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QUANTIZED COOPERATIVE OUTPUT REGULATION OF CONTINUOUS-TIME MULTI-AGENT SYSTEMS OVER SWITCHING GRAPH

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This paper investigates the problem of quantized cooperative output regulation of linear multi-agent systems with switching graphs. A novel dynamic encoding-decoding scheme with a finite communication bandwidth is designed. Leveraging this scheme, a distributed protocol is proposed, ensuring asymptotic convergence of the tracking error under both bounded and unbounded link failure durations. Compared with the existing quantized control work of MASs, the semi-global assumption of initial conditions is not required, and the communication graph is only required to be jointly connected. Finally, two simulation examples demonstrate the effectiveness of the proposed distributed protocol for bounded and unbounded link failure durations.

Keywords: cooperative output regulation, switching graph, jointly connected graph, unbounded link failure duration, multi-agent systems, quantized control

Classification: 90C33, 68W15

1. INTRODUCTION

Cooperative output regulation [9] has become an active and key cooperative control research topic in multi-agent systems (MASs) [8, 21, 25, 1] in recent years because of its widespread application in attitude formation, sensor networks, and other engineering [24, 26, 7, 30]. The objective of cooperative output regulation is to achieve both reference tracking and disturbance rejection by a distributed control protocol. Further, the state or output of the external system is known only to some agents, and each agent can only interact with its neighbors.

In practical communication networks, due to the limited capacity of the communication channels, the finite bandwidth constraint cannot be neglected. Hence, recently, some researchers started to design a suitable quantization coding scheme to study the cooperative control problem of MASs in order to address the communication restrictions and effectively reduce the network bandwidth consumption during data transmission. For example, in [3, 16, 19, 11], the average consensus algorithms were designed based on logarithmical quantizers, which can converge to the average value of the starting state of the node. However, the quantization levels of the logarithmical quantizer are infinite,

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which implies that the required communication bandwidth is thus infinite. Hence, in [6, 10], the authors used the uniform quantizer with finite quantization levels to design consensus algorithms, where the states of all agents can only converge to a small region whose size is dependent on the quantization precision. Using dynamic encoding-decoding schemes based on a decaying scaling function strategy, the quantized consensus problem was solved in [12, 29, 14] and it was shown that the asymptotic consensus can be achieved. However, the communication graph of the MASs is required to be balanced in [12, 29], and the semi-global assumptions on the initial states is required in [12, 14].

In the actual motion of MAS, due to the limitations of communication sensing range and external interference, its network topology will not remain fixed, and the connectivity between nodes and their neighbors often changes with time. Therefore, the study of exchange topology has substantial significance and value. There have been some achievements in relevant research on collaborative output regulation based on switching topology [20, 2, 22, 17, 15], but the influence of quantitative constraints has not been considered. [27] studies the discrete-time distributed quantization problem on the switch graph under the bounded network link failure duration. In fact, this condition is also required in most of the existing quantitative cooperative control of MASs over switch graphs [29, 14, 27, 18, 28, 5, 13]. It requires an upper bound for the link failure duration in order to design the dynamic encoding-decoding strategy with convergent quantization error.

This study considers the global cooperative output regulation problem of continuous-time linear MASs over switching graphs under the communication constraint of constrained bandwidth. The contributions can be summarized as follows:

- The cooperative output regulation problem over switching graphs with a bounded link failure duration is considered. Compared to previous works on quantized control of MASs over switching graphs [12, 14], our result eliminates the semi-global assumption on initial conditions and the requirement for the switching graph to be balanced at all instants t. In addition, the needed bandwidth can be any positive integer, and the number of quantization levels is independent of the link failure time upper bound. This implies that the upper bound of link failure duration is not required in the design of the quantizer for all agents.

- Consideration is given to the cooperative output regulation problem over switching graphs with unbounded link failure duration. In such contexts, the time interval between two successive sampling instances can be indefinite, significantly complicating the analysis of the estimation error that arises due to the constraints imposed by limited bandwidth.

- Our proposed protocol does not need any global information of the communication graph or the initial states.

The remaining sections of this paper are arranged as below.In Section 2, we formulate the quantized cooperative output regulation over switching graph for MASs, and give some definitions. While Section 3 proposes a distributed protocol and provides the main results in Section 4. Then Section 5 provides numerical simulations for illustration. Finally, Section 6 gives some concluding remarks.

2. PROBLEM FORMULATION

2.1. Communication network notations

A weighted graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ can describe the information communication topology among agents, where $\mathcal{V} \triangleq \{0, 1, \ldots, N\}$ is a non-empty finite set representing the node set of n agents, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. Note that an edge of $(j, i) \in \mathcal{E}$ if the information can be exchanged between jth agent and ith agent, while $(j, i) \notin \mathcal{E}$, otherwise. Denote $\mathcal{A} \triangleq [a_{ij}]_{(N+1))\times(N+1)}$ as a weighted adjacency matrix of graph $\mathcal{G}$, where $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The set of neighbors of agent j is defined as $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. The graph Laplacian matrix $\mathcal{L} = [l_{ij}]_{N\times N}$ is defined as $l_{ii} = \sum_{j\neq i}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$. If the graph $\mathcal{G}$ is connected, then all eigenvalues of $\mathcal{L}$ can be arranged in an ascending order $0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \leq \ldots \leq \lambda_N(\mathcal{L})$.

Denote $\mathcal{G}_{\sigma(t)} \triangleq \mathcal{G}(\mathcal{V}, \mathcal{E}_{\sigma(t)})$ as a switching topology, where $\sigma(t) : [0, +\infty) \to \mathcal{P} = \{1, \ldots, p\}$ represents a piece-wise constant switching signal. The notation $\{\mathcal{G}_p : p \in \mathcal{P}\}$ represents the subgraph on agent $\{1, \ldots, N\}$, where $\mathcal{P}$ is the index set of all graphs. The adjacency matrix of the switching graph $\mathcal{G}_{\sigma(t)}$ is denoted as $\mathcal{A}_{\sigma(t)} \triangleq [a_{ij}(t)]_{(N+1)\times(N+1)}$, where $a_{ij}(t)$ represents the weight value between the ith agent and the jth agent at time instant t.

Consider an infinite sequence of nonempty time intervals $[t_0, t_1), [t_1, t_2), \ldots, [t_n, t_{n+1})$, $\ldots$, where $t_0 = 0$ and t_n, $n = 1, 2, \ldots$ represent the switching instants, satisfying $\inf_{n\in\mathbb{N}^+} t_{n+1} - t_n = \tau > 0$. The switching communication graph $\mathcal{G}_{\sigma(t)}$ is deemed jointly connected if, for any given $t \geq 0$, the union graph $\mathcal{G}_{[t,t+T_0)}$ contains a spanning tree. This condition implies the existence of a path from the leader agent to each follower agent in $\bigcup_{n=0}^{n} \mathcal{G}_{\sigma(n)}$. Notably, a jointly connected graph can be disconnected at any specific time instant.

Furthermore, this paper provides a formal definition of "link failure duration", which is central to our discussion.

Definition 2.1. (Link Failure Duration) The link failure duration of the communication process between agent i and agent j,

$$T_{i,j}^{1} = \inf\{t \geq 0 | \int_{t'}^{t'+t} a_{ij}(\tau)\,\mathrm{d}\tau > 0 \text{ holds } \forall\, t' \geq 0\}. \tag{1}$$

A switching graph $\mathcal{G}_{\sigma(t)}$ is considered to have a bounded link failure duration if $\sup\{T_{i,j}^{1}, \forall\, (j, i) \in \mathcal{E}_1\} < \infty$, and an unbounded link failure duration if $\sup\{T_{i,j}^{1}, \forall\, (j, i) \in \mathcal{E}_1\} = \infty$, where $\mathcal{E}_1 = \lim_{t\to\infty} \bigcup_{[t,\infty)} \mathcal{E}_{\sigma(t)}$.

It is noteworthy that in most existing studies on quantized cooperative control of MASs over switching graphs, such as [12, 14, 28, 5], the switching graph $\mathcal{G}_{\sigma(t)}$ is assumed to have a bounded link failure duration. For further details, refer to [12, Assumptions 4], [14, Assumption A5], [28, Assumption 2], and [5, Assumption 6].

2.2. The problem formulation

Consider a group of agents whose dynamics can be described as follows:

$$\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + E_i v(t), \\
e_i(t) &= C_i x_i(t) + D_i u_i(t) + F_i v(t), \\
y_{mi}(t) &= C_{mi} x_i(t) + D_{mi} u_i(t) + F_{mi} v(t),\ i = 1, \dots, N,
\end{aligned} \tag{2}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, $e_i(t) \in \mathbb{R}^{p_i}$ and $y_{mi}(t) \in \mathbb{R}^{p_{mi}}$ represent the state, control input, tracking error and measurement output of agent i, respectively. The matrices A_i, B_i, E_i, C_i, D_i, F_i, C_{mi}, D_{mi} and F_{mi} are of compatible dimensions.

The exogenous signal $v(t) = col(r(t), d(t)), v(t) \in \mathbb{R}^v$ represents the reference input $r(t)$ to be tracked and/or the disturbance $d(t)$ to be rejected. This signal is generated by the following exo-system:

$$\dot{v}(t) = Sv(t), \tag{3}$$

where the matrix $S \in \mathbb{R}^{q \times q}$ has a compatible dimension.

The formal definition for communication bandwidth is given as follows.

Definition 2.2. (Communication Bandwidth) The communication bandwidth in communication process between each pair of agents, or exo-system and agents, is defined as follows:

$$\mathcal{B} = \lim_{t \to \infty} \sup \frac{1}{t} \sum_{t_\varpi \le t} r(\varpi)\ bits/sec, \tag{4}$$

where t_ϖ, $\varpi \in \mathbb{N}$ are the sampling instants, $r(\varpi)$ is the bits of data required to be transmitted at t_ϖ.

Then, the formal problem statement of the cooperative output regulation problem over switching graphs with bounded and unbounded link failure duration is presented in this paper.

Problem 1. Given a MAS composed of equations (2) and (3), design distributed protocols for a switching graph with both bounded and unbounded link failure durations, respectively, such that the following two conditions are ensured for any positive communication bandwidth $\mathcal{B}$:

1. The origin of the closed-loop system is asymptotically stable when $v = 0$.

2. For any initial condition of the system,

$$\lim_{t \to \infty} e_i(t) = 0,\ i = 1, \dots, N. \tag{5}$$

To proceed further, the following assumptions are needed.

Assumption 1. The switching instants t_n, $n = 1, 2, \dots$ satisfy $\inf_{n \in \mathbb{N}^+} t_{n+1} - t_n = \tau_0 > 0$.

Assumption 2. There exists a $T_0 > 0$ such that for any $t \geq 0$, the union graph $\mathcal{G}_{[t,t+T_0)}$ contains a spanning tree.

Assumption 3. All eigenvalues of S have nonpositive real parts.

Assumption 4. The pairs $(A_i,\ B_i),\ i = 1, \dots, N$, are stabilizable.

Assumption 5. The pairs $(C_{mi},\ A_i),\ i = 1, \dots, N$, are detectable.

Assumption 6. The matrix equations

$$\begin{aligned} \Psi_i S &= A_i \Psi_i + B_i \Pi_i + E_i, \\ 0 &= C_i \Psi_i + D_i \Pi_i + F_i,\ i = 1, \dots, N, \end{aligned} \tag{6}$$

have solution pairs (Ψ_i, Π_i).

Remark 2.3. Noting that Assumptions $1-2$, $4-5$ are typical in addressing the linear cooperative control problem of MASs across switching graphs, as referenced in [22, 17, 15]. Equation (6) is referred to as the regulator equation. The existence of this equation is a necessary condition for solving the cooperative output regulation problem. According to [9], the regulator equation (6) is solvable if the following condition is satisfied:

$$\text{Rank} \begin{bmatrix} A_i - \lambda I & B_i \\ C_i & D_i \end{bmatrix} = n_i + p, \tag{7}$$

for any $\lambda \in \sigma(S)$, where $\sigma(S)$ denotes the spectrum of S.

Remark 2.4. It should be noted that Assumption 3 is not required in [8, 21]. Despite the stricter nature of Assumption 3, it still allows for the generation of a wide range of signals, including sinusoidal signals and unbounded signals that exhibit polynomial growth.

3. DESIGN OF DISTRIBUTED PROTOCOL

In this section, the cooperative output regulation problem over switching graphs is addressed, considering both scenarios of bounded and unbounded link failure durations, respectively.

The design of the distributed protocol is outlined as follows:

$$\begin{aligned} u_i(t) &= K_{1i} z_i(t) + K_{2i} \epsilon_i(t),\ i = 1, \dots, N, \\ \dot{\epsilon}_i(t) &= S\epsilon_i(t) + \sum_{j=0}^{N} a_{ij}(t) \Big(\widehat{\epsilon}_j^i(t) - \epsilon_i(t) \Big), \\ \dot{z}_i(t) &= A_i z_i(t) + Bu_i(t) + E_i \epsilon_i(t) \\ &\quad + L_i(C_{mi} z_i(t) + D_{mi} u_i(t) + F_{mi} \epsilon_i(t) - y_i(t)), \end{aligned} \tag{8}$$

where $\epsilon_i(t) \in \mathbb{R}^v$, $\epsilon_0(t) = v(t)$, $\widehat{\epsilon}_j^i(t) \in \mathbb{R}^v$ is the estimation of $\epsilon_j(t)$ by agent i, the gain matrices $K_{1i},\ i = 1, \dots, N$, satisfy that $A_i + B_i K_{1i}$ are Hurwitz, $K_{2i} = \Pi_i - K_{1i}\Psi_i,\ i = 1, \dots, N$, $(\Psi_i, \Pi_i),\ i = 1, \dots, N$, are the solutions of (6), and $L_i,\ i = 1, \dots, N$, satisfy that $A_i + L_i C_{mi}$ are Hurwitz.

3.1. Switching graphs over bounded link failure duration

Due to the limited capacity of actual communication channels, data exchanged between agents must be quantized before transmission, ensuring that only a finite number of bits are sent. The complete transmission process includes a sampler, an encoder Φ_j^i, a decoder Γ_j^i, and a scaling function $l_j^i(\cdot)$. Denote the sampling instants of the communication process from agent j to agent i as $t_j^i(\varpi),\ \varpi \in \mathbb{N}$. The real value $\epsilon_j(t)$ from agent j in the communication channel (j, i) is sampled at the instants $t_j^i(\varpi)$, obtaining $\epsilon_j(t_j^i(\varpi))$.his information is then encoded into a binary sequence $s_j^i(\varpi)$ by the encoder and broadcast over the channel to the neighboring agent i. Upon receiving the finite bits of data $s_j^i(\varpi)$, the ith agent retrieves the estimated value $\hat{\epsilon}_j^i(t)$ of agent $\epsilon_j(t)$ through the decoder. The scaling function $l_j^i(\cdot)$ critical for signal decomposition during encoding and decoding, as well as throughout the transmission process.

Next, the specific forms of the quantizer and scaling function used in this paper are presented. These formulations are universally applicable in scenarios with both bounded and unbounded link failure durations

Let $\mathcal{X} \in \mathbb{R}$ represent the original scalar signal before quantization. The quantizer $q(\cdot)$, a function designed for scalar input values, is defined as

$$q(\mathcal{X}) = \begin{cases} 2^{\alpha-1}, & \mathcal{X} > 2^{\alpha-1}, \\ n-1, & n-1 < \mathcal{X} \le n,\ 2 < n \le 2^{\alpha-1}, \\ 0, & 0 \le \mathcal{X} < 1, \\ -q(-\mathcal{X}), & \mathcal{X} < 0. \end{cases} \tag{9}$$

where $\alpha \in \mathbb{N}^+$ can be chosen as any integer greater than 2, and $n = 1, 2, \ldots, N$. Moreover, we define multi-quantizer as $Q(\cdot) = [q(\cdot), \ldots, q(\cdot)]^T \in \mathbb{R}^m$.

Remark 3.1. The number of the quantization levels in (9) is $2^\alpha + 1$. Since zero level in (9) is not necessary to be transmitted, the quantizer output $s_j^i(n) \in \mathbb{R}^\upsilon$ can be represented by α-bits data.

Let $s_{jk}^i(\varpi)$ be the $k-$th element of $s_j^i(\varpi)$, where $1 \le k \le \upsilon,\ i \in \mathcal{V},\ \varpi \in \mathbb{N}$. The scaling function $l_{jk}^i(\cdot)$ is designed as

$$l_{jk}^i(0) = c,\ j \in \mathcal{V},\ 1 \le k \le \upsilon, \tag{10}$$

$$l_{jk}^i(1) = \begin{cases} 2l_{jk}^i(0),\ \text{if } |s_{jk}^i(0)| = 2^{\alpha-1}, \\ l_{jk}^i(0)/2,\ \text{if } |s_{jk}^i(0)| = 0, \\ l_{jk}^i(0),\ \text{otherwise} \end{cases} \tag{11}$$

$$l_{jk}^i(\varpi+1) = \begin{cases} 2l_{jk}^i(\varpi),\ if\ |s_{jk}^i(\varpi)| = |s_{jk}^i(\varpi-1)| = 2^{\alpha-1}, \\ \phi(t_j^i(\varpi+1)),\ \text{if } |s_{jk}^i(\varpi)| = 2^{\alpha-1}, \\ \qquad\qquad \text{and } |s_{jk}^i(\varpi-1)| < 2^{\alpha-1}, \\ l_{jk}^i(\varpi)/2,\ \text{if } |s_{jk}^i(\varpi)| = 0, \\ l_{jk}^i(\varpi),\ \text{otherwise},\ \varpi \ge 2, \end{cases} \tag{12}$$

where

$$\phi(t) = ce^{-bt^a}, \ t \geq 0, \tag{13}$$

b and c are two positive constants, a is a constant satisfying $0 < a \leq 1/2$. For simplicity, we choose $a = \frac{1}{k_1}$, and k_1 is a positive integer greater than 1. Moreover, we define the vector version of the scaling function $\overline{L}_j^i(\varpi) = \text{diag}(l_{j1}^i(\varpi), \dots, l_{jv}^i(\varpi))$.

The sampler design of switching graphs over bounded link failure duration $t_j^i(\varpi)$ is designed as follows:

$$\begin{aligned} t_j^i(0) &= \inf\{t \geq 0 | a_{ij}(t) > 0\}, \\ t_j^i(\varpi + 1) &= \inf\{t \geq t_j^i(\varpi) + T_{(i,j)}^0 | a_{ij}(t) > 0\}, \end{aligned} \tag{14}$$

with $T_{(i,j)}^0$ being a positive constant satisfying $T_{(i,j)}^0 \geq \frac{v\alpha}{\mathcal{B}}$.

Remark 3.2. Since $T_{(i,j)}^0 \geq \frac{v\alpha}{\mathcal{B}}$, the required bandwidth in the communication process is not greater than $\mathcal{B}$-bit/sec.

Next, we give the design of the encoder Φ_j^i. The encoder Φ_j^i associated with j for the channel $(j, i) \in \mathcal{E}$ and quantizer output is designed follows:

$$s_j^i(\varpi) \triangleq Q\left(\overline{L}_j^{i^{-1}}(\varpi)(e^{-St_j^i(\varpi+1)}\epsilon_j(t_j^i(\varpi + 1)) - e^{-St_j^i(\varpi)}\widehat{\epsilon}_j^i(t_j^i(\varpi))\right). \tag{15}$$

The decoder Γ_j^i associated with i for the $(j, i) \in \mathcal{E}$ with distributed protocol is designed as follows:

$$\begin{cases} \widehat{\epsilon}_j^i(t_j^i(0)) = e^{St_j^i(0)}\phi(t_j^i(0))Q\left(e^{St_j^i(0)}\epsilon_j(0)/\phi(t_j^i(0))\right), \\ \widehat{\epsilon}_j^i(t) = e^{S(t-t_j^i(\varpi))}\widehat{\epsilon}_j(t_j^i(\varpi)), \ t_j^i(\varpi) \leq t < t_j^i(\varpi + 1), \ \varpi \in \mathbb{N}, \\ \widehat{\epsilon}_j^i(t_j^i(\varpi + 1)) = e^{S(t_j^i(\varpi+1)-t_j^i(\varpi))}\widehat{\epsilon}_j^i(t_j^i(\varpi)) + \overline{L}_j^i(\varpi)s_j^i(\varpi). \end{cases} \tag{16}$$

3.2. Switching graphs over unbounded link failure duration

In this subsection, we define the sampling times and outline the design of the encoder and decoder for the problem of a switching graph with an unbounded link fault duration. Under these conditions, the sampling interval of the communication process cannot be bounded, complicating the design of the estimator.

The time sequence $t_m, \ m \in \mathbb{N}$ consists of a set of time instants, designed as a polynomial function of m satisfying the following conditions:

$$t_0 = 0, \ \lim_{m\to\infty} t_m m^{-1/a} = \infty. \tag{17}$$

For simplicity, we choose $t_m = d(m - 1)^{a_1}$, $d > 0$ is a constant, and a_1 is a positive integer greater than $1/a$.

Similarly, the estimated value $\widehat{\epsilon}_j^i(t)$ of the distributed protocol (8) necessitates a more complex encoder and decoder design under unbounded link failure duration conditions.

The design of the encoder is presented first as follows:

$$\begin{cases} \varsigma_j^i(t_j^i(\varpi)) = l(t_j^i(\varpi))Q\Big(e^{-St_j^i(\varpi)}\epsilon_j(t_j^i(\varpi))/l(t_j^i(\varpi))\Big), \varpi = 0, \\ \qquad\qquad \text{or } \varpi > 0, t_j^i(\varpi - 1) < t_m \leq t_j^i(\varpi), m \in \mathbb{N}^+, \\ \widehat{\varsigma}_j^i(t_j^i(\varpi + 1)) = \widehat{\varsigma}_j^i(t_j^i(\varpi)) + \overline{L}_j^i(\varpi)s_j^i(\varpi), \end{cases} \tag{18}$$

where the sampling instant $t_j^i(\varpi)$, $\varpi \in \mathbb{N}$, are designed as in (14). $\varsigma_j^i(t)$ is the intermediate variable, and its estimate is expressed in $\widehat{\varsigma}_j^i(t)$. Then, the quantified output with the sampling instants $t_j^i(\varpi)$ takes the following form:

$$s_j^i(\varpi) = \begin{cases} Q\Big(\overline{L}_j^{i^{-1}}(\varpi)(\min\{e^{-St_j^i(\varpi')}\epsilon_j(t_j^i(\varpi')), \\ \qquad t_m \leq t_j^i(\varpi') \leq t_j^i(\varpi + 1))\} - \widehat{\varsigma}_j^i(t_j^i(\varpi))\Big) \\ \qquad\quad t_m \leq t_j^i(\varpi) < t_j^i(\varpi + 1) < t_{m+1}, m \text{ is odd}, \\ Q\Big(\overline{L}_j^{i^{-1}}(\varpi)(\max\{e^{-St_j^i(\varpi')}\epsilon_j(t_j^i(\varpi')), \\ \qquad t_m \leq t_j^i(\varpi') \leq t_j^i(\varpi + 1))\} - \widehat{\varsigma}_j^i(t_j^i(\varpi))\Big) \\ \qquad\quad t_m \leq t_j^i(\varpi) < t_j^i(\varpi + 1) < t_{m+1}, m \text{ is even}. \end{cases} \tag{19}$$

Secondly, the design of the decoder is given as follows:

$$\begin{cases} \widehat{\epsilon}_j^i(t_j^i(\varpi)) = e^{St_j^i(\varpi)}\varsigma_j^i(t_j^i(\varpi)), \varpi = 0 \\ \qquad\qquad \text{or } \varpi > 0, t_j^i(\varpi - 1) < t_m \leq t_j^i(\varpi), m \in \mathbb{N}^+, \\ \widehat{\epsilon}_j^i(t_j^i(\varpi{+}1)) = e^{St_j^i(\varpi+1)}\min\{\varsigma_j(t_j^i(\varpi)), e^{-St_j^i(\varpi+1)}\epsilon_j(t_j^i(\varpi{+}1))\}, \\ \qquad\qquad t_m \leq t_j^i(\varpi) < t_j^i(\varpi + 1) < t_{m+1}, \ m \text{ is odd} \\ \widehat{\epsilon}_j^i(t_j^i(\varpi{+}1)) = e^{St_j^i(\varpi+1)}\max\{\varsigma_j(t_j^i(\varpi)), e^{-St_j^i(\varpi+1)}\epsilon_j(t_j^i(\varpi{+}1))\}, \\ \qquad\qquad t_m \leq t_j^i(\varpi) < t_j^i(\varpi + 1) < t_{m+1}, \ m \text{ is even} \\ \widehat{\varsigma}_j^i(t_j^i(\varpi + 1)) = \widehat{\varsigma}_j^i(t_j^i(\varpi)) + \overline{L}_j^i(\varpi)s_j^i(\varpi) \\ \widehat{\epsilon}_j^i(t) = e^{S(t - t_j^i(\varpi))}\widehat{\epsilon}_j^i(t_j^i(\varpi)), \ t_j^i(\varpi) < t < t_j^i(\varpi + 1). \end{cases} \tag{20}$$

4. MAIN RESULT

In this section, we provide the proof for theorems related to both bounded and unbounded link failure durations under the designed distributed output regulation protocol (8). These proofs aim to address the quantized cooperative output regulation of continuous-time multi-agent systems (MASs) based on a switching graph.

Prior to presenting the theorems, a technical lemma will be introduced first:

Lemma 4.1. For the states $\Theta_i(t) \in \mathbb{R}^\upsilon$, $i = 1, \ldots, N$, $\widehat{\Theta}_j^i(t)$ is the estimation of $\Theta_i(t)$ with agent i through the formula designed as follows:

$$\begin{cases} \widehat{\Theta}_j^i(0) = \phi(t_j^i(0)) Q\Big(\Theta_j(t_j^i(0))/\phi(t_j^i(0))\Big), \\ \widehat{\Theta}_j^i(t) = \widehat{\Theta}_j^i(t_j^i(\varpi)),\ t_j^i(\varpi) \le t < t_j^i(\varpi+1),\ \varpi \in \mathbb{N}, \\ \widehat{\Theta}_i(t_j^i(\varpi+1)) = \widehat{\Theta}_j(t_j^i(\varpi)) + \overline{L}_j^i(\varpi) s_j^i(\varpi), \end{cases} \tag{21}$$

where the quantizer output

$$s_j^i(\varpi) \triangleq Q\Big(\overline{L}_j^{i^{-1}}(\varpi)\big(\Theta_j^i(t_j^i(\varpi+1)) - \widehat{\Theta}_j^i(t_j^i(\varpi))\big)\Big). \tag{22}$$

Specifically $t_j^i(\varpi)$, $\varpi \in \mathbb{N}$, $Q(\cdot)$, $l_{jk}(\cdot)$ and $\overline{L}_j^i(\varpi)$ stand for the sampling instants, the multi-quantizer, the scaling function and the matrix of the scaling function $l_{jk}(\cdot)$ respectively. And their full forms are given in Section 3.

Under Assumption 1, consider the closed-loop MAS described as follows:

$$\dot{\Theta}_i(t) = \sum_{j=0}^{N} a_{ij}\Big(\widehat{\Theta}_j^i(t) - \Theta_i(t)\Big),\ i \in \mathcal{V}. \tag{23}$$

Then, there exist two positive constants μ and σ such that

$$\|\Theta_i(t) - \Theta_j(t)\| \le \mu e^{-\sigma t^a},\ i, j \in \mathcal{V}. \tag{24}$$

The proof process is detailed in [18], and thus is omitted here.

Lemma 4.2. Given a switching graph $\mathcal{G}_{\sigma(t)}$ with bounded link failure duration, under Assumptions 1−2, consider the following linear MAS

$$\dot{\epsilon}_i(t) = S\epsilon_i(t) + \sum_{j=0}^{N} a_{ij}\Big(\widehat{\epsilon}_j^i(t) - \epsilon_i(t)\Big),\ i \in \mathcal{V}, \tag{25}$$

where $\epsilon_i(t) \in \mathbb{R}^\upsilon$, $\widehat{\epsilon}_j^i(t) \in \mathbb{R}^\upsilon$ is the estimate of $\epsilon_j(t)$ by agent i and generated as in (16), all the eigenvalues of S are in the closed left-half plane. Then,

$$\lim_{t\to\infty} \|\epsilon_i(t) - \epsilon_j(t)\| = 0. \tag{26}$$

Proof. Let $\Theta_i(t) = e^{-St}\epsilon_i(t)$ and $\widehat{\Theta}_i(t) = e^{-St}\widehat{\epsilon}_i(t)$. It follows from (25) that the dynamic of $\Theta_i(t)$ can be described as in (23). And it follows from (18) that $\widehat{\Theta}_i(t)$ is generated as in (21). Then, according to Lemma 4.1 that there exist two positive constants μ and σ such that $\|\Theta_i(t) - \Theta_j(t)\| \le \mu e^{-\sigma t^a}$, $i, j \in \mathcal{V}$. Let $\lambda_1^S, \ldots, \lambda_d^S$ be the distinct eigenvalues of S and let m_i be the corresponding algebraic multiplicity of λ_i^S, $1 \le i \le d$. Then, according to reference [4], there exists a $\zeta > 0$ such that $\|e^{St}\| \le \zeta t^{m-1}$, where $m = \max_{1\le i\le d} m_i$. Then,

$$\begin{aligned} \lim_{t\to\infty} \|\epsilon_i(t) - \epsilon_j(t)\| &\le \lim_{t\to\infty} \|e^{St}\| \|\Theta_i(t) - \Theta_j(t)\| \\ &\le \lim_{t\to\infty} \mu\zeta t^{m-1} e^{-\sigma t^a} = 0. \end{aligned} \tag{27}$$

This completes the proof. □

4.1. Convergence analysis with bounded link failure duration

Theorem 4.3. Given a switching graph $\mathcal{G}_{\sigma(t)}$ with bounded link failure duration, under Assumptions 1 − 6, the cooperative output regulation problem, as stated in Problem 1, is solvable with any positive bandwidth $\mathcal{B}$ using the distributed protocol (8).

Proof. The closed-loop system composed of the MAS (2) and the distributed protocol (8) can be expressed as follows:

$$\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i K_{1i} z_i(t) + B_i K_{2i} \epsilon_i(t),\\
\dot{\epsilon}_i(t) &= S\epsilon_i(t) + \sum_{j\in\mathcal{N}_i} a_{ij}\Big(\widetilde{\epsilon}_j^{\,i}(t) - \epsilon_i(t)\Big),\\
\dot{z}_i(t) &= (A_i + B_i K_{1i}) z_i(t) + B_i K_{2i}\epsilon_i(t) + E_i\epsilon_i(t)\\
&\qquad + L_i(C_{mi}(z_i(t) - x_i(t)) + F_{mi}(\epsilon_i(t) - \epsilon_i(t))),\\
e_i(t) &= C_i x_i(t) + D_i u_i(t) + F_i v(t),\ i = 1,\dots,N.
\end{aligned} \tag{28}$$

Let $\widetilde{x}_i(t) = x_i(t) - \Psi_i v(t)$, $\widetilde{z}_i(t) = z_i(t) - x_i(t)$ and $\widetilde{\epsilon}_i(t) = \epsilon_i(t) - v(t)$. Then, it follows from (6) that the closed-loop system can be rewritten as

$$\begin{aligned}
\dot{\widetilde{x}}_i(t) &= (A_i + B_i K_{1i})\widetilde{x}_i(t) + B_i K_{1i}\widetilde{z}_i(t) + B_i K_{2i}\widetilde{\epsilon}_i(t),\\
\dot{\widetilde{z}}_i(t) &= (A_i + L_i C_{mi})\widetilde{z}_i(t) + (E_i + L_i F_{mi})\widetilde{\epsilon}_i(t),\\
e_i(t) &= (C_i + D_i K_{1i}) x_i(t) + D_i K_{1i}\widetilde{z}_i(t) + D_i K_{2i}\widetilde{\epsilon}_i(t), \quad i = 1,\dots,N.
\end{aligned} \tag{29}$$

According to Lemma 4.2, $\lim_{t\to\infty}\widetilde{\epsilon}_i(t) = 0$. Since $A_i + L_i C_{mi}$ is Hurwitz, $\lim_{t\to\infty}\widetilde{z}_i(t) = 0$. Then, since $A_i + B_i K_{1i}$ is Hurwitz, $\lim_{t\to\infty}\widetilde{x}_i(t) = 0$, that is, $\lim_{t\to\infty} x_i(t) = \Psi_i v(t)$. Then, one has that $\lim_{t\to\infty} e_i(t) = 0$.

This completes the proof. □

Remark 4.4. The required communication bandwidth $\mathcal{B}$ can be chosen as any positive constant. Under Assumption 3, it does not violate the following inequality given in Theorem 1 in [23].

$$\mathcal{B} > \sum \frac{1}{\ln 2}\max\{\mathrm{Re}(\lambda_i(S)), 0\}$$

where $\mathrm{Re}(\lambda_i(S))$ denotes the real part of each eigenvalue of S, denote as $\lambda_i(S)$.

4.2. Convergence analysis with unbounded link failure duration

Theorem 4.5. Given a switching graph $\mathcal{G}_{\sigma(t)}$ with unbounded link failure duration, under Assumptions 1 − 5, the cooperative output regulation problem, as defined in Problem 1, is solvable with any positive bandwidth $\mathcal{B}$ using the distributed protocol (8).

Proof. To prove Theorem 4.5, we first consider the following claim.

Claim 1. In the case of unbounded link failure duration of switch graph, for the states $\Theta_i(t) \in \mathbb{R}^v$, $i = 1, \ldots, N$, $\widehat{\Theta}_j^i(t)$ is the estimation of $\Theta_i(t)$ with agent i through the formula designed as follows:

$$\begin{cases} \varsigma_j^i(t_j^i(\varpi)) = l(t_j^i(\varpi))Q\Big(\epsilon_j(t_j^i(\varpi))/l(t_j^i(\varpi))\Big), \varpi = 0, \\ \qquad\qquad \text{or } \varpi > 0, t_j^i(\varpi-1) < t_m \le t_j^i(\varpi), m \in \mathbb{N}^+, \\ \widehat{\varsigma}_j^i(t_j^i(\varpi+1)) = \widehat{\varsigma}_j^i(t_j^i(\varpi)) + \overline{L}_j^i(\varpi)s_j^i(\varpi), \\ \widehat{\Theta}_j^i(t_j^i(\varpi)) = \varsigma_j^i(t_j^i(\varpi)), \varpi = 0, \\ \qquad\qquad \text{or } n > 0, t_j^i(\varpi-1) < t_m \le t_j^i(\varpi), m \in \mathbb{N}^+, \\ \widehat{\Theta}_j^i(t_j^i(\varpi+1)) = \min\{\varsigma_j(t_j^i(\varpi)), \Theta_j(t_j^i(\varpi+1))\}, \\ \qquad\qquad t_m \le t_j^i(\varpi) < t_j^i(\varpi+1) < t_{m+1},\ m \text{ is odd}, \\ \widehat{\Theta}_j^i(t_j^i(\varpi+1)) = \max\{\varsigma_j(t_j^i(\varpi)), \Theta_j(t_j^i(\varpi+1))\}, \\ \qquad\qquad t_m \le t_j^i(\varpi) < t_j^i(\varpi+1) < t_{m+1},\ m \text{ is even}, \\ \widehat{\varsigma}_j^i(t_j^i(\varpi+1)) = \widehat{\varsigma}_j^i(t_j^i(\varpi)) + \overline{L}_j^i(\varpi)s_j^i(\varpi), \\ \widehat{\Theta}_j^i(t) = \widehat{\Theta}_j^i(t_j^i(\varpi)),\ t_j^i(\varpi) < t < t_j^i(\varpi+1). \end{cases} \tag{30}$$

where the quantizer output

$$s_j^i(\varpi) = \begin{cases} Q\Big(\overline{L}_j^{i\,-1}(\varpi)(\min\{\Theta_j(t_j^i(\varpi')), \\ \qquad t_m \le t_j^i(\varpi') \le t_j^i(\varpi+1))\} - \widehat{\varsigma}_j^i(t_j^i(\varpi))\Big), \\ \qquad\qquad t_m \le t_j^i(\varpi) < t_j^i(\varpi+1) < t_{m+1}, m \text{ is odd}, \\ Q\Big(\overline{L}_j^{i\,-1}(\varpi)(\max\{\Theta_j(t_j^i(\varpi')), \\ \qquad t_m \le t_j^i(\varpi') \le t_j^i(\varpi+1))\} - \widehat{\varsigma}_j^i(t_j^i(\varpi))\Big), \\ \qquad\qquad t_m \le t_j^i(\varpi) < t_j^i(\varpi+1) < t_{m+1}, m \text{ is even}, \end{cases} \tag{31}$$

where the sampling instants $t_j^i(\varpi)$, $\varpi \in \mathbb{N}$, the multi-quantizer $Q(\cdot)$ and the matrix $\overline{L}_j^i(\varpi)$ are designed the same as the case of bounded link failure duration over switching graph.

Under Assumption 1, consider the following closed-loop MASs (23). Then, there exist two positive constants μ' and σ' such that

$$\|\Theta_i(t) - \Theta_j(t)\| \le \mu' e^{-\sigma' t^{1/a_1}},\ i, j \in \mathcal{V}. \tag{32}$$

Let $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{E}})$, where $\overline{\mathcal{V}} = \mathcal{V} \bigcup \{0\}$ and $\overline{\mathcal{E}} \subseteq \overline{\mathcal{V}} \times \overline{\mathcal{V}}$, that is, $\overline{\mathcal{G}}$ be the set of the spanning trees with a leader labeled as 0 and N followers. It can be verified that the number of the set $\overline{\mathcal{G}}$ is a finite positive integer Υ satisfying $\Upsilon \le N$. Define the following function

$$I([t_1, t_2), \overline{\mathcal{G}}_l, \tau) = \begin{cases} 1, & \text{if } \forall\ (i,j) \in \overline{\mathcal{E}}_l, \int_{t_1}^{t_2} a_{ji}(t) \ge \tau, \\ 0, & \text{else}, \end{cases} \tag{33}$$

where $\overline{\mathcal{G}}_l \in \overline{\mathcal{G}},\ 1 \le l \le \Upsilon$. Then, it is easy to get that $t \ge \Upsilon$.

Assume that m is a positive odd number satisfying that for any $\varpi \in \mathbb{N}$,

$$c'(m-1)^{a/a_1}+e_1+e_2\varpi \le d(m+\varpi^{1/a_1}-(m+\varpi-1)^{1/a_1}),$$

where

$$e_1 = ((d_1+d_2)s' + (m-1)^{a/a_1} + \frac{(s'-1)s'}{2}d_3)\Upsilon,$$

$c' = s'c_1\Upsilon$ and $e^2 = s'\lceil \log_2 \gamma' \rceil \Upsilon$, d_1, d_2 and d_3 will be given in (34) and (35). According to drawer principle, there exists a spanning tree $\overline{\mathcal{G}}_{l'} \in \overline{\mathcal{G}}$ and ϖ_1' integers $\varpi_{1,1},\ \ldots, \varpi_{\varpi_1'}$ satisfying $0 \le \varpi_{1,1} < \ldots < \varpi_{\varpi_1'} \le \Upsilon\varpi_1'$ such that $I([t_m+\varpi_{1,s'}T, t_m+(\varpi_{1,s'}+1)T), \overline{\mathcal{G}}_{l'}, \tau) = 1,\ 1 \le s' \le \varpi_1'$.

Let $\overline{\mathcal{E}}_{l'}$ to be the edge set of $\overline{\mathcal{G}}_{l'}$. Let $\mathcal{N}_1^{l'} = \{i \in \mathcal{V} | (0,i) \in \overline{\mathcal{E}}_{l'}\},\ \ldots, \mathcal{N}_s^{l'} = \{i \in \mathcal{V} | (j,i) \in \overline{\mathcal{E}}_{l'}, j \in \mathcal{N}_{s-1}^{l'}\}$, where $s \ge 1$ positive integer.

Let $d_1 = \max\{0, \lceil \log_2 \frac{\widetilde{M}_k}{10c} \rceil\} + \max\{0, \lceil \log_2 \frac{10c}{\widetilde{M}_k} \rceil\}$ $c_1 = \ln(2)b$ and $d_2 = \lceil \ln(2)\Upsilon T \rceil$, then similar to [18, Lemma 4.1], one has that $i_1 \in \mathcal{N}_1^{l'}$, $\Theta_{0k}^{i_1}(t) \le \overline{M}_k - 9\widetilde{M}_k/10,\ t \ge n'_{d_1+d_2 n_1}$. Then we can have the following inequalities, whose proof is similar to the proof of [18, Claim A.1], and is omitted here.

For agent $i \in \overline{N}_s^{l'}$, when $t > \varpi_{d_{1,s}}T,\ 1 \le s \le s'$,

$$\Theta_{ik}(t) \le \overline{M}_k - \frac{4}{5}(\frac{2}{5})^{s-1}\epsilon^s \widetilde{M}_k(0), \tag{34}$$

$$\widehat{\Theta}_{ik}^j(t) \le \overline{M}_k - (\frac{2}{5})^s \epsilon^s \widetilde{M}_k(0),\ j \in \overline{N}_{s+1}^{l'} \bigcap \mathcal{N}_i^1, \tag{35}$$

where $d_{1s} = (d_1+d_2)s + sc_1(m-1)^{a/a_1} + \frac{(s-1)s_1}{2}d_3$, $d_3 = \lceil -\log_2(\frac{2}{5}\epsilon) \rceil$ and ϵ is given in [18, Claim A.1], $\epsilon = 1/d_{max}(1-e^{-d_{max}\tau})$ and $d_{max} = sup_{t\ge 0} d_i(t)$.

Since $t_m + (\Upsilon\varpi_1'+1)T \le t_{m+1}$, then one has that $\Theta_{ik}(t) \le \overline{M}_k - (\frac{2}{5})^{s'}\epsilon^{s'}\widetilde{M}_k(0), t \ge t_{m+1}$.

Let $\gamma' = 1 - (\frac{2}{5})^{s'}\epsilon^{s'}$. And it is noted that $t_{2\varpi+m} - t_{m+2(\varpi-1)} \ge s'\lceil \log_2 \gamma' \rceil$. Then, similar to [18, Claim A.1], one has that for any $\varpi \in \mathbb{N}$

$$\Theta_{i_0 k}(t) - \Theta_0(t) \quad \le \quad \gamma^{\varpi}(\max_{i\in\mathcal{V}} \Theta_{ik}(0) - \Theta_0(0)),\ t \ge t_{m+2\varpi}.$$

Similarity, one has that

$$\Theta_0(t) - \Theta_{i_0 k}(t) \quad \ge \quad \gamma^{\varpi}(\Theta_0(0) - \min_{i\in\mathcal{V}} \Theta_{ik}(0)),\ t \ge t_{m+2\varpi+1}.$$

Based on the above analysis, one has that there exists two positive constants μ' and σ' satisfying (32).

Thus, Claim 1 is verified.

The rest part of the proof is similar to the proof of Theorem 4.3, and thus is omitted here.

This completes the proof. □

5. SIMULATION

In this section, we will present examples of distributed protocols designed for conditions of bounded and unbounded link failure durations as mentioned in Problem 1. These examples aim to demonstrate the effectiveness of the proposed approaches.

Example 1. We provide an example from [17] to verify the proposed distributed protocol (8). This example considers a multi-agent system (MAS) (2) composed of four agents with

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \ A_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \ B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ D_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \ E_2 = \begin{bmatrix} 0 & -0.5 & 0 \\ -0.5 & 0 & 1 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \ E_4 = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

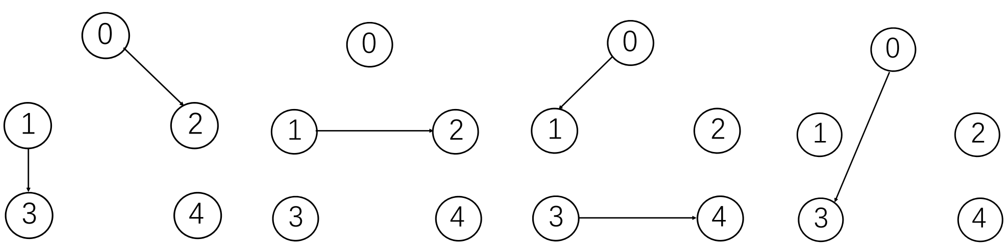
$$F_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \ F_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix},$$

$$F_3 = \begin{bmatrix} -0.5 & -1 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}, \ F_4 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix},$$

$$C_{mi} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ D_{mi} = 0, \ F_{mi} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \ i = 1, 2, 3, 4.$$

The dynamics of the exo-system is given as

$$\dot{v}(t) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v(t). \tag{36}$$



(a) $\mathcal{G}_1$ (b) $\mathcal{G}_2$ (c) $\mathcal{G}_3$ (d) $\mathcal{G}_4$

Fig. 1. Switching graphs $\mathcal{G}_p, p = 1, 2, 3, 4$.

The switching topology $\mathcal{G}_p, p = 1, 2, 3, 4$ is depicted in Figure 1. In this configuration, the exo-system is represented as node 0, while the other agents are depicted as nodes

$1-4$. It is crucial to note that the switching graph $\mathcal{G}_{\sigma(t)}$ is not connected at any time t, yet Assumptions $1-2$ are satisfied. The graph$\mathcal{G}_{\sigma(t)}$ has a bounded link failure duration with $\sup\{T^1_{i,j}, (j,i) \in \mathcal{E}_1\} = 3/4$. It is also assumed that the bandwidth $\mathcal{B}$ is does not exceed 200-bit/sec. The piecewise constant switching signal $\sigma(t)$ for the communication topology is defined as follows:

$$\sigma(t) = \begin{cases} 1, & \text{if } \varpi \le t \le (\varpi + 1/4), \\ 2, & \text{if } (\varpi + 1/4) \le t \le (\varpi + 1/2), \\ 3, & \text{if } (\varpi + 1/2) \le t \le (\varpi + 3/4), \\ 4, & \text{if } (\varpi + 3/4) \le t \le (\varpi + 1), \end{cases} \tag{37}$$

where $\varpi \in \mathbb{N}$. Let

$$K_{11} = \begin{bmatrix} -8 & 32 \end{bmatrix}, K_{12} = \begin{bmatrix} 7 & 32 & -0.5 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -7 & 32 \end{bmatrix}, K_{22} = \begin{bmatrix} 3.5 & 32 & -1 \end{bmatrix},$$

$$K_{31} = \begin{bmatrix} 7 & -32 \end{bmatrix}, K_{32} = \begin{bmatrix} -4.5 & 9 & 0 \end{bmatrix},$$

$$K_{41} = \begin{bmatrix} 8 & -32 \end{bmatrix}, K_{42} = \begin{bmatrix} 33 & -8 & -0.5 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, L_2 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, L_4 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}.$$

Set $\alpha = 6$, $T^0_{(i,j)} = 0.1s$. Given that $\Theta_i(t)$ consists of three elements, $s^i_j(\varpi)$ is encoded using 18 bits at each sampling time. In this scenario, the communication bandwidth $\mathcal{B}$ is limited to 180 bit/sec. The function $\phi(t)$ is designed as $\phi(t) = 2^{-t}$.

The simulation results, which illustrate the tracking errors, are presented in Figure 2. These errors asymptotically converge to zero, demonstrating that the output regulation problem with bounded link failure duration has been effectively addressed.

Example 2. To further validate the framework in a more complex scenario, namely, the cooperative output regulation over switching graphs with unbounded link failure duration, we consider the same dynamic system of the multi-agent system (MAS) as outlined in Example 1. However, for this case, the communication topology's switching signal is redefined as follows:

$$\sigma(t) = \begin{cases} 1, & \text{if } \varpi T \le t \le (\varpi + 1/4)T, \varpi \in \mathbb{N}, \\ 2, & \text{if } (\varpi + 1/4)T \le t \le (\varpi + 3/4)T, \\ & \qquad \varpi \in \{\varpi_1^2, \varpi_1 \in \mathbb{N}\}, \\ 3, & \text{if } (\varpi + 1/4)T \le t \le (\varpi 3/4)T, \\ & \qquad \varpi \in \mathbb{N} \text{ and } \varpi \notin \{\varpi_1^2, \varpi_1 \in \mathbb{N}\}, \\ 4, & \text{if } (\varpi + 3/4)T \le t \le (\varpi + 1)T, \varpi \in \mathbb{N}. \end{cases} \tag{38}$$

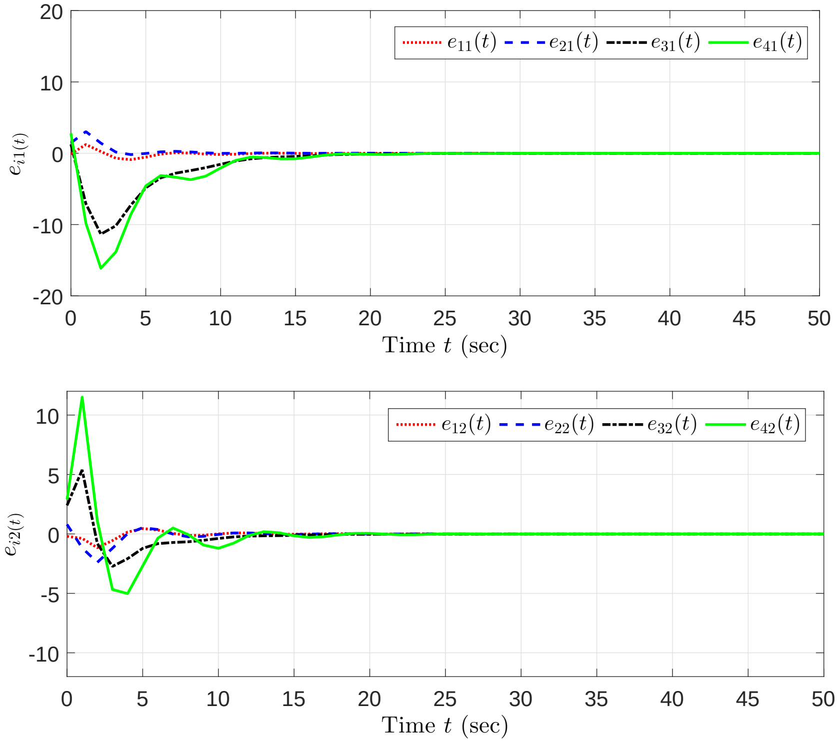


Fig. 2. Trajectories of tracking errors with bounded link failure duration.

The switching topology $\mathcal{G}_p, p = 1, 2, 3, 4$ and the parameters for $K_{i1}, K_{i2}, L_i, i = 1, 2, 3, 4$ and $\alpha, T^0_{(i,j)}, s^i_j(\varpi), \mathcal{B}, \phi(t)$ the values are the same as those used in Example 1 and are thus not repeated here. Notably, the maximum link failure duration for the edge $(1, 2)$, denoted as $T^1_{1,2} = \infty$, is ∞, indicating that the switching graph $\mathbb{G}_{\sigma(t)}$ is characterized by an unbounded link failure duration. The time instants instants $\{t_m, m \in \mathbb{N}^+\}$ is designed as $t_m = (m-1)^3,\ m \in \mathbb{N}^+$.

Simulation results, shown in Figure 3, depict the tracking errors. These errors asymptotically converge to zero, demonstrating that the output regulation problem with unbounded link failure duration has been effectively resolved.

6. CONCLUSION

This paper explores the global cooperative output regulation problem for continuous-time linear multi-agent systems (MASs) on switching graphs, particularly under restricted communication bandwidth conditions. We design distributed protocols that involve sampling and quantizing data to tackle this challenge. This approach differs from current quantitative control work on MASs in that it only requires cooperatively connected agents and does not rely on semi-global initial conditions.

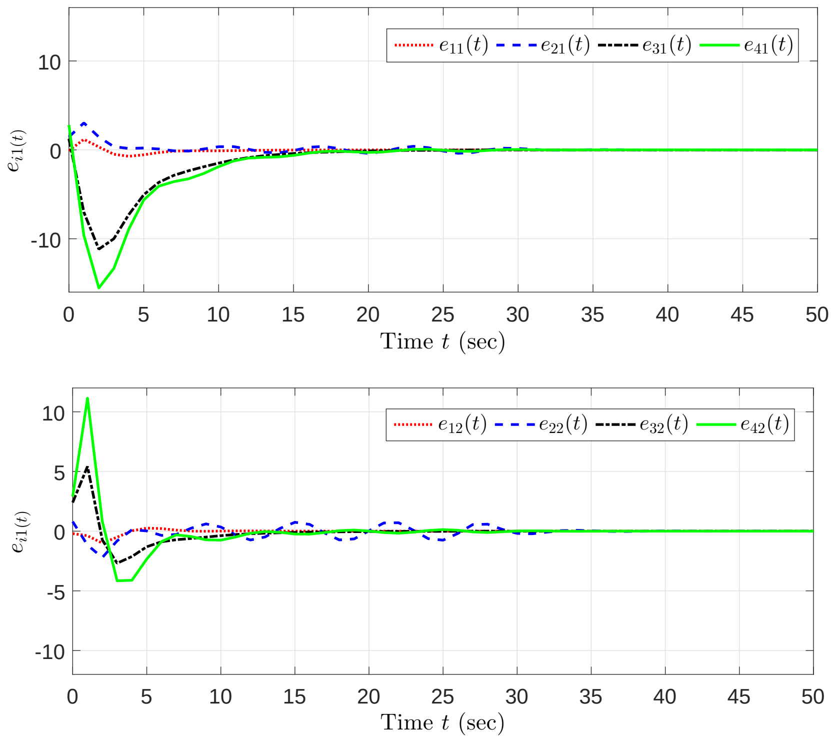


Fig. 3. Trajectories of tracking errors with unbounded link failure duration.

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REFERENCES

[1] A. Abdessameud and A. Tayebi: Distributed output regulation of heterogeneous linear multi-agent systems with communication constraints. Automatica *91* (2018), 152–158. DOI:10.1016/j.automatica.2018.01.020

[2] J. Back and J. Kim: Output feedback practical coordinated tracking of uncertain heterogeneous multi-agent systems under switching network topology. IEEE Trans. Autom. Control *62* (2017), 6399–6406. DOI:10.1109/tac.2017.2651166

[3] F. Ceragioli, C. D. Persis, and P. Frasca: Discontinuities and hysteresis in quantized average consensus. Automatica *47* (2011), 1916–1928. DOI:10.1016/j.automatica.2011.06.020

[4] Chi-Tsong Chen: Linear System Theory and Design. Oxford University Press, Inc., 1995.

[5] D. Dimarogonas and K. Johansson: Stability analysis for multi-agent systems using the incidence matrix: quantized communication and formation control. Automatica *46* (2010), 695–700. DOI:10.1016/j.automatica.2010.01.012

[6] S. Dong, L. Liu, G. Feng, M. Liu, and Z. Wu: Quantized fuzzy cooperative output regulation for heterogeneous nonlinear multiagent systems with directed fixed/switching topologies. IEEE T. Cybernetics *52* (2022), 12393–12402. DOI:10.1109/tcyb.2021.3082164

[7] X. Dong, Z. Yan, R. Zhang, and Y. Zhong: Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying. IEEE T. Ind. Electron. *64* (2017), 5014–5024. DOI:10.1109/tie.2016.2593656

[8] Y. Hong, X. Wang, and Z. Jiang: Distributed output regulation of leader-follower multi-agent systems. Int. J. Robust. Nonlinear Control *23* (2013), 48–66. DOI:10.1002/rnc.1814

[9] J. Huang: Nonlinear output regulation: Theory and application. Soc. Ind. Appl. Math. (2004). DOI:10.1137/1.9780898718683

[10] X. Huang and J. Dong: Reliable leader-to-follower formation control of multiagent systems under communication quantization and attacks. IEEE T. Syst. Man CY-S *50* (2020), 89–99. DOI:10.1109/tsmc.2019.2894946

[11] J. Lavaei and R. M Murray: Quantized consensus by means of gossip algorithm. IEEE Trans. Automat. Control *57* (2012), 19–32. DOI:10.1109/tac.2011.2160593

[12] T. Li, M. Fu, L. Xie, and J. Zhang: Distributed consensus with limited communication data rate. IEEE Trans. Autom. Control *56* (2011), 279–292. DOI:10.1109/tac.2010.2052384

[13] H. Li, S. Liu, Y. C. Soh, and L. Xie: Event-triggered communication and data rate constraint for distributed optimization of multiagent systems. IEEE Trans. Syst. Man Cybern. Syst. *48* (2018), 1908–1919. DOI:10.1109/tsmc.2017.2694323

[14] D. Li, Q. Liu, X. Wang, and Z. Yin: Quantized consensus over directed networks with switching topologies. Syst. Control Lett. *65* (2014), 13–22. DOI:10.1016/j.sysconle.2013.11.013

[15] W. Liu and J. Huang: Cooperative global robust output regulation for a class of nonlinear multi-agent systems with switching network. IEEE Trans. Autom. Control *60* (2015), 1963–1968. DOI:10.1109/tac.2014.2368231

[16] S. Liu, T. Li, L. Xie, M. Fu, and J. Zhang: Continuous-time and sampleddata-based average consensus with logarithmic quantizers. Automatica *49* (2013), 3329–3336. DOI:10.1016/j.automatica.2013.07.016

[17] M. Lu and L. Liu: Distributed feedforward approach to cooperative output regulation subject to communication delays and switching networks. IEEE Trans. Autom. Control *62* (2017), 1999–2005. DOI:10.1109/tac.2016.2594151

[18] J. Ma, X. Yu, L. Liu, H. Ji, and G. Feng: Global cooperative output regulation of linear multiagent systems with limited bandwidth. IEEE Trans. Control Netw. Syst. *9* (2022), 1017–1028. DOI:10.1109/tcns.2021.3128500

[19] Y. Meng, T. Li, and J. Zhang: Coordination over multi-agent networks with unmeasurable states and finite-level quantization. IEEE Trans. Automat. Control *62* (2017), 4647–4653. DOI:10.1109/tac.2016.2627179

[20] Z. Meng, T. Yang, D. Dimarogonas, and K. Johansson: Coordinated output regulation of heterogeneous linear systems under switching topologies. Automatica *53* (2015), 362–368. DOI:10.1016/j.automatica.2015.01.009

[21] Y. Su and J. Huang: Cooperative output regulation of linear multi-agent systems by output feedback. Syst. Control Lett. *61* (2012), 1248–1253. DOI:10.1016/j.sysconle.2012.09.005

[22] Y. Su and J. Huang: Cooperative output regulation with application to multiagent consensus under switching network. IEEE T. Syst. Man CY-S, part B, Cybernetics *42* (2012), 864–875. DOI:10.1109/TSMCB.2011.2179981

[23] S. Tatikonda and S. Mitter, Sanjoy: Control over noisy channels. IEEE Trans. Autom. Control *49* (2004), 1196–1201. DOI:10.1109/TAC.2004.831102

[24] X. Wang: Distributed formation output regulation of switching heterogeneous multi-agent systems. Int. J. Syst. Sci. *44* (2013), 2004–2014. DOI:10.1080/00207721.2012.683831

[25] X. Wang, W. Ni, and Z. Ma: Distributed event-triggered output regulation of multi-agent systems. Int. J. Control *88* (2015), 640–652. DOI:10.1080/00207179.2014.971432

[26] D. Xu, X. Wang, Y. Su, and D. Wang: Formation control in dynamic positioning of multiple offshore vessels via cooperative robust output regulation. In: 2017 56th IEEE Conf. Decis. Control (CDC), Melbourne, IEEE (2017), pp. 4070–4075. DOI:10.1109/CDC.2017.8264256

[27] P. Yi and Y. Hong: Quantized subgradient algorithm and data-rate analysis for distributed optimization. IEEE Trans. Control Netw. Syst. *1* (2014), 380–392. DOI:10.1109/tcns.2014.2357513

[28] K. You and L. Xie: Network topology and communication data rate for consensusability of discrete-time multi-agent systems. IEEE Trans. Autom. Control *56* (2011), 2262–2275. DOI:10.1109/tac.2011.2164017

[29] Q. Zhang and J. Zhang: Quantized data-based distributed consensus under directed time-varying communication topology. SIAM J. Control Optim. *51* (2013), 332–352. DOI:10.1137/100818662

[30] M. Zheng, C. Liu, and F. Liu: Average-consensus tracking of sensor network via distributed coordination control of heterogeneous multi-agent systems. IEEE Control Syst. Lett. *3* (2019), 132–137. DOI:10.1109/lcsys.2018.2856105

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