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NON-FRAGILE OBSERVERS DESIGN FOR NONLINEAR SYSTEMS WITH UNKNOWN LIPSCHITZ CONSTANT

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In this paper, the problem of globally asymptotically stable non-fragile observer design is investigated for nonlinear systems with unknown Lipschitz constant. Firstly, a definition of globally asymptotically stable non-fragile observer is given for nonlinear systems. Then, an observer function of output is derived by an output filter, and a dynamic high-gain is constructed to deal with unknown Lipschitz constant. Even the observer gains contain diverse large disturbances, the observer errors are proven to converge to the origin based on Lyapunov stability theorem and a matrix inequality. Finally, an experimental simulation is provided to confirm the validity of the proposed method.

Keywords: non-fragile, observer, high gain, unknown Lipschitz constant, output filter

Classification: 93C10

1. INTRODUCTION

Since the concept of nonlinear system observer was first proposed in [27], numerous outcomes have been achieved [7, 10, 17]. In the area of observer design, one of the most difficult problems is how to deal with the nonlinear terms. Researchers often assume that the nonlinear terms satisfy the Lipschitz condition. But only few papers have discussed the observer design problem of nonlinear systems with unknown Lipschitz constant [12, 15, 23]. In addition, the unknown Lipschitz constant considered in the literature [23] required to meet some constraints. These observer design methods are all based on LMI (linear matrix inequality) technology.

The application of observer in secure communication has also been studied in [25, 18]. Its principle is to use the state observer to design a receiving system synchronized with the chaotic system, and modulate a digital signal to a certain parameter of the transmission system. At the receiving terminal, the signal is demodulated by using the synchronization error. Moreover, secure communication was also achieved by designing electronic circuits in [22].

It is reported that there often exist observer gain drifts in some industrial applications because of round-off errors in calculation or sensor devices aging [29]. Since a design method of non-fragile observers was firstly proposed in [14], more and more scholars begin to explore the observer design problem in the presence of observer gain disturbances.

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For example, the authors in [29] introduced the following uncertainty linear system

$$\dot{x}(t) = (A + D_1\Delta(t)E)x(t) + \omega_1(t),$$
$$y(t) = (C + D_2\Delta(t)E)x(t) + \omega_2(t),$$

where $\Delta(t)$ is a time-varying matrix of uncertain parameters and $\omega_1(t)$, $\omega_2(t)$ are two zero mean white Gaussian noises. An observer was constructed as

$$\dot{\xi}(t) = A\xi(t) + (G + \Delta G)y(t),$$

where G and ΔG denote the observer gain and the gain drift, respectively. Despite the gain drift ΔG is uncertain due to round-off error and sensor devices aging reasons, the estimation value $\xi(t)$ is still available. Based on an LMI optimization method, a non-fragile observer for nonlinear systems was proposed in [11]. By introducing adaptive technology, the authors in [13] designed an adaptive non-fragile observer. For switching systems, the H_∞ non-fragile observers were studied in [30]. For discrete switching systems, the non-fragile observers were also researched in [28]. However, all the above results are obtained based on the LMI technology.

Although the LMI conditions can be easily tested by computer. However, if the solvable conditions of LMI are not satisfied, the design methods will be failure. Therefore, for nonlinear systems with unknown Lipschitz constants, it is very important to find other methods to design non-fragile observers.

In [1, 2, 3, 8, 9, 20], the high-gain observer design method was proposed for nonlinear systems. The introduced high-gain enables that the observer errors are exponentially convergent. Moreover, the high-gain observers are always achievable. However, there is a blank area to design high-gain observers for nonlinear systems with unknown Lipschitz constant. Although the dynamic high-gain technique is investigated to deal with unknown Lipschitz constant in controller design [19, 21], it is worth of a further study on how to handle the unknown Lipschitz constant in observer design.

In this paper, the problem of non-fragile high-gain observer design is investigated for nonlinear systems with unknown Lipschitz constant. Based on a matrix inequality and a monotone non-decreasing dynamic gain, it is proven that the observer errors are globally asymptotically stable even if there are distinct large disturbances in the observer gains. The paper is structured as follows: Section 2 presents the problem formulation and some important lemmas. The design process of the non-fragile observers is presented for a class of lower triangular nonlinear systems in Section 3. In Section 4, a simulation example demonstrates the effectiveness of the designed method. Section 5 provides a conclusion to the full text.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem description

Consider the following nonlinear system

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + B_0u(t) + f_0(x), \\ y(t) &= C_0x(t), \end{aligned} \tag{1}$$

where $A_0 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$, $B_0 = \begin{pmatrix} 0 & 0 & \cdots & 1 \end{pmatrix}^T$, $C_0 = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}$.

$x(t)$ and $y(t)$ are the state variable and output variable, respectively. The nonlinear function vector $f_0(x) = (f_1(x_1^\iota),\ f_2(x_2^\iota), \dots,\ f_n(x_n^\iota))^T \in \mathbb{R}^n$, where $f_i(x_i^\iota) \in \mathbb{R}$ is acontinuous nonlinear function, and $x_i^\iota = (x_1, \dots, x_i)^T$.

The following assumption is imposed on the nonlinear system (1).

Assumption 2.1. The nonlinear function vector $f_i(x_i^\iota)$ satisfies the following condition

$$|f_i(x_i^\iota) - f_i(\hat{x}_i^\iota)| \le \varrho(|x_1(t) - \hat{x}_1(t)| + \cdots + |x_i(t) - \hat{x}_i(t)|), i = 1, \dots, n,$$

where $\varrho > 0$ is an unknown constant.

Remark 2.2. In order to explain the rationality of Assumption 2.1, introduce the following Duffing oscillator [5],

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -a_0x_1(t) - c_0x_2(t) + x_1^3(t) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{2}$$

where $u(t) = \sin t + \sqrt{2}$. According to [5], if we select $a_0 = c_0 = 5$, then the system (2) exists periodic solutions. However, the authors don't provide a specific expression of the periodic solutions or a boundary of the periodic solutions. That is, the nonlinear term $x_1^3(t)$ satisfies the Lipschitz condition with an unknown Lipschitz constant.

We need the following definition.

Definition 2.3. (Jeong et al. [14]) For nonlinear system (1), construct an observer as,

$$\begin{aligned} \dot{\hat{x}}(t) &= A_0\hat{x}(t) + B_0u(t) + \Omega_0 \cdot \Delta\Omega_0\varphi(y, \hat{y}) + f(\hat{x}), \\ \hat{y}(t) &= C_0\hat{x}(t), \end{aligned} \tag{3}$$

where $\varphi(y, \hat{y})$ is an observer function, $\Omega_0 = \text{diag}\{g_1, \dots, g_n\}$ is the observer gain, $\Delta\Omega_0 = (\Delta g_1, \dots, \Delta g_n)^T$ is unknown multiplicative disturbance arising by electronic components aging or round-off errors in calculation [29].

If there exists two constants $g_{\max}$ and $g_{\min}$ that satisfy $g_{\min} < \Delta g_i < g_{\max}, i = 1, \dots n$, $\forall x(t_0) \in \mathbb{R}^n$ and $\hat{x}(t_0) \in \mathbb{R}^n$, we have

$$\lim_{t\to\infty}(x_i(t) - \hat{x}_i(t)) = 0,\ i = 1, \dots n. \tag{4}$$

Then the system (3) is a globally asymptotically stable non-fragile observer of nonlinear system (1).

Remark 2.4. There are various design methods for the observer function $\varphi(y, \hat{y})$ in (3). For example, we can directly select $\varphi(y, \hat{y}) = y - \hat{y}$ to build nonlinear robust observer [4]. In [6], the observer function is selected as $\varphi_i(y, \hat{y}) = L^i(y - \hat{y})$ (L is the high-gain parameter) to establish the high-gain observer. A dynamic high-gain observer is designed by selecting $\varphi_i(y,\hat{y}) = L^i(t)(y-\hat{y})$ ($L(t)$ is the dynamic high-gain function) in [19]. In order to design a finite-time observer, one can choose $\varphi_i(y,\hat{y}) = |y-\hat{y}|^{\alpha_i} sign(y-\hat{y})$ ($\alpha_i \in (0,1)$) [26]. In this article, we investigate that how to select the appropriate observer function $\varphi(y,\hat{y})$ to design globally asymptotically stable non-fragile high-gain observer for the nonlinear system (1).

Remark 2.5. In order for demonstrating the observer gain sensitivity, introduce the following nonlinear system as an example,

$$\begin{aligned} \dot{x} &= A^*x + B^*u + F^*(x), \\ y &= C^*x, \end{aligned}$$

where $A^* = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$, $B^* = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $F^*(x) = \begin{pmatrix} 0 \\ 0 \\ 2x_4 \\ 9x_5 + \cos(x_4) \\ x_5 \sin(x_5) \end{pmatrix}$

and $C^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix}$.

Design an observer for the nonlinear system,

$$\begin{aligned} \dot{\hat{x}} &= A^*\hat{x} + B^*u + F^*(\hat{x}) + \kappa C^*(x-\hat{x}) \\ \hat{y} &= C^*\hat{x}, \end{aligned}$$

where the observer gain $\kappa = (53.6\ 241.6\ 320.6\ 13.1\ 34.6)^T$.

Next, by letting $e = x - \hat{x}$, the error system becomes,

$$\dot{e} = A_\kappa e + F^*(e),$$

where $A_\kappa = A^* - \kappa C^*$ and $F^*(e) = F^*(x) - F^*(\hat{x})$. Assumethe initial state $(x_1,\ x_2,\ x_3,\ x_4,\ x_5) = (3,\ 1,\ 2,\ 3,\ 2)$ and $(\hat{x}_1,\ \hat{x}_2,\ \hat{x}_3,\ \hat{x}_4,\ \hat{x}_5) = (-4,\ -1,\ 2,\ 1,\ 1)$, and the simulation results are shown in Fig. 1. Obviously, the simulation result shows the error system is stable.

However, if there exists a small disturbance $\Delta = (0.1\ 0.1\ 0.1\ 0.1\ 0.1)^T$ in the observer gain, then the new error system $\varpi = x - \hat{x}$ becomes,

$$\dot{\varpi} = A_{\Delta\kappa}\varpi + F^*(\varpi),$$

where $A_{\Delta\kappa} = A^* - (\kappa + \Delta)C^*$ and $F^*(\varpi) = F^*(x) - F^*(\hat{x})$. The simulation is presented in Fig. 2. It reveals the error system is unstable. Moreover, we have $\frac{\|\Delta\|}{\|\kappa\|} = 0.00075301$, which indicates the error system is very fragile with respect to the observer gain disturbance.

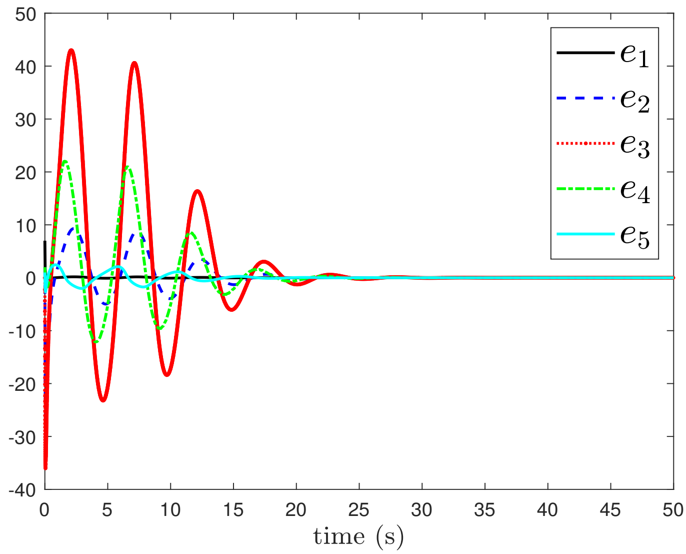

Fig. 1. The trajectories of the estimation error e.

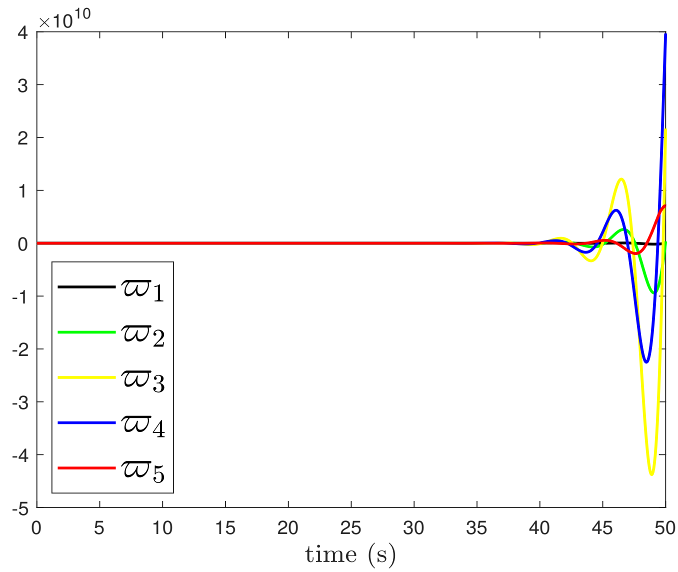

Fig. 2. The trajectories of the estimation error ϖ.

Remark 2.6. Due to the augmented system (7) does not preserve the strict triangular form, it is necessary to figure out whether the augmented system (7) is observable. The observable matrix can be calculated as

$$\begin{pmatrix} C_1 \\ C_1 A_1 \\ \vdots \\ C_1 A_1^{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ L(t)\kappa_0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ L^n(t)\kappa_0^n & L^{n-1}(t)\kappa_0^{n-1} & \cdots & 1 \end{pmatrix}.$$

Obviously, the rank of the observable matrix is $n+1$. Therefore, the augmented system (7) is observable.

Our aim is to design a globally asymptotically stable non-fragile high-gain observer for the nonlinear system (7). That is, the designed observer gains $\kappa_1, \ldots, \kappa_n$ are negative and have unknown observer gain disturbances $\theta_i(t),\ i = 1, \ldots, n$, which are continuous and satisfy the following conditions

$$\begin{array}{l} 0 < \theta_i^{\min} \le 1, \\ 1 \le \theta_i^{\max} < +\infty, \\ \theta_i^{\min} \le \theta_i(t) \le \theta_i^{\max}, \end{array} \tag{5}$$

where $\theta_i^{\min}$ and $\theta_i^{\max}$ are some positive constants.

Remark 2.7. Normally, the observer gain disturbances $\Delta g_i,\ i = 1, \ldots n$ have additive form and multiplicative form. The additive form is able to transform to the multiplicative form by the transformation $g_i + \Delta g_i = g_i(1 + \frac{1}{g_i}\Delta g_i)$. The multiplicative form is also able to transform to the additive form by the transformation $g_i \Delta g_i = g_i + g_i(\Delta g_i - 1)$. We are going to consider the multiplicative form in this paper.

2.2. Important lemmas

Lemma 2.8. Barbalat's lemma [24]: For $t \ge t_0$ ($t_0 \in \mathbb{R}^+$), if $\Phi(t)$ is uniformly continuous and $\int_{t_0}^{t} \Phi(t)\,\mathrm{d}t$ is bounded when $t \to \infty$, then

$$\lim_{t \to +\infty} \Phi(t) = 0.$$

For the convenience of presentation, the definitions of some parameters are provided for later use.

1) Choose the positive constants $b_j,\ j = 2, \ldots, n+1$, such that,

$$\begin{array}{l} (n^2 \Pi_{k=2}^{j} b_k^2 \max\{(\eta_i^{\max} - \eta_{i-1}^{\min})^2, (\eta_{i-1}^{\max} - \eta_i^{\min})^2\})(\alpha_1(\cdot) + \alpha_2(\cdot) + \alpha_3(\cdot)) < 1, \\ j = 2, \ldots, n+1, \end{array}$$

where

$$\begin{array}{l} 0 < \eta_i^{\min} \le 1, \\ 1 \le \eta_i^{\max} < +\infty, \\ \eta_1^{\max} = \eta_1^{\min} = 1, \end{array}$$

and

$$\alpha_1(\cdot) = (\tfrac{\beta_2(\cdot)}{b_2} + 2\beta_3(\cdot)b_2)^2 \max\{(\eta_n^{\max} - 1)^2, (1 - \eta_n^{\min})^2\},$$
$$\alpha_2(\cdot) = 2\textstyle\sum_{i=3}^{n+1}(\Pi_{k=3}^{i} b_k^2(\tfrac{\beta_2(\cdot)}{b_2} + 2\beta_i(\cdot)b_2)^2 \max\{(\eta_i^{\max} - \eta_{i-1}^{\min})^2, (\eta_{i-1}^{\max} - \eta_i^{\min})^2\}),$$
$$\alpha_3(\cdot) = 8\textstyle\sum_{i=3}^{n+1}(\Pi_{k=2}^{i} b_k^2(\beta_i(\cdot) - \beta_{i+1}(\cdot))^2 \max\{(\eta_i^{\max} - 1)^2, (1 - \eta_i^{\min})^2\}),$$

where $\beta_i(b_{i+1}, \ldots, b_{n+1}) = \frac{b_i a_{i-1}}{2k_0 a_i},\ i = 1, \ldots, n+1$ satisfies $\beta_{n+1}(\cdot) = 1$ and $\beta_{n+2}(\cdot) = 0$.

2) The positive constants $a_j,\ j = 1, \ldots, n+1$, can be calculated by,

$$a_n = \tfrac{2a_{n+1}}{b_{n+1}} k_0,$$
$$a_{i-1} = \tfrac{2a_i}{b_i}(k_0 + \tfrac{ia_i}{2a_{i+1}} b_{i+1} + \tfrac{ia_i}{2a_{i+1}b_{i+1}} + \tfrac{1}{2}\textstyle\sum_{j=i}^{n}((\tfrac{b_{j+2}a_{j+1}}{a_{j+2}} + \tfrac{jb_{j+1}a_j}{a_{j+1}})\Pi_{k=i+1}^{j+1} b_k^2)),$$
$$i = 2, \ldots, n,$$

where $a_{n+2} = 1$, $b_{n+2} = 0$, k_0 and a_{n+1} are arbitrary positive constants.

3) The gains $k_i,\ i = 1, \ldots, n+1$, can be calculated by,

$$k_1 = -b_2\tfrac{a_1}{a_2} - \tfrac{a_1}{2b_2a_2} - \alpha_0 k_0,$$
$$k_i = \tfrac{a_1}{a_i}(\tfrac{a_{i-1}}{a_1} b_i k_{i-1} + \tfrac{a_{i-1}}{a_i} b_i \Pi_{k=2}^{i} b_k - \tfrac{a_i}{a_{i+1}}\Pi_{k=2}^{i} b_k),\ i = 2, \ldots, n+1.$$

4) Let

$$A_2 = \begin{pmatrix} k_1 & 1 & 0 & \cdots & 0 \\ k_2\eta_2(t) & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ k_n\eta_n(t) & 0 & 0 & \cdots & 1 \\ k_{n+1}\eta_{n+1}(t) & 0 & 0 & \cdots & 0 \end{pmatrix},$$

and

$$P_0 = \begin{pmatrix} a_1 & 0 & 0 & \cdots & 0 & 0 \\ -b_2a_1 & a_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_n & 0 \\ 0 & 0 & 0 & \cdots & -b_{n+1}a_n & a_{n+1} \end{pmatrix},$$

where $\eta_i^{\min} \le \eta_i(t) \le \eta_i^{\max}$.

The positive definite matrix $\Gamma(\eta(t))$ is produced by

$$\begin{aligned}
\Gamma_{1,1}(\eta(t)) &= \alpha_0 k_0, \\
\Gamma_{1,i}(\eta(t)) &= \Gamma_{i,1}(\eta(t)) = (1 - \eta_i(t))(\frac{a_{i-1}}{a_i} b_i \Pi_{k=2}^{i} b_k - \frac{a_i}{a_{i+1}}\Pi_{k=2}^{i+1} b_k) \\
&\qquad + (\eta_i(t) - \eta_{i-1}(t))(\frac{a_1}{2b_2a_2}\Pi_{k=2}^{i} b_k + \frac{a_{i-1}}{a_i} b_i \Pi_{k=2}^{i} b_k), \\
\Gamma_{i,i}(\eta(t)) &= k_0, \\
\Gamma_{i,j}(\eta(t)) &= 0,\ i \ne j,\ i = 2, \ldots, n+1,\ j = 2, \ldots, n+1,
\end{aligned}$$

where $\Gamma_{i,j}(\eta(t))$ means the ith line and the jth column element of the matrix $\Gamma(\eta(t))$.

From [16], for the matrices A_2, P_0 and $\Gamma(\eta(t))$ defined above, the following lemma can be indicated.

Lemma 2.9. (Koo and Choi [16]) There exists a positive constant λ_0 satisfying

$$A_2^T P + PA_2 = -P_0^T \Gamma(\eta(t)) P_0 \le -\lambda_0 I,$$

where $P = P_0^T P_0$.

For the positive definite matrix P, the following result can be obtained.

Lemma 2.10. For the matrix $Q = \begin{pmatrix} \sigma & 0 & 0 & \cdots & 0 \\ 0 & 1+\sigma & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & n+\sigma \end{pmatrix}$, where σ denotes apositive constant. There exists a positive constant $\bar{\sigma}$ such that when $\bar{\sigma} < \sigma$, the following matrix inequality holds,

$$PQ + QP > 0.$$

Proof. For the system $\dot{\xi} = Q\xi$, introduce the transformation $\psi = P_0\xi$. Then,

$$\begin{aligned} &\dot{\psi}_1 = \sigma\psi, \\ &\dot{\psi}_i = -b_i a_{i-1}(i-2+\sigma)\left(\tfrac{\psi_{i-1}}{a_{i-1}} + \tfrac{1}{a_{i-1}} \textstyle\sum_{j=1}^{i-2} \psi_j \Pi_{k=j+1}^{i-1} b_k\right) \\ &\quad + a_i(i-1+\sigma)\left(\tfrac{\psi_i}{a_i} + \tfrac{1}{a_i} \textstyle\sum_{j=1}^{i-1} \psi_j \Pi_{k=j+1}^{i} b_k\right), \\ &= (i-1+\sigma)\psi_i - (i-2+\sigma) \textstyle\sum_{j=1}^{i-1} \psi_j \Pi_{k=j+1}^{i} b_k + (i-1+\sigma) \textstyle\sum_{j=1}^{i-1} \psi_j \Pi_{k=j+1}^{i} b_k, \\ &= (i-1+\sigma)\psi_i + \textstyle\sum_{j=1}^{i-1} \psi_j \Pi_{k=j+1}^{i} b_k, \ i = 2, \ldots, n+1. \end{aligned}$$

Thus, there exists a positive real $\bar{\sigma}$ such that when $\sigma > \bar{\sigma}$, the following inequality holds.

$$\begin{aligned} &\textstyle\sum_{i=1}^{n+1} \psi_i \dot{\psi}_i = \sum_{i=1}^{n+1} (i-1+\sigma)\psi_i^2 + \sum_{i=1}^{n+1} \psi_i \sum_{j=1}^{i-1} \psi_j \Pi_{k=j+1}^{i} b_k \\ &\ge \textstyle\sum_{i=1}^{n+1} (\sigma - \bar{\sigma})\psi_i^2 > 0. \end{aligned}$$

Therefore,

$$\sum_{i=1}^{n+1} \psi_i \dot{\psi}_i = \frac{1}{2}\frac{d(\psi^T\psi)}{dt} = \frac{1}{2}\xi^T(QP + PQ)\xi > 0.$$

The proof is completed. □

3. THE NON-FRAGILE OBSERVER DESIGN

Consider the following transformation,

$$\dot{\bar{x}}_0(t) = L(t)\kappa_0\bar{x}_0(t) + y(t), \tag{6}$$

where $L(t)$ is a time-varying function to be designed and κ_0 is a negative constant. Thus, the nonlinear system (1) can be augmented as,

$$\begin{aligned} \dot{\bar{x}}(t) &= A_1\bar{x}(t) + B_1u(t) + f(\bar{x}), \\ \bar{y}(t) &= C_1\bar{x}(t), \end{aligned} \tag{7}$$

where $\bar{x}(t) = \begin{pmatrix} \bar{x}_0, & x_1, & \ldots, & x_n \end{pmatrix}^T$, $A_1 = \begin{pmatrix} L(t)\kappa_0 & C_0 \\ 0_{n\times 1} & A_0 \end{pmatrix}$; $B_1 = \begin{pmatrix} 0_{1\times 1} \\ B_0 \end{pmatrix}$, $C_1 = \begin{pmatrix} C_0 & 0_{1\times 1} \end{pmatrix}$ and $f(\bar{x}) = \begin{pmatrix} 0_{1\times 1} \\ f_0(x) \end{pmatrix}$.

Then, the problem of non-fragile observer design for the nonlinear system (1) is transformed into observer design for the augmented nonlinear system (7) with multiplicative gain disturbances. The specific form is as follows.

$$\begin{aligned} \dot{\hat{\bar{x}}}(t) &= A_1\hat{\bar{x}}(t) + B_1u(t) + \Omega(\bar{y}(t) - \hat{\bar{x}}_0(t)) + f(\hat{\bar{x}}), \\ \hat{\bar{y}}(t) &= C_1\hat{\bar{x}}(t), \end{aligned} \tag{8}$$

where $\hat{\bar{x}}(t)$ and $\hat{\bar{y}}(t)$ are the estimation value of $\bar{x}(t)$ and $\bar{y}(t)$, respectively. $\Omega = \begin{pmatrix} 0 \\ -L^2(t)\kappa_1\theta_1(t) \\ \vdots \\ -L^{n+1}(t)\kappa_n\theta_n(t) \end{pmatrix}$, $L(t)$ is the dynamic high-gain. $\kappa_i = k_{n+1-i}$, $(i = 0, \ldots, n+1)$ are the observer gains and $\theta_i(t) = \eta_{n+1-i}(t)$, $(i = 1, \ldots, n)$ are the observer gain disturbances. Note that $\theta_i(t)$, $i = 1, \ldots, n$, are continuous functions and satisfy

$$\begin{aligned} &0 < \theta_i^{\min} \le 1, \\ &1 \le \theta_i^{\max} < +\infty, \\ &\theta_i^{\min} \le \theta_i(t) \le \theta_i^{\max}, \end{aligned} \tag{9}$$

where $\theta_i^{\min}$ and $\theta_i^{\max}$ are positive constants. The dynamic high-gain $L(t)$ is selected as,

$$\dot{L}(t) = \left(\frac{\bar{y}(t) - \hat{\bar{x}}_0(t)}{L^\sigma(t)}\right)^2, \ L(t_0) = 1. \tag{10}$$

The dynamic high-gain $L(t)$ has the following property.

Proposition 3.1. If Assumption 2.1 holds, then the dynamic high-gain $L(t)$ defined in (10) is bounded for all $t \in [0, +\infty)$.

Proof.

Consider the following coordinates transformation,

$$\zeta_i(t) = \frac{\bar{x}_i(t) - \hat{\bar{x}}_i(t)}{L^{i+\sigma}(t)}, \quad i = 0, \ldots, n. \tag{11}$$

Therefore, from (7), (8), (11), we can deduce

$$\begin{aligned}
&\dot{\zeta}_0(t) = L(t)\kappa_0\zeta_0(t) + L(t)\zeta_1(t) - \sigma\tfrac{\dot{L}(t)}{L(t)}\zeta_0(t),\\
&\dot{\zeta}_i(t) = L(t)\theta_i(t)\kappa_i\zeta_0(t) + L(t)\zeta_{i+1}(t) + \tfrac{1}{L^{i+\sigma}(t)}(f_i(x_i^\iota) - f_i(\hat{x}_i^\iota)) - (\sigma+i)\tfrac{\dot{L}(t)}{L(t)}\zeta_i(t),\\
&\qquad\qquad i = 1,\dots,n-1,\\
&\dot{\zeta}_n(t) = L(t)\theta_n(t)\kappa_n\zeta_0(t) + \tfrac{1}{L^{n+\sigma}(t)}(f_n(x_n^\iota) - f_n(\hat{x}_n^\iota)) - (\sigma+n)\tfrac{\dot{L}(t)}{L(t)}\zeta_n(t),
\end{aligned} \tag{12}$$

By representing (12) in compact form, it becomes

$$\dot{\zeta}(t) = L(t)A\zeta(t) + \tilde{f}(\tilde{\bar{x}}) - \tfrac{\dot{L}(t)}{L(t)}Q\zeta(t), \tag{13}$$

where $A = \begin{pmatrix} \kappa_0 & 1 & 0 & \cdots & 0\\ \kappa_1\theta_1(t) & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \kappa_n\theta_n(t) & 0 & 0 & \cdots & 0 \end{pmatrix}$, $\tilde{f}(t,\tilde{\bar{x}}) = \begin{pmatrix} 0\\ \frac{1}{L^{1+\sigma}(t)}(f_1(x_1^\iota) - f_1(\hat{x}_1^\iota))\\ \vdots\\ \frac{1}{L^{n+\sigma}(t)}(f_n(x_n^\iota) - f_n(\hat{x}_n^\iota)) \end{pmatrix}$ and $Q = \begin{pmatrix} \sigma & 0 & 0 & \cdots & 0\\ 0 & 1+\sigma & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & n+\sigma \end{pmatrix}$.

Construct the Lyapunov function $V_1(t) = \zeta^T(t)P\zeta(t)$, and calculate it's derivative along the error system (13). Then, from Lemma 2.9 and Lemma 2.10, it becomes

$$\begin{aligned}
\dot{V}_1(t) &= L(t)\zeta^T(t)(A^TP + PA)\zeta(t) + 2\zeta^T(t)P\tilde{f}(\tilde{\bar{x}}) - \tfrac{\dot{L}(t)}{L(t)}\zeta^T(t)(PQ+QP)\zeta(t)\\
&\le -L(t)\lambda_0\|\zeta(t)\|^2 + 2\zeta^T(t)P\tilde{f}(\tilde{\bar{x}}).
\end{aligned} \tag{14}$$

From Assumption 2.1, it follows that

$$2\zeta^T(t)P\tilde{f}(\tilde{\bar{x}}) \le \|\zeta(t)\|^2 + n^2\varrho^2\|P\|^2\|\zeta(t)\|^2. \tag{15}$$

Substituting (15) into (14) yields

$$\dot{V}_1(t) \le -(L(t)\lambda_0 - 1 - n^2\varrho^2\|P\|^2)\|\zeta(t)\|^2. \tag{16}$$

Now, we prove the boundedness of $L(t)$ on $[0,t_f)$ by contradiction. Assume that $L(t)$ is not bounded on the interval $[0,t_f)$. Then,

$$\lim_{t\to t_f}\sup L(t) = +\infty. \tag{17}$$

Note that $L(t)$ is a monotone nondecreasing function. Then from (17), there exists $t_1 > 0$ such that $L(t)\lambda_0 - 1 - n^2\varrho^2\|P\|^2 > 1,\ \forall t \in [t_1,t_f)$.

Thus, from the differential inequality (16), we can infer that

$$\dot{V}_1(t) \le -\|\zeta(t)\|^2,\ \forall t \in [t_1,t_f).$$

From (10), it follows that

$$\dot{L}(t) = (\frac{\bar{y}(t) - \hat{\bar{x}}_0(t)}{L^{\sigma}(t)})^2 \leq \|\zeta_0(t)\|^2 \leq \|\zeta(t)\|^2. \tag{18}$$

Therefore, from (17) and (18), the following conclusion can be drawn

$$+\infty = L(t_f) - L(t_1) = \int_{t_1}^{t_f} \dot{L}(t)\,\mathrm{d}t \leq \int_{t_1}^{t_f} \|\zeta(t)\|^2\,\mathrm{d}t \leq V_1(\|\zeta(t_1)\|),$$

which is impossible. It reveals the dynamic gain $L(t)$ is bounded on $[t_1, t_f)$ and $\lim_{t\to t_f} L(t)$ is finite. The proof is completed. □

Now, we give our main results.

Theorem 3.2. For the nonlinear system (1), if the observer gain disturbances $\theta_i(t)$ satisfy (9), then the system (6) – (8) is a globally asymptotically stable non-fragile observer of the nonlinear system (1), that is, $\forall x_i(t_0) \in \mathbb{R}$ and $\hat{x}_i(t_0) \in \mathbb{R}$,

$$\lim_{t\to\infty} (x_i(t) - \hat{x}_i(t)) = 0, \ i = 1, \ldots n.$$

P r o o f. Since $\lim_{t\to t_f} L(t)$ is finite, there exists a constant $\bar{L}$ such that

$$\bar{L} > \max\{\frac{n^2\varrho^2\|P\|^2 + 1 + 2\|P\|}{\lambda_0}, L(t)\}.$$

Introduce the coordinates transformation as follows,

$$z_i(t) = \frac{\bar{x}_i(t) - \hat{\bar{x}}_i(t)}{\bar{L}^i}, \quad i = 0, \ldots, n.$$

Thus, the error system becomes

$$\dot{z}(t) = \bar{L}Az(t) + \tilde{g}(\tilde{\bar{x}}) + \bar{L}\Omega_1(t)z_0(t) - \bar{L}\Omega_2(t)z_0(t),$$

where $\tilde{g}(\tilde{\bar{x}}) = \begin{pmatrix} 0 \\ \frac{1}{\bar{L}}(f_1(x_1^{\iota}) - f_1(\hat{x}_1^{\iota})) \\ \vdots \\ \frac{1}{\bar{L}^n}(f_n(x_n^{\iota}) - f_n(\hat{x}_n^{\iota})) \end{pmatrix}$, $\Omega_1(t) = \begin{pmatrix} \frac{L(t)}{\bar{L}}\kappa_0 \\ \frac{L^2(t)}{\bar{L}^2}\kappa_1\theta_1(t) \\ \vdots \\ \frac{L^{n+1}(t)}{\bar{L}^{n+1}}\kappa_n\theta_n(t) \end{pmatrix}$ and

$$\Omega_2(t) = \begin{pmatrix} \kappa_0 \\ \kappa_1\theta_1(t) \\ \vdots \\ \kappa_n\theta_n(t) \end{pmatrix}.$$

Design the Lyapunov function $V_2(t) = z^T(t)Pz(t)$. It is easy to find out

$$\begin{aligned}\dot{V}_2(t) &= \bar{L}z^T(t)(A^TP+PA)z(t)+2z^T(t)P\tilde{g}(\tilde{\bar{x}}) \\ &+2\bar{L}z^T(t)P\Omega_1(t)z_0(t)-2\bar{L}z^T(t)P\Omega_2(t)z_0(t).\end{aligned} \tag{19}$$

By Assumption 2.1 and Lemma 2.9, it follows that

$$\begin{aligned}2z^T(t)P\tilde{g}(\tilde{\bar{x}}) &\le \|z(t)\|^2+n^2\varrho^2\|P\|^2\|z(t)\|^2 \\ 2\bar{L}z^T(t)P\Omega_1(t)z_0(t) &\le \|P\|\|z(t)\|^2+\bar{L}^2\|\Omega_1(t)\|^2z_0^2(t) \\ -2\bar{L}z^T(t)P\Omega_2(t)z_0(t) &\le \|P\|\|z(t)\|^2+\bar{L}^2\|\Omega_2(t)\|^2z_0^2(t)\end{aligned} \tag{20}$$

Substituting (20) into (19) yields

$$\begin{aligned}\dot{V}_2(t) &\le -\bar{L}\lambda_0\|z(t)\|^2+\|z(t)\|^2+n^2\varrho^2\|P\|^2\|z(t)\|^2 \\ &+2\|P\|\|z(t)\|^2+\bar{L}^2\|\Omega_1(t)\|^2z_0^2(t)+\bar{L}^2\|\Omega_2(t)\|^2z_0^2(t) \\ &\le -c_0\|z(t)\|^2+2\bar{L}^{2+2\sigma}\bar{\Omega}^2\dot{L}(t),\end{aligned} \tag{21}$$

where $c_0=\bar{L}\lambda_0-1-n^2\varrho^2\|P\|^2-2\|P\|>0$ and $\bar{\Omega}\ge\|\Omega_2(t)\|\ge\|\Omega_1(t)\|$.

Let λ_P is the minimum eigenvalue of matrix P, then

$$\lambda_p\|z(t)\|^2-z^T(0)Pz(0)\le -c_0\int_{t_0}^t\|z(t)\|^2\,\mathrm{d}t+2\bar{L}^{3+2\sigma}\bar{\Omega}^2L(t).$$

Since $L(t)$ is bounded on $[t_0,t_f)$, we can imply

$$\|z(t)\|^2\le\frac{z^T(0)Pz(0)+2\bar{L}^{3+2\sigma}\bar{\Omega}^2L(t)}{\lambda_p}, \tag{22}$$

and

$$c_0\int_{t_0}^t\|z(t)\|^2\,\mathrm{d}t\le z^T(0)Pz(0)+2\bar{L}^{3+2\sigma}\bar{\Omega}^2L(t). \tag{23}$$

Obviously, from (22) and (23), $\|z(t)\|$ is bounded on $[0,t_f)$ and $\int_{t_0}^t\|z(t)\|\,\mathrm{d}t\le+\infty$. By Lemma 2.8, we can conclude that $\lim\limits_{t\to+\infty}\|z(t)\|=0$, which completes the proof. □

4. EXPERIMENTAL SIMULATIONS

In order to demonstrate the performance of the non-fragile observer, an experimental simulation is given in this section.

For the Duffing oscillator (2) mentioned in [5], by inserting an output filter, it becomes

$$\begin{cases}\dot{\bar{x}}_0(t)=\bar{x}_1(t)+L(t)\kappa_0\bar{x}_0(t),\\ \dot{\bar{x}}_1(t)=\bar{x}_2(t),\\ \dot{\bar{x}}_2(t)=-5\bar{x}_1(t)-5\bar{x}_2(t)+\bar{x}_1^3(t)+u(t),\\ \bar{y}(t)=\bar{x}_0(t).\end{cases} \tag{24}$$

A globally asymptotically stable non-fragile observer can be designed as,

$$\begin{cases}\dot{\hat{\bar{x}}}_0(t)=\hat{\bar{x}}_1(t)+L(t)\kappa_0\hat{\bar{x}}_0(t),\\ \dot{\hat{\bar{x}}}_1(t)=\hat{\bar{x}}_2(t)-L^2(t)\kappa_1\theta_1(t)(\bar{y}(t)-\hat{\bar{x}}_0(t)),\\ \dot{\hat{\bar{x}}}_2(t)=-5\hat{\bar{x}}_1(t)-5\hat{\bar{x}}_2(t)+\hat{\bar{x}}_1^3(t)+u(t)-L^3(t)\kappa_2\theta_2(t)(\bar{y}(t)-\hat{\bar{x}}_0(t)),\\ \dot{L}(t)=(\dfrac{\bar{y}(t)-\hat{\bar{x}}_0(t)}{L(t)})^2,\\ \hat{\bar{y}}(t)=\hat{\bar{x}}_0(t).\end{cases} \tag{25}$$

Choose the initial states as $\bar{x}(0) = (3,\ -15,\ 18)^T$, $\hat{\bar{x}}(0) = (0,\ 0,\ 0)^T$ and select $b_2 = 1$, $b_3 = 0.8$, $a_3 = k_0 = 1$. By Lemma 2, the observer gain vector can be obtained as $\kappa = (-25,\ -185,\ -319)$. Let $\theta_1(t) = 1.1 + 0.2\sin t$, $\theta_2(t) = 0.9 + 0.5\cos t$. The simulation results are shown in Fig. 3.

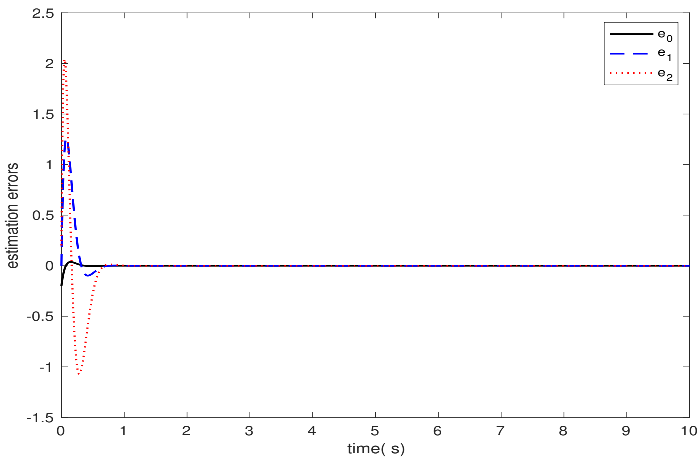

Fig. 3. The trajectories of the estimation errors.

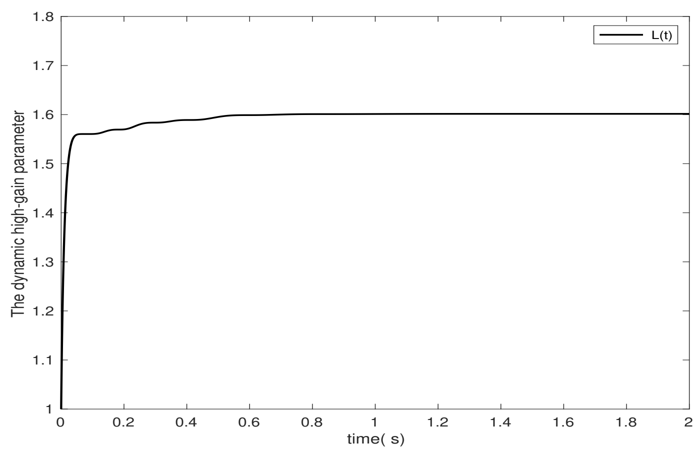

Fig. 4. The trajectory of the dynamic high-gain $L(t)$.

Fig. 4 shows the trajectory of the dynamic high-gain parameter $L(t)$. Obviously, the observer errors asymptotically converge to the origin and the dynamic high-gain is bounded.

In order to demonstrate the superiority of the non-fragile observer, we assume the Lipschitz constant is known. Then, plot a comparison figure with both the non-fragile observer and the following normal high-gain observer,

$$\begin{cases} \dot{\hat{\bar{x}}}_0(t) = \hat{\bar{x}}_1(t) + L\hat{\bar{x}}_0(t), \\ \dot{\hat{\bar{x}}}_1(t) = \hat{\bar{x}}_2(t) - L^2\theta_1(t)(\bar{y}(t) - \hat{\bar{x}}_0(t)), \\ \dot{\hat{\bar{x}}}_2(t) = -5\hat{\bar{x}}_1(t) - 5\hat{\bar{x}}_2(t) + \hat{\bar{x}}_1^3(t) + u(t) - L^3\theta_2(t)(\bar{y}(t) - \hat{\bar{x}}_0(t)), \\ \hat{\bar{y}}(t) = \hat{\bar{x}}_0(t), \end{cases} \tag{26}$$

where $L = 10$ and other parameters keep fixed. Fig. 5 illustrates that our observer design method has better performance.

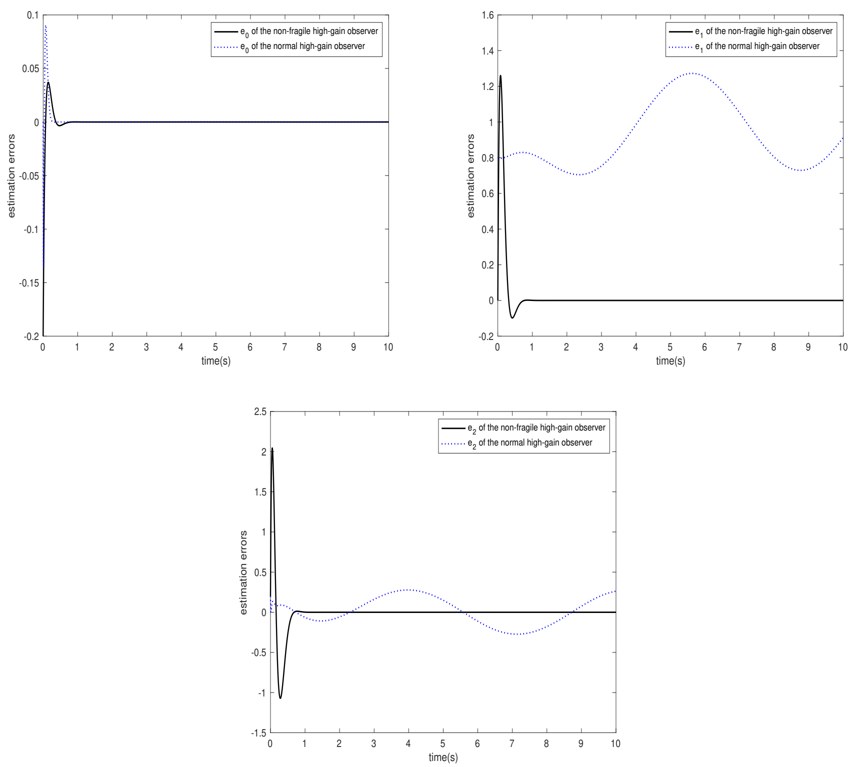

Fig. 5. The comparison between the non-fragile observer and the normal high-gain observer.

5. CONCLUSION

In this paper, we proposed a globally asymptotically stable non-fragile observer for nonlinear systems with unknown Lipschitz constant. The observer errors was proven to converge to the origin asymptotically. In the future, it is interesting to investigate globally asymptotically stable non-fragile observers for nonlinear systems with measurement noise.

DECLARATION OF COMPETING INTEREST

The authors declare that they do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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