KYBERNETIKA — VOLUME *58* (2022), NUMBER 4, PAGES 522–546

LEADER-FOLLOWING CONSENSUS FOR LOWER-TRIANGULAR NONLINEAR MULTI-AGENT SYSTEMS WITH UNKNOWN CONTROLLER AND MEASUREMENT SENSITIVITIES

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In this paper, a novel consensus algorithm is presented to handle with the leader-following consensus problem for lower-triangular nonlinear MASs (multi-agent systems) with unknown controller and measurement sensitivities under a given undirected topology. As distinguished from the existing results, the proposed consensus algorithm can tolerate to a relative wide range of controller and measurement sensitivities. We present some important matrix inequalities, especially a class of matrix inequalities with multiplicative noises. Based on these results and a dual-domination gain method, the output consensus error with unknown measurement noises can be used to construct the compensator for each follower directly. Then, a new distributed output feedback control is designed to enable the MASs to reach consensus in the presence of large controller perturbations. In view of a Lyapunov function, sufficient conditions are presented to guarantee that the states of the leader and followers can achieve consensus asymptotically. In the end, the proposed consensus algorithm is tested and verified by an illustrative example.

Keywords: consensus, lower-triangular, nonlinear multi-agent systems, measurement noises, controller sensitivity, output feedback

Classification: 93C10

1. INTRODUCTION

MASs may be reviewed as a class of large-scale complex networks which are composed of multiple independent but interacting agents in a given distributed configuration. These agents are capable of accomplishing some complex tasks through interaction, cooperation and collective intelligence. Recently, researchers have paid more increasing enthusiasm to the cooperative control problems of MASs because they can be utilized in various fields, for instance, optimization and distributed computing [45], micro-grid [3, 4], power systems [31], formation control of robot [7, 33] and unmanned aerial vehicles [11]. In order to realize cooperative control, the problem of reaching consensus should be solved firstly. That is, an appropriate control law is designed to make the states of all agents converge to same values on the base of the information between each agent and its

DOI: 10.14736/kyb-2022-4-0522

neighbor nodes. At present, researchers have proposed two categories of the consensus problems, i.e., the leaderless consensus problem [8] and the leader-following consensus problem [14, 22, 24, 43]. For example, the problem of distributed consensus control was solved for MASs with unknown control and output directions [22]. In [8], the actuator faults were modelled by a polytopic uncertainty method. Then, robust and reliable consensus control was proposed for linear MASs with time-varying parameters and actuator faults. In the presence of measurement sensitivities, an adaptive distributed controller was investigated to realize the leader-following consensus for MASs via output feedback based on the backstepping control method and the Nussbaum-type function [24]. For the consensus problem of nonlinear MASs, considerable results have been derived [12, 13, 23, 25, 29, 38]. In [29], an adaptive distributed consensus protocol was developed for a class of time-delay nonlinear MASs with external noises. The problems of distributed tracking were also studied for nonlinear MASs with unknown hysteresis [12] and uncertain external disturbances [38]. More recently, the dynamic gain method has been applied to deal with the consensus problem for nonlinear MASs with time delays [13, 23, 25].

The aforementioned works on the consensus problem for MASs are obtained under the assumption that the distributed controller of each agent is not subject to exogenous disturbances or parameter uncertainties. In other words, the proposed consensus protocol can be implemented accurately. In practical, the controller gain may be variable due to the factors, such as round-off errors in the numerical calculation, the aging of the facilities, complex environments and noise disturbances, which may lead to control performance degradation or divergence. In order to overcome this fragility of the controller, various non-fragile consensus protocols have been developed for MASs under the fixed topology [1, 2, 15, 16, 35, 42, 46, 47]. For instance, the authors in [1] proposed a non-fragile sampled-data based control scheme for nonlinear MASs with uncertain Markovian jump parameters and time varying delays. By means of randomly occurring gain fluctuations and an extended dissipative approach, non-fragile controllers was applied such that the nonlinear MASs reached consensus even if there existed semi-Markovian jumping parameters and external disturbances [35]. In [16], a robust non-fragile protocol was proposed to solve the finite-time consensus problem for a class of stochastic nonlinear MASs by state feedback. Non-fragile control protocols such as containment control [47], robust tracking control [15], near-optimal control [42] and H_∞ control [2, 46] have been developed for the consensus problem of MASs. It should be noted that the existing results of non-fragile consensus for nonlinear MASs are all obtained under the framework of LMIs (linear matrix inequalities) or the linear programming.

In this paper, we investigate the leader-following consensus problem of nonlinear MASs in a lower triangular form with unknown output measurement noises and controller sensitivity based on high gain technology. The ideas come from the above works and the works in [18], in which the problem of output feedback stabilization was solved for nonlinear systems with unknown measurement sensitivities. We assume that there exists unknown controller perturbation in the distributed controller of each follower, and construct a matrix A by the controllers' coefficients with multiplicative noises. The matrix A is in Luenberger controllable canonical form. Then, a symmetric positive definite matrix P is also constructed such that $A^T P + PA < 0$. Based on these results and some other important matrix inequalities and high-gain technology, a distributed compensator is proposed to estate the states of the followers, and a new dynamic output feedback controller is designed to guarantee the nonlinear MASs reach consensus. The major contributions are illustrated as follows:

1. Some important matrix inequalities, especially a class of matrix inequalities with multiplicative noises are proposed.
2. A novel consensus protocol is proposed for nonlinear MASs in a lower triangular form with unknown controller and measurement sensitivities. The bounds of the sensitivities are enlarged from 0 to ∞.
3. By constructing a Lyapunov function, sufficient conditions are obtained to ensure that the MASs reach consensus. Compared to the LMIs approach, there always exists a consensus algorithm of the nonlinear lower-triangular MASs.

The remainder of this paper is given as follows. In Section 2, we describe the main issue and introduce some relevant preliminaries. The design of output feedback consensus protocol and consensus analysis are presented in Section 3. Section 4 illustrates the validity of proposed consensus algorithm by means of numerical simulations. At last, the summary is offered in Section 5.

2. PRELIMINARIES

2.1. Graph theory

Consider a set of followers $\mathcal{S} = \{1, \ldots, N\}$, a set of edges $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$, and an undirected graph $\mathcal{G} = (\mathcal{S}, \mathcal{R})$. If $(j, i) \in \mathcal{R}$, then we call that there is an edge between node i to j. The set of neighbors of node i is defined by $E_i = \{j \in \mathcal{S} : (j, i) \in \mathcal{R}, j \neq i\}$. We assume that the graph $\mathcal{G}$ is connected, which means that there exists a path between any two nodes of $\mathcal{G}$.

Let

$$a_{ij} = \begin{cases} 1, \text{if } (i, j) \in \mathcal{R}, \\ 0, \text{otherwise}, \end{cases}$$

and $\Omega_i = \sum_{j \in E_i} a_{ij}$. The adjacency matrix of $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, the degree matrix $\Omega = \mathrm{diag}(\Omega_1, \ldots, \Omega_N)$, the Laplacian matrix $\mathcal{L} = \Omega - \mathcal{A}$ is symmetric, and the augmented graph $\bar{\mathcal{G}}$ is consisted of the graph $\mathcal{G}$, the vertex 0 and edges between the leader and its neighbors, and the matrix $\mathcal{H}$ is defined as $\mathcal{H} = \mathcal{L} + \mathcal{B}$, where $\mathcal{B} = \mathrm{diag}(b_1, \ldots, b_N)$, and

$$b_i = \begin{cases} 1, \text{the leader is a neighbor of node } i, \\ 0, \text{otherwise.} \end{cases}$$

2.2. Problem description

Consider a MAS with $N + 1$ agents which are regarded as a class of nonlinear systems and have a lower-triangular form. The integers 0 and $1, \ldots, N$ are used to index the leader and the N followers, respectively. We give the dynamical equation of ith agent as follows,

$$\left\{\begin{array}{l} \dot{\epsilon}^i(t) = \Delta_0\epsilon^i(t) + \phi\left(\epsilon^i(t)\right) + \Upsilon u^i(t), \\ \nu^i(t) = \theta(t)\Xi\epsilon^i(t),\ i = 0, 1, \ldots, N, \end{array}\right. \tag{1}$$

where

$$\Delta_0 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}, \Upsilon = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \Xi = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix},$$

$\epsilon^i(t) = (\epsilon^i_1(t), \epsilon^i_2(t), \ldots, \epsilon^i_n(t))^T \in \mathbb{R}^n$, $u^i(t) \in \mathbb{R}$ and $\nu^i(t) \in \mathbb{R}$ are the state, the control input and the output of the ith agent, respectively. We make the assumption that only the outputs are measurable and $u^0(t) = 0$, and the continuous nonlinear function $\phi(\epsilon^i(t)) = (\phi_1(\epsilon^i(t)), \phi_2(\epsilon^i(t)), \ldots, \phi_n(\epsilon^i(t)))^T \in \mathbb{R}^n$ possesses the lower-triangular form, i.e., $\phi_m(\epsilon^i(t)) = \phi_m(\epsilon^i_1(t), \epsilon^i_2(t), \ldots, \epsilon^i_m(t)) \in \mathbb{R}$, and $\phi_m(0) = 0$, $m = 1, \ldots, n$. The measurement noise $\theta(t)$ is an unknown continuous bounded function.

Remark 2.1. In practice, we can apply the nonlinear system (1) to model some physical systems, for instance, the electromechanical systems, the synchronous generator, and the single-link robot [26, 34, 39]. Moreover, the system (1) is strict feedback, which is also frequently encountered in the control problems of nonlinear MASs. For some systems, such as mobile manipulators [23] and chemical reactors [13], it is comparatively easy to measure the output compared to the state. In addition, because of manufacturing reasons, sensitivity error $\theta(t)$ in the measurement output of sensors is inevitable [6, 18].

The aim of this paper is illustrated as follows: under a given undirected communication topology, a distributed controller is designed for each follower via output feedback such that all states of followers asymptotically converge to those of the leader. At the same time, the controlled system can remain steady even if the designed controllers have large noise disturbances or implementation errors. In what follows, we propose the definition of consensus for the multi-agent system (1).

Definition 2.2. If the following conditions are satisfied,

$$\lim_{t\to\infty}\left(\epsilon^i(t) - \epsilon^0(t)\right) = 0,\ i = 1, \ldots, N,$$

then, we claim that the nonlinear multi-agent system (1) reaches consensus.

In order to derive our main results, we present the following assumptions and lemmas.

Assumption 2.3. The graph $\mathcal{G}$ is fixed and undirected. Then, for its augmentation $\bar{\mathcal{G}}$, there exists a spanning tree such that the leader is the root.

Assumption 2.4. For any real numbers y_i, z_i, the following inequalities are satisfied,

$$|\phi_h(y_1, y_2, \ldots, y_h) - \phi_h(z_1, z_2, \ldots, z_h)| \leq \tau(|y_1 - z_1| + \cdots + |y_h - z_h|),\ h = 1, \ldots, n,$$

where $\tau > 0$ is a known constant.

Assumption 2.5. The measurement noise $\theta(t)$ is unknown continuous and bounded. There exist known positive constants $0 < \theta_1 \leq 1$ and $1 \leq \theta_2 < \infty$ such that $\theta(t) \in [\theta_1, \theta_2]$, for all $t \geq 0$.

Assumption 2.6. There exist controller sensitivities $\rho^i(t)$, $i = 1, \ldots, N$ with the distributed controllers for all followers, which are assumed to be unknown, continuous and bounded, and satisfy $\rho^i(t) \in \left[\rho_1^i, \rho_2^i\right], \forall t \geq 0$, where $0 < \rho_1^i \leq 1$ and $1 \leq \rho_2^i < \infty$ are two known positive constants.

Remark 2.7. Assumption 2.3 is a general and standard condition for the MASs proposed in many works [10, 14]. Based on this assumption, it is shown that there exists an undirected spanning tree in the communication topology such that the root node is a leader, which ensures that each follower can obtain information from the leader. From Assumption 2.4, the nonlinear terms $\phi_m\left(\epsilon^i(t)\right)$ satisfy the Lipschitz growth condition since $\phi_m(0) = 0$. Note that the Lipschitz constant τ is known. The system uncertainties may be weakened. However, it is still possess representative characteristics in various nonlinear MASs [10, 37]. Moreover, the Assumptions 2.5 and 2.6 are necessary to design our control scheme.

Lemma 2.8. (Krstic and Deng [19]) The following Young's inequality

$$uh \leq \frac{\upsilon^a}{a}|u|^a + \frac{1}{b\upsilon^b}|h|^b,$$

is satisfies for any $(u, h)^T \in \mathbb{R}^2$, where $\upsilon > 0$, $a > 1$, $b > 1$, and $(a-1)(b-1) = 1$.

Lemma 2.9. (Yang and Lin [40]) The following inequality holds,

$$(|u_1| + \cdots + |u_n|)^p \leq n^{p-1}(|u_1|^p + \cdots + |u_n|^p),$$

for any $p \in [1, +\infty)$ and any $u_i \in \mathbb{R}$, $i = 1, \ldots, n$.

Lemma 2.10. (Ni and Cheng [30]) For an undirected and connected graph $\bar{\mathcal{G}}$, we can obtain that the matrix $\mathcal{H}$ is symmetric positive definite. Moreover, the eigenvalues $\lambda_{\mathcal{H}}^i$ $(i = 1, \ldots, N)$ of the matrix $\mathcal{H}$ are positive.

Lemma 2.11. Let $n \times n$ matrices $\Upsilon_{i,k}$ $(i = 1, \ldots, N)$ which are given as follows

$$\Upsilon_{i,k} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\rho^i(t)k_1 & -\rho^i(t)k_2 & \cdots & -\rho^i(t)k_n \end{pmatrix}.$$

The matrices $\Upsilon_{i,k}$ are in Luenberger controllable canonical form. Let $k_1, \ldots, k_n$ be given as

$$\begin{aligned} k_j &= d_j k_{j+1} - d_j \textstyle\prod_{m=j}^{n-1} d_m + \prod_{m=j-1}^{n-1} d_m, \ j = 1, \ldots, n-1, \\ k_n &= d_{n-1} + \tfrac{1}{2} + k_0, \end{aligned} \tag{2}$$

where $k_0 > 0$, and

$$d_j = d_{j-1} + \tfrac{n+1-j}{2} + 1 + \bar{d}_j,\ j = 1, \ldots, n-1, \tag{3}$$

where $d_0 = 0$, $\bar{d}_1 = 0$, and

$$\bar{d}_j = \tfrac{1}{2} \sum_{m=2}^{j-1} \left(\bar{d}_m + 1 + \tfrac{n-m+1}{2}\right)^2 \prod_{k=m}^{j-1} d_k^2 + \tfrac{1}{2} d_1^2 \prod_{k=1}^{j-1} d_k^2,\ j = 2, \ldots, n-1, \tag{4}$$

and

$$\begin{aligned} &\delta_j = d_j \prod_{m=j}^{n-1} d_m - \prod_{m=j-1}^{n-1} d_m,\ j = 1, \ldots, n-1, \\ &\delta_n = d_{n-1} + \tfrac{1}{2}. \end{aligned} \tag{5}$$

Then, there exist two positive constants k_0 and k_*^i such that when $k_0 > k_*^i$ for all $i = 1, \ldots, N$, thus the following inequalities can be yielded,

$$\Upsilon_{i,k}^T Q_k + Q_k \Upsilon_{i,k} \leq -\rho_M^i I, \tag{6}$$

where $\rho_M^i = \frac{\lambda_{\min}(Q_1)}{(\lambda_{\max}(Q_1))^2} \min\left\{k_0 \rho_1^i, 1\right\}$, $Q_1 = Q_d^T Q_d$, I is an $n \times n$ identity matrix and $Q_k = Q_1^{-1}$ is a positive definite matrix, and Q_d is given as

$$Q_d = \begin{pmatrix} 1 & -d_1 & 0 & \cdots & 0 \\ 0 & 1 & -d_2 & \cdots & \vdots \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & -d_{n-1} \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}.$$

Proof. Let $\varepsilon_i = (\varepsilon_{i,1}, \varepsilon_{i,2}, \ldots, \varepsilon_{i,n})^T$ and $\xi_i = (\xi_{i,1}, \xi_{i,2}\ , \ldots,\ \xi_{i,n})^T$. Consider the linear systems $\dot{\varepsilon}_i = \Upsilon_{i,k}^T \varepsilon_i$ $(i = 1, \ldots, N)$, and a linear transformation $\xi_i = Q_d \varepsilon_i$. We have

$$\begin{aligned} &\xi_{i,j} = \varepsilon_{i,j} - d_j \varepsilon_{i,j+1},\ j = 1, \ldots, n-1, \\ &\xi_{i,n} = \varepsilon_{i,n}, \end{aligned}$$

which inverse transformation is as follows,

$$\begin{aligned} &\varepsilon_{i,j} = \xi_{i,j} + \sum_{m=j+1}^{n} \xi_{i,m} \prod_{k=j}^{m-1} d_k,\ j = 1, \ldots, n-1, \\ &\varepsilon_{i,n} = \xi_{i,n}. \end{aligned}$$

Therefore, $\dot{\xi}_i = Q_d \Upsilon_{i,k}^T \varepsilon_i$, that is,

$$\begin{aligned} \dot{\xi}_{i,j} = &\left(d_j k_{j+1} \rho^i(t) - k_j \rho^i(t) - d_j \prod_{k=j}^{n-1} d_k + \prod_{k=j-1}^{n-1} d_k\right) \xi_{i,n} \\ &+ \sum_{m=j+1}^{n-1} \xi_{i,m} \left(\prod_{k=j-1}^{m-1} d_k - d_j \prod_{k=j}^{m-1} d_k\right) + (d_{j-1} - d_j)\xi_{i,j} + \xi_{i,j-1}, \\ &j = 1, \ldots, n-1, \\ \dot{\xi}_{i,n} = &\left(d_{n-1} - \rho^i(t) k_n\right) \xi_{i,n} + \xi_{i,n-1}, \end{aligned} \tag{7}$$

where $\xi_{i,0}=0$. Substituting (2) and (5) into (7) yields,

$$\begin{aligned}\dot{\xi}_{i,j} &= \delta_j\left(\rho^i(t)-1\right)\xi_{i,n}+(d_{j-1}-d_j)\xi_{i,j}+\xi_{i,j-1}\\ &\quad+\textstyle\sum_{m=j+1}^{n-1}\xi_{i,m}\left(\prod_{k=j}^{m-1}d_k(d_{j-1}-d_j)\right),\ j=1,\dots,n-1,\\ \dot{\xi}_{i,n} &= \left(\delta_n\left(1-\rho^i(t)\right)-k_0\rho^i(t)\right)\xi_{i,n}-\tfrac{1}{2}\xi_{i,n}+\xi_{i,n-1},\end{aligned}$$

where $d_0=0$. Note that $\xi_{i,j}\xi_{i,m}\le\frac{1}{2}\left(\xi_{i,j}^2+\xi_{i,m}^2\right)$. Thus,

$$\xi_{i,n}\dot{\xi}_{i,n}\le\left(\delta_n\left(1-\rho^i(t)\right)-k_0\rho^i(t)\right)\xi_{i,n}^2+\frac{1}{2}\xi_{i,n-1}^2. \tag{8}$$

Similarly, we can obtain,

$$\begin{aligned}\xi_{i,j}\dot{\xi}_{i,j} &= \delta_j\left(\rho^i(t)-1\right)\xi_{i,j}\xi_{i,n}+\left(d_{j-1}-d_j+\tfrac{n-j}{2}\right)\xi_{i,j}^2\\ &\quad+\tfrac{1}{2}\textstyle\sum_{m=j+1}^{n-1}\xi_{i,m}^2(d_{j-1}-d_j)^2\prod_{k=j}^{m-1}d_k^2+\tfrac{1}{2}\xi_{i,j-1}^2,\ j=1,\dots,n-1.\end{aligned} \tag{9}$$

From (3), (8) and (9), we have

$$\begin{aligned}\textstyle\sum_{j=1}^n\xi_{i,j}\dot{\xi}_{i,j} &\le \left(\delta_n\left(1-\rho^i(t)\right)-k_0\rho^i(t)\right)\xi_{i,n}^2+\left(\rho^i(t)-1\right)\textstyle\sum_{j=1}^{n-1}\delta_j\xi_{i,j}\xi_{i,n}-\sum_{j=1}^{n-1}\xi_{i,j}^2\\ &\quad-\textstyle\sum_{j=2}^{n-1}\bar{d}_j\xi_{i,j}^2+\tfrac{1}{2}\sum_{j=2}^{n-1}\xi_{i,j}^2\left(\sum_{m=2}^{j-1}\left(\left(\bar{d}_m+1+\tfrac{n-m+1}{2}\right)^2\prod_{k=m}^{j-1}d_k^2\right)\right)\\ &\quad+\tfrac{1}{2}d_1^2\textstyle\sum_{j=2}^{n-1}\xi_{i,j}^2\prod_{k=1}^{j-1}d_k^2.\end{aligned} \tag{10}$$

By substituting (4) into (10), it follows that

$$\begin{aligned}\textstyle\sum_{j=1}^n\xi_{i,j}\dot{\xi}_{i,j} &\le \left(\delta_n\left(1-\rho^i(t)\right)-k_0\rho^i(t)\right)\xi_{i,n}^2+\left(\rho^i(t)-1\right)\textstyle\sum_{j=1}^{n-1}\delta_j\xi_{i,j}\xi_{i,n}-\sum_{j=1}^{n-1}\xi_{i,j}^2\\ &\le -\tfrac{k_0\rho^i(t)}{2}\xi_{i,n}^2-\tfrac{1}{2}\textstyle\sum_{j=1}^{n-1}\xi_{i,j}^2+\left(\delta_n\left(1-\rho^i(t)\right)-\tfrac{k_*^i\rho^i(t)}{2}\right)\xi_{i,n}^2\\ &\quad+\left(\rho^i(t)-1\right)\textstyle\sum_{j=1}^{n-1}\delta_j\xi_{i,j}\xi_{i,n}-\tfrac{1}{2}\sum_{j=1}^{n-1}\xi_{i,j}^2.\end{aligned}$$

We can select k_0 and k_*^i such that $k_0>k_*^i$ $(i=1,\dots,N)$, and $\delta_n\left(1-\rho^i(t)\right)-\frac{k_*^i\rho^i(t)}{2}\le 0$, and

$$\tfrac{2}{(n-1)^2}\left(\tfrac{k_*^i\rho^i(t)}{2}-\delta_n\left(1-\rho^i(t)\right)\right)-\left(1-\rho^i(t)\right)^2\delta_j^2\ge 0,\ j=1,\dots,n-1.$$

Then we can deduce that $\left(\delta_n\left(1-\rho^i(t)\right)-\frac{k_*^i\rho^i(t)}{2}\right)\xi_{i,n}^2+\left(\rho^i(t)-1\right)\sum_{j=1}^{n-1}\delta_j\xi_{i,j}\xi_{i,n}-\frac{1}{2}\sum_{j=1}^{n-1}\xi_{i,j}^2\le 0,\ \forall t\in[0,+\infty)$. Therefore,

$$\textstyle\sum_{j=1}^n\xi_{i,j}\dot{\xi}_{i,j}\le-\tfrac{k_0\rho^i(t)}{2}\xi_{i,n}^2-\tfrac{1}{2}\sum_{j=1}^{n-1}\xi_{i,j}^2\le-\tfrac{1}{2}\min\left\{k_0\rho_1^i,1\right\}\sum_{j=1}^n\xi_{i,j}^2.$$

Note that $\frac{1}{2}\xi_i^T\xi_i=\frac{1}{2}\sum_{j=1}^n\xi_{i,j}^2$, $\xi_i=Q_d\varepsilon_i$ and $\dot{\varepsilon}_i=\Upsilon_{i,k}^T\varepsilon_i$. We have

$$\begin{aligned}\textstyle\sum_{j=1}^n\xi_{i,j}\dot{\xi}_{i,j} &= \tfrac{1}{2}\tfrac{d\left(\xi_i^T\xi_i\right)}{dt}=\tfrac{1}{2}\left(\dot{\varepsilon}_i^TQ_d^TQ_d\varepsilon_i+\varepsilon_i^TQ_d^TQ_d\dot{\varepsilon}_i\right)\\ &= \tfrac{1}{2}\left(\varepsilon_i^T\Upsilon_{i,k}Q_1\varepsilon_i+\varepsilon_i^TQ_1\Upsilon_{i,k}^T\varepsilon_i\right).\end{aligned}$$

Then,

$$\begin{aligned}&\varepsilon_i^T \Upsilon_{i,k} Q_1 \varepsilon_i + \varepsilon_i^T Q_1 \Upsilon_{i,k}^T \varepsilon_i \le -\min\left\{k_0 \rho_1^i, 1\right\} \textstyle\sum_{j=1}^n \xi_{i,j}^2 \\ &\le -\lambda_{\min}(Q_1) \min\left\{k_0 \rho_1^i, 1\right\} \varepsilon_i^T \varepsilon_i.\end{aligned}$$

Further,

$$\Upsilon_{i,k} Q_1 + Q_1 \Upsilon_{i,k}^T \le -\lambda_{\min}(Q_1) \min\left\{k_0 \rho_1^i, 1\right\} I.$$

Let $Q_k = Q_1^{-1}$. Then,

$$\begin{aligned}&Q_1^{-1}\left(\Upsilon_{i,k} Q_1 + Q_1 \Upsilon_{i,k}^T\right) Q_1^{-1} = Q_k \Upsilon_{i,k} + \Upsilon_{i,k}^T Q_k \\ &\le -\lambda_{\min}(Q_1) \min\left\{k_0 \rho_1^i, 1\right\} Q_k^2 \\ &\le -\tfrac{\lambda_{\min}(Q_1)}{(\lambda_{\max}(Q_1))^2} \min\left\{k_0 \rho_1^i, 1\right\} I.\end{aligned}$$

Thus, the conclusions hold. □

Lemma 2.12. Suppose that the conditions of Lemma 2.11 hold. For an $n \times n$ triangular matrix $D_\sigma = diag\{\sigma, 1+\sigma, \ldots, n-1+\sigma\}$, there exists an appropriate positive constant σ_d such that when $\sigma > \sigma_d$, the following matrix inequality holds,

$$c_0 I \le D_\sigma Q_k + Q_k D_\sigma, \tag{11}$$

where $c_0 > 0$ is a real constant.

Proof. Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^T$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T$. Consider a dynamical linear system given by $\dot{\varepsilon} = D_\sigma \varepsilon$, and a linear transformation $\xi = Q_d \varepsilon$. Then, we have

$$\begin{aligned}&\varepsilon_i = \xi_i + \textstyle\sum_{j=i+1}^n \xi_j \prod_{m=i}^{j-1} d_m,\ i = 1, \ldots, n-1, \\ &\varepsilon_n = \xi_n.\end{aligned}$$

Therefore, $\dot{\xi} = Q_d D_\sigma \varepsilon$, that is,

$$\begin{aligned}\dot{\xi}_i &= (i-1+\sigma)\left(\xi_i + \textstyle\sum_{j=i+1}^n \xi_j \prod_{m=i}^{j-1} d_m\right) - d_i(i+\sigma)\left(\xi_{i+1} + \sum_{j=i+2}^n \xi_j \prod_{m=i+1}^{j-1} d_m\right) \\ &= (i-1+\sigma)\xi_i - \textstyle\sum_{j=i+1}^n \xi_j \prod_{m=i}^{j-1} d_m,\ i = 1, \ldots, n-1, \\ \dot{\xi}_n &= (n-1+\sigma)\xi_n.\end{aligned}$$

Note that $\xi_i \xi_j \le \frac{1}{2}(\xi_i^2 + \xi_j^2)$, and $d_i > 1$, $i = 1, \ldots, n-1$. Thus,

$$\begin{aligned}&\textstyle\sum_{i=1}^n \xi_i \dot{\xi}_i = \sum_{i=1}^n (i-1+\sigma)\xi_i^2 - \sum_{i=1}^{n-1} \xi_i \sum_{j=i+1}^n \xi_j \prod_{m=i}^{j-1} d_m \\ &\ge \textstyle\sum_{i=1}^n (\sigma - \sigma_d)\xi_i^2,\end{aligned}$$

where σ_d is a positive constant that is related to d_j in Lemma 2.11. If $\sigma > \sigma_d$, then we get $\sum_{i=1}^n \xi_i \dot{\xi}_i > 0$. Note that

$$\begin{aligned}&\textstyle\sum_{i=1}^n \xi_i \dot{\xi}_i = \frac{1}{2} \frac{d(\xi^T \xi)}{dt} = \frac{1}{2}\left(\dot{\varepsilon}^T Q_d^T Q_d \varepsilon + \varepsilon^T Q_d^T Q_d \dot{\varepsilon}\right) \\ &= \tfrac{1}{2}\left(\varepsilon^T D_\sigma Q_1 \varepsilon + \varepsilon^T Q_1 D_\sigma \varepsilon\right) \\ &\ge \lambda_{\min}(Q_1)(\sigma - \sigma_d)\varepsilon^T \varepsilon.\end{aligned}$$

Then,

$$D_\sigma Q_1 + Q_1 D_\sigma \geq 2\lambda_{\min}(Q_1)(\sigma - \sigma_d)I.$$

Further,

$$\begin{aligned}
&{Q_1}^{-1}(D_\sigma Q_1 + Q_1 D_\sigma){Q_1}^{-1} = Q_k D_\sigma + D_\sigma Q_k \\
&\geq 2\lambda_{\min}(Q_1)(\sigma - \sigma_d)Q_k^2 \\
&\geq \tfrac{2\lambda_{\min}(Q_1)(\sigma-\sigma_d)}{(\lambda_{\max}(Q_1))^2} I = c_0 I.
\end{aligned}$$

Thus, the conclusion holds. □

Lemma 2.13. Consider the following $n \times n$ matrices Δ_i $(i = 1, \dots, N)$

$$\Delta_i = \begin{pmatrix} -\theta(t)\lambda_{\mathcal{H}}^i g_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\theta(t)\lambda_{\mathcal{H}}^i g_{n-1} & 0 & \cdots & 1 \\ -\theta(t)\lambda_{\mathcal{H}}^i g_n & 0 & \cdots & 0 \end{pmatrix},$$

which are in Luenberger observable canonical form. Set $g_1 = q_2 + \frac{1}{2} + g_0$, $g_j = q_j g_{j-1} - q_j \prod_{k=2}^{j} q_k + \prod_{k=2}^{j+1} q_k$, $j = 2, \dots, n$, where g_0 is a positive real number, N positive real numbers g_*^i $(i = 1, \dots, N)$ are given such that $l_1\left(1 - \lambda_{\mathcal{H}}^i \theta(t)\right) - \frac{g_*^i \lambda_{\mathcal{H}}^i \theta(t)}{2} \leq 0$, and the following inequalities,

$$\tfrac{2}{(n-1)^2}\left(\tfrac{g_*^i \lambda_{\mathcal{H}}^i \theta(t)}{2} - l_1\left(1 - \lambda_{\mathcal{H}}^i \theta(t)\right)\right) - \left(1 - \lambda_{\mathcal{H}}^i \theta(t)\right)^2 l_j^2 \geq 0,\ j = 2, \dots, n,$$

hold, where $l_1 = q_2 + \frac{1}{2}$, $l_j = q_j \prod_{k=2}^{j} q_k - \prod_{k=2}^{j+1} q_k$, $q_j = q_{j+1} + \frac{j}{2} + 1 + \bar{q}_j$, $j = 2, \dots, n$, $q_{n+1} = 0$, $\bar{q}_n = 0$, and $\bar{q}_j = \frac{1}{2}\sum_{m=j+1}^{n-1}\left(\bar{q}_m + 1 + \frac{m}{2}\right)^2 \prod_{k=j+1}^{m} q_k^2 + \frac{1}{2} q_n^2 \prod_{k=j+1}^{n} q_k^2$, $j = 2, \dots, n-1$.

Then, if $g_0 > g_*^i (i = 1, \dots, N)$, we have the following inequalities,

$$\Delta_i^T P_1 + P_1 \Delta_i \leq -\theta_M^i I, \tag{12}$$

where $\theta_M^i = \lambda_{\min}(P_1)\min\left\{\lambda_{\mathcal{H}}^i g_0 \theta_1, 1\right\}$, I is an $n \times n$ identity matrix and $P_1 = P_q^T P_q$ is a positive definite matrix, and P_q is given by

$$P_q = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -q_2 & 1 & 0 & \cdots & 0 \\ 0 & -q_3 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & -q_n & 1 \end{pmatrix}.$$

Proof. Based on [18, Lemma 1] and Lemma 2.11, the conclusions can be derived. □

Lemma 2.14. Under the same conditions to Lemma 2.13, for an $n\times n$ triangular matrix $D_\sigma = diag\{\sigma, 1+\sigma, \dots, n-1+\sigma\}$, there exists an appropriate positive constant σ_q such that when $\sigma > \sigma_q$, the following matrix inequality holds,

$$c_1 I \le D_\sigma P_1 + P_1 D_\sigma, \tag{13}$$

where $c_1 > 0$ is a real constant.

Proof. The proof is similar to Lemma 2.12 and is omitted. □

Remark 2.15. In practical, because of round-off errors in the numerical calculation, actuator degradations and sensor faults, there always exist controller gain perturbations when the designed controller is implemented [16, 42]. However, it should be noted that a little variation of the controller may make the existing consensus results null and void. The uncertain term $\rho^i(t)k_j$ in Lemma 2.11 can be rewritten as $k_j + \left(\rho^i(t)-1\right)k_j$, where $\left(\rho^i(t)-1\right)k_j$ is called the nominal controller gain multiplicative perturbation or controller sensitivity error. This form of controller sensitivity is an important technique to reduce the impact of these uncertainties on the closed-loop multi-agent system. It can be seen from Lemma 2.11 that all the parametric uncertainties are lumped in the Luenberger controllable canonical form $\Upsilon_{i,k}$. Due to the variation range of controller sensitivity $\rho^i(t)$ is wide enough, the designed controller may tolerate a relative large level of sensitivity error. In other words, the MASs can work under extreme abominable environments.

3. DISTRIBUTED OUTPUT FEEDBACK CONTROLLER DESIGN

In this section, we design a new distributed controller with unknown controller sensitivities to solve the consensus problem of the multi-agent system (1) when unknown measurement noise exists in the output of each agent by employing an output feedback control scheme. For the purpose of reduction communication burden, we assume that the ith follower only accepts the output information from its neighbors. Therefore, the output consensus error received by the ith follower can be defined as

$$\hat{\xi}^i(\theta) = \sum_{j=1}^{N} a_{ij}\left(\nu^i(t) - \nu^j(t)\right) + b_i\left(\nu^i(t) - \nu^0(t)\right), \tag{14}$$

where a_{ij} and b_i are defined in the graph theory.

Firstly, we construct the following distribute compensator for the ith follower ($i = 1, 2, \dots, N$)

$$\begin{cases} \dot{\hat{\epsilon}}_1^i(t) = \hat{\epsilon}_2^i(t) + \Gamma g_1 \hat{\xi}^i(\theta), \\ \dot{\hat{\epsilon}}_2^i(t) = \hat{\epsilon}_3^i(t) + \Gamma^2 g_2 \hat{\xi}^i(\theta), \\ \quad\vdots \\ \dot{\hat{\epsilon}}_n^i(t) = u^i(t) + \Gamma^n g_n \hat{\xi}^i(\theta), \end{cases} \tag{15}$$

and the high gain Γ is given by

$$\dot{\Gamma}(t) = \Gamma \max\left\{\alpha - \beta\Gamma, 0\right\},\ \Gamma(0) \ge 1, \tag{16}$$

where $\hat{\epsilon}^i(t) = (\hat{\epsilon}^i_1(t), \hat{\epsilon}^i_2(t), \ldots, \hat{\epsilon}^i_n(t))^T \in \mathbb{R}^n$ is the state of the compensator, and α, β are two positive constants.

By introducing the following coordinate transformation, it follows that

$$e^i_m(t) = \frac{\epsilon^i_m(t) - \hat{\epsilon}^i_m(t) - \epsilon^0_m(t)}{\Gamma^{m-1+\sigma}},\ m = 1, \ldots, n, \tag{17}$$

$$z^i_m(t) = \frac{\hat{\epsilon}^i_m(t)}{\Gamma^{m-1+\sigma}\Gamma_0^{m-1}},\ m = 1, \ldots, n, \tag{18}$$

where $\Gamma_0 \geq 1$ is a constant gain.

From (1), (15), (17) and (18), we have

$$\begin{array}{l}\dot{e}^i(t) = \Gamma\Delta_0 e^i(t) - \frac{\dot{\Gamma}}{\Gamma} D_\sigma e^i(t) - \theta(t)\Gamma G\left(\sum_{j=1}^N a_{ij}\left(e^i_1(t) - e^j_1(t)\right) + b_i e^i_1(t)\right) \\ -\theta(t)\Gamma G\left(\sum_{j=1}^N a_{ij}\left(z^i_1(t) - z^j_1(t)\right) + b_i z^i_1(t)\right) + M\bar{\phi}\left(\epsilon^i(t)\right),\ i = 1, 2, \ldots, N,\end{array} \tag{19}$$

where $e^i(t) = \left(e^i_1(t), e^i_2(t), \ldots, e^i_n(t)\right)^T$, $M = diag\left\{\frac{1}{\Gamma^\sigma},\ \frac{1}{\Gamma^{1+\sigma}}, \ldots, \frac{1}{\Gamma^{n-1+\sigma}}\right\}$, $\bar{\phi}\left(\epsilon^i(t)\right) = \phi\left(\epsilon^i(t)\right) - \phi\left(\epsilon^0(t)\right)$, $G = (g_1, g_2, \ldots, g_n)^T$, $D_\sigma = diag\{\sigma, 1+\sigma, \ldots, n-1+\sigma\}$. Define $e(t) = \mathrm{col}\left(e^1(t), e^2(t), \ldots, e^N(t)\right)$, $\bar{\phi}\left(\epsilon(t)\right) = \mathrm{col}\left(\bar{\phi}\left(\epsilon^1(t)\right), \bar{\phi}\left(\epsilon^2(t)\right), \ldots, \bar{\phi}\left(\epsilon^N(t)\right)\right)$, $z_1(t) = \left(z^1_1(t), z^2_1(t), \ldots, z^N_1(t)\right)^T$. Then, the compact form of the system (19) can be obtained,

$$\begin{array}{l}\dot{e}(t) = \Gamma\left(I_N \otimes \Delta_0 - \theta(t)\mathcal{H} \otimes G\Xi\right)e(t) - \frac{\dot{\Gamma}}{\Gamma}(I_N \otimes D_\sigma)e(t) \\ +(I_N \otimes M)\bar{\phi}\left(\epsilon(t)\right) - \theta(t)\Gamma(\mathcal{H} \otimes G)z_1(t).\end{array} \tag{20}$$

Based on the compensator (15), the distributed controllers with unknown controller sensitivities $u^i(t)$ $(i = 1, 2, \ldots, N)$ are designed as

$$u^i(t) = -\Gamma^{n+\sigma}\Gamma_0^n \sum_{j=1}^n \rho^i(t) k_j z^i_j(t), \tag{21}$$

where the controller sensitivities $\rho^i(t)$ $(i = 1, \ldots, N)$ are given in Assumption 2.6. According to the coordinate transformations (17) and (18), we substitute the controllers (21) into the system (15). Then, for $i = 1, 2, \ldots, N$, we have

$$\begin{array}{l}\dot{z}^i(t) = \Gamma_0\Gamma\Delta_0 z^i(t) - \frac{\dot{\Gamma}}{\Gamma} D_\sigma z^i(t) + \theta(t)\Gamma G_Z\left(\sum_{j=1}^N a_{ij}\left(e^i_1(t) - e^j_1(t)\right) + b_i e^i_1(t)\right) \\ +\theta(t)\Gamma G_Z\left(\sum_{j=1}^N a_{ij}\left(z^i_1(t) - z^j_1(t)\right) + b_i z^i_1(t)\right) - \rho^i(t)\Gamma_0\Gamma\Upsilon K z^i(t),\end{array} \tag{22}$$

where $z^i(t) = \left(z^i_1(t), z^i_2(t), \ldots, z^i_n(t)\right)^T$, $G_Z = \left(g_1, \frac{g_2}{\Gamma_0}, \ldots, \frac{g_n}{\Gamma_0^{n-1}}\right)^T$, $K = (k_1, k_2, \ldots, k_n)$ is the output feedback control gain.

Similarly, denote $z(t) = \mathrm{col}\left(z^1(t), z^2(t), \ldots, z^N(t)\right)$, $e_1(t) = \left(e^1_1(t), e^2_1(t), \ldots, e^N_1(t)\right)^T$, $\Upsilon_k = diag\{\Upsilon_{1,k},\ \Upsilon_{2,k}\ , \ldots,\ \Upsilon_{N,k}\}$. Thus, the system (22) can be written in a compact form,

$$\begin{array}{l}\dot{z}(t) = \Gamma_0\Gamma\Upsilon_k z(t) - \frac{\dot{\Gamma}}{\Gamma}(I_N \otimes D_\sigma)z(t) + \theta(t)\Gamma(\mathcal{H} \otimes G_Z)e_1(t) \\ +\theta(t)\Gamma(\mathcal{H} \otimes G_Z)z_1(t).\end{array} \tag{23}$$

Next, based on the results of the Luenberger controllable canonical form $\Upsilon_{i,k}$, sufficient conditions are proposed to ensure consensus of the multi-agent system (1) under the distributed output feedback controller (15) and (21).

Theorem 3.1. Under Assumptions 2.3, 2.4, 2.5 and 2.6, the appropriate gains G and K are chosen by means of Lemmas 2.11 and 2.13, and the parameters α, β, Γ_0 satisfy the following conditions

$$\beta < \frac{\hat{\theta}_M}{2\underline{c}},\ \alpha > \frac{\kappa_6}{\underline{c}}, \tag{24}$$

and

$$\Gamma_0 \geq \max\left\{\frac{\kappa_5^2}{\hat{\theta}_M\hat{\rho}_M}, \frac{\hat{\theta}_M + 2\kappa_4}{\hat{\rho}_M}, 1\right\}. \tag{25}$$

Then, under the distributed output feedback controller (15) and (21), the multi-agent system (1) can reach consensus, where $\underline{c}$, κ_4, κ_5 and κ_6 are four positive real constants, $\hat{\theta}_M = \lambda_{\min}(P_1)\min\{\lambda_0 g_0 \theta_1, 1\}$, $\lambda_0 = \lambda_{\min}(\mathcal{H})$, $\lambda_1 = \lambda_{\max}(\mathcal{H})$, and $\hat{\rho}_M = \frac{\lambda_{\min}(Q_1)}{(\lambda_{\max}(Q_1))^2}\min\{k_0\rho_m, 1\}$ with $\rho_m = \min\left\{\rho_1^1, \rho_1^2, \ldots, \rho_1^N\right\}$.

P r o o f. Consider the following Lyapunov function

$$W(t) = W_1(t) + W_2(t), \tag{26}$$

where $W_1(t) = e(t)^T P e(t) = \sum_{i=1}^N e^i(t)^T P_1 e^i(t)$, $W_2(t) = z(t)^T Q z(t) = \sum_{i=1}^N z^i(t)^T Q_k z^i(t)$. Let $\tilde{e}(t) = (S \otimes I_n)e(t)$, $\tilde{z}(t) = (S \otimes I_n)z(t)$, $P = I_N \otimes P_1$, $Q = I_N \otimes Q_k$.

Calculating the derivative of the function $W_1(t)$ along the closed-loop system (20), (23), we have,

$$\begin{aligned}\dot{W}_1(t) &= \Gamma e(t)^T\left((I_N \otimes \Delta_0 - \theta(t)\mathcal{H} \otimes G\Xi)^T P + P\left(I_N \otimes \Delta_0 - \theta(t)\mathcal{H} \otimes G\Xi\right)\right)e(t)\\ &-\tfrac{\dot{\Gamma}}{\Gamma}e(t)^T\left(\left(I_N \otimes D_\sigma\right)P + P\left(I_N \otimes D_\sigma\right)\right)e(t) + 2e(t)^T P(I_N \otimes M)\bar{\phi}\left(\epsilon(t)\right)\\ &-2\theta(t)\Gamma e(t)^T P\left(\mathcal{H} \otimes G\right)z_1(t).\end{aligned} \tag{27}$$

From the coordinate transformation (17) – (18), Lemma 2.9 and Assumption 2.4, one can obtain that,

$$\begin{aligned}&\left\|M\bar{\phi}\left(\epsilon^i(t)\right)\right\| = \left\|M\left(\phi\left(\epsilon^i(t)\right) - \phi\left(\epsilon^0(t)\right)\right)\right\|\\ &\leq \tau\left(n\sqrt{n}\left\|e^i(t)\right\| + n\textstyle\sum_{m=1}^n \Gamma_0^{m-1}\left|z_m^i(t)\right|\right).\end{aligned} \tag{28}$$

From (28) and Lemma 2.9, it follows that

$$\left\|M\bar{\phi}\left(\epsilon^i(t)\right)\right\|^2 \leq 2\tau^2 n^3 e^i(t)^T e^i(t) + 2\tau^2 n^2 \textstyle\sum_{m=1}^n \Gamma_0^{2m-2} z^i(t)^T z^i(t). \tag{29}$$

Lemma 2.8 and the inequality (29) imply that,

$$\begin{aligned}&2e(t)^T P(I_N \otimes M)\bar{\phi}\left(\epsilon(t)\right) = \textstyle\sum_{i=1}^N 2e^i(t)^T P_1 M\bar{\phi}\left(\epsilon^i(t)\right)\\ &\leq \left(\|P_1\| + 2\tau^2 n^3\|P_1\|\right)e(t)^T e(t) + 2\tau^2 n^2\|P_1\|\textstyle\sum_{m=1}^n \Gamma_0^{2m-2} z(t)^T z(t).\end{aligned} \tag{30}$$

For the symmetric matrix $\mathcal{H}$, we have $S\mathcal{H}S^T = \Lambda = diag\left\{\lambda_{\mathcal{H}}^1, \lambda_{\mathcal{H}}^2, \ldots, \lambda_{\mathcal{H}}^N\right\}$, where S is an orthogonal matrix. Thus

$$\begin{aligned}
&-\Gamma e(t)^T \Big((\theta(t)\mathcal{H}\otimes G\Xi)^T P + P\left(\theta(t)\mathcal{H}\otimes G\Xi\right)\Big) e(t) \\
&= -\Gamma e(t)^T \Big((S^T\otimes I_n)(\theta(t)\Lambda\otimes G\Xi)^T P(S\otimes I_n) \\
&\quad +(S^T\otimes I_n)P\left(\theta(t)\Lambda\otimes G\Xi\right)(S\otimes I_n)\Big) e(t) \\
&= -\Gamma \tilde{e}(t)^T \Big((\theta(t)\Lambda\otimes G\Xi)^T P + P\left(\theta(t)\Lambda\otimes G\Xi\right)\Big) \tilde{e}(t).
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&\Gamma e(t)^T \Big((I_N\otimes\Delta_0 - \theta(t)\mathcal{H}\otimes G\Xi)^T P + P\left(I_N\otimes\Delta_0 - \theta(t)\mathcal{H}\otimes G\Xi\right)\Big) e(t) \\
&= \Gamma e(t)^T \Big((S^T\otimes I_n)(I_N\otimes\Delta_0)^T P(S\otimes I_n) + (S^T\otimes I_n)P(I_N\otimes\Delta_0)(S\otimes I_n)\Big) e(t) \\
&-\Gamma \tilde{e}(t)^T \Big((\theta(t)\Lambda\otimes G\Xi)^T P + P\left(\theta(t)\Lambda\otimes G\Xi\right)\Big) \tilde{e}(t) \\
&= \Gamma \textstyle\sum_{i=1}^N \tilde{e}^i(t)^T \left(\Delta_i^T P_1 + P_1\Delta_i\right)\tilde{e}^i(t).
\end{aligned} \tag{31}$$

Next, we estimate the last term in the inequality (27). From Lemma 2.9 and Assumption 2.5, it follows that

$$\begin{aligned}
&-2\theta(t)\Gamma e(t)^T P\left(\mathcal{H}\otimes G\right) z_1(t) = -2\theta(t)\Gamma\tilde{e}(t)^T P\left(\Lambda\otimes G\Xi\right)\tilde{z}(t) \\
&\le 2\Gamma\theta_2\lambda_1\sqrt{N}\left\|P_1\right\|\left\|G\right\|\left\|\tilde{e}(t)\right\|\left\|\tilde{z}(t)\right\|.
\end{aligned} \tag{32}$$

Note that $\tilde{e}(t)^T\tilde{e}(t) = e(t)^T e(t)$. Substituting (12), (13), (30) – (32) into (27), one can obtain

$$\dot{W}_1(t) \le -\left(\Gamma\hat{\theta}_M + c_1\tfrac{\dot{\Gamma}}{\Gamma} - \kappa_1\right) e(t)^T e(t) + \Gamma\kappa_2\left\|\tilde{e}(t)\right\|\left\|\tilde{z}(t)\right\| + \kappa_3 z(t)^T z(t), \tag{33}$$

where $\kappa_1 = \|P_1\| + 2\tau^2 n^3\|P_1\|$, $\kappa_2 = 2\theta_2\lambda_1\sqrt{N}\|P_1\|\|G\|$, $\kappa_3 = 2\tau^2 n^2\|P_1\|\sum_{m=1}^n \Gamma_0^{2m-2}$.

Along the trajectories of the system (23), the time derivative of $W_2(t)$ is given by,

$$\begin{aligned}
&\dot{W}_2(t) = \Gamma_0\Gamma z(t)^T\left(\Upsilon_k^T Q + Q\Upsilon_k\right) z(t) - \tfrac{\dot{\Gamma}}{\Gamma} z(t)^T\left(\left(I_N\otimes D_\sigma\right)Q + Q\left(I_N\otimes D_\sigma\right)\right) z(t) \\
&+2\theta(t)\Gamma z(t)^T Q\left(\mathcal{H}\otimes G_Z\right) e_1(t) + 2\theta(t)\Gamma z(t)^T Q\left(\mathcal{H}\otimes G_Z\right) z_1(t).
\end{aligned} \tag{34}$$

Note that $\|G_Z\| \le \|G\|$. Using Lemma 2.9 and Assumption 2.5, we have

$$\begin{aligned}
&2\theta(t)\Gamma z(t)^T Q(\mathcal{H}\otimes G_Z)e_1(t) + 2\theta(t)\Gamma z(t)^T Q(\mathcal{H}\otimes G_Z)z_1(t) \\
&= 2\theta(t)\Gamma\tilde{z}(t)^T Q(\Lambda\otimes G_Z\Xi)\tilde{e}(t) + 2\theta(t)\Gamma\tilde{z}(t)^T Q(\Lambda\otimes G_Z\Xi)\tilde{z}(t) \\
&\le 2\Gamma\theta_2\lambda_1\sqrt{N}\left\|Q_k\right\|\left\|G\right\|\left\|\tilde{e}(t)\right\|\left\|\tilde{z}(t)\right\| + 2\Gamma\theta_2\lambda_1\left\|Q_k\right\|\left\|G\right\|\tilde{z}(t)^T\tilde{z}(t).
\end{aligned} \tag{35}$$

Obviously,

$$\Gamma_0\Gamma z(t)^T\left(\Upsilon_k^T Q + Q\Upsilon_k\right) z(t) = \Gamma_0\Gamma\textstyle\sum_{i=1}^N z^i(t)^T\left(\Upsilon_{i,k}^T Q_k + Q_k\Upsilon_{i,k}\right) z^i(t). \tag{36}$$

Since $\tilde{z}(t)^T\tilde{z}(t) = z(t)^T z(t)$, then, substituting (6), (11), (35) and (36) into (34) yields,

$$\dot{W}_2(t) \le -\left(\Gamma_0\Gamma\hat{\rho}_M + c_0\tfrac{\dot{\Gamma}}{\Gamma} - \Gamma\kappa_4\right) z(t)^T z(t) + \Gamma\kappa_4\left\|\tilde{e}(t)\right\|\left\|\tilde{z}(t)\right\|, \tag{37}$$

where $\kappa_4 = 2\theta_2\lambda_1\sqrt{N}\,\|Q_k\|\,\|G\|$. Let $\kappa_5 = \kappa_2 + \kappa_4$, $\kappa_6 = \kappa_1 + \kappa_3$, $\underline{c} = \min\{c_0, c_1\}$. Note that the constant gain Γ_0 satisfies the inequality (25). From (26), (33) and (37), it follows that

$$\begin{aligned}
\dot{W}(t) \leq & -\left(\tfrac{\hat{\theta}_M}{2}\Gamma - \kappa_1\right) e(t)^T e(t) - \left(\tfrac{\hat{\rho}_M}{2}\Gamma_0\Gamma - \Gamma\kappa_4 - \kappa_3\right) z(t)^T z(t) \\
& -\underline{c}\tfrac{\dot{\Gamma}}{\Gamma}\left(e(t)^T e(t) + z(t)^T z(t)\right) - \begin{bmatrix} \|\tilde{e}(t)\| \\ \|\tilde{z}(t)\| \end{bmatrix}^T \begin{bmatrix} \frac{\hat{\theta}_M}{2}\Gamma & -\frac{1}{2}\Gamma\kappa_5 \\ -\frac{1}{2}\Gamma\kappa_5 & \frac{\hat{\rho}_M}{2}\Gamma_0\Gamma \end{bmatrix} \begin{bmatrix} \|\tilde{e}(t)\| \\ \|\tilde{z}(t)\| \end{bmatrix} \\
\leq & -\left(\tfrac{\hat{\theta}_M}{2}\Gamma - \kappa_6\right)\left(e(t)^T e(t) + z(t)^T z(t)\right) - \underline{c}\tfrac{\dot{\Gamma}}{\Gamma}\left(e(t)^T e(t) + z(t)^T z(t)\right).
\end{aligned} \tag{38}$$

The condition (16) and the inequality (38) imply that,

$$\dot{W}(t) \leq -\Gamma\mu\left(e(t)^T e(t) + z(t)^T z(t)\right) - (\underline{c}\alpha - \kappa_6)\left(e(t)^T e(t) + z(t)^T z(t)\right),$$

where $\mu = \frac{\hat{\theta}_M}{2} - \underline{c}\beta$.

From the condition (24), we have $\mu > 0$ and $\underline{c}\alpha - \kappa_6 > 0$. It follows from (16) that $\Gamma(0) \leq \Gamma(t) \leq \Gamma_m$, where $\Gamma_m = \max\left\{\frac{\alpha}{\beta}, \Gamma(0)\right\}$. Then,

$$\dot{W}(t) \leq -\mu\left(e(t)^T e(t) + z(t)^T z(t)\right).$$

Thus,

$$\begin{aligned}
& \lambda_{\min}(P)\|e(t)\|^2 + \lambda_{\min}(Q)\|z(t)\|^2 - \left(e(0)^T P e(0) + z(0)^T Q z(0)\right) \\
& \leq W_1(t) + W_2(t) - W_1(0) - W_2(0) \\
& \leq -\mu\int_0^t \left(\|e(t)\|^2 + \|z(t)\|^2\right)\mathrm{d}t,\ t \in [0, +\infty).
\end{aligned}$$

Therefore,

$$\int_0^t \mu\left(\|e(t)\|^2 + \|z(t)\|^2\right)\mathrm{d}t \leq e(0)^T P e(0) + z(0)^T Q z(0) < +\infty. \tag{39}$$

From the inequality (39), we can obtain that $e(t)$ and $z(t)$ are bounded on $[0, +\infty)$. By the Barbalat's Lemma [17], it follows that $\lim_{t\to\infty} e(t) = 0$ and $\lim_{t\to\infty} z(t) = 0$. Then, (17) and (18) imply that $\lim_{t\to\infty} \hat{\epsilon}^i(t) \to 0$ and $\lim_{t\to\infty} \epsilon^i(t) \to \epsilon^0(t)$, $i = 1, \ldots, N$, that is, the states of followers can asymptotically converge those of the leader. The proof is completed. □

Remark 3.2. In the recent decades, there are many works on consensus schemes for nonlinear MASs with unknown controller sensitivity under a fixed communication topology [1, 2, 16, **?**]. The common characteristic of these schemes is that sufficient conditions of the existence of non-fragile controllers are established in the form of LMIs. Up to now, the controller gain variations are usually assumed to be norm-bounded and generally classified as additive [16, 35] and multiplicative [1, 28]. For example, in [28], control gain perturbations were modelled as $\Delta K = MF(t)NK$ with $\|\Delta K\| \leq \eta_0$, where η_0 is a positive scalar, $F(t)$ represents the parameter uncertainties and satisfies $\|F(t)\| \leq 1$. The non-fragile controllers can tolerate a certain level of gain perturbations, whereas, our newly proposed controller with unknown controller sensitivity can tolerate a relative wide range of gain perturbations. On the other hand, it is worth noting that the existence of non-fragile consensus is dependent on the feasibility of LMI-based sufficient conditions [1, 2, 16, 28, 35]. If the LMI-based sufficient conditions are infeasible, then non-fragile consensus can not be derived. However, there always exists consensus for the nonlinear lower-triangular MASs when the controller has unknown controller sensitivity by using our proposed methods.

Remark 3.3. Until now, the research on heterogeneous MASs has yielded more and more distinguished results. For instance, in the literature [9, 44], the resilient cooperative output regulation problem was investigated for heterogeneous MASs under DoS attacks. The MASs considered in [9] is linear, whose exosystem dynamics are unknown for all agents and can be switched in different time intervals. In [44], the nonlinear dynamics of each agent have uncertain parameters, and remain essentially a lower-triangular form. However, the above works do not take into account the influence of unknown measurement noises. In practical engineering, due to manufacturing reasons, inaccurate measurement of sensors, external disturbances, etc., there always exist measurement noises. In this paper, our main aim is to deal with the consensus problem for a class of lower-triangular nonlinear MASs with unknown measurement sensitivities. We also consider the fragility of the designed controller, such that it has stronger robustness. Beyond the above, the full states tracking consensus is achieved when there exist unknown controller and measurement sensitivities. Whereas, the authors considered the problem of the output of each agent tracking the reference signal in [9, 44]. On the other hand, the regulated output in [9] is globally ultimately bounded.

Remark 3.4. In the literature [6], a dual-domination method originated from [20, 21, 32], was presented to deal with nonlinear terms and unknown measurement noise, and has been widely applied in the works [27, 36, 41]. In this paper, we apply the idea of this method to make the MASs consensus. From (16), (24) and (38), it can be seen that the dynamic gain $\Gamma(t)$ is used to dominate the parameter κ_6 stemming from the growth rate τ, and the constant gain Γ_0 is used to dominate the unknown output measurement noises and the unknown controller sensitivities for all the distributed controllers.

4. NUMERICAL SIMULATIONS

In this section, we present a numerical example to test the feasibility of our newly proposed consensus algorithm. Consider a group of single-link robots [5] with a leader and four followers. In Figure 1, it is shown the topology $\bar{\mathcal{G}}$, where 0 denotes the leader and i ($i = 1, \dots, 4$) denote the four followers. We can verify that Assumption 2.3 holds. The dynamics of each single-link robot is given by

$$M\ddot{q}^i + 0.5mgl\sin(q^i) = F^i,$$

where q^i and F^i denote the angle and input torque of the ith single-link robot, respectively, M, g, m, l are the moment of inertia, the gravity acceleration, the mass and the length of the link, respectively. Then, by defining $\epsilon_1^i = q^i$, $\epsilon_2^i = \dot{q}^i$, $u^i = M^{-1}F^i$, we have

$$\begin{cases} \dot{\epsilon}_1^i(t) = \epsilon_2^i(t), \\ \dot{\epsilon}_2^i(t) = u^i(t) - \frac{mgl\sin\left(\epsilon_1^i(t)\right)}{2M}, \\ \nu^i(t) = \theta(t)\epsilon_1^i(t),\ i = 0, 1, \ldots, 4. \end{cases}$$

The parameters are given as $M = 1$, $m = 1$, $g = 9.8$, $l = 1$. It can be verified that the nonlinear terms satisfy Assumption 2.4 with $\tau = 4.9$. Then the distributed output feedback controller for the ith follower is constructed as

$$\begin{cases} \dot{\hat{\epsilon}}_1^i(t) = \hat{\epsilon}_2^i(t) + \Gamma g_1\hat{\xi}^i(\theta), \\ \dot{\hat{\epsilon}}_2^i(t) = u^i(t) + \Gamma^2 g_2\hat{\xi}^i(\theta), \\ u^i(t) = -\rho^i(t)k_1(\Gamma_0\Gamma)^2\hat{\epsilon}_1^i(t) - \rho^i(t)k_2\Gamma_0\Gamma\hat{\epsilon}_2^i(t), \end{cases}$$

where $\hat{\xi}^i(\theta) = \sum_{j=1}^4 a_{ij}\left(\nu^i(t) - \nu^j(t)\right) + b_i\left(\nu^i(t) - \nu^0(t)\right)$ $(i = 1, 2, 3, 4)$, $u^0(t) = 0$, Γ is given by (16). In order to better illustrate the impact of uncertainties in sensors and distributed controllers on the consensus performance, we conduct the simulation experiment under two cases by setting different values of the measurement noise and controller sensitivities.

Case 1. Set the measurement noise $\theta(t) = 1 + 0.1\sin(10t + 0.1\pi)$. Then, Assumption 2.5 holds with $\theta_1 = 0.9$ and $\theta_2 = 1.1$. The controller sensitivities are given as $\rho^1(t) = 0.8 + 0.4\left|\cos\left(10t + 0.1\pi\right)\right|$, $\rho^2(t) = 0.95 + 0.1\sin\left(20t + 0.2\pi\right)$, $\rho^3(t) = 1.1 + 0.08\cos\left(30t + 0.3\pi\right)$, $\rho^4(t) = 1.05 + 0.05\left|\sin\left(50t + 0.5\pi\right)\right|$, which satisfy the conditions in Assumption 2.6 with $\rho_1^i = \{0.8, 0.85, 1.02, 1.05\}$ and $\rho_2^i = \{1.2, 1.05, 1.18, 1.1\}$. Thus, we have $\rho_m = 0.8$.

Based on the topology diagram illustrated in Figure 1, we can obtain the minimum and maximum eigenvalues of $\mathcal{H}$ as $\lambda_0 = 0.2679$, $\lambda_1 = 3.7321$. According to Lemma 2.11, we have $d_1 = 2$, $\delta_1 = 4$, $\delta_2 = 2.5$. Let $k_0 = 30$, then $K = (61, 32.5)$, $\lambda_{\min}(Q_1) = 0.1716$, $\lambda_{\max}(Q_1) = 5.8284$, $\hat{\rho}_M = 0.0051$. From Lemma 2.13, we have $q_2 = 2$, $l_1 = 2.5$, $l_2 = 4$. Choose $g_0 = 115$, then, $G = (117.5, 231)^T$, $\lambda_{\min}(P_1) = 0.1716$, $\hat{\theta}_M = 0.1716$. Based on Theorem 3.1, the appropriate parameters are given by $\sigma = 4$, $c_0 = 0.02$, $c_1 = 1$, $\alpha = 85$, $\beta = 0.098$, $\Gamma_0 = 69$.

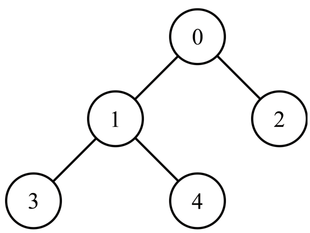

Fig. 1. The connected graph.

The initial conditions are set as $\epsilon^0(0) = (-0.4, 10)^T$, $\epsilon^1(0) = (-0.1, -50)^T$, $\epsilon^2(0) = (0.3, 80)^T$, $\epsilon^3(0) = (-0.5, -100)^T$, $\epsilon^4(0) = (0.2, 150)^T$, and $\hat{\epsilon}^1(0) = (0.5, -50)^T$, $\hat{\epsilon}^2(0) = (0.1, 10)^T$, $\hat{\epsilon}^3(0) = (0.2, -20)^T$, $\hat{\epsilon}^4(0) = (-0.3, 30)^T$, $\Gamma(0) = 1$. The results of the simulation are illustrated in Figures 2 – 7. The trajectories of each agent's states are shown in Figures 2 and 3. Figures 4 and 5 present the trajectories of the state errors between followers and the leader. The dynamic gain is bounded which can be seen in Figure 6. The control inputs are described in Figure 7.

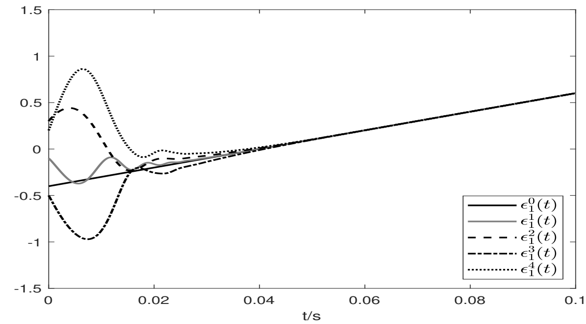

Fig. 2. The trajectories of $\epsilon_1^i(t)$, $i = 0, 1, \ldots, 4$.

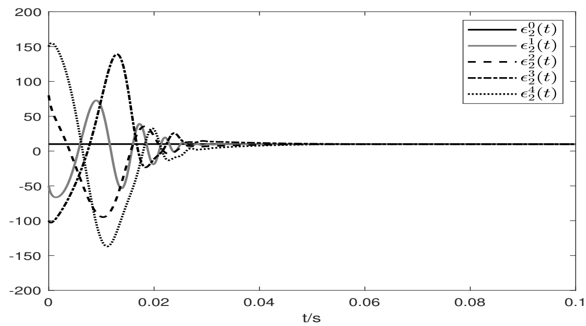

Fig. 3. The trajectories of $\epsilon_2^i(t)$, $i = 0, 1, \ldots, 4$.

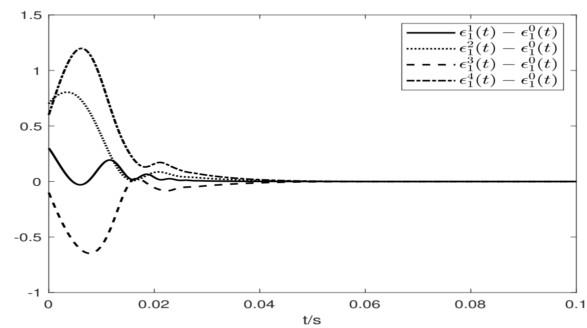

Fig. 4. The trajectories of $\epsilon_1^i(t) - \epsilon_1^0(t)$, $i = 1, \ldots, 4$.

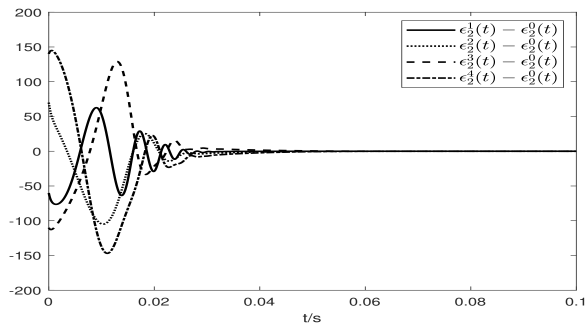

Fig. 5. The trajectories of $\epsilon_2^i(t) - \epsilon_2^0(t)$, $i = 1, \ldots, 4$.

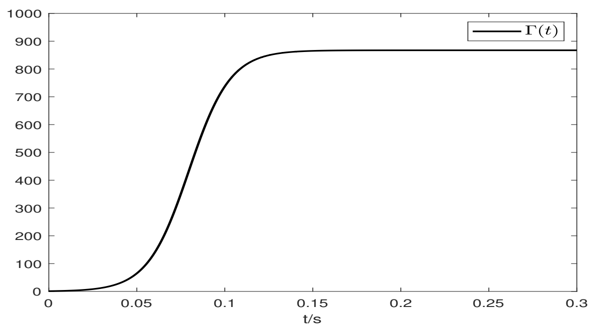

Fig. 6. The trajectory of $\Gamma(t)$.

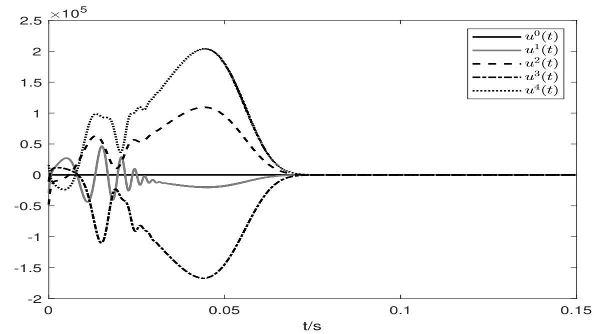

Fig. 7. The trajectories of $u^i(t)$, $i = 0, 1, \ldots, 4$.

Case 2. Let $\theta(t) = 0.15 + 0.05\,|\cos(20t + 0.2\pi)|$, which satisfies Assumption 2.5 with $\theta_1 = 0.15$ and $\theta_2 = 0.2$. The controller sensitivities $\rho^1(t) = 3 + 2.8\,|\sin(10t + 0.1\pi)|$, $\rho^2(t) = 4.5 + 0.5\cos(20t + 0.2\pi)$, $\rho^3(t) = 6 - |\cos(30t + 0.3\pi)|$, $\rho^4(t) = 5 + 1.5\sin(50t + 0.5\pi)$. They satisfy the conditions in Assumption 2.6 with $\rho_1^i = \{3, 4, 5, 3.5\}$ and $\rho_2^i = \{5.8, 5, 6, 6.5\}$. Then we can get $\rho_m = 3$.

Based on Lemma 2.11, we have $d_1 = 2$, $\delta_1 = 4$, $\delta_2 = 2.5$. Set $k_0 = 75$, then $K = (151, 77.5)$, $\lambda_{\min}(Q_1) = 0.1716$, $\lambda_{\max}(Q_1) = 5.8284$, $\hat{\rho}_M = 0.0051$. According to Lemma 2.13, we get $q_2 = 2$, $l_1 = 2.5$, $l_2 = 4$. Select $g_0 = 170$, thus, $G = (172.5, 341)^T$,

$\lambda_{\min}(P_1) = 0.1716$, $\hat{\theta}_M = 0.1716$. Based on Theorem 3.1, the appropriate parameters are given by $\sigma = 4$, $c_0 = 0.02$, $c_1 = 1$, $\alpha = 205$, $\beta = 0.15$, $\Gamma_0 = 45$. The initial conditions are same to those in Case 1. The simulation results are displayed in Figsures 8 – 13. Figsures 8 and 9 represent that the trajectories of each agent's states. The trajectories of the state errors between followers and the leader are described in Figsures 10 and 11. It is evident from Figure 12 that the dynamic gain is bounded. Figure 13 shows that the trajectories of the control inputs.

It is apparent from the simulation results that the states of followers can asymptotically track those of the leader, which demonstrates the validity of our proposed consensus protocol. Furthermore, the values of the measurement noise and controller sensitivities are taken to be close to 1 in Case 1, whereas they are farther away from 1 in Case 2. From Figures 2 and 3, it is easy to see that the trajectories of the agent state consensus have relatively small overshoots but slower convergence rates. However, in Case 2, the overshoots in Figures 8 and 9 are large, but the convergence speeds are faster. Therefore, we can derive the conclusion that if the values of the uncertainties in the sensor and the controller are farther away 1, the overshoot of the dynamic process of the state consensus is larger, and the convergence speed is fast.

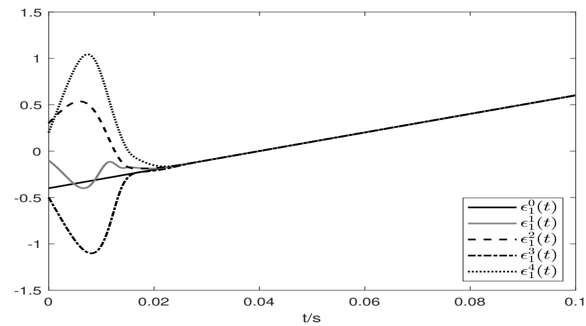

Fig. 8. The trajectories of $\epsilon_1^i(t)$, $i = 0, 1, \ldots, 4$.

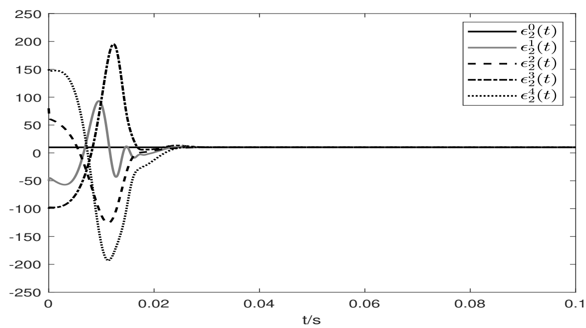

Fig. 9. The trajectories of $\epsilon_2^i(t)$, $i = 0, 1, \ldots, 4$.

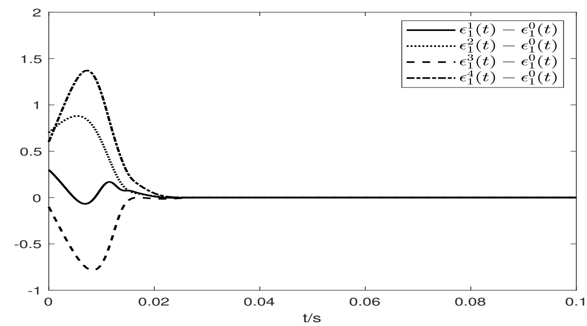

Fig. 10. The trajectories of $\epsilon_1^i(t) - \epsilon_1^0(t)$, $i = 1, \ldots, 4$.

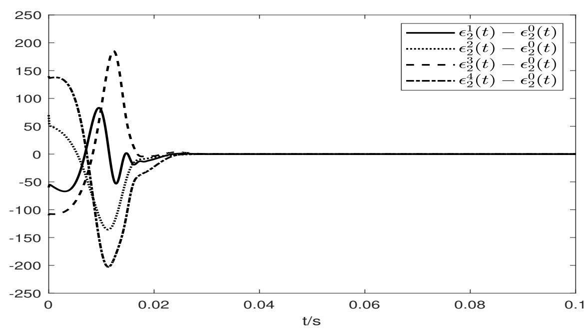

Fig. 11. The trajectories of $\epsilon_2^i(t) - \epsilon_2^0(t)$, $i = 1, \ldots, 4$.

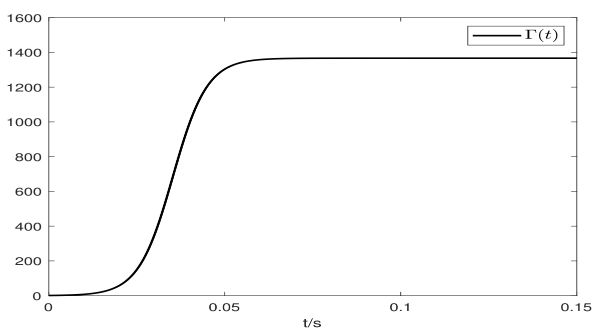

Fig. 12. The trajectory of $\Gamma(t)$.

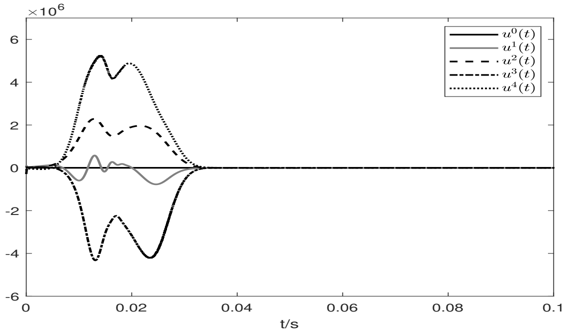

Fig. 13. The trajectories of $u^i(t)$, $i = 0, 1, \ldots, 4$.

5. CONCLUSION

In this paper, we proposed a novel method of the leader-following consensus for a class of lower-triangular nonlinear MASs with unknown measurement noises and controller sensitivity under a fixed undirected graph. By developing some important matrix inequalities and applying a dual-domination gain method, a distributed compensator that only accepts output information with unknown measurement noise was constructed to estimate the state of the corresponding follower. Owing to the presence of unknown controller sensitivity in the control law, a new distributed dynamic output feedback controller was designed, which could tolerate a relative wide range of sensitivity error. Based on the Lyapunov stability theory, sufficient conditions were presented to guarantee that the states of the leader and followers could achieve consensus asymptotically. An illustrative example was also provided to verify the correctness of the proposed consensus algorithm. Further, one possible future investigation is to consider the consensus problem that the output of each agent carries different unknown measurement noises under the sampling and delay mechanism.

ACKNOWLEDGMENTS

This work was supported by Yichang Key Laboratory of Defense and Control of Cyber-Physical Systems (2020XXRH01) and Hubei Key Laboratory of Hydroelectric Machinery Design and Maintenance (2021KJX04).

(Received April 6, 2022)

REFERENCES

[1] M. S. Ali, R. Agalya, V. Shekher, and Y. H. Joo: Non-fragile sampled data control for stabilization of non-linear multi-agent system with additive time varying delays, Markovian jump and uncertain parameters. Nonlinear Analysis: Hybrid Systems *36* (2020), 100830. DOI:10.1016/j.nahs.2019.100830

[2] A. Amini, A. Azarbahram, and M. Sojoodi: H_∞ Consensus of nonlinear multi-agent systems using dynamic output feedback controller: an LMI approach. Nonlinear Dynamics *85* (2016), 3, 1865–1886. DOI:10.1007/s11071-016-2801-6

[3] A. Bidram, A. Davoudi, F. L. Lewis, and J. M. Guerrero: Distributed cooperative secondary control of microgrids using feedback linearization. IEEE Trans. Power Systems *28* (2013), 3, 3462–3470. DOI:10.1109/tpwrs.2013.2247071

[4] H. Cai, G. Hu, F. L. Lewis, and A. Davoudi: A distributed feedforward approach to cooperative control of AC microgrids. IEEE Trans. Power Systems *31* (2016), 5, 4057–4067. DOI:10.1109/tpwrs.2015.2507199

[5] M. Chen, B. Jiang, and W. W. Guo: Fault-tolerant control for a class of nonlinear systems with dead-zone. Int. J. Systems Science *47* (2016), 7, 1689–1699. DOI:10.1080/00207721.2014.945984

[6] C. C. Chen, C. Qian, Z. Y. Sun, and Y. W. Liang: Global output feedback stabilization of a class of nonlinear systems with unknown measurements sensitivity. IEEE Trans. Automat. Control *63* (2018), 7, 2212–2217. DOI:10.1109/tac.2017.2759274

[7] G. Chen and Y. D. Song: Robust fault-tolerant cooperative control of multi-agent systems: A constructive design method. J. Franklin Inst. *352* (2015), 10, 4045–4066. DOI:10.1016/j.jfranklin.2015.05.031

[8] J. Chen, W. Zhang, Y. Y. Cao, and H. Chu: Observer-based consensus control against actuator faults for linear parameter-varying multiagent systems. IEEE Trans. Systems Man Cybernet.: Systems *47* (2017), 7, 1336–1347. DOI:10.1109/tsmc.2016.2587300

[9] C. Deng, D. Zhang, and G. Feng: Resilient practical cooperative output regulation for MASs with unknown switching exosystem dynamics under DoS attacks. Automatica *139* (2022), 110172. DOI:10.1016/j.automatica.2022.110172

[10] H. Du and R. Jia: Synchronization of a class of nonlinear multi-agent systems with sampled-data information. Nonlinear Dynamics *82* (2015), 3, 1483–1492. DOI:10.1007/s11071-015-2255-2

[11] J. A. Fax and R. M. Murray: Information flow and cooperative control of vehicle formations. IEEE Trans. Automat. Control *49* (2004), 9, 1465–1476. DOI:10.1109/tac.2004.834433

[12] Z. Guo, H. Xue, and Y. Pan: Neural networks-based adaptive tracking control of multi-agent systems with output-constrained and unknown hysteresis. Neurocomputing *458* (2021), 24–32. DOI:10.1016/j.neucom.2021.05.079

[13] C. C. Hua, K. Li, and X. P. Guan: Semi-global/global output consensus for nonlinear multiagent systems with time delays. Automatica *103* (2019), 480–489.

[14] C. C. Hua, X. You, and X. P. Guan: Leader-following consensus for a class of high-order nonlinear multi-agent systems. Automatica *73* (2016), 138–144. DOI:10.1016/j.automatica.2016.06.025

[15] B. Kaviarasan, O. M. Kwon, M. J. Park, and R. Sakthivel: Stochastic faulty estimator-based non-fragile tracking controller for multi-agent systems with communication delay. Appl. Math. Comput. *392* (2021), 125704. DOI:10.1016/j.amc.2020.125704

[16] B. Kaviarasan, R. Sakthivel, Y. Li, D. Zhao, and Y. Ren: Non-fragile control protocol for finite-time consensus of stochastic multi-agent systems with input time-varying delay. Int. J. Machine Learning Cybernet. *11* (2020), 2, 325–337. DOI:10.1007/s13042-019-00976-9

[17] K. H. Khalil: Nonlinear Systems. (Third edition.) Prentice-Hall, Upper Saddle River, NJ 2002.

[18] M. S. Koo and H. L. Choi: Output feedback regulation of a class of lower triangular nonlinear systems with arbitrary unknown measurement sensitivity. Int. J. Control Automat. Systems *18* (2020), 9, 2186–2194. DOI:10.1007/s12555-019-0721-1

[19] M. Krstic and H. Deng: Stabilization of Uncertain Nonlinear Systems. Springer, New York 1998.

[20] H. Lei and W. Lin: Universal adaptive control of nonlinear systems with unknown growth rate by output feedback. Automatica *42* (2006), 10, 1783–1789. DOI:10.1016/j.automatica.2006.05.006

[21] H. Lei and W. Lin: Adaptive regulation of uncertain nonlinear systems by output feedback: A universal control approach. Systems Control Lett. *56* (2007), 7–8, 529–537. DOI:10.1016/j.sysconle.2007.03.002

[22] K. Li, C. C. Hua, X. You, and C. K. Ahn: Leader-following asynchronous consensus for multiagent systems with unknown control and output directions. Automatica *132* (2021), 109832. DOI:10.1016/j.automatica.2021.109832

[23] K. Li, C .C. Hua, X. You, and X. P. Guan: Output feedback-based consensus control for nonlinear time delay multiagent systems. Automatica *111* (2020), 108669. DOI:10.1016/j.automatica.2019.108669

[24] K. Li, C. C. Hua, X. You, and X. P. Guan: Distributed output-feedback consensus control of multiagent systems with unknown output measurement sensitivity. IEEE Trans. Automat. Control *66* (2021), 7, 3303–3310. DOI:10.1109/tac.2020.3017711

[25] H. Li, Q. Liu, G. Feng, and X. Zhang: Leader-follower consensus of nonlinear time-delay multiagent systems: A time-varying gain approach. Automatica *126* (2021), 109444. DOI:10.1016/j.automatica.2020.109444

[26] C. Li, S. Tong, and W. Wang: Fuzzy adaptive high-gain-based observer backstepping control for SISO nonlinear systems. Inform. Sci. *181* (2011), 11, 2405–2421. DOI:10.1016/j.ins.2011.01.040

[27] W. Li, X. Yao, and M. Krstic: Adaptive-gain observer-based stabilization of stochastic strict-feedback systems with sensor uncertainty. Automatica *120* (2020), 109112. DOI:10.1016/j.automatica.2020.109112

[28] C. Liu, X. Yue, and Z. Yang: Are nonfragile controllers always better than fragile controllers in attitude control performance of post-capture flexible spacecraft?. Aerospace Sci. Technol. *118* (2021), 107053. DOI:10.1016/j.ast.2021.107053

[29] H. Ma, Z. Wang, D. Wang, D. Liu, P. Yan, and Q. Wei: Neural-network-based distributed adaptive robust control for a class of nonlinear multiagent systems with time delays and external noises. IEEE Trans. Systems Man Cybernet.: Systems *46* (2016), 6, 750–758. DOI:10.1109/tsmc.2015.2470635

[30] W. Ni and D. Cheng: Leader-following consensus of multi-agent systems under fixed and switching topologies. Systems Control Lett. *59* (2010), 3–4, 209–217. DOI:10.1016/j.sysconle.2010.01.006

[31] S. K. Pandey, S. R. Mohanty, and N. Kishor: A literature survey on load-frequency control for conventional and distribution generation power systems. Renewable Sustainable Energy Rev. *25* (2013), 318–334. DOI:10.1016/j.rser.2013.04.029

[32] C. Qian and W. Lin: Output feedback control of a class of nonlinear systems: a non-separation principle paradigm. IEEE Trans. Automat. Control *47* (2002), 10, 1710–1715. DOI:10.1109/tac.2002.803542

[33] W. Ren and N. Sorensen: Distributed coordination architecture for multi-robot formation control. Robotics Autonomous Syst. *56* (2008), 4, 324–333. DOI:10.1016/j.robot.2007.08.005

[34] G. Rigatos, P. Siano, and N. Zervos: Sensorless control of distributed power generators with the derivative-free nonlinear kalman filter. IEEE Trans. Industr. Electronics *61* (2014), 11, 6369–6382. DOI:10.1109/tie.2014.2300069

[35] R. Sakthivel, A. Parivallal, N. H. Tuan, and S. Manickavalli: Nonfragile control design for consensus of semi-Markov jumping multiagent systems with disturbances. Int. J. Adaptive Control Signal Process. *35* (2021), 6, 1039–1061. DOI:10.1002/acs.3245

[36] Z. Y. Sun, J. W. Xing, and Q. Meng: Output feedback regulation of time-delay nonlinear systems with unknown continuous output function and unknown growth rate. Nonlinear Dynamics *100* (2020), 2, 1309–1325. DOI:10.1007/s11071-020-05552-3

[37] X. H. Wang and H. B. Ji: Leader-follower consensus for a class of nonlinear multi-agent systems. Int. J. Control Automat. Systems *10* (2012), 1, 27–35. DOI:10.1007/s12555-012-0104-3

[38] W. Wang, C. Wen, and J. Huang: Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances. Automatica *77* (2017), 133–142. DOI:10.1016/j.automatica.2016.11.019

[39] R. Xu, X. Wang, and Y. Zhou: Observer-based event-triggered adaptive containment control for multiagent systems with prescribed performance. Nonlinear Dynamics *107* (2022), 3, 2345–2362. DOI:10.1007/s11071-021-07064-0

[40] B. Yang and W. Lin: Homogeneous observers, iterative design, and global stabilization of high-order nonlinear systems by smooth output feedback. IEEE Trans. Automat. Control *49* (2004), 7, 1069–1080. DOI:10.1109/tac.2004.831186

[41] G. Yu and Y. Shen: Relative-output-based consensus for nonlinear multi-agent systems with unknown measurement sensitivities. Neurocomputing *382* (2020), 21–31. DOI:10.1016/j.neucom.2019.11.082

[42] Y. Yuan, Z. Wang, P. Zhang, and H. Dong: Nonfragile near-optimal control of stochastic time-varying multiagent systems with control- and state- dependent noises. IEEE Trans. Cybernet. *49* (2019), 7, 2605–2617. DOI:10.1109/tcyb.2018.2829713

[43] C. Zhang, L. Chang, and X. Zhang: Leader-follower consensus of upper-triangular nonlinear multi-agent systems. IEEE/CAA J. Automat. Sinica *1* (2014), 2, 210–217. DOI:10.1109/jas.2014.7004552

[44] D. Zhang, C. Deng, and G. Feng: Resilient cooperative output regulation for nonlinear multi-agent systems under DoS attacks. IEEE Trans. Automat. Control (2022), 1–8. DOI:10.1109/tac.2022.3184388

[45] J. Zhang, D. Qi, and G. Zhao: A new game model for distributed optimization problems with directed communication topologies. Neurocomputing *148* (2015), 278–287. DOI:10.1016/j.neucom.2013.11.054

[46] J. Zhang, J. Song, J. Li, F. Han, and H. Zhang: Observer-based non-fragile H_∞-consensus control for multi-agent systems under deception attacks. Int. J. Systems Sci. *52* (2021), 6, 1223–1236. DOI:10.1080/00207721.2021.1884917

[47] T. Zhou, Q. Liu, and W. Wang: Non-fragile disturbance observer-based containment control of multi-agent systems over switching topologies. IEEE Access *9* (2021), 123430–123437. DOI:10.1109/access.2021.3109984

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