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STABILIZATION OF PARTIALLY LINEAR COMPOSITE STOCHASTIC SYSTEMS VIA STOCHASTIC LUENBERGER OBSERVERS

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The present paper addresses the problem of the stabilization (in the sense of exponential stability in mean square) of partially linear composite stochastic systems by means of a stochastic observer. We propose sufficient conditions for the existence of a linear feedback law depending on an estimation given by a stochastic Luenberger observer which stabilizes the system at its equilibrium state. The novelty in our approach is that all the state variables but the output can be corrupted by noises whereas in the previous works at least one of the state variable should be unnoisy in order to design an observer.

Keywords: stochastic stability, composite stochastic system, feedback law, stochastic observer

Classification: 60H10, 93C10, 93D05, 93E15

1. INTRODUCTION

The purpose of this paper is to investigate the output stabilization (in the sense of exponential stability in mean square) by linear feedback laws given by stochastic Luenberger observers for partially linear composite stochastic systems.

The feedback stabilization of partially linear composite deterministic systems has been studied in the past decades by different authors (see [1, 14, 20, 22] or [18] for example). The stabilization by means of linear state feedback laws of deterministic systems in the form

$$\dot{x} = f(x) + G(x,\xi)\xi,$$
$$\dot{\xi} = A\xi + Bu,$$

where $x \in \mathbb{R}^n$, $\xi \in \mathbb{R}^p$, and $u \in \mathbb{R}^q$ has been handled by Saberi, Kokotovic and Sussmann in [19]. In the latter article, the authors prove, under suitable conditions on the system coefficients, that if the zero dynamics of the nonlinear system

$$\dot{x} = f(x)$$

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is globally exponentially stable, then for every matrix K such that the matrix $A + BK$ is Hurwitz, the linear feedback law $u = K\xi$ renders the original system globally exponentially stable.

The stabilization of composite stochastic systems has been investigated under different sets of hypothesis in [2, 8, 9]. In [8], sufficient conditions for the stabilization (in the sense of exponential stability in mean square) by means of linear feedback laws of partially linear composite stochastic systems in the form

$$dx_t = (f(x_t) + G(x_t, \xi_t)\xi_t)dt + g(x_t)dw_t,$$

$$d\xi_t = (A\xi_t + Bu)dt + C\xi_t dv_t,$$

where $x_t \in \mathbb{R}^n$, $\xi_t \in \mathbb{R}^p$ and $u \in \mathbb{R}^q$ are given. In fact, it is proved, under suitable conditions, that if the equilibrium solution of the nonlinear stochastic differential equation

$$dx_t = f(x_t)dt + g(x_t)dw_t$$

is exponentially stable in mean square, then for every matrix K such that the matrices $A + BK$ and C satisfy a stochastic algebraic Lyapunov inequality, the linear feedback law $u = K\xi$ renders the original system exponentially stable in mean square. The main tool used in this analysis is the stochastic Lyapunov theory developed by Khasminskii in [13].

The concept of observers for deterministic linear systems is due to Luenberger [16, 17]. Despite the fact that the design of observers for deterministic nonlinear systems remains a difficult task to achieve, different authors have proposed observers for some specific classes of nonlinear systems (see [12, 15] or [11] for example). The stabilization of deterministic systems via an observer design has been investigated in the linear case by Luenberger [17] and for some specific cases of nonlinear systems in [23] or [6] for example.

The stabilization of stochastic systems via an observer design has been studied by Tarn and Rasis [21], Wu, Karimi and Shi [25] and Dai, Chung and Hutchison [3]. In [21], the authors propose a Lyapunov based method to design exponentially bounded observers for nonlinear stochastic systems driven by noise with bounded covariance. The stabilization (in the sense of exponential boundedness in mean square) of unstable stochastic systems using an observer feedback is completed. In [3], a new design approach for observers with guaranted stability via a stochastic contraction lemma is given.

The aim of this paper is to achieve the stabilization of the above composite stochastic differential system by making use of a linear feedback law given by a stochastic Luenberger observer when an output

$$y_t = H\xi_t$$

is available. Note that when $C \equiv 0$, a preliminary result in that direction has already been obtained in [7]. The novelty in this work is that the stabilizing feedback law is obtained by means of a stochastic Luenberger observer design whereas in [7] the stabilizing feedback law is given by the usual (deterministic) Luenberger observer. Note that since in our framework we have a "perfect observation" (i. e. an unnoisy output) the usual techniques of nonlinear filtering do not apply and as a consequence the stochastic observer design introduced in this work gives an easy way to obtain an estimate of the signal process in the mean–square sense.

This paper is divided in four sections and is organized as follows. In section two, we give a brief survey of the results proved by Khasminskii [13] on the exponential stability in mean square of the equilibrium solution of a stochastic differential equation. In section three, we introduce the class of input–output partially linear composite stochastic systems we are dealing with in this paper. In section four, we design a stochastic Luenberger observer for the linear part of the system we are dealing with and we prove that this observer renders the linear subsystem exponentially stable in mean square. In section five, we state and prove a stabilization result for the overall system by using the observer design proposed in section three.

2. STOCHASTIC STABILITY

The purpose of this section is to summarize the main results on the exponential stability in mean square for the equilibrium solution of a stochastic differential equation that we need in the sequel. For a complete presentation of the Lyapunov theory of stochastic stability, we refer the reader to the book of Khasminskii [13] for example.

On a complete probability space $(\Omega, \mathcal{F}, P)$, let $(w_t)_{t\geq 0}$ be a standard $\mathbb{R}^m$–valued Wiener process defined on this space and consider the stochastic process $x_t \in \mathbb{R}^n$ solution of the stochastic differential equation written in the sense of Itô,

$$x_t = x_0 + \int_0^t b(x_s)\,\mathrm{d}s + \sum_{k=1}^m \int_0^t \sigma_k(x_s)\,\mathrm{d}w_s^k \tag{1}$$

where the coefficients b and σ_k, $1 \leq k \leq m$, are Lipschitz functionals mapping $\mathbb{R}^n$ into $\mathbb{R}^n$, vanishing at the origin, and with less than linear growth.

If, for any $s \geq 0$ and $x \in \mathbb{R}^n$, $x_t^{s,x}$, $s \leq t$, denotes the solution at time t of the stochastic differential equation (1) starting form the state x at time s, the notion of exponential stability in mean square is defined as follows.

Definition 2.1. The equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (1) is said to be exponentially stable in mean square if there exist positive constants c_1 and c_2 such that

$$E|x_t^{s,x}|^2 \leq c_1|x|^2 e^{-c_2(t-s)},$$

for any $0 \leq s \leq t$.

Remark 2.2. If $\sigma_k \equiv 0$, $1 \leq k \leq m$, the previous definition reduces to that of global exponential stability for deterministic systems.

Furthermore, denoting by $\mathcal{L}$ the infinitesimal generator of the stochastic process solution of the stochastic differential equation (1); that is, the second order differential operator defined for any function ψ in $C^2(\mathbb{R}^n, \mathbb{R})$ by

$$\mathcal{L}\psi(x) = \sum_{i=1}^{n} b^i(x)\frac{\partial \psi}{\partial x_i}(x) + \frac{1}{2}\sum_{i,j=1}^{n} a^{i,j}(x)\frac{\partial^2 \psi}{\partial x_i \partial x_j}(x)$$

where $a^{i,j}(x) = \sum_{k=1}^{m} \sigma_k^i(x)\sigma_k^j(x)$, $1 \leq i,j \leq n$, one can prove the following Lyapunov theorem.

Theorem 2.3. Assume that there exist a Lyapunov function V defined on $\mathbb{R}^n$ (i.e. a proper function V in $C^2(\mathbb{R}^n, \mathbb{R})$ which is positive definite) and three positive constants α_1, α_2 and α_3 such that

$$\alpha_1|x|^2 \leq V(x) \leq \alpha_2|x|^2,$$

and

$$\mathcal{L}V(x) \leq -\alpha_3|x|^2,$$

for any $x \in \mathbb{R}^n$. Then, the equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (1) is exponentially stable in mean square.

In addition, the following converse Lyapunov theorem gives necessary conditions for the existence of a Lyapunov function when the equilibrium solution of the stochastic differential equation (1) is exponentially stable in mean square.

Theorem 2.4. If the equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (1) is exponentially stable in mean square and the coefficients b and σ_k, $1 \leq k \leq m$, have continuous bounded derivatives up to order two, then there exist a Lyapunov function V defined on $\mathbb{R}^n$ and five positive constants α_i, $1 \leq i \leq 5$, such that

$$\alpha_1|x|^2 \leq V(x) \leq \alpha_2|x|^2,$$
$$\mathcal{L}V(x) \leq -\alpha_3|x|^2,$$

and

$$|\nabla V(x)| \leq \alpha_4|x| \quad \text{and} \quad |\nabla^2 V(x)| \leq \alpha_5,$$

for any $x \in \mathbb{R}^n$ where $\nabla V(x) = \left(\frac{\partial V}{\partial x_1}(x), ..., \frac{\partial V}{\partial x_n}(x)\right)^{\tau}$.

3. PROBLEM STATEMENT

The aim of this section is to introduce the class of partially linear composite stochastic systems we are dealing with in this paper. On a complete probability space $(\Omega, \mathcal{F}, P)$, denote by $(x_t, \xi_t, y_t) \in \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q$ the stochastic process solution of the input–output stochastic differential system written in the sense of Itô,

$$x_t = x_0 + \int_0^t (f(x_s) + G(x_s, \xi_s)\xi_s)\,\mathrm{d}s + \int_0^t g(x_s)\,\mathrm{d}w_s \tag{2}$$

$$\xi_t = \xi_0 + \int_0^t (A\xi_s + Bu)\,\mathrm{d}s + \int_0^t C\xi_s\,\mathrm{d}v_s \tag{3}$$

$$y_t = H\xi_t \tag{4}$$

where

1. x_0 and ξ_0 are given in $\mathbb{R}^n$ and $\mathbb{R}^p$, respectively,

2. $(w_t)_{t\geq 0}$ and $(v_t)_{t\geq 0}$ are independent standard Wiener processes defined on the probability space $(\Omega, \mathcal{F}, P)$ with values in $\mathbb{R}^m$ and $\mathbb{R}$, respectively,

3. f and g are functions in $C_b^2(\mathbb{R}^n, \mathbb{R}^n)$ and $C_b^2(\mathbb{R}^n, \mathbb{R}^{n\times m})$, respectively, such that $f(0) = 0$ and $g(0) = 0$,

4. G is a bounded function mapping $\mathbb{R}^n \times \mathbb{R}^p$ into $\mathbb{R}^{n\times p}$,

5. u is an $\mathbb{R}^r$–valued control law,

6. A, B and C are matrices in $\mathcal{M}_{p\times p}(\mathbb{R})$, $\mathcal{M}_{p\times r}(\mathbb{R})$ and $\mathcal{M}_{p\times p}(\mathbb{R})$, respectively, such that the pair (A, B) is stabilizable,

7. H is a matrix in $\mathcal{M}_{q\times p}(\mathbb{R})$ such that the pair (A, H) is completely observable.

Our aim in this paper is twofold. Firstly, we design a stabilizing stochastic observer for the input–output linear stochastic differential system (3)–(4); i. e. by using the output given by (4), we design a stochastic process $\left(\overline{\xi}_t\right)_{t\geq 0}$ with values in $\mathbb{R}^p$ such that the equilibrium solution of the closed–loop system

$$\xi_t = \xi_0 + \int_0^t (A\xi_s + BK\overline{\xi}_s)\,\mathrm{d}s + \int_0^t C\xi_s\,\mathrm{d}v_s$$

$$e_t = \overline{\xi}_t - \xi_t$$

for some matrix K in $\mathcal{M}_{r\times p}(\mathbb{R})$ is exponentially stable in mean square. Secondly, for such a matrix K in $\mathcal{M}_{r\times p}(\mathbb{R})$, we prove that the feedback law $u = K\overline{\xi}_t$ renders the original stochastic system exponentially stable in mean square.

Remark 3.1. The novelty in this paper lies in the fact that we are dealing with a stochastic differential system with a "perfect observation" (i. e. equation (4) defining the output of the system is unnoisy) for which the usual techniques of stochastic filtering do not apply. As a consequence, to overcome this difficulty, we design in the following section a stochastic Luenberger observer which gives a straightforward method to compute a good estimate in the mean square sense for the system process given by (3).

4. A STOCHASTIC LUENBERGER OBSERVER

The purpose of this section is to design a stabilizing observer for the input–output linear stochastic differential system (3) – (4). This new stochastic Luenberger observer design gives an alternative to filtering when dealing with stochastic systems with a perfect observation since in this framework the computation of the filter is a difficult task to reach.

Denote by $\left(\overline{\xi}_t\right)_{t\geq 0}$ the stochastic process with values in $\mathbb{R}^p$ defined by

$$\overline{\xi}_t = \overline{\xi}_0 + \int_0^t \left(A\overline{\xi}_s + Bu\right) \mathrm{d}s + \int_0^t L\left(y_s - H\overline{\xi}_s\right) \mathrm{d}s \tag{5}$$

where $\overline{\xi}_0$ is given in $\mathbb{R}^p$ and L in a matrix in $\mathcal{M}_{p\times q}(\mathbb{R})$.

Remark 4.1. The process $\left(\overline{\xi}_t\right)_{t\geq 0}$ given by (5) is the Luenberger observer (see [16] for example) associated with the input–output deterministic linear system,

$$\begin{aligned} \dot{\xi}_t &= A\xi_t + Bu \\ y_t &= H\xi_t. \end{aligned}$$

Then, setting for every $t \geq 0$, $e_t = \overline{\xi}_t - \xi_t$, one can deduce from (3)–(4) and (5) with the feedback law $u = K\overline{\xi}_t$ where K is a matrix in $\mathcal{M}_{r\times p}(\mathbb{R})$, that the stochastic process $(\xi_t, e_t)_{t\geq 0}$ solves the stochastic differential system

$$d\xi_t = (A+BK)\xi_t dt + BKe_t dt + C\xi_t dv_t \tag{6}$$

$$de_t = (A-LH)e_t dt - C\xi_t dv_t. \tag{7}$$

Therefore, if $\lambda_{\max}(K,L)$ denotes the largest eigenvalue of the symmetric matrix

$$\int_0^{+\infty} \exp\left(tM(K,L)^\tau\right) \begin{pmatrix} C^\tau C & 0 \\ 0 & 0 \end{pmatrix} \exp\left(tM(K,L)\right) \mathrm{d}t$$

where $M(K,L)$ is the block matrix $\begin{pmatrix} A+BK & BK \\ 0 & A-LH \end{pmatrix}$, one gets the following result.

Theorem 4.2. Assume that K and L are matrices in $\mathcal{M}_{r\times p}(\mathbb{R})$ and $\mathcal{M}_{p\times q}(\mathbb{R})$, respectively, such that the matrices $A+BK$ and $A-LH$ are stable (i. e. all their eigenvalues have negative real parts) and $\lambda_{\max}(K,L) < \frac{1}{2}$, then the equilibrium solution of the stochastic differential system (6) – (7) is exponentially stable in mean square.

P r o o f. First, note that if the matrices K and L in $\mathcal{M}_{r\times p}(\mathbb{R})$ and $\mathcal{M}_{p\times q}(\mathbb{R})$, respectively, are chosen such that the matrices $A+BK$ and $A-LH$ are stable then the matrix $M(K,L)$ is also stable and, since $\lambda_{\max}(K,L) < \frac{1}{2}$, the hypothesis of Theorem 4.1 proved by Wonham in [24] are satisfied.

As a consequence, if Q is a symmetric and positive definite matrix in $\mathcal{M}_{2p\times 2p}(\mathbb{R})$ there exists a symmetric and positive definite matrix P in $\mathcal{M}_{2p\times 2p}(\mathbb{R})$ solution of the stochastic algebraic Lyapunov equation

$$M(K,L)^\tau P + PM(K,L) + \begin{pmatrix} C & 0 \\ -C & 0 \end{pmatrix}^\tau P \begin{pmatrix} C & 0 \\ -C & 0 \end{pmatrix} = -Q. \tag{8}$$

Therefore, the function V mapping $\mathbb{R}^{2p}$ into $\mathbb{R}$ defined for any $\begin{pmatrix} \xi \\ e \end{pmatrix} \in \mathbb{R}^{2p}$ by

$$V\begin{pmatrix}\xi\\e\end{pmatrix} = < P\begin{pmatrix}\xi\\e\end{pmatrix},\begin{pmatrix}\xi\\e\end{pmatrix} >$$

is a Lyapunov function such that

$$\alpha_1\left|\begin{pmatrix}\xi\\e\end{pmatrix}\right|^2 \leq V\begin{pmatrix}\xi\\e\end{pmatrix} \leq \alpha_2\left|\begin{pmatrix}\xi\\e\end{pmatrix}\right|^2$$

where α_1 and α_2 are respectively the smallest and largest eigenvalue of the symmetric and positive definite matrix P.

Moreover, if $\mathcal{L}_1$ is the infinitesimal generator of the stochastic process solution of the stochastic differential system (6) – (7), one gets

$$\begin{aligned}\mathcal{L}_1 V\begin{pmatrix}\xi\\e\end{pmatrix} &= \left\langle (M(K,L)^\tau P + PM(K,L))\begin{pmatrix}\xi\\e\end{pmatrix},\begin{pmatrix}\xi\\e\end{pmatrix}\right\rangle \\ &\quad + \left\langle \begin{pmatrix}C & 0\\-C & 0\end{pmatrix}^\tau P\begin{pmatrix}C & 0\\-C & 0\end{pmatrix}\begin{pmatrix}\xi\\e\end{pmatrix},\begin{pmatrix}\xi\\e\end{pmatrix}\right\rangle\end{aligned}$$

which yields

$$\mathcal{L}_1 V\begin{pmatrix}\xi\\e\end{pmatrix} = - < Q\begin{pmatrix}\xi\\e\end{pmatrix},\begin{pmatrix}\xi\\e\end{pmatrix} > \leq -\alpha_3\left|\begin{pmatrix}\xi\\e\end{pmatrix}\right|^2$$

where α_3 is the smallest eigenvalue of the matrix Q and hence, according with the stochastic Lyapunov theorem (Theorem 2.3) the equilibrium solution of the stochastic differential system (6) – (7) is exponentially stable in mean square. □

Remark 4.3. Assumption $\lambda_{\max}(K,L) < \frac{1}{2}$ is quite restrictive but needed to ensure the existence of a solution for the stochastic algebraic Lyapunov equation (8) according with Theorem 4.1 in [24]. Indeed, to our knowledge the are no result in the literature involving a less restrictive assumption in order to prove the existence of a solution for a stochastic algebraic Lyapunov equation.

To conclude this section, note that the above analysis remains valid if the stochastic differential equation (3) is driven by a multidimensional Wiener process.

5. A STABILIZATION RESULT

In this section, we prove that the stochastic Luenberger observer obtained in the previous section permits to stabilize the original system introduced in section three.

Theorem 5.1. Assume that the equilibrium solution $x_t \equiv 0$ of the nonlinear stochastic differential equation

$$x_t = x_0 + \int_0^t f(x_s)\,\mathrm{d}s + \int_0^t g(x_s)\,\mathrm{d}w_s \tag{9}$$

is exponentially stable in mean square. Then the control law u defined on $\mathbb{R}^q$ by $u = K\overline{\xi}_t$ where $\left(\overline{\xi}_t\right)_{t\geq 0}$ is the stochastic Luenberger observer given by (5) where K and L are matrices in $\mathcal{M}_{r\times p}(\mathbb{R})$ and $\mathcal{M}_{p\times q}(\mathbb{R})$, respectively, such that the matrices $A+BK$ and $A-LH$ are stable and $\lambda_{\max}(K,L)<\frac{1}{2}$ is a stabilizing feedback law for the stochastic differential system (2) – (3).

P r o o f. Since the equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (9) is exponentially stable in mean square, one can deduce from the converse stochastic Lyapunov theorem (Theorem 2.4) that there exist a Lyapunov function V defined on $\mathbb{R}^n$ and five positive constants α_i, $1\le i\le 5$, such that

$$\alpha_1|x|^2 \le V(x) \le \alpha_2|x|^2 \tag{10}$$

$$\mathcal{L}_2 V(x) \le -\alpha_3|x|^2 \tag{11}$$

and

$$|\nabla V(x)| \le \alpha_4|x| \ \text{ and } \ |\nabla^2 V(x)| \le \alpha_5 \tag{12}$$

for all $x\in\mathbb{R}^n$, where $\mathcal{L}_2$ denotes the infinitesimal generator of the stochastic process solution of the stochastic differential equation (9).

Applying Itô's formula to $V(x_t)$ where x_t is the solution of the stochastic differential equation (2) and taking the expectation in the resulting equality yields,

$$\frac{d}{dt}E(V(x_t)) = E(\mathcal{L}_2 V(x_t)) + E(\nabla V(x_t)G(x_t,\xi_t)\xi_t)). \tag{13}$$

Hence, from Young's inequality and estimates (10) and (11) one gets,

$$\frac{d}{dt}E(V(x_t)) \le -\frac{\alpha_3}{\alpha_2}E(V(x_t)) + cE\left(|\nabla V(x_t)|^2\right) + \frac{1}{c}||G||_\infty^2 E\left(|\xi_t|^2\right) \tag{14}$$

where $c>0$ and $||G||_\infty = \sup_{(x,\xi)\in\mathbb{R}^n\times\mathbb{R}^p}|G(x,\xi)|$.

Moreover, since according with Theorem 4.2, the equilibrium solution of the stochastic differential system (6)–(7) is exponentially stable in mean square it yields,

$$E(|\xi_t|^2) \le \beta_1 \left|\begin{pmatrix} \xi_0 \\ \bar{\xi}_0 - \xi_0 \end{pmatrix}\right|^2 \mathrm{e}^{-\beta_2 t} \tag{15}$$

for some positive constants β_1 and β_2 and hence,

$$\frac{d}{dt}E(V(x_t)) \le \left(-\frac{\alpha_3}{\alpha_2} + c\frac{\alpha_4^2}{\alpha_1}\right)E(V(x_t)) + \frac{1}{c}\beta_1||G||_\infty^2 \left|\begin{pmatrix} \xi_0 \\ \bar{\xi}_0 - \xi_0 \end{pmatrix}\right|^2 \mathrm{e}^{-\beta_2 t}.$$

As a consequence, since the constant β_2 in (15) can be chosen small enough so that $\beta_2 < \frac{\alpha_3}{2\alpha_2}$, setting $c = \frac{\alpha_1}{\alpha_4^2}\left(\frac{\alpha_3}{\alpha_2} - 2\beta_2\right)$ and applying Gronwall's lemma to the previous inequality yields

$$E(V(x_t)) \leq E(V(x_0))\mathrm{e}^{-2\beta_2 t} + \frac{1}{c}\frac{\beta_1}{\beta_2}||G||_\infty^2 \left|\begin{pmatrix} \xi_0 \\ \bar{\xi}_0 - \xi_0 \end{pmatrix}\right|^2 (\mathrm{e}^{-2\beta_2 t} + 2\mathrm{e}^{-\beta_2 t})$$

and therefore, taking (10) into account, one gets

$$E\left(|x_t|^2\right) \leq \left(\frac{\alpha_2}{\alpha_1}|x_0|^2 + \frac{3}{c}\frac{\beta_1}{\alpha_1\beta_2}||G||_\infty^2 \left|\begin{pmatrix} \xi_0 \\ \bar{\xi}_0 - \xi_0 \end{pmatrix}\right|^2\right)\mathrm{e}^{-\beta_2 t}. \tag{16}$$

Hence, the equilibrium solution of the closed–loop system deduced from the original system with the feedback law $u = K\bar{\xi}_t$ is exponentially stable in mean square and consequently, the control law $u = K\bar{\xi}_t$ is a stabilizing feedback for the composite stochastic system (2)–(4). □

Example 5.2. Assume that the processes ξ_t and y_t are given by

$$d\xi_t = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\xi_t + \begin{pmatrix} 0 \\ 1 \end{pmatrix}u\right)\mathrm{d}t + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\xi_t \mathrm{d}v_t$$

and

$$y_t = \begin{pmatrix} 0 & 1 \end{pmatrix}\xi_t$$

and that the equilibrium solution $x_t \equiv 0$ of the nonlinear stochastic differential equation (9) is exponentially stable in mean square. Then, it is easy to verify that the hypothesis of Theorem 5.1 are satisfied with $K = \begin{pmatrix} -17/16 & -1/2 \end{pmatrix}$ and $L = \begin{pmatrix} 17/16 \\ 1/2 \end{pmatrix}$ and therefore the overall system is stabilizable by means of the feedback law u given by $u = K\bar{\xi}_t$ where $\left(\bar{\xi}_t\right)_{t\geq 0}$ is the stochastic Luenberger observer given by (5).

6. CONCLUSION

In this paper we have investigated the stabilization (in the sense of exponential stability in mean square) of partially linear composite stochastic systems by means of a stochastic observer. The technique presented in section 4 to design the stochastic Luenberger observer may be extended to obtain stochastic observers for nonlinear stochastic differential systems by generalizing to the stochastic context the methods developed among others by Gauthier and Kupka [11] or Hu [12] for example. This task is under investigation and will be the subject of future publications. Based on the research works of Zhou, Xiao and Lu [28] or Ghanes, De Leon and Barbot [10] for example, the technique exposed in this paper can be extended to stochastic differential systems with time–delays and may then be applied to the study of delayed neural networks introduced by Zhang, Han and Wang [27] or Zhang, Han, Ge and Zhang [26] for example. Note also that the stochastic observers introduced in this paper may be used when studying specific cases of communication protocols like those exposed by Ding, Wang and Han [5] or Ding, Han, Wang and Ge [4].

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