KYBERNETIKA — VOLUME *59* (2023), NUMBER 3, PAGES 512–526

GENERALIZED SYNCHRONIZATION-BASED PARTIAL TOPOLOGY IDENTIFICATION OF COMPLEX NETWORKS

Xueqin Zhang, Yunru Zhu, and Yuanshi Zheng

In this paper, partial topology identification of complex networks is investigated based on synchronization method. We construct the response networks consisting of nodes withsimplerdynamics than that in the drive networks. By constructing Lyapunov function, sufficient conditions are derived to guaranteepartial topology identificationby designing suitable controllers and parameters update laws.Several numerical examples are provided to illustrate the effectiveness of the theoretical results.

Keywords: complex network, partial topology identification, generalized outer synchronization

Classification: 93D15, 93C05

1. INTRODUCTION

In recent years, there has been an increasing amount of research on complex networks, including the synchronization and control of complex networks, etc. [11, 14, 21, 26, 31–33, 35–37]. Most studies on complex networks have an implicit assumption that the network topology is known. However, in many real-world situations, the network topology is often unknown or difficult to measure. Therefore, topology identification of complex networks is very important and meaningful.

Different methods for topology identification of complex networks have been studied. In [9], Bayesian estimation was used to study the relationship between yeast proteins. In 2006, a synchronization-based topology identification method was proposed [27]. After that, many interesting topology identification results based on synchronization have been obtained [20, 22]. In [20], the topology identification of complex networks with delayed weighted coupling was studied. In [22], the authors investigated the identification of coupled neural networks with multiple state couplings or multiple delay state couplings. Based on the dynamic evolution of the network, a new method for identifying discrete dynamic network topology was proposed [7]. To identify the topology of networks with time-varying node systems, an adaptive method was proposed [30]. In [8], the topology of complex networks was identified for the first time from an optimization perspective. In [25], Granger causality test was proposed for network with stochastic

DOI: 10.14736/kyb-2023-3-0512

perturbations. In [23], the piecewise approximation technique was employed in the partial Granger causality test to detect interactions among nonlinear time series affected by hidden variables. Since there are some engineered complex dynamic networks including multi-layer networks, for example, communication rumor and communication epidemic spreading networks, multi-layer network has become an important research directions [1,2,5,6,10,16–18,28]. Therefore, topology identification of multi-layer networks is necessary. [17] considered the topology identification of two-layer undirected networks using an auxiliary system method. Wang et al. [18] considered the topology identification for two-layer networks with coupled time delays and stochastic perturbations using stochastic differential correlation tools. In addition to the above methods, many other methods have been proposed for topology identification of complex networks, such as compressive sensing [19], node knockout [13], recurrence [12, 15], et al.

Many of the methods discussed above are used to identify the entire topology of complex networks, but we may only be interested in the partial topology of the network at times. For example, in an interpersonal network, one only wants to know the relationship among a small number of people, not the whole network, or in an infectious disease network, we are only interested in a part of the network that is closely connected to the source of infection. However, there are few scholars on the partial topology identification of complex networks, so it is very necessary to identify the partial topology of complex networks. [38] firstly puts forward the method of identifying partial network topology. On the basis of [38], [4] considers the case with stochastic perturbations and time delay. It is noted that in the method of [4, 38], the node dynamics of the response network need to be the same as the node dynamics of the drive network, which will be very costly when the node dynamics of the drive network are very complex or has a high dimension. In order to solve this problem, generalized synchronization method is employed in the paper, where the node dynamics of the constructed response network can be simpler than that of the drive network. Firstly, a response network is constructed in which the node dynamics are simpler than those in the drive network. Secondly, an effective controller and an adaptive updating law are designed to achieve partial generalized outer synchronization and unknown topology identification. By constructing Lyapunov function, the sufficient conditions for achieving topology identification are given. Finally, the effectiveness and feasibility of the proposed method are verified by numerical simulations.

The rest of the paper is organized as follows. In Section II, we introduce the network models and some important preliminaries. Section III gives our main results. In Section IV, several numerical simulations are provided to illustrate the effectiveness of the theoretical findings. Finally, Section V gives some conclusions.

Notation: we use R^n and $R^{n\times m}$ to denote the n-dimensional Euclidean space and the set of all the $n\times m$-dimensional real matrices, respectively, $||x||$ denotes the 2-norm of a vector. Let $\Phi : R^n \to R^m$ be a continously differentiable vector mapping function of $u = (u_1, u_2, \dots, u_n)^T$ to $v = \Phi(u) = (v_1, v_2, \dots, v_m)^T$. Then the Jacobian Matrix can be defined

$$D\Phi(u) = \begin{bmatrix} \frac{\partial v_1}{\partial u_1} & \cdots & \frac{\partial v_1}{\partial u_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial v_m}{\partial u_1} & \cdots & \frac{\partial v_m}{\partial u_n} \end{bmatrix}.$$

2. PROBLEM DESCRIPTION

Consider a complex network consisting of N dynamical nodes described as follows:

$$\dot{x}_i = f_i(x_i) + \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma_1 x_j, \qquad i = 1, 2, \dots, N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in R^n$ is the state of ith node, $f_i : R^n \to R^n$ is the dynamics of the ith node, Γ_1 is the inner coupling matrix, and $\varepsilon(\varepsilon > 0)$ is the coupling strength. If there exists an edge from node $j(j \neq i)$ to node i, then $a_{ij} > 0$, otherwise, $a_{ij} = 0$. $A = (a_{ij})_{N\times N}$ is the unknow weight configuration matrix, and the diagonal elements of matrix A are defined as

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}, \qquad i = 1, 2, \dots, N.$$

For simplicity, network(1) is referred to as the drive network.

In this paper, partial topology identification of the network is considered. Without loss of generality, we identify the couplings among $s(1 \le s \le N)$ nodes in a complex network. The response network is constructed as follows:

$$\dot{y}_i = g_i(y_i) + \varepsilon \sum_{j=1}^{s} b_{ij}\Gamma_2 y_j + \varepsilon D\Phi_i(x_i) \sum_{j=s+1}^{N} b_{ij}\Gamma_1 x_j + u_i, \qquad i = 1, 2, \dots, s, \tag{2}$$

where $y_i = (y_{i1}, y_{i2}, \dots, y_{im})^T \in R^m$ is the state of ith node, $g_i : R^m \to R^m$ is the dynamics of the ith node, $\Phi_i(x_i) : R^n \to R^m$ is a continously differentiable vector mapping function, Γ_2 is the inner coupling matrix, and $B_{s\times N} = (b_{ij})_{s\times N}$ is the estimation of the unknown weight configuration matrix $A_{s\times N}$.

Definition 2.1. The network (1) and network (2) are said to achieve partial generalized outer synchronization, if

$$\lim_{t\to\infty} \sum_{i=1}^{s} ||y_i(t) - \Phi_i(x_i(t))|| = 0.$$

When $s = N$, the two networks achieve generalized outer synchronization. In addition, if $\Phi_i(x_i(t)) = x_i(t)$, the two networks have achieved partial complete outer synchronization.

Assumption 2.2. Suppose there exists a positive constant α_i such that the function g_i satisfies the following inequality

$$||g_i(z_1) - g_i(z_2)|| \le \alpha_i ||z_1 - z_2||$$

for any z_1 and z_2.

Assumption 2.3. Suppose that $D\Phi_i(x_i)\Gamma_1 x_1, D\Phi_i(x_i)\Gamma_1 x_2, \dots, D\Phi_i(x_i)\Gamma_1 x_N (i = 1, 2, \dots, s)$ are linearly independent on the orbit $\{x_i\}_{i=1}^N$ of the outer synchronization manifold $\{y_i = \Phi_i(x_i)\}_{i=1}^s$.

3. MAIN RESULTS

Using the partial generalized outer synchronization method and designing controllers and update laws to identify the partial network topology is mainly introduced in this section.

Theorem 3.1. Suppose Assumption 2.2 and 2.3 hold, and the designed controllers and update laws are as follows:

$$\begin{aligned} u_i &= D\Phi_i(x_i)f_i(x_i) - g_i(\Phi_i(x_i)) - \varepsilon \sum_{j=1}^{s} b_{ij}\Gamma_2 y_j \\ &\quad + \varepsilon D\Phi_i(x_i) \sum_{j=1}^{s} b_{ij}\Gamma_1 x_j - k_i e_i, \\ \dot{b}_{ij} &= -\varepsilon e_i^T D\Phi_i(x_i)\Gamma_1 x_j, \\ \dot{k}_i &= e_i^T e_i, \end{aligned} \tag{3}$$

where $e_i = y_i - \Phi_i(x_i)$, $1 \leq i \leq s$, $1 \leq j \leq N$. Then the network (1) and network (2) achieve partial generalized outer synchronization, and the unknown weight configuration matrix $A_{s\times N}$ can be identified by the estimation matrix $B_{s\times N}$ in the response network.

Proof. According to (1) and (2), the error system can be written as:

$$\begin{aligned} \dot{e}_i &= g_i(y_i) + \varepsilon \sum_{j=1}^{s} b_{ij}\Gamma_2 y_j + \varepsilon D\Phi_i(x_i) \sum_{j=s+1}^{N} b_{ij}\Gamma_1 x_j \\ &\quad + D\Phi_i(x_i)f_i(x_i) - g_i(\Phi_i(x_i)) - \varepsilon \sum_{j=1}^{s} b_{ij}\Gamma_2 y_j \\ &\quad + \varepsilon D\Phi_i(x_i) \sum_{j=1}^{s} b_{ij}\Gamma_1 x_j - k_i e_i - D\Phi_i(x_i)f_i(x_i) - \varepsilon D\Phi_i(x_i) \sum_{j=1}^{N} a_{ij}\Gamma_1 x_j \\ &= g_i(y_i) - g_i(\Phi_i(x_i)) + \varepsilon D\Phi_i(x_i) \sum_{j=1}^{N} (b_{ij} - a_{ij})\Gamma_1 x_j - k_i e_i, \end{aligned} \tag{4}$$

where $1 \leq i \leq s$, $1 \leq j \leq N$.

Consider the Lyapunov candidate function as follows:

$$V = \frac{1}{2}\sum_{i=1}^{s} e_i^T e_i + \frac{1}{2}\sum_{i=1}^{s}\sum_{j=1}^{N} (b_{ij} - a_{ij})^2 + \frac{1}{2}\sum_{i=1}^{s} (k_i - k^*)^2, \tag{5}$$

where k^* is a positive constant. Then we can obtain

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{s} e_i^T \dot{e}_i + \sum_{i=1}^{s}\sum_{j=1}^{N}(b_{ij}-a_{ij})\dot{b}_{ij} + \sum_{i=1}^{s}(k_i-k^*)\dot{k}_i \\ &= \sum_{i=1}^{s} e_i^T (g_i(y_i) - g_i(\Phi_i(x_i)) + \varepsilon D\Phi_i(x_i)\sum_{j=1}^{N}(b_{ij}-a_{ij})\Gamma_1 x_j - k_i e_i) \\ &+ \sum_{i=1}^{s}\sum_{j=1}^{N}(b_{ij}-a_{ij})(-\varepsilon e_i^T D\Phi_i(x_i)\Gamma_1 x_j) + \sum_{i=1}^{s}(k_i-k^*)e_i^T e_i \\ &= \sum_{i=1}^{s} e_i^T (g_i(y_i) - g_i(\Phi_i(x_i))) - \sum_{i=1}^{s} k^* e_i^T e_i \\ &\le \alpha e^T e - k^* e^T e \\ &= (\alpha - k^*) e^T e,\end{aligned}$$

where $e = (e_1^T, e_2^T, \ldots, e_s^T)$, $\alpha = \max_{1\le i\le s}\{\alpha_i\}$. Let $k^* = \alpha + 1$, one gets $\dot{V} \le -e^T e$. Namely, $\dot{V} \le 0$. Let M be the set of all points satisfying $\dot{V} = 0$. Obviously, $M = \{(e_i, b_{ij}, k_i) || e_i = 0, i = 1, 2, \ldots, s\}$. Based on the LaSalle's invariance principle, it has that $\lim_{t\to\infty} y_i(t) - \Phi_i(x_i(t)) = 0$, which implies that the network (1) and network (2) achieve partial generalized outer synchronization. According to (3) and error system (4), we further get

$$M = \{(e_i, b_{ij}, k_i) || e_i = 0, \dot{b}_{ij} = 0, \dot{k}_i = 0, D\Phi_i(x_i)\sum_{j=1}^{N}(b_{ij}(t) - a_{ij})\Gamma_1 x_j = 0\}.$$

Under Assumption 2.3, $D\Phi_i(x_i)\sum_{j=1}^{N}(b_{ij}(t)-a_{ij})\Gamma_1 x_j = 0$ if and only if $b_{ij}(t) - a_{ij} = 0$. According to the LaSalle's invariance principle, we have $\lim_{t\to\infty} b_{ij}(t) - a_{ij} = 0$, that is, the unknown weight configuration matrix $A_{s\times N}$ can be identified by the estimation matrix $B_{s\times N}$ in the response network. The proof is completed. □

Remark 3.2. In our method, if we take $s = N$, the whole topology can be identified. Hence, our work is more general compared with the work just on the whole topology identification [29].

Remark 3.3. For large scale networks, when identifying the partial topology, the constructed response network can be composed of nodes with simpler dynamics. It is especially suitable for complex or high-dimensional node dynamics in the drive network.

Remark 3.4. The unknown weight configuration matrix A does not need to be symmetric or irreducible, nor does the estimation matrix B.

The response network (2) has the virtue of using only a part of nodes's state information. However, it results in the more complex controller u_i, shown in (3). In order to make the controller simple, we modify the response network (2) as follows:

$$\dot{y}_i = g_i(y_i) + \varepsilon D\Phi_i(x_i)\sum_{j=1}^{N} b_{ij}\Gamma_1 x_j + u_i, \qquad i = 1, 2, \ldots, s. \tag{6}$$

Theorem 3.5. Suppose Assumption 2.2 and 2.3 hold, and the designed controllers and update laws are as follows:

$$\begin{aligned} u_i &= D\Phi_i(x_i)f_i(x_i) - g_i(\Phi_i(x_i)) - k_i e_i, \\ \dot{b}_{ij} &= -\varepsilon e_i^T D\Phi_i(x_i)\Gamma_1 x_j, \\ \dot{k}_i &= e_i^T e_i, \end{aligned} \tag{7}$$

where $1 \leq i \leq s$, $1 \leq j \leq N$. Then network (1) and network (6) achieve partial generalized outer synchronization, and the unknown weight configuration matrix $A_{s\times N}$ can be identified by the estimation matrix $B_{s\times N}$ in the response network.

Proof. The error system is as follows:

$$\begin{aligned} \dot{e}_i &= g_i(y_i) + \varepsilon D\Phi_i(x_i)\sum_{j=1}^{N} b_{ij}\Gamma_1 x_j + D\Phi_i(x_i)f_i(x_i) \\ &\quad - g_i(\Phi_i(x_i)) - k_i e_i - D\Phi_i(x_i)f_i(x_i) - \varepsilon D\Phi_i(x_i)\sum_{j=1}^{N} a_{ij}\Gamma_1 x_j \\ &= g_i(y_i) - g_i(\Phi_i(x_i)) + \varepsilon D\Phi_i(x_i)\sum_{j=1}^{N}(b_{ij} - a_{ij})\Gamma_1 x_j - k_i e_i, \end{aligned}$$

where $1 \leq i \leq s$, $1 \leq j \leq N$.

Consider the Lyapunov candidate function as follows:

$$V = \frac{1}{2}\sum_{i=1}^{s} e_i^T e_i + \frac{1}{2}\sum_{i=1}^{s}\sum_{j=1}^{N}(b_{ij} - a_{ij})^2 + \frac{1}{2}\sum_{i=1}^{s}(k_i - k^*)^2.$$

The rest of the proof is the same as Theorem 3.1, so it is omitted. □

In response network (2), if $m = n$, we can take $\Phi_i(x_i) = x_i$, and the inner coupling matrix $\Gamma_2 = \Gamma_1$. Thus, the response network (2) can be rewritten as follows:

$$\dot{y}_i = g_i(y_i) + \varepsilon\sum_{j=1}^{s} b_{ij}\Gamma_1 y_j + \varepsilon\sum_{j=s+1}^{N} b_{ij}\Gamma_1 x_j + u_i, \qquad i = 1, 2, \ldots, s. \tag{8}$$

Then the following corollary can be obtained.

Corollary 3.6. Suppose Assumption 2.2 and 2.3 hold, and the controllers and update laws are designed as follows:

$$\begin{aligned} u_i &= f_i(x_i) - g_i(x_i) - k_i e_i, \\ \dot{b}_{ij} &= -\varepsilon e_i^T \Gamma_1 y_j, \qquad 1 \leq j \leq s, \\ \dot{b}_{ij} &= -\varepsilon e_i^T \Gamma_1 x_j, \qquad s+1 \leq j \leq N, \\ \dot{k}_i &= e_i^T e_i, \end{aligned} \tag{9}$$

where $1 \le i \le s$. Then network (1) and network (8) achieve partial complete outer synchronization, and the unknow partial topology can be identified.

For network (8), if take $g_i = f_i (i = 1, 2, \ldots, s)$, the following corollary is obtained.

Corollary 3.7. Suppose Assumption 2.2 and 2.3 hold, and the controllers and update laws are designed as follows:

$$\begin{aligned} u_i &= -k_i e_i, \\ \dot{b}_{ij} &= -\varepsilon e_i^T \Gamma_1 y_j, \qquad 1 \le j \le s, \\ \dot{b}_{ij} &= -\varepsilon e_i^T \Gamma_1 x_j, \qquad s+1 \le j \le N, \\ \dot{k}_i &= e_i^T e_i. \end{aligned} \tag{10}$$

Then network (1) and network (8) achieve partial complete outer synchronization, and the unknow partial topology can be identified.

4. SIMULATION EXAMPLE

In this section, several simulation examples are used to illustrate the validity of our theoretical results. For simplicity, let Γ_1 and Γ_2 be identity matrix.

Example 1: Suppose there are 6 nodes in the unknown drive network, but we are only interested in coupling the first 3 nodes.

Consider a drive network consisiting of 6 nodes as follows:

$$\dot{x}_i = f_i(x_i) + 0.1 \sum_{j=1}^{6} a_{ij} x_j, \qquad i = 1, 2, 3, 4, 5, 6, \tag{11}$$

where

$$A = \begin{bmatrix} -6 & 3 & 2 & 1 & 0 & 0 \\ 2 & -5 & 1 & 1 & 1 & 0 \\ 0 & 4 & -5 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 2 & 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

and the following hyperchaotic Lü system [3] is chosen as the node dynamics :

$$f_i(x_i) = \begin{bmatrix} 36(x_{i2} - x_{i1}) + x_{i4} \\ -x_{i1}x_{i3} + 20x_{i2} \\ x_{i1}x_{i2} - 3x_{i3} \\ x_{i1}x_{i3} + 1.3x_{i4} \end{bmatrix}.$$

The response network consists of 3 nodes is as follow:

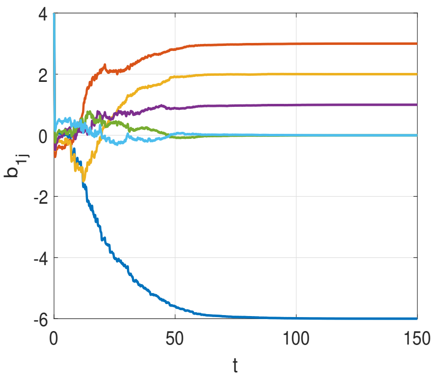
$$\dot{y}_i = g_i(y_i) + 0.1 \sum_{j=1}^{3} b_{ij} y_j + 0.1 D\Phi_i(x_i) \sum_{j=4}^{6} b_{ij} x_j + u_i, \qquad i = 1, 2, 3, \tag{12}$$

where the following Lorenz system [34] is chosen as the node dynamics :

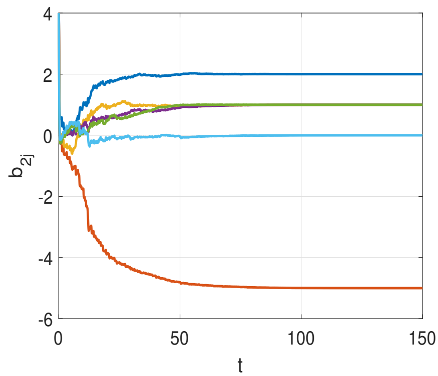
$$g_i(y_i) = \begin{bmatrix} 10(y_{i2} - y_{i1}) \\ 28y_{i1} - y_{i2} - y_{i1}y_{i3} \\ y_{i1}y_{i2} - \frac{8}{3}y_{i3} \end{bmatrix}.$$

Therefore, the dynamic dimensions of the nodes in the drive network and the response network are not the same. Let $\Phi_i(x_i) = (0.2ix_{i1}, x_{i2} + 2 + 0.2x_{i4}, x_{i3} - 1)^T$, one has

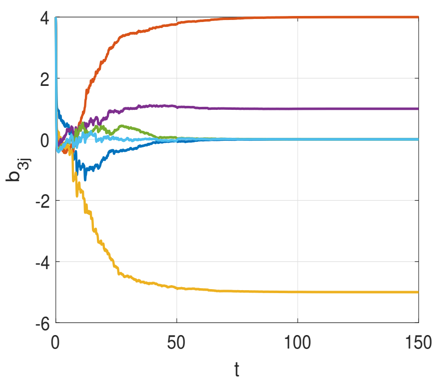
$$D\Phi_i(x_i) = \begin{bmatrix} 0.2i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$



(a) Time evolution of $b_{1j}(1 \le j \le 6)$



(b) Time evolution of $b_{2j}(1 \le j \le 6)$



(c) Time evolution of $b_{3j}(1 \le j \le 6)$

Fig. 1. Identification of partial topology for drive network(11) where the response network is (12).

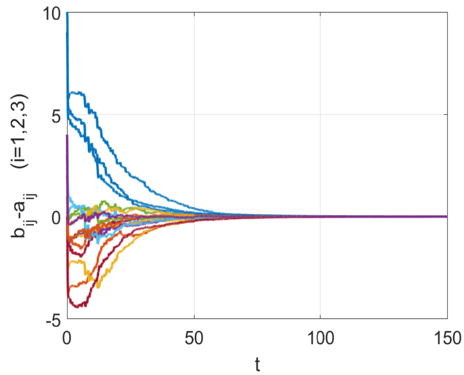
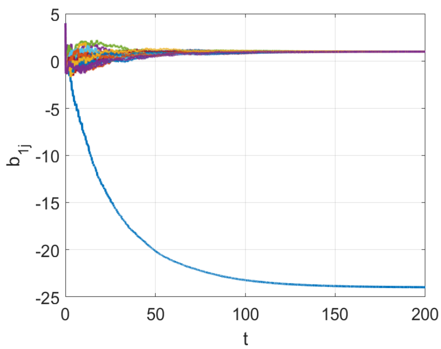
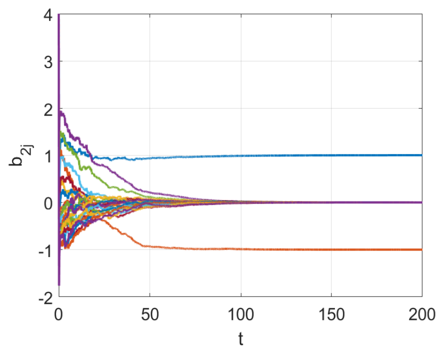


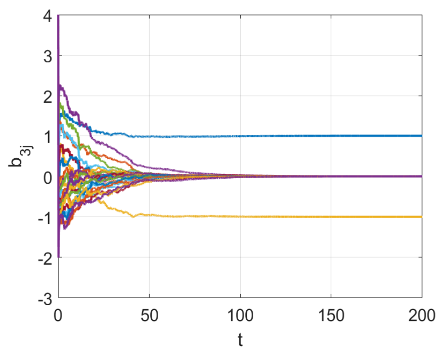
Fig. 2. The partial identification error of (11) and (12).



(a) Time evolution of $b_{1j}(1 \le j \le 25)$



(b) Time evolution of $b_{2j}(1 \le j \le 25)$



(c) Time evolution of $b_{3j}(1 \le j \le 25)$

Fig. 3. Identification of partial topology for drive network(13) where the response network is (14).

Here we use the controllers and update laws (3). Figure 1 shows that $B_{3\times 6}$ estimates the partial topology of the drive network. From Figure 2, we can see that the identification error of the partial network topology tends to zero.

Example 2: A typical star network with 25 nodes is used to verify our conclusions. That is, $a_{11} = -24, a_{ii} = -1(2 \leq i \leq 25), a_{i1} = 1(2 \leq i \leq 25), a_{1j} = 1(2 \leq j \leq 25)$, and the rest of a_{ij} is 0. We are only interested in coupling the first 3 nodes. The drive network is as follows:

$$\dot{x}_i = f_i(x_i) + 0.1 \sum_{j=1}^{25} a_{ij} x_j, \qquad i = 1, 2, \ldots, 25, \tag{13}$$

and unlike Example 1, the response network is as follows:

$$\dot{y}_i = g_i(y_i) + 0.1 D \Phi_i(x_i) \sum_{j=1}^{25} b_{ij} \Gamma_1 x_j + u_i, \qquad i = 1, 2, 3. \tag{14}$$

The node dynamics of the response network and drive network are the same as in Example 1, and $\Phi_i(x_i)$ is the same as in Example 1.

Controllers and update laws (7) is used here. Figure 3 shows that $B_{3\times 25}$ estimates the partial topology of the drive network. It can be seen from Figure 4 that the identification error of the partial network tends to zero.

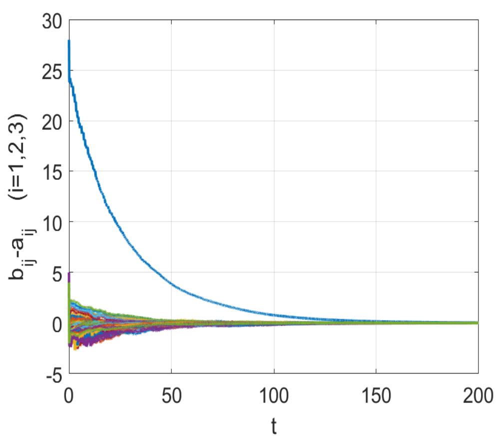


Fig. 4. The partital identification error of (13) and (14).

Example 3: Here, we consider the case that the response network and the drive network achieve partial complete outer synchronization, i. e.,$\Phi_i(x_i) = x_i$. The drive network consists of 6 nodes. The response network form is the same as 8. Similarly, consider the coupling of the first 3 nodes.

The following Lü system [24] is chosen as the node dynamics of the drive network:

$$f_i(x_i) = \begin{bmatrix} 36(x_{i2} - x_{i1}) \\ -x_{i1}x_{i3} + 20x_{i2} \\ x_{i1}x_{i2} - 3x_{i3} \end{bmatrix},$$

and the node dynamics in the response network and A are the same as in Example 1.

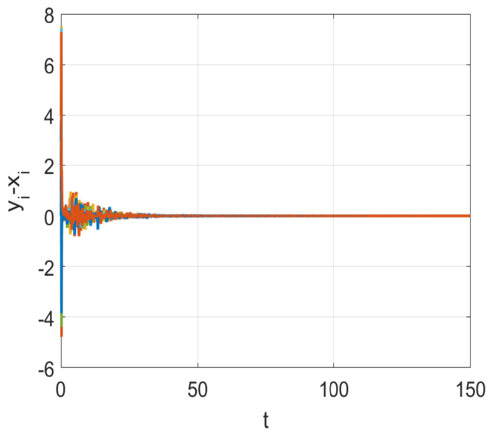
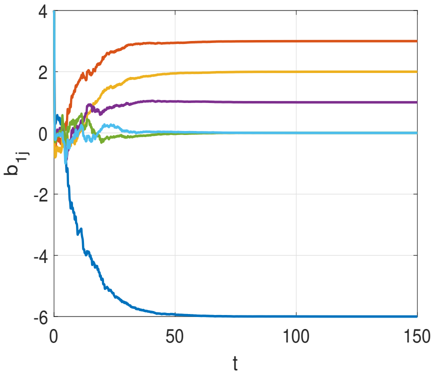


Fig. 5. The partital synchronization error of (11) and (12).

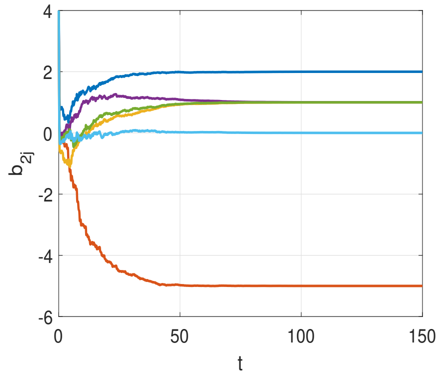
Due to $\Phi_i(x_i) = x_i$, one has

$$D\Phi_i(x_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

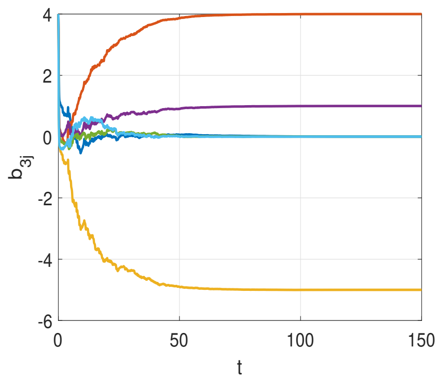
Here we use the controllers and update laws (9). As can be seen from Figure 5, the partial synchronization error of the two networks tends to zero. Figure 6 shows that $B_{3\times 6}$ estimates the partial topology of the drive network.



(a) Time evolution of $b_{1j}(1 \leq j \leq 6)$



(b) Time evolution of $b_{2j}(1 \leq j \leq 6)$



(c) Time evolution of $b_{3j}(1 \leq j \leq 6)$

Fig. 6. Identification of partial topology for drive network(11) where the response network is (12).

5. CONCLUSION

This paper studied the partial topology identification problem for complex networks. A response network with node of simpler dynamics than that in the drive network was constructed. Suitable controllers and parameters update laws were designed for partial generalized outer synchronization. Based on Lyapunovfunction, sufficient conditions were obtained to guarantee partial topology identification of complex networks. This method is cost-effective and can also identify the whole topology of complex networks. Future works will also pay attention to partial topology identification problem because of practical requirements.

ACKNOWLEDGEMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 62273267, the Natural Science Basic Research Program of Shaanxi, P. R. China under Grants 2022JC-46 and 2022JM-343.

(Received October 8, 2022)

REFERENCES

[1] F. Battiston, V. Nicosia, and V. Latora: Structural measures for multiplex network. Phys. Rev. X *89* (2014), 3, 032804. DOI:10.1103/PhysRevE.89.032804

[2] S. Boccaletti, G. Bianconi, R. Criado, C. I. Del Genio, J. Gómez-Gardeñes, M. Romance, I. Sendiña-Nadal, Z. Wang, and M. Zanin: The structure and dynamical of multilayer networks. Phys. Rep. *554* (2014), 1, 1–122. DOI:10.1016/j.physrep.2014.07.001

[3] A. Chen, J. Lu, J. Lü, and S. Yu: Generating hyperchaotic Lü attractor via state feedback control. Physica A: Statist. Mech. Appl. *364* (2006), 103–110. DOI:10.1016/j.physa.2005.09.039

[4] C. Chen, J. Zhou, F. Qu, C. Song, and S. Zhu: Identifying partial topology of complex networks with stochastic perturbations and time delay. Comm. Nonlinear Sci. Numer. Simul. *115* (2022), 106779. DOI:10.1016/j.cnsns.2022.106779

[5] M. D. Domenico, A. Solé-Ribalta, E. Cozzo, M. Kivelä, Y. Moreno, M. A. Porter, S. Gómez, and A. Arenas: Mathematical formulation of multilayer networks. Phys. Rev. E *3* (2013), 4, 041022. DOI:10.1103/physrevx.3.041022

[6] S. Gomze, A. Díaz-Guilera, J. Gomez-Gardeñes, C. J. Pérez-Vicente, Y. Moereno, and A. Arenas: Diffusion dynamics on multiplex networks. Phys. Rev. Lett. *110* (2013), 2, 028701. DOI:10.1103/physrevlett.110.028701

[7] S. J. Guo and X. C. Fu: Identification the topology of networks with discrete-time dynamics. J. Phys. A: Math. Theoret.*43* (2010), 29, 295101. DOI:10.1088/0022-3727/43/29/295101

[8] T. He, X. Lu, X. Wu, J. Lu, and X. Zheng: Optimization-based structure identification of dynamical networks. Phys. A: Statist. Mech. Appl. *392* (2013), 4, 1038–1049. DOI:10.1016/j.physa.2012.11.014

[9] R. Jansen, H. Yu, D. Greenbaum, Y. Kluger, N. J. Krogan, S. Chung, A. Emili, M. Snyder, J. F. Greenblatt, and M. Gerstein: A bayesian networks approach for predicting protein interaction from genomic data. Science *302* (2003), 5644, 449–453. DOI:10.1126/science.1087361

[10] M. Kivelä, A. Arenas, M. Barthelemy, J. P. Gleeson, Y. Moreno, and A. M. Porter: Multilayer networks. J. Complex Networks *2* (2014), 3, 203–271. DOI:10.1093/comnet/cnu016

[11] G. M. Mahmod and E. E. Mahmoud: Phase and antiphase synchronization of two identical hyperchaotic complex nonlinear systems. Nonlinear Dynamics *61* (2010), 141–152. DOI:10.1007/s11071-009-9637-2

[12] N. Marwan, M. C. Romano, M. Thiel, and J. Kurths: Recurrence plots for the analysis of complex system. Phys. Rep. *438* (2007), 5 – 6, 237–329. DOI:10.1016/j.physrep.2006.11.001

[13] M. Nabi-Abdolyousefi and M. Mesbahi: Network identification via node knockout. IEEE Trans. Automat. Control *57* (2012), 12, 3214–3219. DOI:10.1109/TAC.2012.2200376

[14] Y. Qian: Finite-time topological identification of complex network with time delay and stochastic disturbance. Kybernetika *57* (2021), 3, 534–545. DOI:10.14736/kyb-2021-3-0534

[15] M. C. Romano, M. Thiel, J. Kurths, and C. Grebogi: Estimation of the direction of the coupling by conditional probabilities of recurrence. Phys. Rev. E *76* (2007), 3, 036211. DOI:10.1103/physreve.76.036211

[16] A. Solé-Ribalta, M. D. Domenico, N. E. Kouvaris, A. Díaz-Guilera, S. Gómez, and A. Arenas: Spectral properties of the laplacian of multiplex networks. Phys. Rev. E *88* (2013), 3, 032807. DOI:10.1103/physreve.88.032807

[17] Y. Wang, X. Wu, and J. Lu: Topology identification of two-layer unidirectional complex dynamical networks based on auxiliary system approach. In: Chinese Control, Decision Conference 2016, pp. 883–887.

[18] Y. Wang, X. Wu, J. Lü, J. Lu, and R. M. D'Souza: Topology identification in two-layer complex dynamical networks. IEEE Trans. Network Sci. Engrg. *7* (2018), 1, 538–548. DOI:10.1109/tnse.2018.2885163

[19] W. X. Wang, R. Yang, Y. C. Lai, V. Kovanis, and C. Grebogi: Predicting catastrophes in nonlinear dynamical systems by compressive sensing. Phys. Rev. Lett. *106* (2011), 15, 154101. DOI:10.1103/physrevlett.106.154101

[20] X. Wu: Synchronization-based topology identification of weighted general complex dynamical networks with time-varying coupling delay. Phys. A: Statist. Mech. Appl. *387* (2008), 4, 997–1008. DOI:10.1515/9783110949261.997

[21] J. Wu and L. Jiao: Synchronization in complex dynamical networks with nonsymmetric coupling. Phys. D *237* (2008), 2487–2489. DOI:10.1016/j.physd.2008.03.002

[22] H. Y. Wu, L. Wang, L. H. Zhao, and J. L. Wang: Topology identification of coupled neural networks with multiple weights. Neurocomputing *457* (2021), 254–264. DOI:10.1016/j.neucom.2021.06.019

[23] X. Wu, W. Wang, and W. X. Zheng: Inferring topologies of complex networks with hidden variables. Phys. Rev. E *86* (2012), 4, 046106. DOI:10.1103/physreve.86.046106

[24] X. Wu, W. X. Zheng, and J. Zhou: Generalized outer synchronization between complex dynamical networks. Chaos: Interdisciplin. J. Nonlinear Sci. *19* (2009), 1, 013109. DOI:10.1063/1.3072787

[25] X. Wu, C. Zhou, G. Chen, and J. Lu: Detecting the topologies of complex networks with stochastic perturbations. Chaos: Interdisciplin. J. Nonlinear Sci. *21* (2011), 4, 043129. DOI:10.1063/1.3664396

[26] J. Wu, Y. Zhu, Y. Zheng, and H. Wang: Resilient bipartite consensus of second-order multi-agent systems with event-triggered communication. IEEE Systems J. *17* (2023), 1, 146–153. DOI:10.1109/JSYST.2021.3132623

[27] D. Yu, M. Righero, and L. Kocarev: Estimating topology of networks. Phys. Rev. Lett. *97* (2006), 18, 188701. DOI:10.1103/physrevlett.97.188701

[28] Z. Yuan, C. Zhao, W., X. Wang, Z. Di, and Y. C. Lai: Exact controllability of multiplex networka. New J. Phys. *16* (2014), 10, 103036. DOI:10.1088/1367-2630/16/10/103036

[29] S. Zhang, X. Wu, J. A. Lu, H. Feng, and J. Lü: Recovering structures of complex dynamical networks based on generalized outer synchronization. IEEE Trans. Circuits Systems l: Regular Papers *61* (2014), 11, 3216–3224. DOI:10.1109/TCSI.2014.2334951

[30] J. Zhao, Q. Li, J. A. Lu, and Z. P. Jiang: Topology identification of complex dynamical networks. Nonlinear Sci. *20* (2010), 2, 023119. DOI:10.1063/1.3421947

[31] Q. Zhao, Y. Zheng and Y. Zhu: Consensus of hybrid multi-agent systems with heterogeneous dynamics. International Journal of Control *56* (2020), 1, 189–212.

[32] Y. Zheng, Q. Zhao, J. Ma, and L. Wang: Second-order consensus of hybrid multi-agent systems. Systems Control Lett. *125* (2019), 51–58. DOI:10.1016/j.sysconle.2019.01.009

[33] L. Zhou, J. Liu, Y. Zheng, F. Xiao, and J. Xi: Game-based consensus of hybrid multi-agent systems. IEEE Trans. Cybernet. 2022. DOI:10.1109/TCYB.2022.3215 619

[34] J. Zhou and J. Lu: Topology identification of weighted complex dynamical networks. Phys. A: Statist. Mech. Appl. *386* (2007), 1, 481–491. DOI:10.1016/j.physa.2007.07.050

[35] Y. Zhu, S. Li, J. Ma, and Y. Zheng: Bipartite consensus in networks of agents with antagonistic interactions and quantization. IEEE Trans. Circuits Syst. II: Express Briefs *65* (2018), 12, 2012–2016. DOI:10.1109/tcsii.2018.2811803

[36] Y. Zhu, L. Zhou, Y. Zheng, J. Liu, and S. Chen: Sampled-data based resilient consensus of heterogeneous multiagent systems. Int. J. Robust Nonlinear Control *30* (2020), 17, 7370–7381. DOI:10.1002/rnc.5179

[37] P. Zhu, G. Wei, and J. Li: On hybrid consensus-based extended Kalman filtering with random link failures over sensor networks. Kybernetika *386* (2007), 1, 481–491. DOI:10.1016/j.physa.2007.07.050

[38] S. Zhu, J. Zhou, and J. Lu: Identification partial topology of complex dynamical networks via a pinning mechanism. Chaos: Interdiscipl. J. Nonlinear Sci. *28* (2018), 4, 043108. DOI:10.1063/1.5009946

Xueqin Zhang, Shaanxi Key Laboratory of Space Solar Power Station System, School of Mechano-electronic Engineering, Xidian University, Xi'an, 710071. P. R. China.
e-mail: zhang_xueqin98@163.com

Yunru Zhu, Shaanxi Key Laboratory of Space Solar Power Station System, School of Mechano-electronic Engineering, Xidian University, Xi'an, 710071. P. R. China.
e-mail: zhuuruu@163.com

Yuanshi Zheng, Shaanxi Key Laboratory of Space Solar Power Station System, School of Mechano-electronic Engineering, Xidian University, Xi'an, 710071. P. R. China.
e-mail: zhengyuanshi2005@163.com