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DELAY DEPENDENT COMPLEX-VALUED BIDIRECTIONAL ASSOCIATIVE MEMORY NEURAL NETWORKS WITH STOCHASTIC AND IMPULSIVE EFFECTS: AN EXPONENTIAL STABILITY APPROACH

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This paper investigates the stability in an exponential sense of complex-valued Bidirectional Associative Memory (BAM) neural networks with time delays under the stochastic and impulsive effects. By utilizing the contracting mapping theorem, the existence and uniqueness of the equilibrium point for the proposed complex-valued neural networks are verified. Moreover, based on the Lyapunov – Krasovskii functional construction, matrix inequality techniques and stability theory, some novel time-delayed sufficient conditions are attained in linear matrix inequalities (LMIs) form, which ensure the exponential stability of the trivial solution for the addressed neural networks. Finally, to illustrate the superiority and effects of our theoretical results, two numerical examples with their simulations are provided via MATLAB LMI control toolbox.

Keywords: Complex-valued neural networks, Linear matrix inequality, Lyapunov–Krasovskii functional, BAM neural networks, Exponential stability, Impulsive effects, Stochastic noise, discrete delays, distributed delays, leakage delays, mixed time delays

Classification: 34Dxx, 92B20, 93Exx

Notations: Throughout this paper, $\mathbb{Z}_+$ represents the set of all positive integers and $\mathbb{C}^n$ denotes the n-dimensional complex spaces equipped with the Euclidean norm $\|\cdot\|$; I represents the unitary matrix with appropriate dimensions; $\mathbb{C}^n$, $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times m}$ denotes respectively, the set of all n-dimensional complex-valued vectors, the set of all $n\times n$ real-valued matrices and the set of all $n\times m$ complex-valued matrices respectively. The notation $\mathcal{C}^T$ and $\mathcal{C}^{-1}$ mean the transpose of $\mathcal{C}$ and the inverse of a square matrix, respectively. $\lambda_{\max}(\mathcal{C})$ or $\lambda_{\min}(\mathcal{C})$ stand for the maximum eigenvalue or the minimum eigenvalue of Hermitian matrix $\mathcal{C}$, respectively. $\mathcal{A}^*$ indicates the complex conjugate transpose of complex-valued matrix $\mathcal{A}$. For any interval $\mathcal{T}\subseteq\mathbb{R}$, set $\mathcal{U}\subseteq\mathcal{C}^k$ $(1\le k\le n)$, $C(\mathcal{T},\mathcal{U})=\{\varphi:\mathcal{T}\longrightarrow\mathcal{U}$ is continuous$\}$ and $PC^1(\mathcal{T},\mathcal{U})=\{\varphi:\mathcal{T}\longrightarrow\mathcal{U}$ is continuously differentiable everywhere except at finite number of points t, at which $\varphi(t^+),\varphi(t^-)$, $\dot\varphi(t^+)$ and $\dot\varphi(t^-)$ exist and $\varphi(t^+)=\varphi(t)$, $\dot\varphi(t^+)=\dot\varphi(t)$, where $\dot\varphi$ refers to the deriva-

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tive of $\varphi\}$. For any $x = (x_1, x_2, \dots, x_n) \in \mathbb{C}^n$, f or any $t \in \mathbb{R}^+$, z_t and $\tilde{z}_t$ are defined by $z_t = z(t+s)$, $z_{t^-} = z(t^- + s)$, $s \in [-\sigma_1, 0]$ and $\tilde{z}_t = \tilde{z}(t+s)$, $\tilde{z}_{t^-} = \tilde{z}(t^- + s)$, $s \in [-\sigma_2, 0]$, respectively. The notation $\mathcal{M} > \mathcal{N}$ means that $\mathcal{M}$ and $\mathcal{N}$ are Hermitian matrices and that $\mathcal{M} - \mathcal{N}$ is positive definite. i shows the imaginary unit, i. e., $i = \sqrt{-1}$. $|a|$ indicates the module of complex number $a \in \mathbb{C}$ and $\|z\|$ denote the norm of $z \in \mathbb{C}^n$, i. e., $\|z\| = \sqrt{z^* z}$. If $\mathcal{A} \in \mathbb{C}^{n\times n}$, denote by $\|\mathcal{A}\|$ its operator norm, i. e., $\|\mathcal{A}\| = \sup\{\|\mathcal{A}x\| : \|x\| = 1\} = \sqrt{\lambda_{\max}(\mathcal{A}^*\mathcal{A})}$. For $\mathcal{A} = (a_{ij})_{n\times n} \in \mathbb{C}^{n\times n}$, $|\mathcal{A}| = |a_{ij}|$ stands for the modulus of matrix $\mathcal{A}$.

1. PREPARATORY KNOWLEDGE AND MODEL DESCRIPTION

Neural networks containing complex variables is a developing field of focus research that has garnered a lot of interest from scholars in recent years [12, 34, 36, 62]. As is well known, state variables, connection weight matrices, and neuron activation functions in complex-valued neural networks (CVNNs) are expressed in terms of complex variables. Therefore, one way to think of real-valued neural networks (RVNNs) is as the foundation for complex-valued neural networks. It appears that the attributes of CVNNs are more intricate and unique than those of RVNNs. Thus, it is now essential and required to take complex variables into account while designing neural networks. Complex-valued neural networks are widely used in dynamical systems for a variety of purposes, including speech synthesis, filtering, imaging, computer vision, quantum devices, optoelectronics, imaging, and electronic power grids. Furthermore, because of the amplifiers' limited switching rate, time delays will commonly occur during the transmission and storage of data in real neural networks. Time delay facts in neural networks (NNs) can therefore result in persistent oscillations, subpar performance, splitting, or instability, all of which are commonly seen in many technical, physical, and neural-based systems. As a result, a lot of study has been done on the stability issue of neural networks with temporal delays; see [7, 30, 32, 35, 40]. Cao and Wang looked at the stability requirements for time-delayed recurrent neural networks in [17]. Song, Zhao and Liu in [44] proposed the time delay dependent stability conditions for CVNNs as follows:

$$\mathrm{d}z(t) \;=\; [-Az(t) + W_0 f_1(z(t)) + W_1 g_1(z(t - \mu_1(t))) + \hat{I}]\,\mathrm{d}t,\ t \;>\; 0.$$

Economists and mathematicians have made the most contributions to M-matrix theory. In mathematics, M-matrices are used to set restrictions on eigenvalues and convergence requirements for iterative techniques that solve big sparse systems of linear equations. Because they are naturally occurring in some discretizations of differential operators, like the Laplacian, M-matrices are extensively researched in the field of scientific computing. The analysis of linear complementarity issue solutions also involves M-matrices. In computational mechanics, linear and quadratic programming, and the challenge of determining the equilibrium point of a bimatrix game, linear complementarity problems can occur. Finally, the study of finite Markov chains in operations research, such as queuing theory, and probability theory involves M-matrices. Economists have concurrently investigated M-matrices in relation to Leontief's input-output analysis, general equilibrium stability, and gross substitutability in economic systems. In economic literature, the term "Hawkins – Simon condition" refers to the need of positivity of all principal minors. M-matrices are related to Hurwitz matrices in engineering and are present in the Lyapunov stability and feedback control problems in control theory. M-matrices are used in the research of population dynamics in computational biology.

Dynamical analysis networks is an emerging branch of science that combines the study of network science and network theory with classical social network analysis, link analysis, social simulation, and multi-agent systems. A collection of graphs corresponds to a function of time (represented as a subset of the real numbers) in dynamic networks; a graph exists for every time point. This is similar to how dynamical systems are defined, where the function is from time to an ambient space, but time is transformed into interactions between pairs of vertices in place of ambient space.

Nonlinear differential equations are frequently used in neural network architectures. Thus, it is not possible for us to determine the answers immediately. To solve the NNs, a Lyapunov – Krasovskii functional (LKF) containing the parameters of the resultant neural networks is built. Moreover, the solution of the Lyapunov – Krasovskii functional requires the use of certain lemmas and inequalities relevant to stability theory. The terminology mentioned above allows us to determine whether or not the NNs are stable without the need for solutions.

Furthermore, because of the multitude of parallel paths with varying axon lengths and diameters, neural networks frequently have a spatial scope. As a result, propagation delays are allocated throughout time [26, 38, 39, 58]. It makes sense that the NNs would take into account both discrete and distributed time delays as a result of the time delay. Numerous research studies on neural networks with mixed time delays have been covered in the literature as of late, including [1, 4, 54] and its references. A review of papers on discrete time and distributed time delays for neural networks was conducted by the authors in [6, 48, 53]. Furthermore, the neural system's negative feedback state variables exhibit a unique kind of temporal delay known as leakage delay. Neural networks' stability behaviour is significantly affected by temporal delays of this kind in the forgetting (leakage) term, which are difficult to manage. Therefore it becomes sense to take leakage delays into account while designing neural networks [21, 29, 43]. Researchers have studied complex-valued neural networks with leakage terms and stability issues during the last few decades [2, 20]. The leakage term in the state variable was addressed by Li and Cao in [14], and the stability performance of neural networks with time delays was examined. Further, the researchers in [42] considered the following CVNNs with leakage delays and checked the stability analysis in Lagrange sense

$$\begin{aligned} \mathrm{d}z(t) \quad = \quad & \Big[- Az(t-\sigma_1) + W_0 f_1(z(t)) + W_1 g_1(z(t-\mu_1(t))) + W_2 \int_{t-\rho_1(t)}^{t} h_1(z(s))\,\mathrm{d}s \\ & + \hat{I} \Big]\,\mathrm{d}t,\ t > 0, \end{aligned}$$

where $\rho_1(t)$ denotes the time varying delay in distributed term which are bounded with $0 < \rho_1(t) < \rho_1$, $\dot{\rho}_1(t) \leq \rho_1 < 1$, where ρ_1 is a constant.

As is well known, Kosko first developed bidirectional associative memory neural networks (BAMNNs) in [18, 19] with the capacity for memory and information association. A particular kind of recurrent neural network, or RNN, known as a BAMNN is made up of neurons stacked in two layers: a M layer and a N layer. Each layer's neurons are completely linked to every other layer's neuron. Yet, because of its effective uses in artificial intelligence, image processing, parallel computing, pattern recognition, automatic control, and associative memories, it has garnered a lot of interest. Specifically, BAMNNs are designed to expose only one asymptotic or exponential stable trivial solution for a given external input. Hence, a great number of adequate criteria have been put forth to ensure the asymptotic or exponential stability of the suggested neural networks; for instance, refer to [11, 13, 31, 41]. Recently, a large number of researchers have explored in great detail the exponential stability of BAMNNs with time delays [10].

However, many physical processes in real-world situations are characterised by sudden changes at specific times. Additionally, it is likely that a wide range of evolutionary processes exhibit the hurried consequence (impulsive effect), which causes states to change quickly at specific times [27, 33, 45]. Moreover, BAMNNs can experience sudden fluctuations that impair the systems' dynamic performance. In order to properly analyze the stability of NNs, the impulsive effects must be considered. It is positive that there has been much discussion in recent years regarding the stability of neural networks with impulsive occurrences in the practical use of neural networks. The reader may consult [25, 63] and the references therein for more information on how to cope with the impulsive effects while examining the stability behavior of NNs. Chen et al. examined the CVNNs with impulses, leakage, and mixed time delays in [2] in the following ways:

$$\begin{aligned}
\mathrm{d}z(t) &= \Big[- Az(t-\sigma_1) + W_0 f_1(z(t)) + W_1 g_1(z(t-\mu_1)) + W_2 \int_{t-\rho_1}^{t} h_1(z(s))\,\mathrm{d}s \\
&\qquad + \hat{I}\Big]\,\mathrm{d}t;\ t \neq t_k, t > 0, \\
\Delta z(t_k) &= U_k(z(t_k^-), z_{t_k^-});\ t = t_k,\ k \text{ in } \mathbb{Z}_+.
\end{aligned}$$

The delay-dependent stability criteria of impulsive BAM neural networks were thoroughly examined by Li et al. in [23]. In actuality, external disturbances can cause a dynamical system to malfunction regularly. These disturbances might be regarded as random because they often involve a great deal of uncertainty. As mentioned in [9], stochastic (random) fluctuations resulting from the release of neurotransmitters and other probabilistic causes in the actual nervous system, as well as the use of artificial neural networks, produce the noisy process of synaptic transmission. Consequently, it makes sense to take stochastic noises into account while designing NNs with temporal delays. Many unique results on the stability criteria of neural networks with stochastic disturbances and temporal delays have recently been presented in [24, 46, 47, 61], using various methods. Ganesh Kumar, Syed Ali, et al. addressed the time delay criterion for BAM neural networks with stochastic effects in [5]. In the meanwhile, Wang and Huang's description of the discrete time delay CVBAMNNs in [50]. The addressed neural networks in [50] is given by

$$\begin{aligned}
\mathrm{d}z(t) &= [-Az(t) + W_0 f_1(\tilde{z}(t)) + W_1 g_1(\tilde{z}(t-\mu_1)) + \hat{I}]\,\mathrm{d}t, \\
d\tilde{z}(t) &= [-D\tilde{z}(t) + V_0 f_2(z(t)) + V_1 g_2(z(t-\mu_2)) + J]\,\mathrm{d}t,\ t\ >\ 0,
\end{aligned}$$

where μ_1 and μ_2 are constant scalars.

Inspired by the above discussions, our current research work is motivated by the fact that the problem of exponential stability analysis of complex-valued BAM neural networks (CVBAMNNs) with mixed time delays has not yet been fully studied due to the combination of stochastic and impulsive perturbations. In order to close this gap, we examine the exponential stability problem for CVBAMNNs with temporal delays and random, impulsive disturbances in this study. Some unique time delay dependent stability criteria are derived, which guarantee the exponential stability of the trivial solution. These criteria are based on the Lyapunov – Krasovskii functional (LKF), stochastic analysis theory, matrix theory, complex analysis theory, and LMIs technique. And the MATLAB LMI control toolbox makes it simple to verify. Additionally, two numerical examples are provided to help understand the benefits and relevance of the suggested research project. The main contributions of this paper are outlined as follows:

1. Impulsive effects, Mixed time-delays, leakage delay and stochastic noises are taken into account in the stability analysis of CVBAMNNs via exponential sense.
2. In this paper, more general assumptions are equipped for complex-valued activation functions. Hence our planned work can serve an extensive class of neural networks.
3. Depend on the contraction mapping theorem, we analyzed the existence and uniqueness of the trivial solution for the addressed complex-valued BAMNNs.
4. Based on novel Lyapunov – Krasovskii functional, some sufficient conditions for exponential stability of stochastic BAM-type complex-valued neural networks are evolving. Furthermore, compared to the existing results, the obtained conclusions are distinct and advanced.

The rest of the paper is structured as follows. In Section 1, a formulation of the investigated stochastic complex-valued BAM neural networks model and some preliminary information are presented. Within Section 2, the existence and uniqueness of the equilibrium point are examined through the application of the contraction mapping theorem, Lyapunov – Krasovskii functionals, and integral inequality technique. Additionally, a few time delay-dependent novel conditions are achieved for the exponential stability of addressing neural networks. The obtained findings are expressed in terms of LMIs, which are readily confirmed through the use of the MATLAB LMI control toolbox through two numerical examples. These examples demonstrate the efficacy of the derived exponential stability criterion in Section 3. In Section 4, conclusions are drawn.

By the above encouraged arguments, we consider the following complex-valued BAM neural networks (CVBAMNNs) with mixed time delays, leakage time delays and stochastic *and* impulsive effects as

$$\begin{aligned}
\mathrm{d}z(t) &= \Big[- Az(t-\sigma_1) + W_0 f_1(\tilde{z}(t)) + W_1 g_1(\tilde{z}(t-\mu_1(t))) + W_2 \int_{t-\rho_1}^{t} h_1(\tilde{z}(s))\,\mathrm{d}s \\
&\qquad + \hat{I} \Big]\,\mathrm{d}t + \delta_1(z(t-\sigma_1), \tilde{z}(t), \tilde{z}(t-\mu_1(t))), t)\,\mathrm{d}\omega_1(t);\ \ t \neq t_k, t > 0, \\
z(t) &= B_k h_k(z(t^-)) + C_k l_k(z(t^- - \mu_1(t^-))) + \bar{I}_k = U_k(z(t_k^-), z_{t_k^-});\ \ t = t_k,\ k \text{ in } \mathbb{Z}_+ \\
\mathrm{d}\tilde{z}(t) &= \Big[- D\tilde{z}(t-\sigma_2) + V_0 f_2(z(t)) + V_1 g_2(z(t-\mu_2(t))) + V_2 \int_{t-\rho_2}^{t} h_2(z(s))\,\mathrm{d}s
\end{aligned}$$

$$+J\Big] \mathrm{d}t + \delta_2(\tilde{z}(t-\sigma_2), z(t), z(t-\mu_2(t))), t) \,\mathrm{d}\omega_2(t);\ t \neq t_k, t > 0,$$

$$\tilde{z}(t) \quad = \quad E_k m_k(\tilde{z}(t^-)) + Q_k o_k(\tilde{z}(t^- - \mu_2(t^-))) + \bar{J}_k = Y_k(\tilde{z}(t_k^-), \tilde{z}_{t_k^-});\ t = t_k,\ k \text{ in } \mathbb{Z}_+, \tag{1}$$

where $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T \in \mathbb{C}^n$ and $\tilde{z}(t) = (\tilde{z}_1(t), \tilde{z}_2(t), \ldots, \tilde{z}_n(t))^T \in \mathbb{C}^n$ denotes the neurons state vectors at time t; $f_1(\cdot), f_2(\cdot), g_1(\cdot), g_2(\cdot), h_1(\cdot), h_2(\cdot)$ are the neuron activation functions whose elements consist of complex-valued nonlinear functions, where $f_1(z(t)) = (f_{11}(z(t)), f_{12}(z(t)), \ldots, f_{1n}(z(t)))^T \in \mathbb{C}^n$, $f_2(\tilde{z}(t)) = (f_{21}(\tilde{z}(t)), f_{22}(\tilde{z}(t)), \ldots, f_{2n}(\tilde{z}(t)))^T \in \mathbb{C}^n$, $g_1(z(t-\mu_1(t))) = (g_{11}(z(t-\mu_1(t))), g_{12}(z(t-\mu_1(t))), \ldots, (g_{1n}(z(t-\mu_1(t))))^T$, $g_2(\tilde{z}(t-\mu_2(t))) = (g_{21}(\tilde{z}(t-\mu_2(t))), g_{22}(\tilde{z}(t-\mu_2(t))), \ldots, (g_{2n}(\tilde{z}(t-\mu_2(t))))^T$, $h_1(z(t)) = (h_{11}(z(t)), h_{12}(z(t)), \ldots, (h_{1n}(z(t)))^T$ and $h_2(\tilde{z}(t)) = (h_{21}(\tilde{z}(t)), h_{22}(\tilde{z}(t)), \ldots, (h_{2n}(\tilde{z}(t)))^T$ all are in $\mathbb{C}^n$; $\mu_1(t)\&\mu_2(t)$ corresponds to the transmission discrete delays and satisfies $0 \leq \mu_1(t) \leq \mu_1$ and $0 \leq \mu_2(t) \leq \mu_2$, respectively; μ_1, μ_2, $\sigma_1, \sigma_2, \rho_1$ and ρ_2 are positive constants; $A = dia\{a_i\}, D = dia\{d_j\} \in \mathbb{R}^{n\times n}$, $i,j = 1,2,\ldots,n$ are the self-feedback connection weight matrices, where $a_i, d_j > 0$, $i,j = 1,2,\ldots,n$; $W_0, V_0 \in \mathbb{C}^{n\times n}$ are the connection weight matrices, $W_1, V_1 \in \mathbb{C}^{n\times n}$ represent the discretely delayed connection weight matrices and $W_2, V_2 \in \mathbb{C}^{n\times n}$ denote the distributively delayed connection weight matrices; $\hat{I}, J \in \mathbb{C}^n$ are the constant input vectors; $\delta_1 : \mathbb{C}^n \times \mathbb{C}^n \times \mathbb{C}^n \times \mathbb{R}^+ \longrightarrow \mathbb{C}^n$ and $\delta_2 : \mathbb{C}^n \times \mathbb{C}^n \times \mathbb{C}^n \times \mathbb{R}^+ \longrightarrow \mathbb{C}^n$ denote the stochastic disturbances $\omega_1(t) = (\omega_{11}(t), \omega_{12}(t), \ldots, \omega_{1n}(t))^T$ and $\omega_2(t) = (\omega_{21}(t), \omega_{22}(t), \ldots, \omega_{2n}(t))^T$ are n-dimensional respective Brownian motions defined on a complete probability space $(\mathbb{A}, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets) and $\mathbb{E}\{\mathrm{d}\omega_1(t)\} = \mathbb{E}\{\mathrm{d}\omega_2(t)\} = 0$, $\mathbb{E}\{\mathrm{d}\omega_1^2(t)\} = \mathbb{E}\{\mathrm{d}\omega_2^2(t)\} = dt$; B_k, C_k, E_k and Q_k are the impulses gain matrices; $\bar{I}_k$ and $\bar{J}_k$ represents external impulsive constant inputs; $h_k(z(t^-)) = (h_{k_1}(z(t^-)), h_{k_2}(z(t^-)), \ldots, h_{k_n}(z(t^-)))^T \in \mathbb{C}^n$, $m_k(\tilde{z}(t^-)) = (m_{k_1}(\tilde{z}(t^-)), m_{k_2}(\tilde{z}(t^-)), \ldots, m_{k_n}(\tilde{z}(t^-)))^T \in \mathbb{C}^n$ and $l_k(z(t-\mu_1(t^-))) = (l_{k_1}(z(t-\mu_1(t^-))), l_{k_2}(z(t-\mu_1(t^-))), \ldots, l_{k_n}(z(t-\mu_1(t^-))))^T \in \mathbb{C}^n$, $o_k(\tilde{z}(t-\mu_2(t^-))) = (o_{k_1}(\tilde{z}(t-\mu_2(t^-))), o_{k_2}(\tilde{z}(t-\mu_2(t^-))), \ldots, o_{k_n}(\tilde{z}(t-\mu_2(t^-))))^T \in \mathbb{C}^n$ be impulsive perturbations; t_k are called impulsive moments and satisfies $0 \leq t_1 < t_2 < \ldots$, $\lim_{k\to+\infty} t_k = +\infty$, $z(t_k^+) = \lim_{t\to t_k^+} z(t)$, $z(t_k^-) = \lim_{t\to t_k^-} z(t)$ and $\tilde{z}(t_k^+) = \lim_{t\to t_k^+} \tilde{z}(t)$, $\tilde{z}(t_k^-) = \lim_{t\to t_k^-} \tilde{z}(t)$. Without loss of generality, we assume that $\lim_{t\to t_k^+} z(t) = z(t_k)$ and $\lim_{t\to t_k^+} \tilde{z}(t) = \tilde{z}(t_k)$, which means that the solution of (1) is right continuous at time t_k.

The initial conditions of complex-valued BAM neural networks (1) are given by

$$\begin{aligned} z(s) &= \phi(s), s \in [t_0 - \mu_1, t_0], \\ \tilde{z}(s) &= \psi(s), s \in [t_0 - \mu_2, t_0], \end{aligned} \tag{2}$$

where $\phi(s)$ and $\psi(s) \in \mathbb{C}^n$ are continuous in $[t_0 - \mu_1, t_0]$ and $[t_0 - \mu_2, t_0]$, respectively.

Throughout this paper, we make the following assumptions:

Assumption 1 If $(z^*, \tilde{z}^*)$ is an equilibrium point of neural networks (1), then the impulsive jumps of (1) satisfy the following condition

$$(z^*, \tilde{z}^*) \quad = \quad B_k h_k(z^*) + C_k l_k(z^*) + E_k m_k(\tilde{z}^*) + Q_k o_k(\tilde{z}^*) + \hat{T}, k \in \mathbb{Z}^+, \hat{T} = \overline{I}_k + \overline{J}_k. \tag{3}$$

Assumption 2 The neuron activation functions $f_1(\cdot)$, $g_1(\cdot)$ and $h_1(\cdot)$ satisfy the Lipschitz continuity condition in the complex domain, i. e., there exist a positive diagonal matrices $F_1 = diag(F_{11}, F_{12}, \ldots, F_{1n})$, $G_1 = diag(G_{11}, G_{12}, \ldots, G_{1n})$ and $H_1 = diag(H_{11}, H_{12}, \ldots, H_{1n})$ such that

$$\begin{aligned} \|f_{1i}(u_1) - f_{1i}(u_2)\| &\leq F_{1i}|u_1 - u_2|, \\ \|G_{1i}(v_1) - G_{1i}(v_2)\| &\leq G_{1i}|v_1 - v_2|, \\ \&\|H_{1i}(w_1) - H_{1i}(w_2)\| &\leq H_{1i}|w_1 - w_2|, \forall u_1, u_2, v_1, v_2, w_1, w_2 \in \mathbb{C}, i = 1, 2, \ldots, n. \end{aligned}$$

Similarly, $f_2(\cdot)$, $g_2(\cdot)$ and $h_2(\cdot)$ are also satisfying the above conditions in a similar way.

Assumption 3 $U_k(\cdot), Y_k(\cdot) : \mathbb{C}^n \times \mathbb{C}^n \longrightarrow \mathbb{C}^n, k \in \mathbb{Z}_+$, are some continuous functions.

Assumption 4 There exist some positive diagonal matrices $H_k = diag(H_{k_1}, H_{k_2}, \ldots, H_{k_n})$, $L_k = diag(L_{k_1}, L_{k_2}, \ldots, L_{k_n})$, $M_k = diag(M_{k_1}, M_{k_2}, \ldots, M_{k_n})$ and $O_k = diag(O_{k_1}, O_{k_2}, \ldots, O_{k_n})$ such that

$$\begin{aligned} |h_{k_i(u_1)} - h_{k_i(u_2)}| &\leq H_{k_i}|u_1 - u_2|, \\ |l_{k_i(u_1)} - l_{k_i(u_2)}| &\leq L_{k_i}|u_1 - u_2|, \\ |m_{k_j(v_1)} - m_{k_j(v_2)}| &\leq M_{k_j}|v_1 - v_2|, \\ |o_{k_j(v_1)} - o_{k_j(v_2)}| &\leq O_{k_j}|v_1 - v_2|, \ \forall u_1, u_2, v_1, v_2 \in \mathbb{C}, i, j = 1, 2, \ldots, n, \ k \in \mathbb{Z}^+. \end{aligned}$$

Assumption 5 (Li et al. [22]) The noise intensity function matrices $\delta_1(x(t-\sigma_1), y(t), y(t-\mu_1(t)))$ & $\delta_2(y(t-\sigma_2), x(t), x(t-\mu_2(t)))$, $i \in \mathbb{N}$ in (1) satisfies

$$\begin{aligned} & trace\Big[\delta_1^T(x(t-\sigma_1), y(t), y(t-\mu_1(t)))S\delta_1(x(t-\sigma_1), y(t), y(t-\mu_1(t)))\Big] \\ & \quad \leq \quad x^T(t-\sigma_1)R_1x(t-\sigma_1) + y^T(t)R_2y(t) + y^T(t-\mu_1(t))R_3y(t-\mu_1(t)). \end{aligned}$$

$$\begin{aligned} & trace\Big[\delta_2^T(y(t-\sigma_2), x(t), x(t-\mu_2(t)))T\delta_2(y(t-\sigma_2), x(t), x(t-\mu_2(t)))\Big] \\ & \quad \leq \quad y^T(t-\sigma_2)\hat{R}_1y(t-\sigma_2) + x^T(t)\hat{R}_2x(t) + x^T(t-\mu_2(t))\hat{R}_3x(t-\mu_2(t)). \end{aligned}$$

for all $x(\cdot)$ and $y(\cdot) \in \mathbb{R}^n$, where R_1, R_2, R_3, $\hat{R}_1$, $\hat{R}_2$, & $\hat{R}_3$, are positive definite matrices with appropriate dimensions.

Remark 1.1 It should be pointed out that the assumptions on neurons activation functions are weaker, when compared with those generally used in the literature. Specifically, the differentiability and boundedness of the activation functions are not mandatory in this manuscript.

Remark 1.2 Assuming that the activation functions in [28, 60] may be represented by dividing their real and imaginary components, the writers talked about how stable CVNNs with delays would be. Alternatively, the obtained stability criteria cannot be applied if the activation functions in [12, 59] cannot be revealed by the separation of their real and imaginary components. Therefore, regardless of whether the activation function in this paper can separate, the stability requirement provided for complex-valued stochastic BAMNNs is valid.

Before ending this section, we introduce some definition and lemmas, which will play an important role in the derivation of the main results below.

Definition 1.3 (Zhang et al. [57]) *Exponentially stable*

The equilibrium point of neural networks (1) is said to be exponentially stable, if there exists constants $M > 0$ and $\eta > 0$ such that for $t \geq 0$

$$\|x(t)\|^2 + \|y(t)\|^2 \leq Me^{-\eta t},$$

where $\|.\|$ denotes the Euclidean norm.

Lemma 1.4 (Park and Kwon [35]) Let $a, b \in \mathbb{C}^n$ and $G \in C^{n\times n}$ be a positive definite Hermitian matrix, then

$$a^T b + ab^T \leq a^T G^{-1} a + b^T G b.$$

Lemma 1.5 (Gi [8]) For any constant symmetric positive-definite matrix $M \in \mathbb{R}^{n\times n}$ with appropriate dimension, a scalar $\eta > 0$ and the vector function $\omega(\cdot) : [\alpha, \beta] \to \mathbb{R}^n$, the integrations in the following are well defined, then

$$\left[\int_\alpha^\beta \omega(s)\,\mathrm{d}s\right]^T M \left[\int_\alpha^\beta \omega(s)\,\mathrm{d}s\right] \leq (\beta - \alpha) \int_\alpha^\beta \omega^T(s) M \omega(s)\,\mathrm{d}s.$$

Lemma 1.6 (Park and Kwon [35]) A given matrix $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} > 0$, where $L_{11}^T = L_{11}$, $L_{22}^T = L_{22}$, is equivalent to any one of the following conditions:

(i) $L_{22} > 0$, $L_{11} - L_{12} L_{22}^T L_{12}^T > 0$;

(ii) $L_{11} > 0$, $L_{22} - L_{12}^T L_{11}^{-1} L_{12} > 0$.

Remark 1.7 The neural networks (1) is more advanced than the existing works in the available literature, see for instance [3, 52, 56]. Hence our research work is distinct from previous ones in the sense of novelty.

2. EXPONENTIAL STABILITY FOR DETERMINISTIC SYSTEMS

In this section, we investigate the existence, uniqueness and also the exponential stability of the equilibrium point of complex-valued stochastic impulsive BAM neural networks with delay components.

Theorem 2.1 Under the Assumptions 1−,4, the solution $(z, \tilde{z}) = (z(t, 0, \phi), \tilde{z}(t, 0, \psi))$ of CVBAMNNs (1)−(2) uniquely exists on $[-\varrho, \infty) \times [-\varrho, \infty)$.

Proof.
Let us start the proof now. For u and $v \in C([0, t_1], \mathbb{C}^n)$, set

$$\begin{aligned} \|u\|^* &= \max_{t\in[0,t_1]} \Big\{e^{-\tilde{\omega}_1 t} \max_{s\in[0,t]} \{\|u(s)\|\}\Big\}, \\ \|v\|^* &= \max_{t\in[0,t_1]} \Big\{e^{-\tilde{\omega}_2 t} \max_{s\in[0,t]} \{\|v(s)\|\}\Big\}, \end{aligned}$$

where

$$\begin{aligned} \tilde{\omega}_1 &= \sqrt{\sum_{j=1}^n d_j^2} + F_1\sqrt{n}\Bigg\{\sqrt{\sum_{i=1}^n\sum_{j=1}^n w_{0ij}^2}\Bigg\} + G_1\sqrt{n}\Bigg\{\sqrt{\sum_{i=1}^n\sum_{j=1}^n w_{1ij}^2}\Bigg\} + 1, \\ \tilde{\omega}_2 &= \sqrt{\sum_{i=1}^n a_i^2} + F_2\sqrt{n}\Bigg\{\sqrt{\sum_{i=1}^n\sum_{j=1}^n v_{0ij}^2}\Bigg\} + G_2\sqrt{n}\Bigg\{\sqrt{\sum_{i=1}^n\sum_{j=1}^n v_{1ij}^2}\Bigg\} + 1. \end{aligned} \tag{4}$$

And then it is easy to see that $C([0, t_1], \mathbb{C}^n)$ is a Banach space endowed with the norm $\|.\|^*$. For any (u,v) $\in C([0, t_1], \mathbb{C}^n) \times C([0, t_1], \mathbb{C}^n)$ define an integral operator $\mathfrak{J}_1$ by

$$\begin{aligned} (\mathfrak{J}_1(u, v))(t) &= \phi(0) + \int_0^t \Big\{ - Au(s - \sigma_1) + W_0 f_1(v(s)) + W_1 g_1(v(s - \mu_1(s))) + W_2 \\ &\quad \times \int_{t-\rho_1}^t h_1(v(s))\,\mathrm{d}s + \hat{I}\Big\}\,\mathrm{d}s + \psi(0) + \int_0^t \Big\{ - Dv(s - \sigma_2) + V_0 f_2(u(s)) \\ &\quad + V_1 g_2(u(s - \mu_2(s))) + V_2 \int_{t-\rho_2}^t h_2(u(s))\,\mathrm{d}s + J\Big\}\,\mathrm{d}s, t \in [0, t_1]; \end{aligned} \tag{5}$$

where $u(s) = \phi(s)$ & $v(s) = \psi(s)$, $s \in [-\varrho, 0]$.

It is obviously to see that $\mathfrak{J}_1$ maps $C([0, t_1], \mathbb{C}^n) \times C([0, t_1], \mathbb{C}^n)$ into $C([0, t_1], \mathbb{C}^n) \times C([0, t_1], \mathbb{C}^n)$.

From excellence of Assumptions 1−4 and the properties of the Euclidean norm, for any (u,v), $(\tilde{u}, \tilde{v}) \in C([0, t_1], \mathbb{C}^n) \times C([0, t_1], \mathbb{C}^n)$, where

$$\begin{aligned} (u(t), v(t)) &= (u_1(t), u_2(t), \ldots, u_n(t), v_1(t), v_2(t), \ldots, v_n(t))^T. \\ (\tilde{u}(t), \tilde{v}(t)) &= (\tilde{u}_1(t), \tilde{u}_2(t), \ldots, \tilde{u}_n(t), \tilde{v}_1(t), \tilde{v}_2(t), \ldots, \tilde{v}_n(t))^T. \end{aligned}$$

$\|(\mathfrak{J}_1(u, v))(t) - (\mathfrak{J}_1(\tilde{u}, \tilde{v}))(t)\|$

$$\begin{aligned} &= \| - \int_0^t A[u(s - \sigma_1) - \tilde{u}(s - \sigma_1)]\,\mathrm{d}s + \int_0^t W_0[f_1(v(s)) - f_1(\tilde{v}(s))]\,\mathrm{d}s \\ &\quad + \int_0^t W_1[g_1(v(s - \mu_1(s))) - g_1(\tilde{v}(s - \mu_1(s)))]\,\mathrm{d}s + \int_0^t W_2 \end{aligned}$$

$$
\begin{aligned}
& \times \int_{t-\rho_1}^{t} h_1(v(s)) - h_1(\tilde{v}(s))\,\mathrm{d}s\mathrm{d}t\| + \| - \int_0^t D[v(s-\sigma_2) - \tilde{v}(s \\
& -\sigma_2)]\,\mathrm{d}s + \int_0^t V_0[f_2(u(s)) - f_2(\tilde{u}(s))]\,\mathrm{d}s + \int_0^t V_1[g_2(u(s-\mu_2(s))) \\
& -g_2(\tilde{u}(s-\mu_2(s)))]\,\mathrm{d}s + \int_0^t V_2 \int_{t-\rho_2}^{t} h_2(u(s)) - h_2(\tilde{u}(s))\,\mathrm{d}s\mathrm{d}t\| \\
\leq\ & \int_0^t \|A[u(s-\sigma_1) - \tilde{u}(s-\sigma_1)]\|\,\mathrm{d}s + \int_0^t \|W_0[f_1(v(s)) - f_1(\tilde{v}(s))]\|\,\mathrm{d}s \\
& + \int_0^t \|W_1[g_1(v(s-\mu_1(s))) - g_1(\tilde{v}(s-\mu_1(s)))]\|\,\mathrm{d}s + \int_0^t \|W_2 \\
& \times \int_{t-\rho_1}^{t} h_1(v(s)) - h_1(\tilde{v}(s))\,\mathrm{d}s\|\mathrm{d}t + \int_0^t \|D[v(s-\sigma_2) - \tilde{v}(s-\sigma_2)]\| \\
& \times \mathrm{d}s + \int_0^t \|V_0[f_2(u(s)) - f_2(\tilde{u}(s))]\|\,\mathrm{d}s + \int_0^t \|V_1[g_2(u(s-\mu_2(s))) \\
& -g_2(\tilde{u}(s-\mu_2(s)))]\|\,\mathrm{d}s + \int_0^t \|V_2 \int_{t-\rho_2}^{t} h_2(u(s)) - h_2(\tilde{u}(s))\,\mathrm{d}s\|\mathrm{d}t \\
\leq\ & \sqrt{\sum_{i=1}^{n} a_i^2} \int_0^t \|[u(s-\sigma_1) - \tilde{u}(s-\sigma_1)]\|\,\mathrm{d}s + F_1\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{0ij}^2} \\
& \times \int_0^t \|v(s) - \tilde{v}(s)\|\,\mathrm{d}s + G_1\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{1ij}^2} \int_0^t \|v(s-\mu_1(s)) \\
& -\tilde{v}(s-\mu_1(s))\|\,\mathrm{d}s + H_1\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{2ij}^2} \int_0^t [\| \int_{t-\rho_1}^{t} [v(s) - \tilde{v}(s)]\| \\
& \times \mathrm{d}s]\mathrm{d}t + \sqrt{\sum_{j=1}^{n} d_j^2} \int_0^t \|[v(s-\sigma_2) - \tilde{v}(s-\sigma_2)]\|\,\mathrm{d}s + F_2\sqrt{n} \\
& \times \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{0ij}^2} \int_0^t \|u(s) - \tilde{u}(s)\|\,\mathrm{d}s + G_2\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{1ij}^2} \int_0^t \|u(s \\
& -\mu_2(s)) - \tilde{u}(s-\mu_2(s))\|\,\mathrm{d}s + H_2\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{2ij}^2} \int_0^t [\| \int_{t-\rho_2}^{t} [u(s) \\
& -\tilde{u}(s)]\|\,\mathrm{d}s]\mathrm{d}t. \qquad (6)
\end{aligned}
$$

Now

$$
\begin{aligned}
\int_{t-\rho_1}^{t} \|[v(s) - \tilde{v}(s)]\|\,\mathrm{d}s &\leq \|(v(t) - \tilde{v}(t)) - (v(t-\rho_1) - \tilde{v}(t-\rho_1))\| \\
&\leq \|(v(t) - \tilde{v}(t))\| - \|v(t-\rho_1) - \tilde{v}(t-\rho_1)\|, \qquad (7)
\end{aligned}
$$

$$\text{and} \int_{t-\rho_2}^{t} \|[u(s)-\tilde{u}(s)]\|\,\mathrm{d}s \le \|(u(t)-\tilde{u}(t))-(u(t-\rho_2)-\tilde{u}(t-\rho_2))\| \le \|(u(t)-\tilde{u}(t))\|-\|u(t-\rho_2)-\tilde{u}(t-\rho_2)\|. \quad (8)$$

Substitute (7) and (8) in (6), we have
$\|(\mathfrak{J}_1(u,v))(t) - (\mathfrak{J}_1(\tilde{u},\tilde{v}))(t)\|$

$$\begin{aligned}
&\le \sqrt{\sum_{i=1}^{n} a_i^2}\int_0^t \max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\,\mathrm{d}s + F_1\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{0ij}^2}\\
&\quad\times\int_0^t \max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s + G_1\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{1ij}^2}\\
&\quad\times\int_0^t \max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s + H_1\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{2ij}^2}\\
&\quad\times\Big[\int_0^t \max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s - \int_0^t \max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s\Big]\\
&\quad+\sqrt{\sum_{j=1}^{n} d_j^2}\int_0^t \max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s + F_2\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{0ij}^2}\\
&\quad\times\int_0^t \max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\,\mathrm{d}s + G_2\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{1ij}^2}\\
&\quad\times\int_0^t \max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\,\mathrm{d}s + H_2\sqrt{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} v_{2ij}^2}\\
&\quad\times\Big[\int_0^t \max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\,\mathrm{d}s - \int_0^t \max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\,\mathrm{d}s\Big]\\
&= (\tilde{\omega}_1-1)\int_0^t \max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s + (\tilde{\omega}_2-1)\int_0^t \max_{r\in[0,s]}\|u(r)\\
&\quad-\tilde{u}(r)\|\,\mathrm{d}s\\
&= (\tilde{\omega}_1-1)\int_0^t e^{\tilde{\omega}_1 s}e^{-\tilde{\omega}_1 s}\max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\,\mathrm{d}s + (\tilde{\omega}_2-1)\int_0^t e^{\tilde{\omega}_2 s}\\
&\quad\times e^{-\tilde{\omega}_2 s}\max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\,\mathrm{d}s
\end{aligned}$$

$$\begin{aligned}
&\le (\tilde{\omega}_1-1)\int_0^t e^{\tilde{\omega}_1 s}\max_{s\in[0,t_1]}\Big\{e^{-\tilde{\omega}_1 s}\max_{r\in[0,s]}\|v(r)-\tilde{v}(r)\|\Big\}\,\mathrm{d}s + (\tilde{\omega}_2-1)\\
&\quad\times\int_0^t e^{\tilde{\omega}_2 s}\max_{s\in[0,t_1]}\Big\{e^{-\tilde{\omega}_2 s}\max_{r\in[0,s]}\|u(r)-\tilde{u}(r)\|\Big\}\,\mathrm{d}s
\end{aligned}$$

$$\begin{aligned} &= (\tilde{\omega}_1 - 1)\int_0^t e^{\tilde{\omega}_1 s}\,\mathrm{d}s\|v-\tilde{v}\|^* + (\tilde{\omega}_2 - 1)\int_0^t e^{\tilde{\omega}_2 s}\,\mathrm{d}s\|u-\tilde{u}\|^* \\ &\leq \frac{\tilde{\omega}_1 - 1}{\tilde{\omega}_1}e^{\tilde{\omega}_1 t}\|v-\tilde{v}\|^* + \frac{\tilde{\omega}_2 - 1}{\tilde{\omega}_2}e^{\tilde{\omega}_2 t}\|u-\tilde{u}\|^*, \end{aligned} \tag{9}$$

which implies that,

$$\begin{aligned} \max_{s\in[0,t]}\{\|(\mathfrak{J}_1(u,v))(s) - (\mathfrak{J}_1(\tilde{u},\tilde{v}))(s)\|\} &\leq \max_{s\in[0,t]}\left\{\frac{\tilde{\omega}_1 - 1}{\tilde{\omega}_1}e^{\tilde{\omega}_1 s}\|v-\tilde{v}\|^* + \frac{\tilde{\omega}_2 - 1}{\tilde{\omega}_2}\right. \\ &\qquad \left.\times e^{\tilde{\omega}_2 s}\|u-\tilde{u}\|^*\right\} \\ &\leq \frac{\tilde{\omega}_1 - 1}{\tilde{\omega}_1}e^{\tilde{\omega}_1 t}\|v-\tilde{v}\|^* + \frac{\tilde{\omega}_2 - 1}{\tilde{\omega}_2}e^{\tilde{\omega}_2 t}\|u-\tilde{u}\|^*. \end{aligned} \tag{10}$$

Put $\hat{\omega} = \max\{\tilde{\omega}_1, \tilde{\omega}_2\}$, then (10) becomes

$$\max_{s\in[0,t]}\{\|(\mathfrak{J}_1(u,v))(s) - (\mathfrak{J}_1(\tilde{u},\tilde{v}))(s)\|\} \leq \frac{\hat{\omega}-1}{\hat{\omega}}e^{\hat{\omega}t}\{\|v-\tilde{v}\|^* + \|u-\tilde{u}\|^*\}.$$

It follows that

$$e^{-\hat{\omega}t}\max_{s\in[0,t]}\{\|(\mathfrak{J}_1(u,v))(s) - (\mathfrak{J}_1(\tilde{u},\tilde{v}))(s)\|\} \leq \frac{\hat{\omega}-1}{\hat{\omega}}\{\|v-\tilde{v}\|^* + \|u-\tilde{u}\|^*\}.$$

Therefore, we obtain

$$\|(\mathfrak{J}_1(u,v))(s) - (\mathfrak{J}_1(\tilde{u},\tilde{v}))(s)\|^* \leq \frac{\hat{\omega}-1}{\hat{\omega}}\{\|v-\tilde{v}\|^* + \|u-\tilde{u}\|^*\},$$

for any (u,v) & $(\tilde{u},\tilde{v}) \in C([0,t_1,\mathbb{C}^n]) \times C([0,t_1,\mathbb{C}^n])$.

By using the contraction mapping theorem, we know that there exists a unique fixed point $(u_1^*, v_1^*) \in C([0,t_1,\mathbb{C}^n]) \times C([0,t_1,\mathbb{C}^n])$ such that $\mathfrak{J}_1(u_1^*, v_1^*) = (u_1^*, v_1^*)$. Hence, we get that $(u_1^*(t_1), v_1^*(t_1))$ exists finitely. This implies that $B_k h_k(u_1^*(t_1)) + C_k l_k(u_1^*(t_1 - \mu_1(t_1))) + E_k m_k(v_1^*(t_1)) + Q_k o_k(v_1^*(t_1 - \mu_2(t_1))) + \hat{T}$ also exists finitely, since Assumption 3 holds. Then we replace $(u_1^*(t_1), v_1^*(t_1))$ with $B_k h_k(u_1^*(t_1)) + C_k l_k(u_1^*(t_1 - \mu_1(t_1))) + E_k m_k(v_1^*(t_1)) + Q_k o_k(v_1^*(t_1 - \mu_2(t_1))) + \hat{T}$ and define $\zeta_1 = B_k h_k(u_1^*(t_1)) + C_k l_k(u_1^*(t_1 - \mu_1(t_1))) + E_k m_k(v_1^*(t_1)) + Q_k o_k(v_1^*(t_1 - \mu_2(t_1))) + \hat{T}$ for later use.

For $u, v \in C([t_1, t_2], \mathbb{C}^n)$, set

$$\begin{aligned} \|u\|^* &= \max_{t\in[t_1,t_2]}\left\{e^{-\tilde{\omega}_1(t-t_1)}\max_{s\in[t_1,t]}\{\|u(s)\|\}\right\}, \\ \|v\|^* &= \max_{t\in[t_1,t_2]}\left\{e^{-\tilde{\omega}_2 t}\max_{s\in[t_1,t]}\{\|v(s)\|\}\right\}, \end{aligned}$$

where $\tilde{\omega}_1, \tilde{\omega}_2$ is defined in (4), Clearly, $C([t_1,t_2],\mathbb{C}^n)$ is also a Banach space endowed with the norm $\|.\|^*$. For any $(u,v) \in C([t_1,t_2],\mathbb{C}^n) \times C([t_1,t_2],\mathbb{C}^n)$, define the integral operator $\mathfrak{J}_2$ by,

$$\begin{aligned}(\mathfrak{J}_2(u,v))(t) \quad = \quad & \zeta_1 + \int_{t_1}^{t} \Big\{ - Au(s-\sigma_1) + W_0 f_1(v(s)) + W_1 g_1(v(s-\mu_1(s))) + W_2 \\ & \times \int_{t-\rho_1}^{t} h_1(v(s))\,\mathrm{d}s + \hat{I}\Big\}\,\mathrm{d}s + \int_{t_1}^{t} \Big\{ - Dv(s-\sigma_2) + V_0 f_2(u(s)) + V_1 \\ & \times g_2(u(s-\mu_2(s))) + V_2 \int_{t-\rho_2}^{t} h_2(u(s))\,\mathrm{d}s + J\Big\}\,\mathrm{d}s, \text{ for } t \in [t_1,t_2];\end{aligned} \tag{11}$$

where

$$u(s) = \begin{cases} \phi(s), s \in [-\varrho, 0], \\ u_1^*(s), s \in [0, t_1). \end{cases} \quad \& \ v(s) = \begin{cases} \psi(s), s \in [-\varrho, 0], \\ v_1^*(s), s \in [0, t_1). \end{cases}$$

It follows from the above definition, we see that $\mathfrak{J}_2$ maps $C([t_1,t_2],\mathbb{C}^n) \times C([t_1,t_2],\mathbb{C}^n)$ into $C([t_1,t_2],\mathbb{C}^n)$
$\times\, C([t_1,t_2],\mathbb{C}^n)$. Similarly, it can be proved that

$$\|(\mathfrak{J}_2(u,v))(s) - (\mathfrak{J}_2(\tilde{u},\tilde{v}))(s)\|^* \quad \leq \quad \frac{\hat{\omega}-1}{\hat{\omega}}\{\|u-\tilde{u}\|^* + \|v-\tilde{v}\|^*\}, \tag{12}$$

for any (u,v) & $(\tilde{u},\tilde{v}) \in C([t_1,t_2],\mathbb{C}^n) \times C([t_1,t_2],\mathbb{C}^n)$. Again using the contraction mapping theorem, we obtain that there exists a unique fixed point $(u_2^*, v_2^*) \in C([t_1,t_2],\mathbb{C}^n) \times C([t_1,t_2],\mathbb{C}^n)$ such that $\mathfrak{J}_2(u_2^*, v_2^*) = (u_2^*, v_2^*)$.

Moreover, we know that $(u_2^*(t_2), v_2^*(t_2))$ exists finitely, which implies that $B_k h_k(u_2^*(t_2))$ $+\, C_k l_k(u_2^*(t_2-\mu_1(t_2))) + E_k m_k(v_2^*(t_2)) + Q_k o_k(v_2^*(t_2-\mu_2(t_2))) + \hat{T}$ also exists finitely, in view of Assumption 3. Then we replace $(u_2^*(t_2), v_2^*(t_2))$ with $B_k h_k(u_2^*(t_2)) + C_k l_k(u_2^*(t_2-\mu_1(t_2))) + E_k m_k(v_2^*(t_2)) + Q_k o_k(v_2^*(t_2-\mu_2(t_2))) + \hat{T}$ and define $\zeta_2 = B_k h_k(u_2^*(t_2)) + C_k l_k(u_2^*(t_2-\mu_1(t_2))) + E_k m_k(v_2^*(t_2)) + Q_k o_k(v_2^*(t_2-\mu_2(t_2))) + \hat{T}$ for later use.

With analogous arguments, one may deduce that $C([t_{p-1},t_p],\mathbb{C}^n)$, $p \in \mathbb{Z}_+$, is a Banach space endowed with a similar norm, $\|.\|^*$. Furthermore, the operator J_p which maps $C([t_{p-1},t_p],\mathbb{C}^n) \times C([t_{p-1},t_p],\mathbb{C}^n)$ into $C([t_{p-1},t_p],\mathbb{C}^n) \times C([t_{p-1},t_p],\mathbb{C}^n)$ has a unique fixed point. $(u_p^*, v_p^*) \in C([t_{p-1},t_p], \mathbb{C}^n) \times C([t_{p-1},t_p],\mathbb{C}^n)$ such that $\mathfrak{J}_p(u_p^*, v_p^*)$ $= (u_p^*, v_p^*)$, where

$$\begin{aligned}(\mathfrak{J}_p(u,v))(t) \quad = \quad & \zeta_{p-1} + \int_{t_{p-1}}^{t} \Big\{ - Au(s-\sigma_1) + W_0 f_1(v(s)) + W_1 g_1(v(s-\mu_1(s))) + W_2 \\ & \times \int_{t-\rho_1}^{t} h_1(v(s))\,\mathrm{d}s + \hat{I}\Big\}\,\mathrm{d}s + \int_{t_{p-1}}^{t} \Big\{ - Dv(s-\sigma_2) + V_0 f_2(u(s)) + V_1 \\ & \times g_2(u(s-\mu_2(s))) + V_2 \int_{t-\rho_2}^{t} h_2(u(s))\,\mathrm{d}s + J\Big\}\,\mathrm{d}s, \text{ for } t \in [t_{p-1},t_p];\end{aligned} \tag{13}$$

in which $\zeta_{p-1} = B_k h_k(u^*_{p-1}(t_{p-1})) + C_k l_k(u^*_{p-1}(t_{p-1} - \mu_1(t_{p-1}))) + \overline{I}_k + E_k m_k(v^*_{p-1}(t_{p-1})) + Q_k\ o_k(v^*_{p-1}(t_{p-1} - \mu_2(t_{p-1}))) + \overline{J}_k$ and

$$u(s) = \begin{cases} \phi(s), s \in [-\varrho, 0], \\ u^*_1(s), s \in [0, t_1). \\ \vdots \\ u^*_{p-1}(s),\ s\ \in\ [t_{p-2}, t_{p-1}). \end{cases}$$

and

$$v(s) = \begin{cases} \psi(s), s \in [-\varrho, 0], \\ v^*_1(s), s \in [0, t_1). \\ \vdots \\ v^*_{p-1}(s),\ s\ \in\ [t_{p-2}, t_{p-1}). \end{cases}$$

In view of impulsive properties, we finally define

$$u(t) = \begin{cases} \phi(t), t \in [-\varrho, 0], \\ u^*_1(t), t \in [0, t_1). \\ \vdots \\ u^*_n(t),\ t\ \in\ [t_{n-1}, t_n),\ n\ \in\ \mathbb{Z}_+. \\ \vdots \end{cases}$$

and

$$v(t) = \begin{cases} \psi(t), t \in [-\varrho, 0], \\ v^*_1(t), t \in [0, t_1). \\ \vdots \\ v^*_n(t), t \in [t_{n-1}, t_n), n \in \mathbb{Z}_+ \\ \vdots \end{cases}$$

Thus, it is easy to verify that $(u(t), v(t)) = (u(t, 0, \phi), v(t, 0, \psi))$ is the unique solution of neural networks $(1)-(2)$ defined on $[-\varrho, \infty) \times [-\varrho, \infty)$. The proof of this theorem is completed. □

Remark 2.2 The impulsive impacts on complex-valued NNs for stability requirements were examined by the authors in [37]. The stability performance of time-delayed CVNNs in the sense of exponential was covered by X. Liu and T. Chen in [28]. The problem of exponential stability for complex-valued neural networks is only addressed in the aforementioned references with discrete time delays and impulsive effects; BAM-type, complex variables, leakage delays, distributed time delays, impulsive effects, and stochastic noise have not been considered, nor have any studies looked into exponentially stable at a time. As a result, this work takes a very demanding and advanced approach to considering the facts mentioned above.

Theorem 2.3 Assume that Assumptions 1 − 5 holds. The equilibrium point of the system (1) − (2) is exponentially stable if, for given η, $\widetilde{\eta} > 0$, there exists positive definite Hermitian matrices S, T, P_1, P_2, Q_1, Q_2, positive diagonal matrices R, E, F, $\widetilde{F}$ and positive scalars η^*, λ, μ, μ_1 and μ_2 such that the LMIs are satisfied as follows:

$$S < \lambda I, \tag{14}$$

$$T < \mu I, \tag{15}$$

$$K_1^T S K_1 - S \leq 0, \tag{16}$$

$$\hat{K}_1^T T \hat{K}_1 - T \leq 0, \tag{17}$$

$$\Omega = \begin{bmatrix} \Omega_{11} & SW_0 & 0 & 0 & \Omega_{15} & 0 & 0 & 0 & 0 & 0 \\ * & -L_1 & 0 & 0 & \Omega_{25} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & \Omega_{45} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & \Omega_{56} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{1010} \end{bmatrix}_{10\times 10} < 0, \tag{18}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & TV_0 & 0 & 0 & \Xi_{15} & 0 & 0 & 0 & 0 & 0 \\ * & -L_2 & 0 & 0 & \Xi_{25} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \Xi_{1010} \end{bmatrix}_{10\times 10} < 0, \tag{19}$$

where

$$\Omega_{11} = -SA - A^T S + SW_1 R^{-1} W_1^T S + SW_2 \hat{R}^{-1} W_2^T S + \mu \hat{R}_2, \quad \Omega_{33} = G_1 R G_1 + \lambda R_3,$$
$$\Omega_{15} = A^T SA - \eta SA, \quad \Omega_{25} = -A^T SW_0, \quad \Omega_{56} = -A^T SW_1, \quad \Omega_{45} = -A^T SW_2,$$
$$\Omega_{77} = \lambda R_1, \quad \Omega_{55} = \eta A^T SA, \quad \Omega_{99} = e^{-\eta t}(L_1^T F L_1) + \lambda R_2, \quad \Omega_{44} = \hat{R} - e^{-\eta \rho_1} Q_1,$$
$$\Omega_{88} = P_1 - e^{-\eta t} F, \quad \Omega_{66} = (1-\mu_1) e^{-\eta \mu_1} P_1, \quad \Xi_{77} = \mu \hat{R}_1, \quad \Xi_{25} = -D^T TV_0,$$
$$\Xi_{11} = -TD - D^T T + TV_1 E^{-1} V_1^T T + TV_2 \hat{E}^{-1} V_2^T T + \lambda R_2, \quad \Xi_{33} = G_2 E G_2 + \mu \hat{R}_3,$$
$$\Xi_{15} = D^T TD - \widetilde{\eta} TD, \quad \Xi_{88} = P_2 - e^{-\widetilde{\eta} t} \widetilde{F}, \quad \Xi_{44} = \hat{E} - e^{-\widetilde{\eta} \rho_2} Q_2, \quad \Xi_{55} = \widetilde{\eta} D^T TD,$$
$$\Xi_{99} = e^{-\widetilde{\eta} t}(L_2^T \widetilde{F} L_2) + \mu \hat{R}_2, \quad \Xi_{1010} = \rho_2^2 Q_2, \quad \Omega_{1010} = \rho_1^2 Q_1, \quad \Xi_{45} = -D^T TV_2,$$
$$\Xi_{56} = -D^T TV_1, \quad \Xi_{66} = (1-\mu_2) e^{-\widetilde{\eta} \mu_2} P_2.$$

Proof. Let $(z^*, \tilde{z}^*)$ be an equilibrium point of CVBAMNNs (1). Next, we transform the equilibrium $(z^*, \tilde{z}^*)$ to the origin by taking $x(t) = z(t) - z^*$, $y(t) = \tilde{z}(t) - \tilde{z}^*$. Subsequently, the system (1) can be rewritten as

$$\begin{aligned}
\mathrm{d}x(t) &= \Big[- Ax(t-\sigma_1) + W_0\bar{f}_1(y(t)) + W_1\bar{g}_1(y(t-\mu_1(t))) + W_2\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big] \\
&\qquad \times \mathrm{d}t + \bar{\delta}_1(x(t-\sigma_1), y(t), y(t-\mu_1(t))), t)\mathrm{d}\omega_1(t);\ t > 0, t \neq t_k, \\
x(t) &= B_k\beta_k(x(t^-)) + C_k\gamma_k(x(t^- - \mu_1(t^-))) = U_k^*(x(t_k^-), x_{t_k^-});\ t = t_k,\ k \in \mathbb{Z}_+. \\
\mathrm{d}y(t) &= \Big[- Dy(t-\sigma_2) + V_0\bar{f}_2(x(t)) + V_1\bar{g}_2(x(t-\mu_2(t))) + V_2\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big] \\
&\qquad \times \mathrm{d}t + \bar{\delta}_2(y(t-\sigma_2), x(t), x(t-\mu_2(t))), t)\mathrm{d}\omega_2(t);\ t > 0, t \neq t_k, \\
y(t) &= E_k\hat{\beta}_k(y(t^-)) + Q_k\hat{\gamma}_k(y(t^- - \mu_2(t^-))) = Y_k^*(y(t_k^-), y_{t_k^-});\ t = t_k,\ k \in \mathbb{Z}_+
\end{aligned} \tag{20}$$

where $\bar{f}_1(y(t)) = f_1(\tilde{z}(t) + \tilde{z}^*) - f_1(\tilde{z}^*)$, $\bar{f}_2(x(t)) = f_2(z(t) + z^*) - f_2(z^*)$, $\bar{g}_1(y(t-\mu_1(t))) = g_1(\tilde{z}(t-\mu_1(t)) + \tilde{z}^*) - g_1(\tilde{z}^*)$, $\bar{g}_2(x(t-\mu_2(t))) = g_2(z(t-\mu_2(t)) + z^*) - g_2(z(t-\mu_2(t))^*)$, $\bar{h}_1(y(t)) = h_1(\tilde{z}(t) + \tilde{z}^*) - h_1(\tilde{z}^*)$, $\bar{h}_2(x(t)) = h_2(z(t) + z^*) - h_2(z^*)$, $\bar{\delta}_1(x(t-\sigma_1), y(t), y(t-\mu_1(t))), t) = \delta_1(x(t-\sigma_1) + z^*, y(t) + \tilde{z}^*, y(t-\mu_1(t)) + \tilde{z}^*, t) - \delta_1(z^*, \tilde{z}^*, \tilde{Z}^*, t)$, $\bar{\delta}_2(y(t-\sigma_2), x(t), x(t-\mu_2(t))), t) = \delta_2(y(t-\sigma_2) + \tilde{z}^*, x(t) + z^*, x(t-\mu_2(t)) + z^*, t) - \delta_2(\tilde{z}^*, z^*, Z^*, t)$, $\beta_k(x(t^-)) = h_k(x(t^-) + z^*) - h_k(z^*)$, $\hat{\beta}_k(y(t^-)) = m_k(y(t^-) + \tilde{z}^*) - m_k(\tilde{z}^*)$, $\gamma_k(x(t^- - \mu_1(t^-))) = l_k(x(t^- - \mu_1(t^-)) + z^*) - l_k(z^*)$, $\hat{\gamma}_k(y(t^- - \mu_2(t^-))) = o_k(y(t^- - \mu_2(t^-)) + \tilde{z}^*) - o_k(\tilde{z}^*)$.

Also, the initial condition combined with neural networks (20) is given by

$$x(s) = \phi(s) - z^* \text{ and } y(s) = \psi(s) - \tilde{z}^*,\ s \in [t_0 - \mu, t_0], \tag{21}$$

Construct a Lyapunov−Krasovskii functional candidate for model (20) as

$$V(x(t), y(t), t) = \sum_{l=1}^{3} V_l(x(t), y(t), t), \tag{22}$$

where

$$\begin{aligned}
V_1(x(t), y(t), t) &= e^{\eta t}\Big[x(t) - A\int_{t-\sigma_1}^{t} x(s)\,\mathrm{d}s\Big]^T S\Big[x(t) - A\int_{t-\sigma_1}^{t} x(s)\,\mathrm{d}s\Big] + e^{\tilde{\eta} t}\Big[y(t) - D \\
&\qquad \times \int_{t-\sigma_2}^{t} y(s)\,\mathrm{d}s\Big]^T T\Big[y(t) - D\int_{t-\sigma_2}^{t} y(s)\,\mathrm{d}s\Big]\bar{h}_2(x(s)) \\
V_2(x(t), y(t), t) &= \int_{t-\mu_1(t)}^{t} e^{\eta s}\bar{g}_1^T(y(s))P_1\bar{g}_1(y(s))\,\mathrm{d}s + \int_{t-\mu_2(t)}^{t} e^{\tilde{\eta} s}\bar{g}_2^T(x(s))P_2\bar{g}_2(x(s))\,\mathrm{d}s \\
V_3(x(t), y(t), t) &= \rho_1\int_{-\rho_1}^{0}\int_{t+\alpha}^{t} e^{\eta s}\bar{h}_1^T(y(s))Q_1\bar{h}_1(y(s))\,\mathrm{d}s\mathrm{d}\alpha + \rho_2\int_{-\rho_2}^{0}\int_{t+\beta}^{t} e^{\tilde{\eta} s}\bar{h}_2^T(x(s)) \\
&\qquad \times Q_2\bar{h}_2(x(s))\,\mathrm{d}s\mathrm{d}\beta.
\end{aligned}$$

From Assumption 5 and the conditions (14) and (15), we have

$$
\begin{aligned}
&trace\Big[\bar{\delta}_1^T(x(t-\sigma_1), y(t), y(t-\mu_1(t)))S\bar{\delta}_1(x(t-\sigma_1), y(t), y(t-\mu_1(t)))\Big] \\
&\quad\le \lambda\Big[x^T(t-\sigma_1)R_1x(t-\sigma_1) + y^T(t)R_2y(t) + y^T(t-\mu_1(t))R_3y(t-\mu_1(t))\Big]. \\
&trace\Big[\bar{\delta}_2^T(y(t-\sigma_2), x(t), x(t-\mu_2(t)))T\bar{\delta}_2(y(t-\sigma_2), x(t), x(t-\mu_2(t)))\Big] \\
&\quad\le \mu\Big[y^T(t-\sigma_2)\hat{R}_1y(t-\sigma_2) + x^T(t)\hat{R}_2x(t) + x^T(t-\mu_2(t))\hat{R}_3x(t-\mu_2(t))\Big].
\end{aligned}
$$

It is easy to prove that NNs (20) is equivalent to the following forms

$$
\begin{aligned}
d\Big[x(t) - A\int_{t-\sigma_1}^t x(s)\,\mathrm{d}s\Big] &= \Big[-Ax(t) + W_0\bar{f}_1(y(t)) + W_1\bar{g}_1(y(t-\mu_1(t))) \\
&\quad + W_2\int_{t-\sigma_1}^t \bar{h}_1(y(s)) \\
&\quad \times \mathrm{d}s\Big]\,\mathrm{d}t + \bar{\delta}_1(x(t-\sigma_1), y(t), y(t-\mu_1(t)), t)d\bar{\omega}_1(t), \\
d\Big[y(t) - D\int_{t-\sigma_2}^t y(s)\,\mathrm{d}s\Big] &= \Big[-Dy(t) + V_0\bar{f}_2(x(t)) + V_1\bar{g}_2(x(t-\mu_2(t))) \\
&\quad + V_2\int_{t-\sigma_2}^t \bar{h}_2(x(s)) \\
&\quad \times \mathrm{d}s\Big]\,\mathrm{d}t + \bar{\delta}_2(y(t-\sigma_2), x(t), x(t-\mu_2(t)), t)\,\mathrm{d}\bar{\omega}_2(t).
\end{aligned}
$$

Now, we consider the case of $t \neq t_k$. Calculating the derivative of V(t) along the solutions of (20), we obtain from Lemma 1.4 that $\mathcal{L}V_1(x(t), y(t), t)$

$$
\begin{aligned}
&= \eta e^{\eta t}\Big[x(t) - A\int_{t-\sigma_1}^t x(s)\,\mathrm{d}s\Big]^T S\Big[x(t) - A\int_{t-\sigma_1}^t x(s)\,\mathrm{d}s\Big] + e^{\eta t}\Bigg\{2\Big[x(t) - A \\
&\quad \times \int_{t-\sigma_1}^t x(s)\,\mathrm{d}s\Big]^T Sd\Big[x(t) - A\int_{t-\sigma_1}^t x(s)\,\mathrm{d}s\Big] + trace\Big[\bar{\delta}_1^T(x(t-\sigma_1), y(t), \\
&\quad y(t-\mu_1(t)), t)S\bar{\delta}_1(x(t-\sigma_1), y(t), y(t-\mu_1(t)), t)\Big]\Bigg\} + \widetilde{\eta}e^{\widetilde{\eta}t}\Big[y(t) - D \\
&\quad \times \int_{t-\sigma_2}^t y(s)\,\mathrm{d}s\Big]^T T\Big[y(t) - D\int_{t-\sigma_2}^t y(s)\,\mathrm{d}s\Big] + e^{\widetilde{\eta}t}\Bigg\{2\Big[y(t) - D\int_{t-\sigma_2}^t y(s)\,\mathrm{d}s\Big]^T \\
&\quad \times Td\Big[y(t) - D\int_{t-\sigma_2}^t y(s)\,\mathrm{d}s\Big] + trace\Big[\bar{\delta}_2^T(y(t-\sigma_2), x(t), x(t-\mu_2(t)), t) \\
&\quad \times T\bar{\delta}_2(y(t-\sigma_2), x(t), x(t-\mu_2(t)), t)\Big]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\leq\ & \eta e^{\eta t}\bigg[x^T(t)Sx(t)-x^T(t)SA\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s-A^TS\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^Tx(t)\\
&+\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TA^TSA\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)\bigg]+e^{\eta t}\bigg\{2\Big[x(t)-A\int_{t-\sigma_1}^{t}x(s)\\
&\times\mathrm{d}s\Big]^TS\Big[-Ax(t)+W_0\bar{f}_1(y(t))+W_1\bar{g}_1(y(t-\mu_1(t)))+W_2\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\\
&\times\mathrm{d}s\Big]+\lambda[x^T(t-\sigma_1)R_1x(t-\sigma_1)+y^T(t)R_2y(t)+y^T(t-\mu_1(t))R_3\\
&\times y(t-\mu_1(t))]\bigg\}+\widetilde{\eta}e^{\widetilde{\eta}t}\bigg[y^T(t)Ty(t)-y^T(t)TD\int_{t-\sigma_1}^{t}y(s)\,\mathrm{d}s-D^TT\\
&\times\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^Ty(t)+\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TD^TTD\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)\bigg]+e^{\widetilde{\eta}t}\bigg\{2\\
&\times\Big[y(t)-D\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big]^TT\Big[-Dy(t)+V_0\bar{f}_2(x(t))+V_1\bar{g}_2(x(t-\mu_2(t)))\\
&+V_2\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big]+\mu[y^T(t-\sigma_2)\hat{R}_1y(t-\sigma_2)+x^T(t)\hat{R}_2x(t)\\
&+x^T(t-\mu_2(t))\hat{R}_3x(t-\mu_2(t))]\bigg\}\\
\leq\ & \eta e^{\eta t}\bigg[x^T(t)Sx(t)-x^T(t)SA\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s-A^TS\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^Tx(t)\\
&+\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TA^TSA\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)\bigg]+e^{\eta t}\bigg\{-2x^T(t)SAx(t)+2x^T(t)\\
&\times SW_0\bar{f}_1(y(t))+2W_1x^T(t)S\bar{g}_1(y(t-\mu_1(t)))+2x^T(t)SW_2\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\\
&\times\mathrm{d}s+2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSAx(t)-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSW_0\bar{f}_1(y(t))\\
&-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSW_1\bar{g}_1(y(t-\mu_1(t)))-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^T\\
&\times SW_2\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)+\lambda[x^T(t-\sigma_1)R_1x(t-\sigma_1)+y^T(t)R_2y(t)\\
&+y^T(t-\mu_1(t))R_3y(t-\mu_1(t))]\bigg\}+\widetilde{\eta}e^{\widetilde{\eta}t}\bigg[y^T(t)Ty(t)-y^T(t)TD\int_{t-\sigma_2}^{t}y(s)\\
&\times\mathrm{d}s-D^TT\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^Ty(t)+\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TD^TTD\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)\bigg]
\end{aligned}
$$

$$
\begin{aligned}
&+e^{\widetilde{\eta}t}\bigg\{-2y^T(t)TDy(t)+2y^T(t)TV_0\bar{f}_2(x(t))+2V_1y^T(t)T\bar{g}_2(x(t-\mu_2(t)))\\
&+2y^T(t)TV_2\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s+2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTDy(t)-2D^T\\
&\times\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTV_0\bar{f}_2(x(t))-2D^T\Big(\int_{t-\sigma_2}^{t}(s)\,\mathrm{d}s\Big)^TTV_1\bar{g}_2(x(t-\mu_2(t)))\\
&-2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTV_2\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)+\mu[y^T(t-\sigma_2)\hat{R}_1y(t-\sigma_2)\\
&+x^T(t)\hat{R}_2x(t)+x^T(t-\mu_2(t))\hat{R}_3x(t-\mu_2(t))]\bigg\}\\
\leq\;&\eta e^{\eta t}\bigg[x^T(t)Sx(t)-x^T(t)SA\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s-A^TS\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^Tx(t)\\
&+\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TA^TSA\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)\bigg]+e^{\eta t}\bigg[x^T(t)(-SA-A^TS)x(t)\\
&+x^TSW_0\bar{f}_1(y(t))+\bar{f}_1^T(y(t))SW_0^Tx(t)+\bar{g}_1^T(y(t-\mu_1(t)))R\bar{g}_1(y(t-\mu_1(t)))\\
&+x^T(t)(SW_2\hat{R}^{-1}W_2^TS)x(t)+\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\mathrm{d}s\Big)^T\hat{R}\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)\\
&+2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSAx(t)-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSW_0\bar{f}_1(y(t))-2A^T\\
&\times\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)SW_1\bar{g}_1(y(t-\mu_1(t)))-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSW_2\\
&\times\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)+x^T(t-\sigma_1)\lambda R_1x(t-\sigma_1)+y^T(t)\lambda R_2y(t)\\
&+y^T(t-\mu_1(t))\lambda R_3y(t-\mu_1(t))\bigg]+e^{\widetilde{\eta}t}\bigg[y^T(t)Ty(t)-y^T(t)TD\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s
\end{aligned}
$$

$$
\begin{aligned}
&-D^TT\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^Ty(t)+\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TD^TTD\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)\bigg]+e^{\widetilde{\eta}t}\\
&\times\bigg[y^T(t)(-TD-D^TT)y(t)+y^TTV_0\bar{f}_2(x(t))+\bar{f}_2^T(x(t))TV_0^Ty(t)\\
&+\bar{g}_2^T(x(t-\mu_2(t)))E\bar{g}_2(x(t-\mu_2(t)))+y^T(t)(TV_2\hat{E}^{-1}V_2^TT)y(t)\\
&+\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)^T\hat{E}\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)+2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTDy(t)\\
&-2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTV_0\bar{f}_2(x(t))-2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTV_1
\end{aligned}
$$

$$\begin{aligned}
&\times\bar{g}_2(x(t-\mu_2(t))) - 2D^T\Big(\int_{t-\sigma_2}^{t} y(s)\,\mathrm{d}s\Big)^T TV_2\Big(\int_{t-\rho_2}^{t} \bar{h}_2(x(s))\,\mathrm{d}s\Big)\\
&+y^T(t-\sigma_2)\mu\hat{R}_1 y(t-\sigma_2) + x^T(t)\mu\hat{R}_2 x(t) + x^T(t-\mu_2(t))\mu\hat{R}_3\\
&\times x(t-\mu_2(t))\Bigg]. \qquad (23)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{L}V_2(x(t),y(t),t)\\
&\quad\le e^{\eta t}\bar{g}_1^T(y(t))P_1\bar{g}_1(y(t)) - (1-\mu_1)e^{\eta(t-\mu_1(t))}\bar{g}_1^T(y(t-\mu_1(t)))P_1\\
&\qquad\times\bar{g}_1(y(t-\mu_1(t))) + e^{\widetilde{\eta}t}\bar{g}_2^T(x(t))P_2\bar{g}_2(x(t)) - (1-\mu_2)e^{\widetilde{\eta}(t-\mu_2(t))}\\
&\qquad\times\bar{g}_2^T(x(t-\mu_2(t)))P_2\bar{g}_2(x(t-\mu_2(t))), \qquad (24)
\end{aligned}$$

$$\begin{aligned}
&\mathcal{L}V_3(x(t),y(t),t)\\
&\quad\le \rho_1\int_{-\rho_1}^{0} e^{\eta t}\bar{h}_1^T(y(t))Q_1\bar{h}_1(y(t))\,\mathrm{d}s - \rho_1\int_{t-\rho_1}^{t} e^{\eta s}\bar{h}_1^T(y(s))Q_1\bar{h}_1(y(s))\,\mathrm{d}s\\
&\qquad+\rho_2\int_{-\rho_2}^{0} e^{\widetilde{\eta}t}\bar{h}_2^T(x(t))Q_2\bar{h}_2(y(t))\,\mathrm{d}s - \rho_2\int_{t-\rho_2}^{t} e^{\widetilde{\eta}s}\bar{h}_2^T(x(s))Q_2\bar{h}_2(x(s))\\
&\qquad\times\mathrm{d}s. \qquad (25)
\end{aligned}$$

Since R, E, L_1 and L_2 are real-valued positive diagonal matrices. By Assumption 2, we can get that

$$\begin{aligned}
\bar{g}_1^T(y(t-\mu_1(t)))R\bar{g}_1(y(t-\mu_1(t))) &\le y^T(t-\mu_1(t))G_1RG_1y(t-\mu_1(t)), \quad (26)\\
\bar{g}_2^T(x(t-\mu_2(t)))E\bar{g}_2(x(t-\mu_2(t))) &\le x^T(t-\mu_2(t))G_2EG_2x(t-\mu_2(t)), \quad (27)\\
y^T(t)L_1^TFL_1y(t) - \bar{g}_1^T(y(t))F\bar{g}_1(y(t)) &\ge 0, \quad (28)\\
x^T(t)L_2^T\widetilde{F}L_2x(t) - \bar{g}_2^T(x(t))\widetilde{F}\bar{g}_2(x(t)) &\ge 0, \quad (29)
\end{aligned}$$

and

$$\begin{aligned}
&\rho_1\int_{-\rho_1}^{0} e^{\eta t}\bar{h}_1^T(y(t))Q_1\bar{h}_1(y(t))\,\mathrm{d}s - \rho_1\int_{t-\rho_1}^{t} e^{\eta s}\bar{h}_1^T(y(s))Q_1\bar{h}_1(y(s))\,\mathrm{d}s\\
&\quad\le \rho_1^2 e^{\eta t}\bar{h}_1^T(y(t))Q_1\bar{h}_1(y(t)) - \rho_1 e^{\eta(t-\rho_1)}\int_{t-\rho_1}^{t} \bar{h}_1^T(y(s))Q_1\bar{h}_1(y(s))\,\mathrm{d}s\\
&\quad\le \rho_1^2 e^{\eta t}\bar{h}_1^T(y(t))Q_1\bar{h}_1(y(t)) - e^{\eta(t-\rho_1)}\Big(\int_{t-\rho_1}^{t} \bar{h}_1(y(s))\,\mathrm{d}s\Big)^T Q_1\Big(\int_{t-\rho_1}^{t} \bar{h}_1(y(s))\,\mathrm{d}s\Big).\\
&\qquad (30)
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\rho_2\int_{-\rho_2}^{0} e^{\widetilde{\eta}t}\bar{h}_2^T(x(t))Q_2\bar{h}_2(x(t))\,\mathrm{d}s - \rho_2\int_{t-\rho_2}^{t} e^{\widetilde{\eta}s}\bar{h}_2^T(x(s))Q_2\bar{h}_2(x(s))\,\mathrm{d}s\\
&\quad\le \rho_2^2 e^{\widetilde{\eta}t}\bar{h}_2^T(x(t))Q_2\bar{h}_2(x(t)) - e^{\widetilde{\eta}(t-\rho_2)}\Big(\int_{t-\rho_2}^{t} \bar{h}_2(x(s))\,\mathrm{d}s\Big)^T Q_2\Big(\int_{t-\rho_2}^{t} \bar{h}_2(x(s))\,\mathrm{d}s\Big).\\
&\qquad (31)
\end{aligned}$$

Hence, it follows from (23) – (31) that

$$
\begin{aligned}
&\mathcal{L}V(x(t),y(t),t)\\
\leq\ & e^{\eta t}\Big[-x^T(t)\eta SA\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)-\eta A^TS\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^Tx(t)+\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^T\\
&\times\eta A^TSA\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)\Big]+e^{\eta t}\{x^T(t)[-SA-A^TS+SW_1R^{-1}W_1^TS+\eta S\\
&+SW_2\hat{R}^{-1}W_2^TS]x(t)+x^T(t)SW_0\bar{f}_1(y(t))+\bar{f}_1^T(y(t))SW_0^Tx(t)+y^T(t\\
&-\mu_1(t))[G_1RG_1+\lambda R_3]y(t-\mu_1(t))+\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)^T\hat{R}\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\\
&\times\mathrm{d}s\Big)+2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSAx(t)-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSW_0\bar{f}_1(y(t))\\
&-2A^T\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^TSW_1\bar{g}_1(y(t-\mu_1(t)))-2A^T\Big(\int_{t-\sigma_1(t)}^{t}x(s)\,\mathrm{d}s\Big)^TSW_2\\
&\times\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)+x^T(t-\sigma_1)\lambda R_1x(t-\sigma_1)\}+e^{\widetilde{\eta}t}\Big[-y^T(t)\widetilde{\eta}TD\\
&\times\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)-\widetilde{\eta}D^TT\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^Ty(t)+\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^T\widetilde{\eta}D^TT\\
&\times D\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)\Big]+e^{\widetilde{\eta}t}\{y^T(t)[-TD-D^TT+TV_1E^{-1}V_1^TT+\widetilde{\eta}T\\
&+TV_2\hat{E}^{-1}V_2^TT]y(t)+y^T(t)TV_0\bar{f}_2(x(t))+\bar{f}_2^T(x(t))TV_0^Ty(t)+x^T(t-\mu_2(t))\\
&\times[G_2EG_2+\mu\hat{R}_3]x(t-\mu_2(t))+\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)^T\hat{E}\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)\\
&+2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTDy(t)-2D^T\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTV_0\bar{f}_2(x(t))-2D^T\\
&\times\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^TTV_1\bar{g}_2(x(t-\mu_2(t)))-2D^T\Big(\int_{t-\sigma_2(t)}^{t}y(s)\,\mathrm{d}s\Big)^TTV_2\\
&\times\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)+y^T(t-\sigma_2)\mu\hat{R}_1y(t-\sigma_2)\}+e^{\eta t}\bar{g}_1^T(y(t))P_1\bar{g}_1(y(t))\\
&-(1-\mu_1)e^{\eta(t-\mu_1(t))}\bar{g}_1^T(y(t-\mu_1(t)))P_1\bar{g}_1(y(t-\mu_1(t)))+e^{\widetilde{\eta}t}\bar{g}_2^T(x(t))P_2\\
&\times\bar{g}_2(x(t))-(1-\mu_2)e^{\widetilde{\eta}(t-\mu_2(t))}\bar{g}_2^T(x(t-\mu_2(t)))P_2\bar{g}_2(x(t-\mu_2(t)))+\rho_1^2e^{\eta t}\\
&\times\bar{h}_1^T(y(t))Q_1\bar{h}_1(y(t))-e^{\eta(t-\rho_1)}\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)^TQ_1\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)\\
&+\rho_2^2e^{\widetilde{\eta}t}\bar{h}_2^T(x(t))Q_2\bar{h}_2(x(t))-e^{\widetilde{\eta}(t-\rho_2)}\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)^TQ_2\\
&\times\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big). \qquad (32)
\end{aligned}
$$

$$\mathcal{L}V(x(t),y(t),t) \leq e^{\eta t}\xi^T(t)\Omega\xi(t)+e^{\widetilde{\eta}t}\zeta^T(t)\Xi\zeta(t), \tag{33}$$

where,

$$\begin{aligned}\xi(t) &= \Big[x^T(t),\bar{f}_1^T(y(t)),y^T(t-\mu_1(t)),\Big(\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big)^T,\Big(\int_{t-\sigma_1}^{t}x(s)\,\mathrm{d}s\Big)^T,\\ &\qquad \bar{g}_1^T(y(t-\mu_1(t))),x^T(t-\sigma_1),\bar{g}_1^T(y(t)),y^T(t),\bar{h}_1^T(y(t))\Big].\\ \zeta(t) &= \Big[y^T(t),\bar{f}_2^T(x(t)),x^T(t-\mu_2(t)),\Big(\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big)^T,\Big(\int_{t-\sigma_2}^{t}y(s)\,\mathrm{d}s\Big)^T,\\ &\qquad \bar{g}_2^T(x(t-\mu_2(t))),y^T(t-\sigma_2),\bar{g}_2^T(x(t)),x^T(t),\bar{h}_2^T(x(t))\Big].\end{aligned}$$

where Ω and Ξ are given by (18) and (19), respectively.

According to the conditions (18), (19) and (32), we can get that $\Omega<0$, $\Xi<0$; t $\in [t_{k-1},t_k)$, k $\in \mathbb{Z}_+$. Suppose $t=t_k$, we obtain

$$\begin{aligned}&V_1(x(t_k),y(t_k),t_k)-V_1(x(t_k^-),y(t_k^-),t_k^-)\\ &\quad = e^{\eta t_k}\Big\{\Big[B_k\beta_k(x(t_k^-))+C_k\gamma_k(x(t_k^- -\mu_1(t_k^-)))-A\int_{t_k-\sigma_1}^{t_k}x(s)\,\mathrm{d}s\Big]^T S\Big[B_k\beta_k(x(t_k^-))\\ &+C_k\gamma_k(x(t_k^- -\mu_1(t_k^-)))-A\int_{t_k-\sigma_1}^{t_k}x(s)\,\mathrm{d}s\Big]\Big\}+e^{\widetilde{\eta}t_k}\Big\{\Big[E_k\hat{\beta}_k(y(t_k^-))+Q_k\hat{\gamma}_k(y(t_k^-\\ &-\mu_2(t_k^-)))-D\int_{t_k-\sigma_2}^{t_k}y(s)\,\mathrm{d}s\Big]^T T\Big[E_k\hat{\beta}_k(y(t_k^-))+Q_k\hat{\gamma}_k(y(t_k^- -\mu_2(t_k^-)))-D\\ &\times\int_{t_k-\sigma_2}^{t_k}y(s)\,\mathrm{d}s\Big]\Big\}-e^{\eta t_k^-}\Big\{\Big[B_k\beta_k(x(t_k))+C_k\gamma_k(x(t_k-\mu_1(t_k)))-A\int_{t_k^- -\sigma_1}^{t_k^-}x(s)\,\mathrm{d}s\Big]^T\\ &\times S\Big[B_k\beta_k(x(t_k))+C_k\gamma_k(x(t_k-\mu_1(t_k)))-A\int_{t_k^- -\sigma_1}^{t_k^-}x(s)\,\mathrm{d}s\Big]\Big\}-e^{\widetilde{\eta}t_k^-}\Big\{\Big[E_k\hat{\beta}_k(y(t_k))\\ &+Q_k\hat{\gamma}_k(y(t_k-\mu_2(t_k)))-D\int_{t_k^- -\sigma_2}^{t_k^-}y(s)\,\mathrm{d}s\Big]^T T\Big[E_k\hat{\beta}_k(y(t_k))+Q_k\hat{\gamma}_k(y(t_k-\mu_2(t_k)))\\ &-D\int_{t_k^- -\sigma_2}^{t_k^-}y(s)\,\mathrm{d}s\Big]\Big\}.\end{aligned} \tag{34}$$

It is easy to compute that

$$\beta_k^T(x(t_k^-))B_k^TSB_k\beta_k(x(t_k^-)) \leq \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|B_k\|^2\max_{1\leq i\leq n}\{H_{k_i}^2\}x^T(t_k^-)Sx(t_k^-). \tag{35}$$

$$\hat{\beta}_k^T(y(t_k^-))E_k^TTE_k\hat{\beta}_k(y(t_k^-)) \leq \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|E_k\|^2\max_{1\leq j\leq n}\{M_{k_j}^2\}y^T(t_k^-)Ty(t_k^-). \tag{36}$$

$$\begin{aligned}&\gamma_k^T(x(t_k^- - \mu_1(t_k^-)))C_k^T S C_k \gamma_k(x(t_k^- - \mu_1(t_k^-)))\\&\qquad\qquad\leq \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|C_k\|^2 \max_{1\leq i\leq n}\{L_{k_i}^2\}x^T(t_k^- - \mu_1(t_k^-)).\end{aligned} \tag{37}$$

$$\begin{aligned}&\hat{\gamma}_k^T(y(t_k^- - \mu_2(t_k^-)))Q_k^T T Q_k \hat{\gamma}_k(y(t_k^- - \mu_2(t_k^-)))\\&\qquad\qquad\leq \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|Q_k\|^2 \max_{1\leq j\leq n}\{O_{k_j}^2\}y^T(t_k^- - \mu_2(t_k^-)).\end{aligned} \tag{38}$$

Thus we have,
$V_1(x(t_k),y(t_k),t_k) - V_1(x(t_k^-),y(t_k^-),t_k^-)$

$$\begin{aligned}
&\leq e^{\eta t_k}\Bigg\{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|B_k\|^2 \max_{1\leq i\leq n}\{H_{k_i}^2\}x^T(t_k^-)Sx(t_k^-) + \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|C_k\|^2 \max_{1\leq i\leq n}\{L_{k_i}^2\}\\
&\quad \times x^T(t_k^- - \mu_1(t_k^-))Sx(t_k^- - \mu_1(t_k^-)) + \Big(\int_{t_k-\sigma_1}^{t_k} x(s)\,\mathrm{d}s\Big)^T(A^T SA)\Big(\int_{t_k-\sigma_1}^{t_k} x(s)\,\mathrm{d}s\Big)\Bigg\}\\
&\quad - e^{\eta t_k^-}\Bigg\{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|B_k\|^2 \max_{1\leq i\leq n}\{H_{k_i}^2\}x^T(t_k)Sx(t_k) + \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|C_k\|^2 \max_{1\leq i\leq n}\{L_{k_i}^2\}\\
&\quad \times x^T(t_k - \mu_1(t_k))Sx(t_k - \mu_1(t_k)) + \Big(\int_{t_k^- -\sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big)^T(A^T SA)\Big(\int_{t_k^- -\sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big)\Bigg\}\\
&\quad e^{\widetilde{\eta} t_k}\Bigg\{\frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|E_k\|^2 \max_{1\leq j\leq n}\{M_{k_j}^2\}y^T(t_k^-)Ty(t_k^-) + \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|Q_k\|^2 \max_{1\leq j\leq n}\{O_{k_j}^2\}\\
&\quad \times y^T(t_k^- - \mu_2(t_k^-))Ty(t_k^- - \mu_2(t_k^-)) + \Big(\int_{t_k-\sigma_2}^{t_k} y(s)\,\mathrm{d}s\Big)^T(D^T TD)\Big(\int_{t_k-\sigma_2}^{t_k} y(s)\,\mathrm{d}s\Big)\Bigg\}\\
&\quad - e^{\widetilde{\eta} t_k^-}\Bigg\{\frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|E_k\|^2 \max_{1\leq j\leq n}\{M_{k_j}^2\}y^T(t_k)Ty(t_k) + \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|Q_k\|^2 \max_{1\leq j\leq n}\{O_{k_j}^2\}\\
&\quad \times y^T(t_k - \mu_2(t_k))Ty(t_k - \mu_2(t_k)) + \Big(\int_{t_k^- -\sigma_2}^{t_k^-} y(s)\,\mathrm{d}s\Big)^T(D^T TD)\Big(\int_{t_k^- -\sigma_2}^{t_k^-} y(s)\,\mathrm{d}s\Big)\Bigg\}.
\end{aligned} \tag{39}$$

Put

$$\begin{aligned}
\mathcal{A}_1 &= \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|B_k\|^2 \max_{1\leq i\leq n}\{H_{k_i}^2\}, \quad \mathcal{B}_1 = \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}\|C_k\|^2 \max_{1\leq i\leq n}\{L_{k_i}^2\},\\
\mathcal{A}_2 &= \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|E_k\|^2 \max_{1\leq j\leq n}\{M_{k_j}^2\}, \quad \mathcal{B}_2 = \frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\|Q_k\|^2 \max_{1\leq j\leq n}\{O_{k_j}^2\}.
\end{aligned}$$

Then, $V_1(x(t_k),y(t_k),t_k) - V_1(x(t_k^-),y(t_k^-),t_k^-)$

$$= e^{\eta t_k^-}\Bigg\{\mathcal{A}_1 x^T(t_k^-)K_1^T S K_1 x(t_k^-) - \mathcal{A}_1 x^T(t_k^-)Sx(t_k^-) + \mathcal{B}_1 x^T(t_k^- - \mu_1(t_k^-))K_1^T S$$

$$\times K_1x(t_k^- - \mu_1(t_k^-)) - \mathcal{B}_1x^T(t_k^- - \mu_1(t_k^-))Sx(t_k^- - \mu_1(t_k^-)) + \Big(\int_{t_k^- - \sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big)^T$$
$$\times (A^TK_1^TSK_1A)\Big(\int_{t_k^- - \sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big) - \Big(\int_{t_k^- - \sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big)^T(A^TSA)\Big(\int_{t_k^- - \sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big)\Big\}$$
$$\times e^{\widetilde{\eta}t_k^-}\Big\{\mathcal{A}_2y^T(t_k^-)\hat{K}_1^TT\hat{K}_1y(t_k^-) - \mathcal{A}_2y^T(t_k^-)Ty(t_k^-) + \mathcal{B}_2y^T(t_k^- - \mu_2(t_k^-))\hat{K}_1^TT\hat{K}_1$$
$$\times y(t_k^- - \mu_2(t_k^-)) - \mathcal{B}_2y^T(t_k^- - \mu_2(t_k^-))Ty(t_k^- - \mu_2(t_k^-)) + \Big(\int_{t_k^- - \sigma_2}^{t_k^-} y(s)\,\mathrm{d}s\Big)^T(D^T$$
$$\times \hat{K}_1^TT\hat{K}_1D)\Big(\int_{t_k^- - \sigma_2}^{t_k^-} y(s)\mathrm{d}s\Big) - \Big(\int_{t_k^- - \sigma_2}^{t_k^-} y(d)\,\mathrm{d}s\Big)^T(D^TTD)\Big(\int_{t_k^- - \sigma_2}^{t_k^-} y(s)\,\mathrm{d}s\Big)\Big\}$$
$$= e^{\eta t_k^-}\{\mathcal{A}_1x^T(t_k^-)[K_1^TSK_1 - S]x(t_k^-)\} + e^{\eta t_k^-}\{\mathcal{B}_1x^T(t_k^- - \mu_1(t_k^-))[K_1^TSK_1 - S]$$
$$\times x(t_k^- - \mu_1(t_k^-))\} + e^{\eta t_k^-}\Big\{\Big(\int_{t_k^-} x(s)\,\mathrm{d}s\Big)^TA^T[K_1^TSK_1 - S]A\Big(\int_{t_k^- - \sigma_1}^{t_k^-} x(s)\,\mathrm{d}s\Big)\Big\}$$
$$+ e^{\widetilde{\eta}t_k^-}\{\mathcal{A}_2y^T(t_k^-)[\hat{K}_1^TT\hat{K}_1 - T]y(t_k^-)\} + e^{\widetilde{\eta}t_k^-}\{\mathcal{B}_2y^T(t_k^- - \mu_2(t_k^-))[\hat{K}_1^TT\hat{K}_1 - T]$$
$$\times y(t_k^- - \mu_2(t_k^-))\} + e^{\widetilde{\eta}t_k^-}\Big\{\Big(\int_{t_k^-}^{t_k^-} y(s)\,\mathrm{d}s\Big)^TD^T[\hat{K}_1^TT\hat{K}_1 - T]D\Bigg(\int_{t_k^- - \sigma_2}^{t_k^-} y(s)\,\mathrm{d}s\Big)\Big\}$$
$$\leq 0.$$

$$V_1(x(t_k), y(t_k), t_k) \leq V_1(x(t_k^-), y(t_k^-), t_k^-), t = t_k, k \in \mathbb{Z}_+. \tag{40}$$

Therefore, we can deduce that

$$V(x(t_k), y(t_k), t_k) \quad \leq \quad V(x(t_k^-), y(t_k^-), t_k^-), t = t_k, k \in \mathbb{Z}_+. \tag{41}$$

By the equations (40) and (41), we know that V is monotonically non increasing for $t \in [T, \infty)$, which implies that

$$V(t) \quad \leq \quad V(T), \quad t \geq T. \tag{42}$$

From the definition of V(t) in (22) and (42), we get that

$$e^{\eta t}\lambda_{\min}(S)\|x(t)\|^2 + e^{\widetilde{\eta}t}\lambda_{\min}(T)\|y(t)\|^2 \quad \leq \quad V(t) \leq V_0 < \infty, t \geq 0, \tag{43}$$

where $V_0 = \max_{0\leq s\leq T} V(s)$. Put $e^{\eta^*t}\lambda_{\min}(\Lambda) = \min\{e^{\eta t}\lambda_{\min}(S), e^{\widetilde{\eta}t}\lambda_{\min}(T)\}$. Then (43) implies that

$$e^{\eta^*t}\lambda_{\min}(\Lambda)\{\|x(t)\|^2 + \|y(t)\|^2\} \quad \leq \quad V(t) \leq V_0 < \infty, \tag{44}$$

where $V_0 = \max_{0\leq s\leq T} V(s)$.

$$\|x(t)\|^2 + \|y(t)\|^2 \quad \leq \quad \frac{V_0}{\lambda_{\min}(\Lambda)}e^{-\eta^*t}, t \geq 0,$$

$$\|x(t)\|^2 + \|y(t)\|^2 \quad \leq \quad \chi e^{-\eta^* t}, t \geq 0, \tag{45}$$

where $\chi = \frac{V_0}{\lambda_{\min}(\Lambda)}$. It means that the unique equilibrium point (x^*, y^*) of neural networks (20) is exponentially stable. Hence, the neural networks (1)−(2) is exponentially stable. This completes the proof. □

Remark 2.4 As a result, the existence and uniqueness of the trivial solution for CVBAMNNs (1-2) have been proposed in Theorem 2.1 through contraction mapping theorem. For consequence, in Theorem 2.2, the time-delay dependent exponential stability criteria for neural networks (1) be entrenched by utilizing the novel Lyapunov−Krasovskii functionals.

Remark 2.5 Suppose, the stochastic disturbances are not appeared in neural networks (20) then the time-delayed neural networks (20) reduces to the following system

$$\begin{aligned}
\mathrm{d}x(t) &= \Big[- Ax(t-\sigma_1) + W_0 \bar{f}_1(y(t)) + W_1 \bar{g}_1(y(t-\mu_1(t))) + W_2 \int_{t-\rho_1}^{t} \bar{h}_1(y(s))\,\mathrm{d}s \Big] \\
&\quad \times \mathrm{d}t;\ t \neq t_k, t > 0, \\
x(t) &= B_k \beta_k(x(t^-)) + C_k \gamma_k(x(t^- - \mu_1(t^-))) = U_k^*(x(t_k^-), x_{t_k^-});\ t = t_k,\ k\ in\ \mathbb{Z}_+. \\
\mathrm{d}y(t) &= \Big[- Dy(t-\sigma_2) + V_0 \bar{f}_2(x(t)) + V_1 \bar{g}_2(x(t-\mu_2(t))) + V_2 \int_{t-\rho_2}^{t} \bar{h}_2(x(s))\,\mathrm{d}s \Big] \\
&\quad \times \mathrm{d}t; t \neq t_k, t > 0, \\
y(t) &= E_k \hat{\beta}_k(y(t^-)) + Q_k \hat{\gamma}_k(y(t^- - \mu_2(t^-))) = Y_k^*(y(t_k^-), y_{t_k^-});\ t = t_k,\ k \text{ in } \mathbb{Z}_+.
\end{aligned} \tag{46}$$

Then by Theorem 2.3, it is easy to obtain the following Corollary 2.6.

Corollary 2.6 Assume that Assumptions 1−5 hold. The equilibrium point of the system (46) is exponentially stable if, for given η, $\widetilde{\eta} > 0$, there exist positive definite Hermitian matrices S, T, P_1, P_2, Q_1, Q_2, positive diagonal matrices R, E, F, $\widetilde{F}$ and positive scalars η^*, λ, μ, μ_1 and μ_2 such that the following LMIs are satisfied

$$K_1^T S K_1 - S \ \leq\ 0, \tag{47}$$
$$\hat{K}_1^T T \hat{K}_1 - T \ \leq\ 0, \tag{48}$$

$$\Omega = \begin{bmatrix}
\Omega_{11}^* & SW_0 & 0 & 0 & \Omega_{15} & 0 & 0 & 0 & 0 \\
* & -L_1 & 0 & 0 & \Omega_{25} & 0 & 0 & 0 & 0 \\
* & * & \Omega_{33}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Omega_{44} & \Omega_{45} & 0 & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & \Omega_{56} & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{88}^* & 0 \\
* & * & * & * & * & * & * & * & \Omega_{99}
\end{bmatrix}_{9\times 9} < 0, \tag{49}$$

$$\Xi = \begin{bmatrix} \Xi_{11}^* & TV_0 & 0 & 0 & \Xi_{15} & 0 & 0 & 0 & 0 \\ * & -L_2 & 0 & 0 & \Xi_{25} & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33}^* & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88}^* & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} \end{bmatrix}_{9\times 9} < 0, \tag{50}$$

where

$\Omega_{11} = -SA - A^TS + SW_1R^{-1}W_1^TS + SW_2\hat{R}^{-1}W_2^TS, \quad \Omega_{33}^* = G_1RG_1, \quad \Xi_{99} = \rho_2^2Q_2,$
$\Omega_{15} = A^TSA - \eta SA, \quad \Omega_{25} = -A^TSW_0, \quad \Omega_{56} = -A^TSW_1, \quad \Omega_{45} = -A^TSW_2,$
$\Omega_{55} = \eta A^TSA, \quad \Omega_{88}^* = e^{-\eta t}(L_1^TFL_1), \quad \Omega_{44} = \hat{R} - e^{-\eta\rho_1}Q_1, \quad \Xi_{45} = -D^TTV_2,$
$\Omega_{77} = P_1 - e^{-\eta t}F, \quad \Omega_{66} = (1-\mu_1)e^{-\eta\mu_1}P_1, \quad \Xi_{55} = \widetilde{\eta}D^TTD, \quad \Xi_{56} = -D^TTV_1,$
$\Xi_{11}^* = -TD - D^TT + TV_1E^{-1}V_1^TT + TV_2\hat{E}^{-1}V_2^TT, \quad \Xi_{33}^* = G_2EG_2, \quad \Xi_{25} = -D^TTV_0,$
$\Xi_{15} = D^TTD - \widetilde{\eta}TD, \quad \Xi_{77} = P_2 - e^{-\widetilde{\eta}t}\widetilde{F}, \quad \Xi_{44} = \hat{E} - e^{-\widetilde{\eta}\rho_2}Q_2, \quad \Omega_{99} = \rho_1^2Q_1,$
$\Xi_{88}^* = e^{-\widetilde{\eta}t}(L_2^T\widetilde{F}L_2), \quad \Xi_{66} = (1-\mu_2)e^{-\widetilde{\eta}\mu_2}P_2.$

Proof.

Similar to the proof of Theorem 2.3, we can easily derive this Corollary. Its proof is straightforward and hence omitted. □

Remark 2.7 In case, the impulsive effects do not present in (20) then the proposed neural network system (20) can be reduced to the following form

$$\begin{aligned} \mathrm{d}x(t) &= \Big[-Ax(t-\sigma_1) + W_0\bar{f}_1(y(t)) + W_1\bar{g}_1(y(t-\mu_1(t))) + W_2\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big] \\ &\quad \times \mathrm{d}t + \bar{\delta}_1(x(t-\sigma_1), y(t), y(t-\mu_1(t))), t)\mathrm{d}\omega_1(t); \\ \mathrm{d}y(t) &= \Big[-Dy(t-\sigma_2) + V_0\bar{f}_2(x(t)) + V_1\bar{g}_2(x(t-\mu_2(t))) + V_2\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big] \\ &\quad \times \mathrm{d}t + \bar{\delta}_2(y(t-\sigma_2), x(t), x(t-\mu_2(t))), t)\mathrm{d}\omega_2(t);\ t > 0. \end{aligned} \tag{51}$$

Then the following Corollary 2.6 can be obtained from Theorem 2.3.

Corollary 2.8 Under the Assumptions 1−5, the equilibrium point of the NNs (51) is exponentially stable if for given η, $\widetilde{\eta} > 0$, there exist positive definite Hermitian matrices S, T, P_1, P_2, Q_1, Q_2, Positive diagonal matrices R, E, F, $\widetilde{F}$ and positive scalars η^*, λ, μ, μ_1 and μ_2 such that the following LMIs are satisfied

$$S < \lambda\, I, \tag{52}$$

$$T < \mu\, I, \tag{53}$$
$$\Omega \text{ and } \Xi < 0, \tag{54}$$

where the values of Ω and Ξ are derived in Theorem 2.3.

Remark 2.9 If the delays in leakage term are not considered then the neural network system (20) turns into to the following CVBAMNNs

$$\begin{aligned}
\mathrm{d}x(t) &= \Big[-Ax(t) + W_0\bar{f}_1(y(t)) + W_1\bar{g}_1(y(t-\mu_1(t))) + W_2\int_{t-\rho_1}^{t}\bar{h}_1(y(s))\,\mathrm{d}s\Big] \\
&\quad \times\mathrm{d}t + \bar{\delta}_1(x(t), y(t), y(t-\mu_1(t))), t)\mathrm{d}\omega_1(t);\ t > 0, t \neq t_k, \\
x(t) &= B_k\beta_k(x(t^-)) + C_k\gamma_k(x(t^- - \mu_1(t^-))) = U_k^*(x(t_k^-), x_{t_k^-});\ t = t_k,\ k \in \mathbb{Z}_+. \\
\mathrm{d}y(t) &= \Big[-Dy(t) + V_0\bar{f}_2(x(t)) + V_1\bar{g}_2(x(t-\mu_2(t))) + V_2\int_{t-\rho_2}^{t}\bar{h}_2(x(s))\,\mathrm{d}s\Big] \\
&\quad \times\mathrm{d}t + \bar{\delta}_2(y(t), x(t), x(t-\mu_2(t))), t)\mathrm{d}\omega_2(t);\ t > 0, t \neq t_k, \\
y(t) &= E_k\hat{\beta}_k(y(t^-)) + Q_k\hat{\gamma}_k(y(t^- - \mu_2(t^-))) = Y_k^*(y(t_k^-), y_{t_k^-});\ t = t_k,\ k \in \mathbb{Z}_+.
\end{aligned} \tag{55}$$

For system (55), by Theorem 2.2, we have the following Corollary 2.11.

Remark 2.10 According to Remark 2.7, the real and imaginary parts of the trajectory responses for neural networks (55) are depicted in Figure 9 and Figure 10, respectively. Therefore, by the state trajectories in Example 3.2, we can easily verified that the neural networks without leakage term (55) is stable in the sense of exponential.

Corollary 2.11 Suppose that Assumptions $1-5$ hold. The equilibrium point of the system (55) is exponentially stable if for given η, $\widetilde{\eta} > 0$, there exist positive definite Hermitian matrices S, T, P_1, P_2, Q_1, Q_2, positive diagonal matrices R, E, F, $\widetilde{F}$ and positive scalars η^*, λ, μ, μ_1 and μ_2 such that the following LMIs are satisfied

$$S < \lambda\, I, \tag{56}$$
$$T < \mu\, I, \tag{57}$$
$$K_1^T S K_1 - S \ \leq\ 0, \tag{58}$$
$$\hat{K}_1^T T \hat{K}_1 - T \ \leq\ 0, \tag{59}$$

$$\Omega = \begin{bmatrix}
\Omega_{11} & SW_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & -L_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Omega_{44} & 0 & 0 & 0 & 0 \\
* & * & * & * & \Omega_{55} & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{66} & 0 & 0 \\
* & * & * & * & * & * & \Omega_{77} & 0 \\
* & * & * & * & * & * & * & \Omega_{88}
\end{bmatrix}_{8\times 8} < 0, \tag{60}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & TV_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -L_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 \\ * & * & * & * & * & * & * & \Xi_{88} \end{bmatrix}_{8\times 8} < 0, \tag{61}$$

where

$$\Omega_{11} = -SA - A^T S + SW_1 R^{-1} W_1^T S + SW_2 \hat{R}^{-1} W_2^T S + \mu \hat{R}_2, \quad \Omega_{33} = G_1 R G_1 + \lambda R_3,$$
$$\Omega_{88} = \rho_1^2 Q_1, \quad \Omega_{77} = e^{-\eta t}(L_1^T F L_1) + \lambda R_2, \quad \Omega_{44} = \hat{R} - e^{-\eta \rho_1} Q_1, \quad \Xi_{88} = \rho_2^2 Q_2,$$
$$\Omega_{66} = P_1 - e^{-\eta t} F, \quad \Omega_{55} = (1-\mu_1) e^{-\eta \mu_1)} P_1, \quad \Xi_{77} = e^{-\widetilde{\eta} t}(L_2^T \widetilde{F} L_2) + \mu \hat{R}_2,$$
$$\Xi_{11} = -TD - D^T T + TV_1 E^{-1} V_1^T T + TV_2 \hat{E}^{-1} V_2^T T + \lambda R_2, \quad \Xi_{33} = G_2 E G_2 + \mu \hat{R}_3,$$
$$\Xi_{66} = P_2 - e^{-\widetilde{\eta} t} \widetilde{F}, \quad \Xi_{44} = \hat{E} - e^{-\widetilde{\eta} \rho_2} Q_2, \quad \Xi_{55} = (1-\mu_2) e^{-\widetilde{\eta} \mu_2} P_2.$$

Remark 2.12 The maximum admissible upper bounds of discrete time delays are provided in Table 1 together with some current findings in order to validate the benefits of this addressed neural networks (1). The maximum permitted time delays are determined based on observation. Thus, what actually occurs indicates that the little delays in our brains that cause the storage & passage of memories are not impacted because the neural networks that are built for this manner have enormous delays and are in stable state. In this manner, we use this kind of neural network technique to online network connections, ultimately obtaining the information without causing any harm over an extended period of time.

3. NUMERICAL EXAMPLES

In this section, providing two numerical examples with their simulations to illustrate the effectiveness and benefits of the proposed criteria.

Example 3.1. Consider a two-dimensional complex-valued BAM neural networks with leakage, mixed time delays and stochastic, impulsive effects (20) with the following associated parameters

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2.5 \end{pmatrix}, \; W_0 = \begin{pmatrix} 0.3+0.03i & 0.2+0.003i \\ -0.3+0.03i & 0.4+0.02i \end{pmatrix}, \; W_1 = \begin{pmatrix} -0.2+0.02i & 0.5+0.06i \\ 0.3+0.03i & 0.2+0.04i \end{pmatrix},$$

$$C_k = \begin{pmatrix} 0.4+0.02i & 0.2+0.06i \\ 0.1+0.02i & 0.2+0.03i \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad V_0 = \begin{pmatrix} 0.4+0.02i & 0.2+0.004i \\ -0.1+0.02i & 0.3+0.05i \end{pmatrix},$$

$$W_2 = \begin{pmatrix} 0.3+0.02i & 0.2+0.03i \\ -0.1+0.02i & 0.4+0.01i \end{pmatrix}, \quad B_k = \begin{pmatrix} 0.1+0.03i & 0.3+0.05i \\ 0.2+0.04i & 0.4+0.06i \end{pmatrix},$$

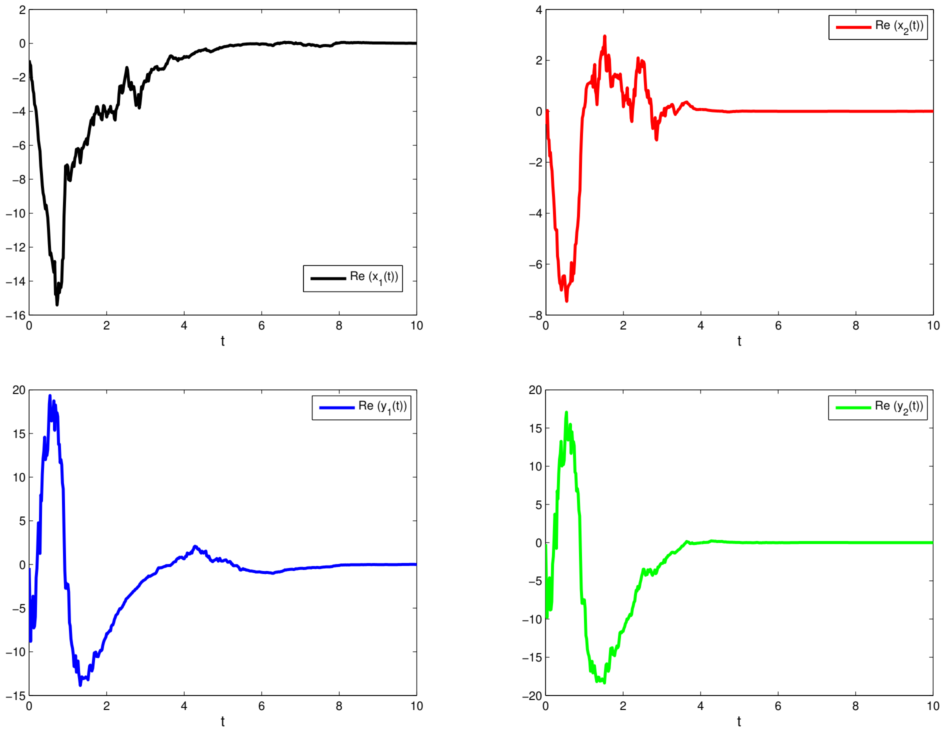

Fig. 1. The real part in the state responses $x_1(t)$, $x_2(t)$, $y_1(t)$ and $y_2(t)$ of (20).

$$V_1 = \begin{pmatrix} -0.3+0.01i & 0.3+0.07i \\ 0.2+0.04i & 0.1+0.03i \end{pmatrix}, \quad V_2 = \begin{pmatrix} 0.2+0.03i & 0.4+0.02i \\ -0.2+0.03i & 0.5+0.02i \end{pmatrix},$$

$$E_k = \begin{pmatrix} 0.2+0.05i & 0.5+0.01i \\ 0.3+0.02i & 0.3+0.05i \end{pmatrix}, \quad Q_k = \begin{pmatrix} 0.1+0.02i & 0.3+0.05i \\ 0.3+0.03i & 0.4+0.02i \end{pmatrix},$$

$$\bar{I} \;=\; [0.05\ 0.03], \quad \bar{J} = [0.02\ 0.04],$$

$\sigma_1 = 0.1$, $\sigma_2 = 0.3$, $\mu_1(t) = 0.3 * \sin(t) + 2.173$, $\mu_2(t) = 0.5 * \cos(t) + 1.973$, $\mu_1 = \mu_2 = 2.473$, $\rho_1 = 0.6, \rho_2 = 1.3$. Let

$$\begin{aligned} \delta_1(z(t-\sigma_1), \tilde{z}(t), \tilde{z}(t-\mu_1(t)), t) &= \begin{pmatrix} 0.3 * z(t-\sigma_1) & 0.2 * (\tilde{z}(t) + \tilde{z}(t-\mu_1(t))) \\ 0.2 * \tilde{z}(t) & 0.2 * z(t-\sigma_1) \end{pmatrix}, \\ \delta_2(\tilde{z}(t-\sigma_2), z(t), z(t-\mu_2(t)), t) &= \begin{pmatrix} 0.1 * \tilde{z}(t-\sigma_2) & 0.3 * (z(t) - z(t-\mu_2(t))) \\ 0.5 * z(t) & 0.3 * \tilde{z}(t-\sigma_2) \end{pmatrix}, \end{aligned}$$

$$h_k(\cdot) = 0.4 * tan(\cdot), \quad m_k(\cdot) = 0.02 * \sinh(\cdot), \quad l_k(\cdot) = 0.3 * tan(\cdot), \quad o_k(\cdot) = 0.04 * \sin(\cdot).$$

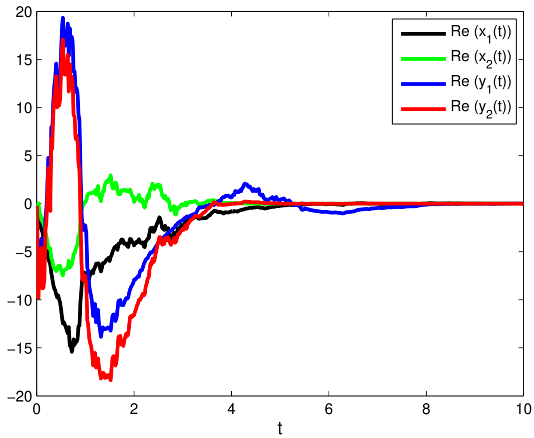

Fig. 2. Real part of the state trajectories for concerned complex-valued BAM neural networks (20).

Methods	$\mu_1 = \mu_2 > 0$	Status
In [15]	0.8696	feasible
In [16]	0.8784	feasible
In [49]	1.96	feasible
In [51]	0.8643	feasible
In [55]	0.2642	feasible
Theorem 2.3	2.473	feasible

Tab. 1. Maximum allowable upper bounds μ_1 & μ_2.

The following activation functions are playing a key role in neural networks (1):

$$\begin{aligned} f_1(\cdot) &= g_1(\cdot) = h_1(\cdot) = 0.01 * \tanh(\cdot) \\ \textit{and } f_2(\cdot) &= g_2(\cdot) = h_2(\cdot) = 0.01 * \sin(\cdot). \end{aligned}$$

From Theorem 2.3, we can conclude that the complex-valued BAM neural networks (1) − (2) is exponentially stable. Moreover, the simulation results for complex system (20) narrates in Figure 1 – Figure 4. Also, the impulsive effects of neural networks (20) is depicted in Figure 5 and Figure 6. By the above figures one can easily see that the effectiveness of our proposed theoretical experiments.

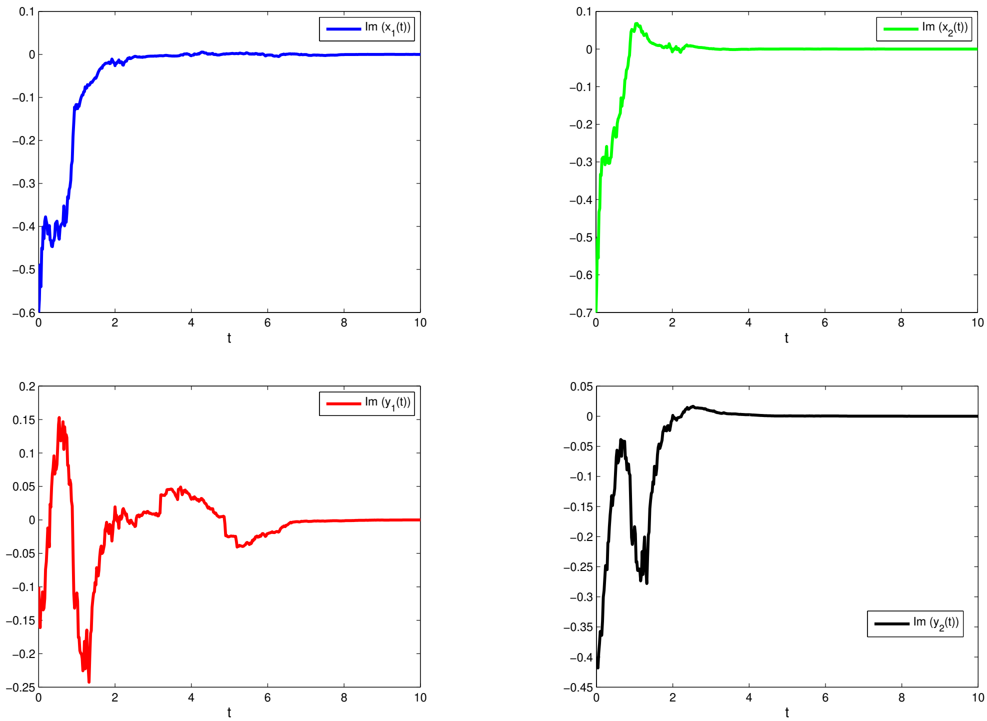

Fig. 3. The imaginary part in the state responses $x_1(t)$, $x_2(t)$, $y_1(t)$, $y_2(t)$ of (20).

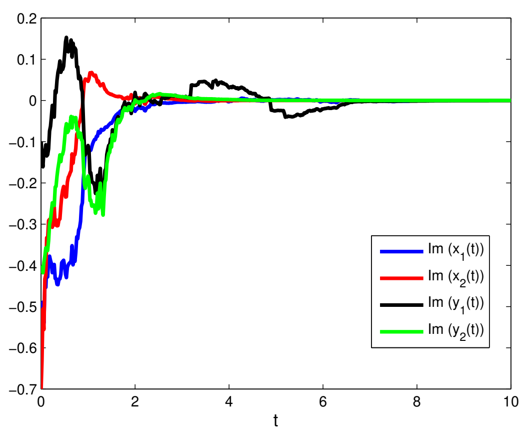

Fig. 4. Imaginary part of the state trajectories for concerned complex-valued BAM neural networks (20).

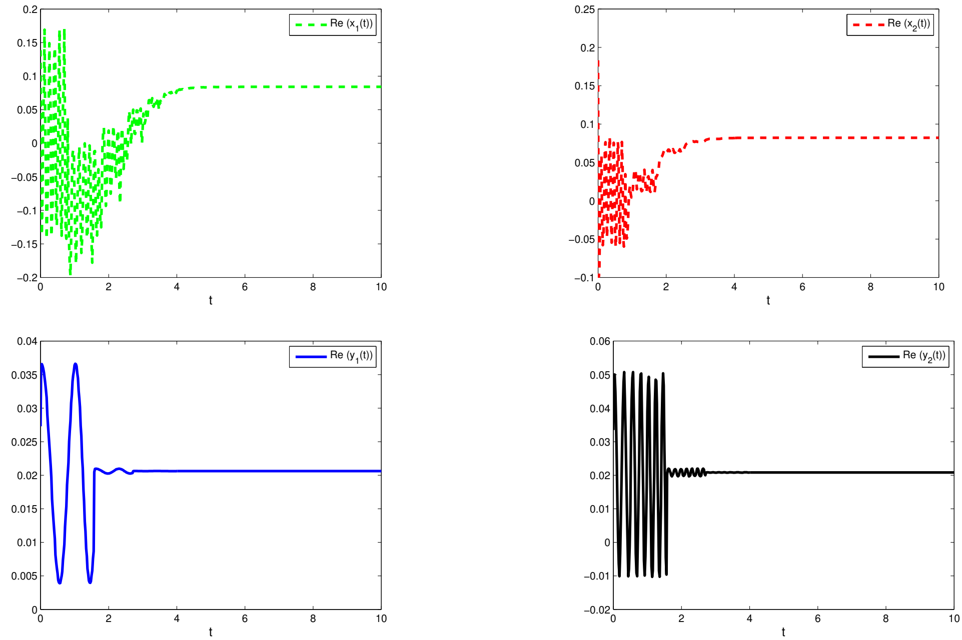

Fig. 5. The impulsive effects behavior of real part of the state trajectories in (20).

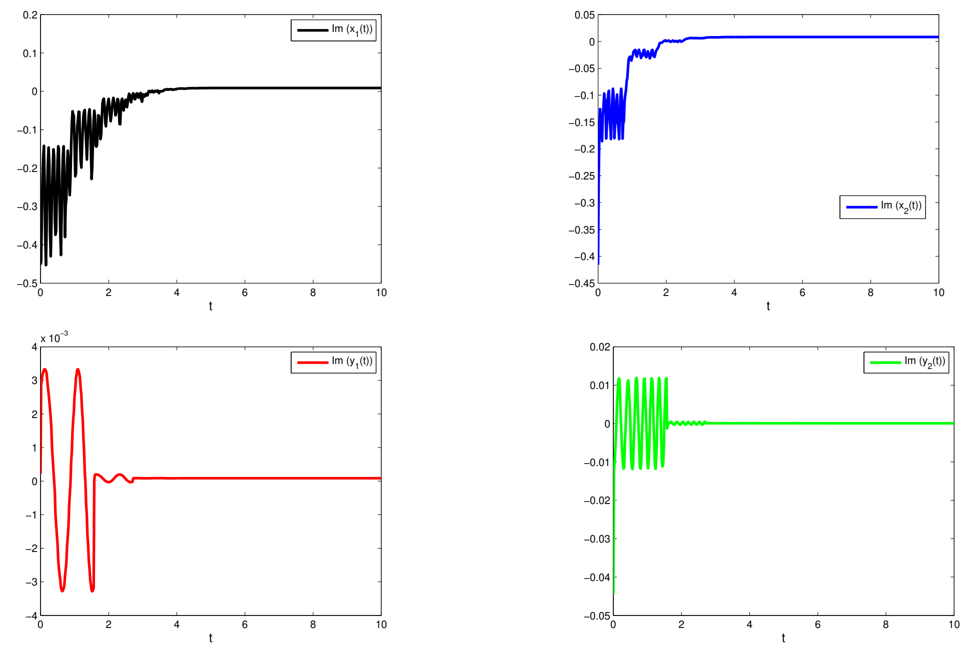

Fig. 6. The impulsive effects behavior of imaginary part of the state trajectories in (20).

Example 3.2. Consider a two-neuron time-delayed impulsive effects on complex-valued BAMNNs without stochastic noises (46) with the associated parameters as follows:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2.5 \end{pmatrix}, \quad W_0 = \begin{pmatrix} 0.4 + 0.02i & 0.1 + 0.05i \\ -0.2 + 0.04i & 0.1 + 0.06i \end{pmatrix}, \quad W_1 = \begin{pmatrix} -0.3 + 0.04i & 0.2 + 0.05i \\ 0.1 + 0.07i & 0.3 + 0.06i \end{pmatrix},$$

$$C_k = \begin{pmatrix} 0.1 + 0.09i & 0.4 + 0.03i \\ 0.2 + 0.01i & 0.3 + 0.03i \end{pmatrix}, \quad D = \begin{pmatrix} 1.6 & 0 \\ 0 & 2.3 \end{pmatrix}, \quad V_0 = \begin{pmatrix} 0.2 + 0.05i & 0.1 + 0.06i \\ -0.3 + 0.07i & 0.2 + 0.08i \end{pmatrix},$$

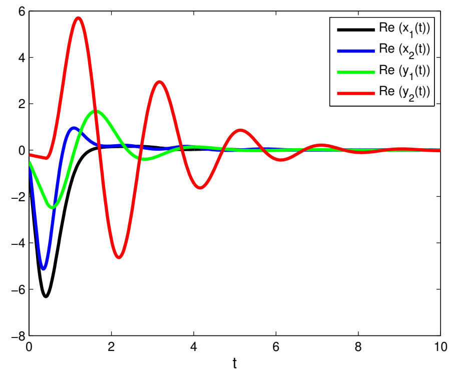

Fig. 7. Time responses and state trajectories of real part of the CVBAMNNs (46) without stochastic noises.

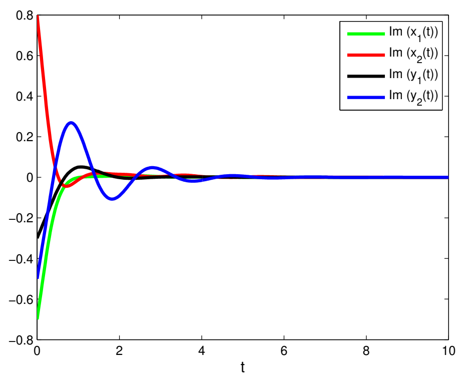

Fig. 8. Time responses and state trajectories of imaginary part of the CVBAMNNs (46) without stochastic noises.

$$W_2 = \begin{pmatrix} 0.2+0.03i & 0.1+0.06i \\ -0.2+0.08i & 0.2+0.04i \end{pmatrix}, \quad B_k = \begin{pmatrix} 0.3+0.07i & 0.1+0.04i \\ 0.3+0.05i & 0.2+0.08i \end{pmatrix},$$

$$V_1 = \begin{pmatrix} -0.5+0.03i & 0.4+0.06i \\ 0.3+0.05i & 0.2+0.05i \end{pmatrix}, \quad V_2 = \begin{pmatrix} 0.3+0.06i & 0.2+0.04i \\ -0.1+0.04i & 0.3+0.08i \end{pmatrix},$$

$$E_k = \begin{pmatrix} 0.4+0.07i & 0.1+0.06i \\ 0.2+0.08i & 0.4+0.09i \end{pmatrix}, \quad Q_k = \begin{pmatrix} 0.2+0.06i & 0.4+0.06i \\ 0.1+0.02i & 0.2+0.05i \end{pmatrix},$$

$$\bar{I} = [0.04\ 0.07], \quad \bar{J} = [0.03\ 0.05],$$

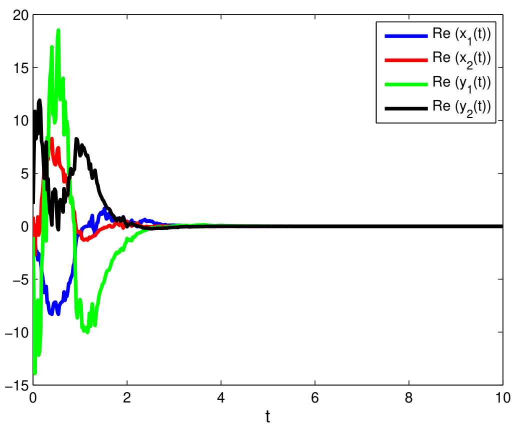

Fig. 9. Real part of the state trajectories for neural networks (46) without leakage.

$\sigma_1 = 0.2$, $\sigma_2 = 0.4$, $\mu_1(t) = 0.4 * \sin(t) + 0.4$, $\mu_2(t) = 0.7 * \cos(t) + 1.4$, Let

$$\delta_1(z(t-\sigma_1), \tilde{z}(t), \tilde{z}(t-\mu_1(t)), t) = \begin{pmatrix} 0.3 * z(t-\sigma_1) & 0.2 * (\tilde{z}(t) + \tilde{z}(t-\mu_1(t))) \\ 0.2 * \tilde{z}(t) & 0.2 * z(t-\sigma_1) \end{pmatrix},$$

$$\delta_2(\tilde{z}(t-\sigma_2), z(t), z(t-\mu_2(t)), t) = \begin{pmatrix} 0.1 * \tilde{z}(t-\sigma_2) & 0.3 * (z(t) - z(t-\mu_2(t))) \\ 0.5 * z(t) & 0.3 * \tilde{z}(t-\sigma_2) \end{pmatrix},$$

Taking the functions $h_k(\cdot)$, $m_k(\cdot)$, $l_k(\cdot)$, $o_k(\cdot)$. and also the activation functions $f_1(\cdot)$, $g_1(\cdot)$, $h_1(\cdot)$, $f_2(\cdot)$, $g_2(\cdot)$ and $h_2(\cdot)$ are same as in Example 3.1.

Hence by Corollary 2.6, the complex-valued BAM neural networks without stochastic disturbances (46) is exponentially stable, which is further clarified by the simulation outcomes shown in Figure 7 and Figure 8. The real and imaginary trajectories for (46) in above figures demonstrate the reality of our developed methods.

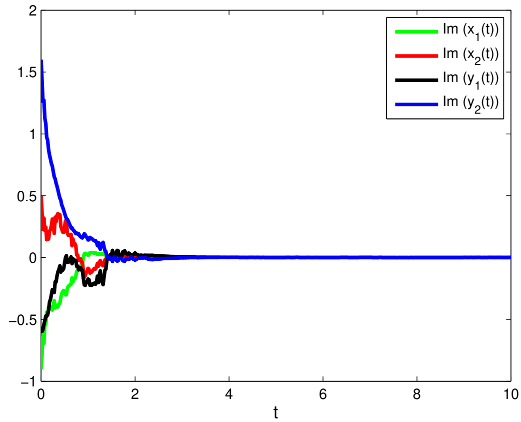

Fig. 10. Imaginary part of the state responses for concerned NNs (46) without leakage.

4. CONCLUSIONS

In this study, we have addressed the leakage, temporal delays, and stochastic, impulsive effects exponential stability problem for complex-valued BAM neural networks. We guarantee the existence and uniqueness of the equilibrium point by utilizing the contraction mapping theorem, the M-matrix technique, and fixed point theory. Furthermore, several new-brand sufficient conditions with less conservatism have been obtained in terms of LMIs to ensure the exponential stability for CVBAMNNs with proposed external disturbances by utilizing the stability theory, inequality techniques, Schur complement lemma, and construction of an appropriate Lyapunov − Krasovskii functional. The remainder of this work consists of two numerical examples with simulations that support the effectiveness of the implemented approach and allow us to verify the validity of our stated theoretical findings.

To the best of our knowledge, there are no results on the passivity of time-delayed complex-valued Cohen-Grossberg neural networks via Wirtinger based inequality, which might be our future research work.

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