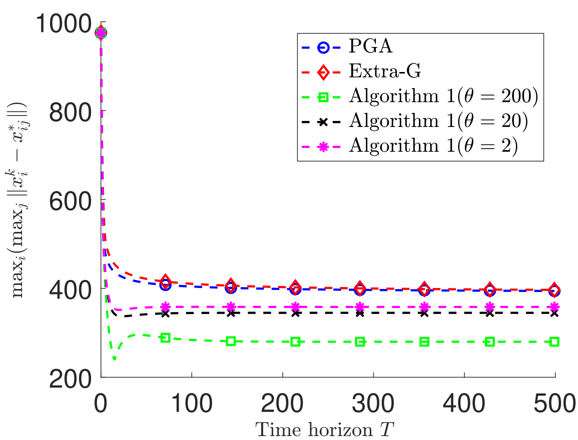

Fig. 3. Comparison of Algorithm 1 with the projected gradient method (PGA) and the extra-gradient method (Extra-G). The trajectories are derived by maximizing with 5 player paths).

In Figure 4, when there is no deceptive strategy θ, the vertical axis is not the lowest point, and it can be seen from Figure 4 that selecting a suitable deception strategy θ can improve the player's cognitive stability and make them less skeptical of their own cognition compared to the case without adopting a deceptive strategy.

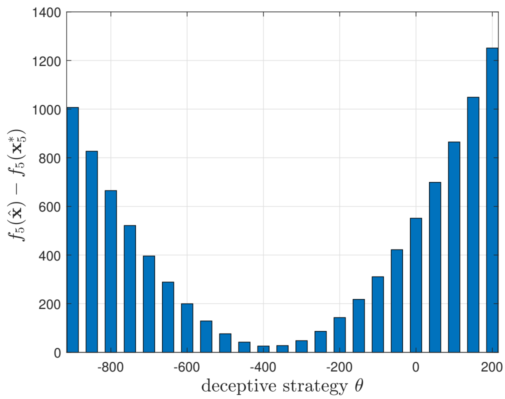

Fig. 4. Comparison of Algorithm 1 with different deceptive strategy θ. The trajectories are derived by maximizing with 5 player paths).

4.2. Deceptive analysis

The cost function of player i can be written as

$$f_i(\mathbf{x}) = x_i^2 - \left(0.04\sum_{i=1}^{5} x_i + 5 + 2\hat{x}_i\right) x_i.$$

The cost functions of other players under the perspective of player 5 are as follows:

$$f_{15}(\mathbf{x}) = {x_1}^2 + \left(0.04\sum_{i=1}^{5} x_1 + 40\right) x_1,$$

$$f_{25}(\mathbf{x}) = {x_2}^2 + \left(0.04\sum_{i=1}^{5} x_2 + 50\right) x_2,$$

$$f_{35}(\mathbf{x}) = {x_3}^2 + \left(0.04\sum_{i=1}^{5} x_3 + 60\right) x_3,$$

$$f_{45}(\mathbf{x}) = {x_4}^2 + \left(0.04\sum_{i=1}^{5} x_4 + 70\right) x_4,$$

$$f_5(\mathbf{x}) = {x_5}^2 + \left(0.04\sum_{i=1}^{5} x_5 + 5\right) x_5.$$

As shown in Figure 5, when $\theta = 0$, the cost function value of player 5 under the Nash equilibrium on the vertical axis is not the lowest for deceiving player 5. This is because the Nash equilibrium is a strategy combination, which means that when adopting such a strategy, players will not only change their own strategy to reduce costs, but it is not necessarily the best choice for each player.

Below is the condition under which the deceptive player is willing to adopt a deceptive strategy. $f_5(\hat{\mathbf{x}})$ is the cost function after adopting the deception strategy and $f_5(\mathbf{x}_5^*)$ is the optimal cost function for player 5's own cognition. When $f_5(\hat{\mathbf{x}}) < f_5(\mathbf{x}_5^*)$, i.e. $f_5(\hat{\mathbf{x}}) - f_5(\mathbf{x}_5^*) < 0$, the deceptive player is willing to engage in deceptive behavior. Figure 5 illustrates the relationship between the difference and the deceptive strategy θ. It can be seen that when the deceptive strategy θ meets certain conditions, player 5 can reduce its own costs, which means that the deceptive player 5 will sample the deceptive strategy within this range for deception.

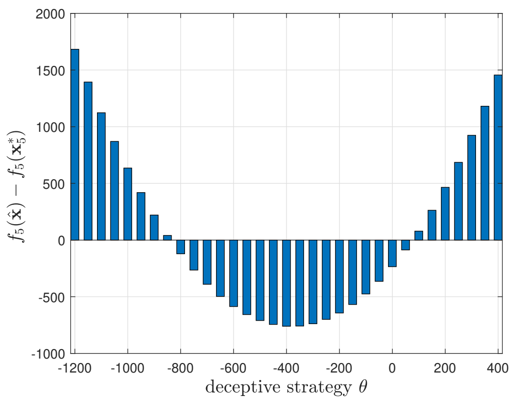

Fig. 5. $f_5(\hat{\mathbf{x}}) - f_5(\mathbf{x}_5^*)$ vs deceptive θ ($T = 50$).

5. CONCLUSION

This paper has explored the distributed aggregative game problem with misinformation, with each player holding a biased estimate of the cost function pertaining to other players in the game. We have formulated this issue as a hypergame and proposed a distributed deception algorithm. Then we have provided convergence analysis for our proposed algorithm, where the metric ϵ correlates with the deceptive strategy. Moreover, for the distributed quadratic game problem, we have outlined the conditions under which the deceptive player is inclined to adopt deceptive strategy. Specifically, we have determined the range of deceptive strategy values capable of reducing costs. In addition, for a given tolerance ϵ, we have proposed the optimal deceptive strategy from the perspective of the deceptive player. Finally, we have validated the effectiveness of the algorithm via numerical experiments.

ACKNOWLEDGEMENT

This work is supported by the National Natural Science Foundation of China under Grant 62173250.

(Received August 14, 2024)

REFERENCES

[1] N. Abuzainab and W. Saad: A multiclass mean-field game for thwarting misinformation spread in the internet of battlefield things. IEEE Trans. Commun. *66* (2018), 12, 6643–6658. DOI:10.1109/TCOMM.2018.2866859

[2] H. Chen, Y. Li, R. H. Louie, and B. Vucetic: Autonomous demand side management based on energy consumption scheduling and instantaneous load billing: An aggregative game approach. IEEE Trans. Smart Grid *5* (2014), 4, 1744–1754. DOI:10.1109/TSG.2014.2311122

[3] J. Chen and Q. Zhu: nterdependent strategic security risk management with bounded rationality in the internet of things. IEEE Trans. Inform. Forensics Security *14* (2019), 11, 2958–2971. DOI:10.1109/TIFS.2019.2911112

[4] Z. Cheng, G. Chen, and Y. Hong: Single-leader-multiple-followers stackelberg security game with hypergame framework. IEEE Trans. Inform. Forensics Security *17* (2022), 954–969. DOI:10.1109/TIFS.2022.3155294

[5] J. P. Hespanha, Y. S. Ateskan, H. Kizilocak et al.: Deception in non-cooperative games with partial information. In: Proc. 2nd DARPA-JFACC Symposium on Advances in Enterprise Control, Citeseer 2000, pp. 1–9.

[6] S. Huang, J. Lei, and Y. Hong: A linearly convergent distributed Nash equilibrium seeking algorithm for aggregative games. IEEE Trans. Automat. Control *68* (2022), 3, 1753–1759. DOI:10.1109/TAC.2022.3154356

[7] L. Huang and Q. Zhu: Duplicity games for deception design with an application to insider threat mitigation. IEEE Trans. Inform. Forensics Security *16* (2021), 4843–4856. DOI:10.1109/TIFS.2021.3118886

[8] S. Jelassi, C. Domingo-Enrich, D. Scieur, A. Mensch, and J. Bruna: Extragradient with player sampling for faster Nash equilibrium finding. In: Proc. International Conference on Machine Learning 2020.

[9] R. Jin, X. He, and H. Dai: On the security-privacy tradeoff in collaborative security: A quantitative information flow game perspective. IEEE Trans. Inform. Forensics Security *14* (2019), 12, 3273–3286. DOI:10.1109/TIFS.2019.2914358

[10] B. Johansson, T. Keviczky, M. Johansson, and K. H. Johansson: Subgradient methods and consensus algorithms for solving convex optimization problems. In: 47th IEEE Conference on Decision and Control, IEEE 2008, pp. 4185–4190. DOI:10.1109/CDC.2008.4739339

[11] J. Koshal, A. Nedić, and U. V. Shanbhag: Distributed algorithms for aggregative games on graphs. Oper. Res. *64* (2016), 3, 680–704. DOI:10.1287/opre.2016.1501

[12] N. S. Kovach, A. S. Gibson, and G. B. Lamont: Hypergame theory: a model for conflict, misperception, and deception. Game Theory (2015).

[13] J. Lei and U. V. Shanbhag: Asynchronous schemes for stochastic and misspecified potential games and nonconvex optimization. Operations Research —mi 68 (2020), 6, 1742–1766. DOI:10.1287/opre.2019.1946

[14] S. Liang, P. Yi, Y. Hong, and K. Peng: Exponentially convergent distributed Nash equilibrium seeking for constrained aggregative games. Autonomous Intell. Systems *2* (2022), 1, 6. DOI:10.1155/2022/1036942

[15] J. Ma, Z. Yang, and Z. Chen: Distributed Nash equilibrium tracking via the alternating direction method of multipliers. Kybernetika *59* (2023), 4, 612–632. DOI:10.14736/kyb-2023-4-0612

[16] Y. Meng, M. Broom, and A. Li: Impact of misinformation in the evolution of collective cooperation on networks. J. Royal Soc. Interface *20* (2023), 206, 20230295. DOI:10.1098/rsif.2023.0295

[17] Y. Meng, S. P. Cornelius, Y. Y. Liu, and A. Li: Dynamics of collective cooperation under personalised strategy updates. Nature Commun. *15* (2024), 1, 3125. DOI:10.56337/sbm.1350842

[18] A. Nedic, A. Ozdaglar, and P. A. Parrilo: Constrained consensus and optimization in multi-agent networks. IEEE Trans. Automat. Control *55* (2010), 4, 922–938. DOI:10.1109/TAC.2010.2041686

[19] K. C. Nguyen, T. Alpcan, and T. Basar: Security games with incomplete information. In: 2009 IEEE International Conference on Communications, pp. 1–6. DOI:10.1109/icc.2009.5199443

[20] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros: Distributed computation of generalized Nash equilibria in quadratic aggregative games with affine coupling constraints. In: 55th IEEE Conference on Decision and Control, IEEE 2016, pp. 6123–6128. DOI:10.1109/cdc.2016.7799210

[21] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros: Nash and wardrop equilibria in aggregative games with coupling constraints. IEEE Trans. Automat. Control *64* (2018), 4, 1373–1388. DOI:10.1109/TAC.2018.2849946

[22] J. Pawlick, E. Colbert, and Q. Zhu: Modeling and analysis of leaky deception using signaling games with evidence. IEEE Trans. Inform. Forensics Security *14* (2018), 7, 1871–1886. DOI:10.1109/TIFS.2018.2886472

[23] Y. Sasaki: Preservation of misperceptions–stability analysis of hypergames. In: Proc. 52nd Annual Meeting of the ISSS-2008, Madison 2008.

[24] Y. Sasaki: Generalized Nash equilibrium with stable belief hierarchies in static games with unawareness. Ann. Oper. Res. *256* (2017), 271–284. DOI:10.1007/s10479-016-2266-5

[25] G. Scutari, D. P. Palomar, F. Facchinei, and J.-S. Pang: Convex optimization, game theory, and variational inequality theory. IEEE Signal Process. Magazine *27* (2010), 3, 35–49. DOI:10.1109/MSP.2010.936021

[26] M. Wang, K. W. Hipel, and N. M. Fraser: Modeling misperceptions in games. Behavioral Sci. *33* (1988), 3, 207–223. DOI:10.1002/bs.3830330305

[27] J. Wang, J. F. Zhang, and X. He: Differentially private distributed algorithms for stochastic aggregative games. Automatica *142* (2022), 110440. DOI:10.1016/j.automatica.2022.110440

[28] G. Xu, G. Chen, H. Qi, and Y. Hong: Efficient algorithm for approximating Nash equilibrium of distributed aggregative games. IEEE Trans. Cybernet. *53* (2023), 7, 4375–4387. DOI:10.1109/TCYB.2022.3175831

[29] T. Yilmaz and Ö. Ulusoy: Misinformation propagation in online social networks: game theoretic and reinforcement learning approaches. IEEE Trans. Comput. Social Systems (2022). DOI:10.1109/tcss.2022.3208793

[30] S. Yu, Q. Sun, and Z. Yang: Recent advances on false information governance. Control Theory Technol. *21* (2023), 1, 110–113. DOI:10.1007/s11768-023-00126-1

[31] H. Zhang, H. Qin, and G. Chen: Bayesian Nash equilibrium seeking for multi-agent incomplete-information aggregative games. Kybernetika (2023), 575–591, 09 2023. DOI:10.14736/kyb-2023-4-0575

[32] T. Y. Zhang and D. Ye: False data injection attacks with complete stealthiness in cyber–physical systems: A self-generated approach. Automatica *120* (2020), 109117. DOI:10.1016/j.automatica.2020.109117

Meng Yuan, Department of Control Science and Engineering, Tongji University, Shanghai 201804. P. R. China.
e-mail: 2011256@tongji.edu.cn

Zhaoyang Cheng, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, SE-10044 Stockholm. Sweden.
e-mail: chengzhaoyang61@gmail.com

Te Ma, PLA Academy of Military Science and Intelligent Gaming and Decision-making Laboratory, Beijing. P. R. China.
e-mail: matt_rocket@163.com