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GLOBAL STABILITY OF CLIFFORD-VALUED TAKAGI–SUGENO FUZZY NEURAL NETWORKS WITH TIME-VARYING DELAYS AND IMPULSES

Ramalingam Sriraman and Asha Nedunchezhian

In this study, we consider the Takagi–Sugeno (T-S) fuzzy model to examine the global asymptotic stability of Clifford-valued neural networks with time-varying delays and impulses. In order to achieve the global asymptotic stability criteria, we design a general network model that includes quaternion-, complex-, and real-valued networks as special cases. First, we decompose the n-dimensional Clifford-valued neural network into $2^m n$-dimensional real-valued counterparts in order to solve the noncommutativity of Clifford numbers multiplication. Then, we prove the new global asymptotic stability criteria by constructing an appropriate Lyapunov-Krasovskii functionals (LKFs) and employing Jensen's integral inequality together with the reciprocal convex combination method. All the results are proven using linear matrix inequalities (LMIs). Finally, a numerical example is provided to show the effectiveness of the achieved results.

Keywords: global stability, T-S fuzzy, Clifford-valued neural networks, Lyapunov–Krasovskii functionals, impulses

Classification: 92B20, 34D08, 35R12, 03E72

1. INTRODUCTION

Since the 1990s, neural networks (NNs) have received more and more attention because of their wide range of applications in various areas such as associative memory, pattern recognition, signal processing, optimal control, and so on [7, 14, 24, 36]. However, real-valued NNs have several challenges when dealing with multidimensional information. In particular, real-valued NNs are not appropriate for solving the symmetry detection and XOR problems, but complex-valued NNs are well-suited for solving these issues [12, 26]. Furthermore, it has been demonstrated that complex-valued NNs perform better than real-valued NNs when dealing with complex signals [38, 42]. On the other hand, in numerous applications including color imaging, high-dimensional geometrical affine transformation, and quantum physics, complex- and real-valued NNs cannot be used directly, while quaternion-valued signals can be used to address these problems directly [15, 23, 25]. Therefore, the investigation of quaternion-valued NNs has received rapid

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advances in recent years. In this regard, several dynamical investigations of quaternion-, complex-valued NNs have been published [35, 37, 38, 42].

W. K. Clifford first introduced Clifford algebra [9] by extending complex and quaternion algebras. Clifford networks are generalizations of quaternion-, complex-, and real-valued networks. Clifford-valued NNs have been successfully applied in various domains, e. g. cognitive computing, computer and robot vision, control problems, and other areas [13, 28]. Recent studies have proven that Clifford-valued NNs are better than complex- and real-valued NNs; thus they have been a popular research topic in recent years [3, 4, 6, 17, 19, 21, 29, 30, 31, 34, 43]. By converting Clifford networks into multidimensional real networks, the issue of global asymptotic stability analysis for a class of Clifford-valued NNs with impulsive and time delays was analyzed in [31]. The Lyapunov function approach and the LMI method were used to study global stability with respect to a class of Clifford-valued neutral-type NNs, which involved discrete time delays [30]. The weighted pseudo almost automorphic issue pertaining to a class of fuzzy Clifford-valued cellular NNs with neutral type and mixed time delays was explored in [3]. However, Clifford-valued NN dynamics are difficult to investigate fully due to the non-commutativity of Clifford numbers multiplication. Given the extensive history of Clifford-valued NNs and their present state of development, it is vital to properly examine their global stability analysis, which is the motivation for our current research.

On the other hand, the fuzzy logic theory has been used to describe the nonlinear systems in various mathematical modeling, which has been a substantial influence on NNs dynamics [39]. As such, T-S fuzzy model is a potential strategy for accounting for ambiguity while approximating a complicated nonlinear system. Furthermore, T-S fuzzy NNs perform better than normal NNs in terms of their capacity to deal with ambiguous input and reflect nonlinear dynamics. There are some research works on the dynamics of T-S fuzzy NNs have been published [1, 2, 5, 16, 33, 41]. In 2010, the author of [1] discussed T-S fuzzy delayed Hopfield NNs and proposed new delay-dependent state estimation criteria. In 2011, Balasubramanian et al [5] proposed a sufficient criterion for global stability of T-S fuzzy cellular NNs with leakage delays. In 2018, Jian and Wan [16] investigated the global exponential convergence of fuzzy complex-valued NNs with time-varying delays and impulsive effects.

Time delays are unavoidable in signal transmission between neurons in many real-world systems. There is no doubt that the presence of time delays can lead to poor system performance, including oscillation, instability, bifurcation, and so on [8, 20, 32]. As a result, time delays must be analyzed in order to determine the stability of NNs. Recently, there have been extensive studies about the stability of NNs with various time delays have been published [3, 4, 8, 17, 19, 20, 32, 34]. On the other hand, impulsive differential equations are advantageous due to their potential applications in a variety of sectors, such as biological systems, chemical processes, and others [10, 18]. Similar to time delays, impulses cause stable systems to become unstable [11, 22]. As a result, the dynamical analysis of impulsive NNs with time delays is crucial, and many researchers have investigated this challenge [11, 18, 22, 40].

Based on the above motivation, our aim in this paper is to investigate the global asymptotic stability of T-S fuzzy Clifford-valued NNs. The major contributions of our research are as follows: (1) This is the first study to examine the global asymptotic stability of T-S fuzzy Clifford-valued NNs with time-varying delays and impulses. (2) We established new sufficient conditions that ensure the global asymptotic stability of the equilibrium point for the considered networks by constructing an appropriate LKFs and by applying Jensen's integral inequality together with reciprocal convex combination method. All the results are presented in the form of LMIs which can be easily verified using the MATLAB LMI toolbox. (3) The methods discussed in this paper is generic and adaptable to investigating various dynamics of other T-S fuzzy Clifford-valued NNs.

The structure of this paper is as follows. Section 2 explains the proposed NNs. Section 3 introduces the new stability criteria, while Section 4 provides the numerical analysis. Conclusions are shown in Section 5.

2. MATHEMATICAL FORMULATION AND PROBLEM DEFINITION

2.1. Notations

In the rest of this paper, $\mathbb{R}^n$, $\mathbf{A}^n$, $\mathbb{R}^{n\times m}$, $\mathbf{A}^{n\times m}$ denote the n-dimensional real vector space, n-dimensional real Clifford vector space, the set of all $n\times m$ real matrices and the set of all $n\times m$ real Clifford matrices, respectively. The Clifford algebra over $\mathbb{R}$ is defined as $\mathbf{A}$ with m generators. The transpose of matrix and conjugate transpose of matrix are represented by T and $*$, respectively. A positive or negative definite matrix is denoted by $\boldsymbol{\mathcal{Q}} > 0$ or $\boldsymbol{\mathcal{Q}} < 0$, respectively. The norm of $\mathbb{R}^n$ is defined as $\|y\| = \sum_{i=1}^{n}|y_i|$, we denote $\|M\| = \max_{1\le i\le n}\{\sum_{j=1}^{n}|m_{ij}|\}$ for $M = (m_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$, $\|y\|_{\mathbf{A}} = \sum_{A\in\Gamma}|y^A|$ for $y = \sum_{A\in\Gamma}y^A e_A \in \mathbf{A}$ and $\|W\|_{\mathbf{A}} = \max_{1\le i\le n}\{\sum_{j=1}^{n}|w_{ij}|_{\mathbf{A}}\}$ for $W = (w_{ij})_{n\times n} \in \mathbf{A}^{n\times n}$. For $\omega \in \boldsymbol{\mathscr{C}}([-\tau, 0], \mathbf{A}^n)$, we denote the norm $\|\omega\|_\tau \le \sup_{-\tau\le s\le 0}\|\omega(t+s)\|$.

2.2. Clifford Algebra

Clifford algebra over $\mathbb{R}^m$ is defined as

$$\mathbf{A} = \Big\{ \sum_{A\subseteq\{1,2,\dots,m\}} a^A e_A,\ a^A \in \mathbb{R}\Big\},$$

where $e_A = e_{r_1}e_{r_2}\dots e_{r_\nu}$ with $A = \{r_1, r_2, \dots, r_\nu\},\ 1 \le r_1 < r_2 < \dots < r_\nu \le m$.

The Clifford generators are also known as $e_\emptyset = e_0 = 1$ and $e_r = e_{\{r\}},\ r = 1, 2, \dots, m$, and they meet the following criteria

$$\begin{cases} e_ie_j + e_je_i = 0,\ i \ne j, \quad i, j = 1, 2, \dots, m, \\ e_i^2 = -1,\ i = 1, 2,\ \dots, m. \end{cases}$$

When an element represents the product of many Clifford generators, we combine the associated subscripts for convenience, e. g. $e_4e_5e_6e_7 = e_{4567}$.

Let $\Gamma = \{\emptyset, 1, 2, \dots, A, \dots, 12\dots m\}$, we have

$$\mathbf{A} = \Big\{ \sum_A a^A e_A,\ a^A \in \mathbb{R}\Big\},$$

where $\sum_A$ denotes $\sum_{A\in\Gamma}$ and $\mathbf{A}$ is isomorphic to $\mathbb{R}^{2^m}$.

The involution of y for every Clifford number $y = \sum_A y^A e_A$ is defined by

$$\bar{y} = \sum_A y^A \bar{e}_A,$$

where $\bar{e}_A = (-1)^{\frac{\varrho[A](\varrho[A]+1)}{2}} e_A$, and

$$\varrho[A] = \begin{cases} 0, & \text{if } A = \emptyset, \\ \nu, & \text{if } A = r_1 r_2 \dots r_\nu. \end{cases}$$

We can deduce from the definition that $e_A \bar{e}_A = \bar{e}_A e_A = 1$. When describing a Clifford-valued function $y = \sum_A y^A e_A : \mathbb{R} \to \mathbf{A}$, where $y^A : \mathbb{R} \to \mathbb{R}$, $A \in \Gamma$, and its derivative is given by $\frac{dy(t)}{dt} = \sum_A \frac{dy^A(t)}{dt} e_A$.

In the following, $e_B \bar{e}_A = (-1)^{\frac{\varrho[A](\varrho[A]+1)}{2}} e_B e_A$, we can write $e_B \bar{e}_A = e_C$ or $e_B \bar{e}_A = -e_C$, where e_C is a basis of Clifford algebra $\mathbf{A}$. For illustrate, $e_{r_1 r_2} \bar{e}_{r_2 r_3} = -e_{r_1 r_2} e_{r_2 r_3} = -e_{r_1} e_{r_2} e_{r_2} e_{r_3} = -e_{r_1}(-1) e_{r_3} = e_{r_1} e_{r_3} = e_{r_1 r_3}$. As a result, we can determine a unique corresponding basis e_C with respect to a given $e_B \bar{e}_A$. Define

$$\varrho[B.\bar{A}] = \begin{cases} 0, & \text{if } e_B \bar{e}_A = e_C, \\ 1, & \text{if } e_B \bar{e}_A = -e_C, \end{cases}$$

and then, $e_B \bar{e}_A = (-1)^{\varrho[B.\bar{A}]} e_C$.

Furthermore, for any $\mathscr{G} \in \mathbf{A}$, there is a unique $\mathscr{G}^C$ that fulfills $\mathscr{G}^{B.\bar{A}} = (-1)^{\varrho[B.\bar{A}]} \mathscr{G}^C$ for $e_B \bar{e}_A = (-1)^{\varrho[B.\bar{A}]} e_C$. Thus

$$\mathscr{G}^{B.\bar{A}} e_B \bar{e}_A = \mathscr{G}^{B.\bar{A}} (-1)^{\varrho[B.\bar{A}]} e_C = (-1)^{\varrho[B.\bar{A}]} \mathscr{G}^C (-1)^{\varrho[B.\bar{A}]} e_C = \mathscr{G}^C e_C.$$

and $\mathscr{G} = \sum_C \mathscr{G}^C e_C \in \mathbf{A}$.

2.3. Problem definition

Consider the following Clifford-valued NN model with time-varying delays:

$$\dot{y}_i(t) = -d_i y_i(t) + \sum_{j=1}^n a_{ij} g_j(y_j(t)) + \sum_{j=1}^n b_{ij} g_j(y_j(t - \tau_j(t))) + u_i, \; t \geq 0, \tag{1}$$

where $i = 1, \dots, n$; $y_i(t) = (y_1(t), \dots, y_n(t))^T \in \mathbf{A}^n$ represents the neuron state vector; $d_i \in \mathbb{R}^+$ is the rate at which it resets its potential to the resting state in isolation; $a_{ij}, b_{ij} \in \mathbf{A}$ denotes the connection weight matrix and delayed connection weight matrix between cells i and j, respectively. $g_j(\cdot) : \mathbf{A} \to \mathbf{A}$ is the Clifford-valued neuron activation function; u_i is the external input vector. The transmission delay is denoted by $\tau_j(t) \in \mathbb{R}^+$.

The initial condition of (1) is as follows:

$$y_i(t) = \omega_i(t), \; t \in [-\tau, 0], \; i = 1, 2, \dots, n. \tag{2}$$

We can express NN (1) as a vector model, as follows.

$$\dot{y}(t) = -\mathscr{D}y(t) + \mathscr{A}g(y(t)) + \mathscr{B}g(y(t-\tau(t))) + U,\ t \geq 0, \tag{3}$$

where $y(t) = (y_1(t), \ldots, y_n(t))^T \in \mathbf{A}^n$, $\mathscr{D} = diag(d_1, \ldots, d_n) \in \mathbf{R}^{n\times n}$, $\mathscr{A} = (a_{ij})_{n\times n} \in \mathbf{A}^{n\times n}$, $\mathscr{B} = (b_{ij})_{n\times n} \in \mathbf{A}^{n\times n}$, $U = (u_1, \ldots, u_n)^T \in \mathbf{A}^n$, $g(y(t)) = (g_1(y_1(t)), \ldots, g_n(y_n(t)))^T \in \mathbf{A}^n$, $g(y(t-\tau(t))) = (g_1(y_1(t-\tau(t))), \ldots, g_n(y_n(t-\tau(t))))^T \in \mathbf{A}^n$.

(A1) In NN (3), the transmission delay $\tau(t)$ is a continuous and differentiable function that satisfies

$$0 \leq \tau(t) \leq \tau,\ \dot{\tau}(t) \leq \mu < 1,$$

where τ and μ are real constants.

(A2) For every $j = 1, 2, \ldots, n$, the neuron activation function $g_j(\cdot) \in \mathscr{C}(\mathbf{A}, \mathbf{A})$ is bounded and Lipschitz continuous. There exist positive real numbers k_j

$$|g_j(\ell) - g_j(\tilde{\ell})|_{\mathbf{A}} \leq k_j|\ell - \tilde{\ell}|_{\mathbf{A}},\ j = 1, 2, \ldots, n,$$

for all $\ell, \tilde{\ell} \in \mathbf{A}$. There exist constant $\mathscr{K} > 0$, such that $|g_j(\ell)|_{\mathbf{A}} \leq \mathscr{K}$, $j = 1, 2, \ldots, n$.

It is obvious from assumption **(A2)** that,

$$(g(\ell) - g(\tilde{\ell}))^*(g(\ell) - g(\tilde{\ell})) \leq (\ell - \tilde{\ell})^* \mathscr{K}^T \mathscr{K} (\ell - \tilde{\ell}), \tag{4}$$

where $\mathscr{K} = diag\{k_1, \ldots, k_n\}$.

Remark 2.1. In general, Clifford-valued networks aim to investigate new capabilities and improve accuracy by addressing issues that cannot be resolved with quaternion-valued, complex-valued, and real-valued NNs. As of now, Clifford-valued networks are the most generalized forms of quaternion-valued, complex-valued, and real-valued network models. For example, Clifford-valued networks (1) can be viewed as a general case of real-valued ($m = 0$), complex-valued ($m = 1$), and quaternion-valued ($m = 2$) network models.

Definition 2.2. (Song et al. [38]) Under assumption **(A2)**, there exist an equilibrium point $y^* \in \mathbf{A}^n$ for the Clifford-valued NNs (3) if it meets the following condition

$$-\mathscr{D}y^* + \mathscr{A}g(y^*) + \mathscr{B}g(y^*) + U = 0.$$

For convenience, we use the transformation $z(t) = y(t) - y^*$ to shift the equilibrium point to the origin. As such, NN (3) can be re-written as

$$\dot{z}(t) = -\mathscr{D}z(t) + \mathscr{A}h(z(t)) + \mathscr{B}h(z(t-\tau(t))),\ t \geq 0, \tag{5}$$

where $z(t)$ is the state vector, $\phi(t) = \omega(t) - y^*$ is the initial condition, and the transformed activation function $h(z(\cdot)) = g(y(\cdot) + y^* + U) - g(y^* + U)$ satisfies

$$|h_j(\ell) - h_j(\tilde{\ell})|_{\mathbf{A}} \leq k_j|\ell - \tilde{\ell}|_{\mathbf{A}}, \ \forall \ \ell, \tilde{\ell} \in \mathbf{A}, \ \ell \neq \tilde{\ell}, \ j = 1, 2, \ldots, n.$$

To describe a nonlinear system, the continuous fuzzy system was introduced in [39] and this concept well discussed in [41]. In addition, the T-S fuzzy model was devised to design complex-valued NNs in [16]. Based on [34, 39, 41], the T-S fuzzy Clifford-valued NNs with time delays can be described, as follows.

Plant Rule p:
If $\{\chi_1(t)$ is $\varpi_{p1}\}$, $\{\chi_2(t)$ is $\varpi_{p2}\}, \ldots, \{\chi_g(t)$ is $\varpi_{pg}\}$.
Then

$$\begin{cases} \dot{z}(t) = \ -\boldsymbol{\mathscr{D}}_p z(t) + \boldsymbol{\mathscr{A}}_p h(z(t)) + \boldsymbol{\mathscr{B}}_p h(z(t - \tau(t))), \ t \geq 0, \\ z(t) = \phi(t), \ t \in [-\tau, 0], \end{cases} \tag{6}$$

where the premise variables are $\chi_r(t), \ r = 1, \ldots, g$, the fuzzy sets are $\varpi_{pr}, \ p = 1, \ldots, m, \ r = 1, \ldots, g$ and m is the total of If-Then rules.

The final output of T-S fuzzy Clifford-valued NN can be achieved by inferring from the fuzzy NN model (6), as follows

$$\begin{cases} \dot{z}(t) = \ \sum_{p=1}^{m} \varphi_p(\chi(t)) \Big\{ -\boldsymbol{\mathscr{D}}_p z(t) + \boldsymbol{\mathscr{A}}_p h(z(t)) + \boldsymbol{\mathscr{B}}_p h(z(t - \tau(t))) \Big\}, \ t \geq 0, \\ z(t) = \phi(t), \ t \in [-\tau, 0], \end{cases} \tag{7}$$

where $\chi(t) = (\chi_1(t), \ldots, \chi_g(t))^T$, $\varphi_p(\chi(t)) = \frac{w_p(\chi(t))}{\sum_{p=1}^{m} w_p(\chi(t))}$ and $w_p(\chi(t)) = \prod_{r=1}^{g} \varpi_{pr}(\chi(t))$. The term $\varpi_{pr}(\chi(t))$ is the grade membership of $\chi_r(t)$ in ϖ_{pr}. It is assumed that $w_p(\chi(t)) \geq 0, \ p = 1, \ldots, m$ and $\sum_{p=1}^{m} w_p(\chi(t)) > 0$ for all $t \geq 0$. From the fuzzy set theory, we have $\varphi_p(\chi(t)) \geq 0, \ p = 1, \ldots, m$ and $\sum_{p=1}^{m} \varphi_p(\chi(t)) = 1$ for all $t \geq 0$.

If Clifford-valued NNs incorporated with impulse effects, the model (7) becomes

$$\begin{cases} \dot{z}(t) = \ \sum_{p=1}^{m} \varphi_p(\chi(t)) \Big\{ -\boldsymbol{\mathscr{D}}_p z(t) + \boldsymbol{\mathscr{A}}_p h(z(t)) + \boldsymbol{\mathscr{B}}_p h(z(t - \tau(t))) \Big\}, \ t \geq 0, \ t \neq t_k, \\ \triangle z(t_k) = z(t_k^+) - z(t_k^-) = \boldsymbol{\mathscr{I}}_k(z(t_k^-)), \ t = t_k, \ k \in \mathbb{Z}_+, \\ z(t) = \phi(t), \ t \in [-\tau, 0], \end{cases} \tag{8}$$

where $\triangle z(t_k) = z(t_k^+) - z(t_k^-)$ is the impulse at moments t_k and $z(t_k^+)$ and $z(t_k^-)$ denotes the right and left hand limits of $z(t_k)$, respectively. In addition, $\boldsymbol{\mathscr{I}}_k \in \mathbb{R}^{n \times n}$ denotes the impulsive matrix and the impulse time t_k satisfies $0 = t_1 < t_2 < \ldots \ t_k < \ldots \to \infty$ and $\inf_{k \in \mathbb{Z}_+} \{t_k - t_{k-1}\} > 0$.

3. MAIN RESULTS

First, we use $e_A\bar{e}_A = \bar{e}_A e_A = 1$ and $e_B\bar{e}_A e_A = e_B$ to rewrite the Clifford-valued NNs (9) into real-valued NNs. From the definition, it is easy to obtain a unique $\mathscr{G}^C$ fulfilling $\mathscr{G}^C e_C h^A e_A = (-1)^{\varrho(B.\bar{A})}\mathscr{G}^C h^A e_B = \mathscr{G}^{B.\bar{A}} h^A e_B$, which implies the following real-valued NN model transformation.

$$\begin{cases} \dot{z}^A(t) = \sum_{p=1}^m \varphi_p(\chi(t))\Big\{ -\boldsymbol{\mathscr{D}}_p z^A(t) + \sum_B \boldsymbol{\mathscr{A}}_p^{A\cdot\bar{B}} h^A(z(t)) + \sum_B \boldsymbol{\mathscr{B}}_p^{A\cdot\bar{B}} h^A(z(t-\tau(t)))\Big\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad t \geq 0,\ t \neq t_k, \\ \triangle z^A(t_k) = z^A(t_k^+) - z^A(t_k^-) = \boldsymbol{\mathscr{I}}_k(z^A(t_k^-)), \qquad t = t_k,\ k \in \mathbb{Z}_+,\ A \in \Gamma. \end{cases} \tag{9}$$

The initial condition of (9) is as follows

$$z^A(t) = \phi^A(t),\ t \in [-\tau, 0], \tag{10}$$

where

$$\begin{aligned} z^A(t) &= (z_1^A(t), \dots, z_n^A(t))^T,\ z(t) = \sum_A z^A(t) e_A, \\ h^A(z(t)) &= (h_1^A(z(t)), \dots, h_n^A(z(t)))^T, \\ h^A(z(t-\tau(t))) &= (h_1^A(z(t-\tau(t))), \dots, h_n^A(z(t-\tau(t))))^T, \\ h(z(t)) &= \sum_B h^B(z^{C_1}(t), \dots, z^{C_n}(t)) e_B = \sum_B h^B(z(t)) e_B, \\ h(z(t-\tau(t))) &= \sum_B h^B(z^{C_1}(t-\tau(t)), \dots, z^{C_n}(t-\tau(t))) e_B = \sum_B h^B(z(t-\tau(t))) e_B, \\ \boldsymbol{\mathscr{A}}_p &= \sum_C \boldsymbol{\mathscr{A}}_p^C e_C,\ \boldsymbol{\mathscr{A}}_p^{A.\bar{B}} = (-1)^{\varrho[A.\bar{B}]} \boldsymbol{\mathscr{A}}_p^C, \\ \boldsymbol{\mathscr{B}}_p &= \sum_C \boldsymbol{\mathscr{B}}_p^C e_C,\ \boldsymbol{\mathscr{B}}_p^{A.\bar{B}} = (-1)^{\varrho[A.\bar{B}]} \boldsymbol{\mathscr{B}}_p^C, \\ e_A\bar{e}_B &= (-1)^{\varrho[A.\bar{B}]} e_C. \end{aligned}$$

According to Clifford algebra, NN (9) can be expressed as a new real-valued NN. Let

$$\begin{aligned} \check{Y}(t) &= \big((z^0(t))^T, (z^1(t))^T, \dots, (z^A(t))^T, \dots, (z^{12\dots m}(t))^T\big)^T \in \mathbb{R}^{2^m n}, \\ \check{H}(\check{Y}(t)) &= \big((h^0(z(t)))^T, (h^1(z(t)))^T, \dots, (h^A(z(t)))^T, \dots, (h^{12\dots m}(z(t)))^T\big)^T \in \mathbb{R}^{2^m n} \\ \check{H}(\check{Y}(t-\tau(t))) &= \big((h^0(z(t-\tau(t))))^T, (h^1(z(t-\tau(t))))^T, \dots, (h^A(z(t-\tau(t))))^T, \\ &\qquad \dots, (h^{12\dots m}(z(t-\tau(t))))^T\big)^T \in \mathbb{R}^{2^m n}, \end{aligned}$$

$$\check{\boldsymbol{\mathscr{D}}}_p = \begin{pmatrix} \boldsymbol{\mathscr{D}}_p & 0 & \dots & 0 \\ 0 & \boldsymbol{\mathscr{D}}_p & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \boldsymbol{\mathscr{D}}_p \end{pmatrix}_{2^m n \times 2^m n},\ \check{\boldsymbol{\mathscr{I}}}_k = \begin{pmatrix} \boldsymbol{\mathscr{I}}_k & 0 & \dots & 0 \\ 0 & \boldsymbol{\mathscr{I}}_k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \boldsymbol{\mathscr{I}}_k \end{pmatrix}_{2^m n \times 2^m n},$$

$$\check{\mathscr{A}}_p = \begin{pmatrix} \mathscr{A}_p^0 & \dots & \mathscr{A}_p^{\overline{A}} & \dots & \mathscr{A}_p^{\overline{12\dots m}} \\ \mathscr{A}_p^1 & \dots & \mathscr{A}_p^{1.\overline{A}} & \dots & \mathscr{A}_p^{1.\overline{12\dots m}} \\ \vdots & \dots & \vdots & \ddots & \vdots \\ \mathscr{A}_p^{12\dots m} & \dots & \mathscr{A}_p^{12\dots m.\overline{A}} & \dots & \mathscr{A}_p^{12\dots m.\overline{12\dots m}} \end{pmatrix}_{2^m n \times 2^m n},$$

$$\check{\mathscr{B}}_p = \begin{pmatrix} \mathscr{B}_p^0 & \dots & \mathscr{B}_p^{\overline{A}} & \dots & \mathscr{B}_p^{\overline{12\dots m}} \\ \mathscr{B}_p^1 & \dots & \mathscr{B}_p^{1.\overline{A}} & \dots & \mathscr{B}_p^{1.\overline{12\dots m}} \\ \vdots & \dots & \vdots & \ddots & \vdots \\ \mathscr{B}_p^{12\dots m} & \dots & \mathscr{B}_p^{12\dots m.\overline{A}} & \dots & \mathscr{B}_p^{12\dots m.\overline{12\dots m}} \end{pmatrix}_{2^m n \times 2^m n}.$$

Then, NN (9) can be written as

$$\begin{cases} \dot{\check{Y}}(t) = \sum_{p=1}^{m} \varphi_p(\chi(t)) \Big\{ -\check{\mathscr{D}}_p \check{Y}(t) + \check{\mathscr{A}}_p \check{H}(\check{Y}(t)) + \check{\mathscr{B}}_p \check{H}(\check{Y}(t-\tau(t))) \Big\}, \ t \geq 0, \ t \neq t_k, \\ \triangle \check{Y}(t_k) = \check{Y}(t_k^+) - \check{Y}(t_k^-) = \check{\mathscr{I}}_k(\check{Y}(t_k^-)), \ t = t_k, \ k \in \mathbb{Z}_+, \end{cases} \tag{11}$$

with the initial condition,

$$\check{Y}(t) = \check{\Phi}(t), \ t \in [-\tau, 0], \tag{12}$$

where $\check{\Phi}(t) = [(\phi^0(t))^T, (\phi^1(t))^T, \dots, (\phi^A(t))^T, \dots (\phi^{12\dots m}(t))^T]^T \in \mathbb{R}^{2^m n}$.

Furthermore, (4) can be written in the following form:

$$(\check{H}(\ell) - \check{H}(\tilde{\ell}))^T (\check{H}(\ell) - \check{H}(\tilde{\ell})) \leq (\ell - \tilde{\ell})^T \check{\mathscr{K}} (\ell - \tilde{\ell}), \tag{13}$$

where $\check{\mathscr{K}} = \begin{pmatrix} \mathscr{K}^T\mathscr{K} & 0 & \dots & 0 \\ 0 & \mathscr{K}^T\mathscr{K} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathscr{K}^T\mathscr{K} \end{pmatrix}_{2^m n \times 2^m n}$.

To explore the global asymptotic stability, we further assume that the impulsive function $\check{\mathscr{I}}_k(\check{Y}(t_k^-))$ satisfies the following assumption:

(A3) $\triangle \check{Y}(t_k) = \check{\mathscr{I}}_k(\check{Y}(t_k^-)) = -\check{\mathscr{J}}_k \check{Y}(t_k^-), \ k \in \mathbb{Z}_+$.

where $\check{\mathscr{J}}_k = \begin{pmatrix} \mathscr{J}_k & 0 & \dots & 0 \\ 0 & \mathscr{J}_k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathscr{J}_k \end{pmatrix}_{2^m n \times 2^m n}$ and $\mathscr{J}_k \in \mathbb{R}^{n \times n}$.

Lemma 3.1. (Balasubramaniam et al. [5]) Let $\mathscr{O}_1$, $\mathscr{O}_2$, $\mathscr{O}_3$ be given matrices such that $\mathscr{O}_3 > 0$, then

$$\begin{bmatrix} \mathscr{O}_2 & \mathscr{O}_1^T \\ \mathscr{O}_1 & -\mathscr{O}_3 \end{bmatrix} < 0 \Leftrightarrow \mathscr{O}_2 + \mathscr{O}_1^T \mathscr{O}_3^{-1} \mathscr{O}_1 < 0.$$

Lemma 3.2. (Balasubramaniam [5]) Given matrix $0 < \boldsymbol{\mathscr{O}} = \boldsymbol{\mathscr{O}}^T \in \mathbb{R}^{2^m n \times 2^m n}$, the following inequality is true for any continuously differentiable function $\check{Y}(\alpha)$ in $[\xi_1, \xi_2] \in \mathbb{R}^{2^m n}$

$$-(\xi_2 - \xi_1) \int_{t-\xi_2}^{t-\xi_1} \check{Y}^T(\alpha) \boldsymbol{\mathscr{O}} \check{Y}(\alpha)\,\mathrm{d}\alpha \le - \bigg[\int_{t-\xi_2}^{t-\xi_1} \check{Y}(\alpha)\,\mathrm{d}\alpha \bigg]^T \boldsymbol{\mathscr{O}} \bigg[\int_{t-\xi_2}^{t-\xi_1} \check{Y}(\alpha)\,\mathrm{d}\alpha \bigg].$$

Lemma 3.3. (Park et al. [27]) Given matrix $0 < \boldsymbol{\mathscr{O}} = \boldsymbol{\mathscr{O}}^T \in \mathbb{R}^{2^m n \times 2^m n}$, any matrices $\boldsymbol{\mathscr{X}} \in \mathbb{R}^{2^n \times 2^m n}$, any vector $\zeta_1, \zeta_2 \in \mathbb{R}^{2^m n}$, and any $\vartheta \in (0,1)$, such that $\begin{pmatrix} \boldsymbol{\mathscr{O}} & \boldsymbol{\mathscr{X}} \\ \boldsymbol{\mathscr{X}}^T & \boldsymbol{\mathscr{O}} \end{pmatrix} > 0$, the following LMI holds

$$\frac{1}{\vartheta} \zeta_1^T \boldsymbol{\mathscr{O}} \zeta_1 + \frac{1}{1-\vartheta} \zeta_2^T \boldsymbol{\mathscr{O}} \zeta_2 \ge \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix}^T \begin{pmatrix} \boldsymbol{\mathscr{O}} & \boldsymbol{\mathscr{X}} \\ \boldsymbol{\mathscr{X}}^T & \boldsymbol{\mathscr{O}} \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix}.$$

Theorem (3.4) presents the global asymptotic stability criterion for the NNs (11).

Theorem 3.4. Suppose Assumptions **(A1)**−**(A3)** are satisfied, NN (11) is globally asymptotic stable if there exist positive definite symmetric matrices $\boldsymbol{\mathscr{P}}$, $\boldsymbol{\mathscr{Q}}_1$, $\boldsymbol{\mathscr{Q}}_2$, $\boldsymbol{\mathscr{R}}_1$ and $\begin{pmatrix} \boldsymbol{\mathscr{S}}_{11} & \boldsymbol{\mathscr{S}}_{12} \\ \star & \boldsymbol{\mathscr{S}}_{22} \end{pmatrix} > 0$, any matrix $\boldsymbol{\mathscr{X}}$ and scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$ such that the following LMIs hold for $p = 1, 2, \dots, m$:

$$\begin{pmatrix} \boldsymbol{\mathscr{P}} & (I - \check{\boldsymbol{\mathscr{J}}}_k)^T \boldsymbol{\mathscr{P}} \\ \star & \boldsymbol{\mathscr{P}} \end{pmatrix} \ge 0, \ k \in \mathbb{Z}_+, \tag{14}$$

$$\Xi_p = \begin{pmatrix} (\Theta^p_{i,j})_{5\times 5} & \tau \boldsymbol{\mathscr{R}}_1 \Pi^T & \sqrt{\tau} \boldsymbol{\mathscr{S}}_{22} \Pi^T \\ \star & -\boldsymbol{\mathscr{R}}_1 & 0 \\ \star & \star & -\boldsymbol{\mathscr{S}}_{22} \end{pmatrix} < 0, \tag{15}$$

where $\Theta^p_{1,1} = -2\boldsymbol{\mathscr{P}} \check{\boldsymbol{\mathscr{D}}}_p + \boldsymbol{\mathscr{Q}}_1 - \boldsymbol{\mathscr{R}}_1 + \epsilon_1 \check{\boldsymbol{\mathscr{K}}}$, $\Theta^p_{1,2} = \boldsymbol{\mathscr{R}}_1^T - \boldsymbol{\mathscr{X}} + \boldsymbol{\mathscr{S}}_{12}^T$, $\Theta^p_{1,3} = \boldsymbol{\mathscr{X}}$, $\Theta^p_{1,4} = \boldsymbol{\mathscr{P}} \check{\boldsymbol{\mathscr{A}}}_p$, $\Theta^p_{1,5} = \boldsymbol{\mathscr{P}} \check{\boldsymbol{\mathscr{B}}}_p$, $\Theta^p_{2,2} = -(1-\mu)\boldsymbol{\mathscr{Q}}_1 - \boldsymbol{\mathscr{R}}_1 - \boldsymbol{\mathscr{R}}_1 + \boldsymbol{\mathscr{X}} + \boldsymbol{\mathscr{X}}^T + \tau \boldsymbol{\mathscr{S}}_{11} - 2\boldsymbol{\mathscr{S}}_{12}^T + \epsilon_2 \check{\boldsymbol{\mathscr{K}}}$, $\Theta^p_{2,3} = \boldsymbol{\mathscr{R}}_1 - \boldsymbol{\mathscr{X}}$, $\Theta^p_{3,3} = -\boldsymbol{\mathscr{R}}_1$, $\Theta^p_{4,4} = \boldsymbol{\mathscr{Q}}_2 - \epsilon I$, $\Theta^p_{5,5} = -(1-\mu)\boldsymbol{\mathscr{Q}}_2 - \epsilon_2 I$, $\Pi = \big[-\check{\boldsymbol{\mathscr{D}}}_p^T \ \ 0 \ \ 0 \ \ \check{\boldsymbol{\mathscr{A}}}_p^T \ \ \check{\boldsymbol{\mathscr{B}}}_p^T \big]$.

P r o o f. Construct the following LKF for NN model (11):

$$\boldsymbol{\mathscr{V}}(t) = \sum_{\ell=1}^{5} \boldsymbol{\mathscr{V}}_\ell(t), \tag{16}$$

where

$$\begin{aligned}
\boldsymbol{\mathscr{V}}_1(t) &= \check{Y}^T(t) \boldsymbol{\mathscr{P}} \check{Y}(t), \\
\boldsymbol{\mathscr{V}}_2(t) &= \int_{t-\tau(t)}^{t} \check{Y}^T(s) \boldsymbol{\mathscr{Q}}_1 \check{Y}(s)\,\mathrm{d}s + \int_{t-\tau(t)}^{t} \check{H}^T(\check{Y}(s)) \boldsymbol{\mathscr{Q}}_2 \check{H}(\check{Y}(s))\,\mathrm{d}s, \\
\boldsymbol{\mathscr{V}}_3(t) &= \tau \int_{t-\tau}^{t} (s - (t-\tau)) \dot{\check{Y}}^T(s) \boldsymbol{\mathscr{R}}_1 \dot{\check{Y}}(s)\,\mathrm{d}s,
\end{aligned}$$

$$\mathscr{V}_4(t) = \int_0^t \int_{u-\tau(u)}^{u} \begin{pmatrix} \check{Y}(u-\tau(u)) \\ \dot{\check{Y}}(s) \end{pmatrix}^T \begin{pmatrix} \boldsymbol{\mathscr{S}}_{11} & \boldsymbol{\mathscr{S}}_{12} \\ \star & \boldsymbol{\mathscr{S}}_{22} \end{pmatrix} \begin{pmatrix} \check{Y}(u-\tau(u)) \\ \dot{\check{Y}}(s) \end{pmatrix} \mathrm{d}s\mathrm{d}u,$$

$$\mathscr{V}_5(t) = \int_{t-\tau}^{t} (s-(t-\tau))\dot{\check{Y}}^T(s)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(s)\,\mathrm{d}s.$$

Computing the upper right derivative of $\mathscr{V}(t) = \sum_{\ell=1}^{5}\mathscr{V}_\ell(t)$ with $\sum_{p=1}^{m}\varphi_p(\chi(t)) = 1$ along the trajectories of model (11) with interval $t \in [t_{k-1}, t_k),\ k \in \mathbb{Z}_+$, we have

$$D^+\mathscr{V}(t) = \sum_{\ell=1}^{5} D^+\mathscr{V}_\ell(t), \tag{17}$$

where

$$D^+\mathscr{V}_1(t) = \sum_{p=1}^{m}\varphi_p(\chi(t))\Big\{2\check{Y}^T(t)\boldsymbol{\mathscr{P}}\big[-\check{\boldsymbol{\mathscr{D}}}_p\check{Y}(t) + \check{\boldsymbol{\mathscr{A}}}_p\check{H}(\check{Y}(t)) + \check{\boldsymbol{\mathscr{B}}}_p\check{H}(\check{Y}(t-\tau(t)))\big]\Big\}, \tag{18}$$

$$\begin{aligned} D^+\mathscr{V}_2(t) &= \check{Y}^T(t)\boldsymbol{\mathscr{Q}}_1\check{Y}(t) - (1-\dot{\tau}(t))\check{Y}^T(t-\tau(t))\boldsymbol{\mathscr{Q}}_1\check{Y}(t-\tau(t)) \\ &\quad + \check{H}^T(\check{Y}(t))\boldsymbol{\mathscr{Q}}_2\check{H}(\check{Y}(t)) - (1-\dot{\tau}(t))\check{H}^T(\check{Y}(t-\tau(t)))\boldsymbol{\mathscr{Q}}_2\check{H}(\check{Y}(t-\tau(t))), \end{aligned} \tag{19}$$

$$D^+\mathscr{V}_3(t) = \tau^2\dot{\check{Y}}^T(t)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(t) - \tau\int_{t-\tau}^{t}\dot{\check{Y}}^T(s)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(s)\,\mathrm{d}s. \tag{20}$$

The first integral term in (20) can be defined as

$$-\tau\int_{t-\tau}^{t}\dot{\check{Y}}^T(s)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(s)\,\mathrm{d}s = -\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}^T(s)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t}\dot{\check{Y}}^T(s)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(s)\,\mathrm{d}s. \tag{21}$$

By applying Lemma (3.2) in the following forms

$$\begin{aligned} -\tau\int_{t-\tau}^{t}\dot{\check{Y}}^T(s)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(s)\,\mathrm{d}s \le &- \frac{\tau}{\tau-\tau(t)}\Big[\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T\boldsymbol{\mathscr{R}}_1\Big[\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ &- \frac{\tau}{\tau(t)}\Big[\int_{t-\tau(t)}^{t}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T\boldsymbol{\mathscr{R}}_1\Big[\int_{t-\tau(t)}^{t}\dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ = &- \Big[\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T\boldsymbol{\mathscr{R}}_1\Big[\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ &- \frac{\tau(t)}{\tau-\tau(t)}\Big[\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T\boldsymbol{\mathscr{R}}_1\Big[\int_{t-\tau}^{t-\tau(t)}\dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ &- \Big[\int_{t-\tau(t)}^{t}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T\boldsymbol{\mathscr{R}}_1\Big[\int_{t-\tau(t)}^{t}\dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ &- \frac{\tau-\tau(t)}{\tau(t)}\Big[\int_{t-\tau(t)}^{t}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T\boldsymbol{\mathscr{R}}_1\Big[\int_{t-\tau(t)}^{t}\dot{\check{Y}}(s)\,\mathrm{d}s\Big]. \end{aligned} \tag{22}$$

If $\begin{pmatrix} \mathscr{R}_1 & \mathscr{X} \\ \mathscr{X}^T & \mathscr{R}_1 \end{pmatrix} \geq 0$, by Lemma (3.3), the following inequality true:

$$\begin{pmatrix} \sqrt{\frac{\tau(t)}{\tau-\tau(t)}}\Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ \sqrt{\frac{\tau-\tau(t)}{\tau(t)}}\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \end{pmatrix}^T \begin{pmatrix} \mathscr{R}_1 & \mathscr{X} \\ \mathscr{X}^T & \mathscr{R}_1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{\tau(t)}{\tau-\tau(t)}}\Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ \sqrt{\frac{\tau-\tau(t)}{\tau(t)}}\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \end{pmatrix} \geq 0, \quad (23)$$

which implies

$$\begin{aligned} -\frac{\tau(t)}{\tau-\tau(t)}\Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{R}_1 \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] - \frac{\tau-\tau(t)}{\tau(t)}\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \\ \times \mathscr{R}_1 \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \leq -\Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{X} \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ -\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{X}^T \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]. \quad (24) \end{aligned}$$

From (22) and (24), once can obtain that

$$\begin{aligned} -\tau \int_{t-\tau}^{t} \dot{\check{Y}}^T(s)\mathscr{R}_1\dot{\check{Y}}(s)\,\mathrm{d}s \\ \leq -\Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{R}_1 \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] - \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \\ \times \mathscr{R}_1 \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] - \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{X} \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ -\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{X}^T \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]. \quad (25) \end{aligned}$$

From (20) – (25), once can obtain that

$$\begin{aligned} D^+\mathscr{V}_3(t) = \tau^2\dot{\check{Y}}^T(t)\mathscr{R}_1\dot{\check{Y}}(t) - \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{R}_1 \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ -\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{R}_1 \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ -\Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{X} \Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big] \\ -\Big[\int_{t-\tau(t)}^{t} \dot{\check{Y}}(s)\,\mathrm{d}s\Big]^T \mathscr{X}^T \Big[\int_{t-\tau}^{t-\tau(t)} \dot{\check{Y}}(s)\,\mathrm{d}s\Big], \quad (26) \end{aligned}$$

$$D^+\mathscr{V}_4(t) = \int_{t-\tau(t)}^{t} \begin{pmatrix} \check{Y}(t-\tau(t)) \\ \dot{\check{Y}}(s) \end{pmatrix}^T \begin{pmatrix} \mathscr{S}_{11} & \mathscr{S}_{12} \\ \star & \mathscr{S}_{22} \end{pmatrix} \begin{pmatrix} \check{Y}(t-\tau(t)) \\ \dot{\check{Y}}(s) \end{pmatrix} \mathrm{d}s$$

$$
\begin{aligned}
&= \tau(t)\check{Y}^T(t-\tau(t))\boldsymbol{\mathscr{S}}_{11}\check{Y}(t-\tau(t)) + 2\check{Y}^T(t)\boldsymbol{\mathscr{S}}_{12}^T\check{Y}(t-\tau(t)) \\
&\quad - 2\check{Y}^T(t-\tau(t))\boldsymbol{\mathscr{S}}_{12}^T\check{Y}(t-\tau(t)) + \int_{t-\tau(t)}^{t} \dot{\check{Y}}^T(s)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(s)\,\mathrm{d}s, \qquad (27)
\end{aligned}
$$

$$
D^+\boldsymbol{\mathscr{V}}_5(t) = \tau\dot{\check{Y}}^T(t)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(t) - \int_{t-\tau(t)}^{t} \dot{\check{Y}}^T(s)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(s)\,\mathrm{d}s. \qquad (28)
$$

There exist positive scalar $\epsilon_1 > 0, \epsilon_2 > 0$. By assumption **(A2)**, we have

$$
0 \le \epsilon_1[\check{Y}^T(t)\boldsymbol{\check{\mathscr{K}}}\check{Y}(t) - \check{H}^T(\check{Y}(t))\check{H}(\check{Y}(t))], \qquad (29)
$$

$$
0 \le \epsilon_2[\check{Y}^T(t-\tau(t))\boldsymbol{\check{\mathscr{K}}}\check{Y}(t-\tau(t)) - \check{H}^T(\check{Y}(t-\tau(t)))\check{H}(\check{Y}(t-\tau(t)))]. \qquad (30)
$$

Combining (18) – (30), we have

$$
\begin{aligned}
D^+\boldsymbol{\mathscr{V}}(t) \le \sum_{p=1}^{m} \varphi_p(\chi(t))\Big\{ & 2\check{Y}^T(t)\boldsymbol{\mathscr{P}}\big[-\boldsymbol{\check{\mathscr{D}}}_p\check{Y}(t) + \boldsymbol{\check{\mathscr{A}}}_p\check{H}(\check{Y}(t)) + \boldsymbol{\check{\mathscr{B}}}_p\check{H}(\check{Y}(t-\tau(t)))\big] \\
& + \check{Y}^T(t)\boldsymbol{\mathscr{Q}}_1\check{Y}(t) - (1-\dot{\tau}(t))\check{Y}^T(t-\tau(t))\boldsymbol{\mathscr{Q}}_1\check{Y}(t-\tau(t)) \\
& + \check{H}^T(\check{Y}(t))\boldsymbol{\mathscr{Q}}_2\check{H}(\check{Y}(t)) - (1-\dot{\tau}(t))\check{H}^T(\check{Y}(t-\tau(t)))\boldsymbol{\mathscr{Q}}_2\check{H}(\check{Y}(t-\tau(t))) \\
& + \tau^2\dot{\check{Y}}^T(t)\boldsymbol{\mathscr{R}}_1\dot{\check{Y}}(t) \\
& - [\check{Y}(t-\tau(t)) - \check{Y}(t-\tau)]^T\boldsymbol{\mathscr{R}}_1[\check{Y}(t-\tau(t)) - \check{Y}(t-\tau)] \\
& - [\check{Y}(t) - \check{Y}(t-\tau(t))]^T\boldsymbol{\mathscr{R}}_1 \times [\check{Y}(t) - \check{Y}(t-\tau(t))] \\
& - [\check{Y}(t-\tau(t)) - \check{Y}(t-\tau)]^T\boldsymbol{\mathscr{X}}[\check{Y}(t) - \check{Y}(t-\tau(t))] \\
& - [\check{Y}(t) - \check{Y}(t-\tau(t))]^T\boldsymbol{\mathscr{X}}^T[\check{Y}(t-\tau(t)) - \check{Y}(t-\tau)] \\
& + \tau\check{Y}^T(t-\tau(t))\boldsymbol{\mathscr{S}}_{11}\check{Y}(t-\tau(t)) + \check{Y}^T(t)\boldsymbol{\mathscr{S}}_{12}^T\check{Y}(t-\tau(t)) \\
& - 2\check{Y}^T(t-\tau(t))\boldsymbol{\mathscr{S}}_{12}^T\check{Y}(t-\tau(t)) + \int_{t-\tau(t)}^{t} \dot{\check{Y}}^T(s)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(s)\,\mathrm{d}s \\
& + \tau\dot{\check{Y}}^T(t)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(t) - \int_{t-\tau(t)}^{t} \dot{\check{Y}}^T(s)\boldsymbol{\mathscr{S}}_{22}\dot{\check{Y}}(s)\,\mathrm{d}s \\
& + \epsilon_1[\check{Y}^T(t)\boldsymbol{\check{\mathscr{K}}}\check{Y}(t) - \check{H}^T(\check{Y}(t))\check{H}(\check{Y}(t))] \\
& + \epsilon_2[\check{Y}^T(t-\tau(t))\boldsymbol{\check{\mathscr{K}}}\check{Y}(t-\tau(t)) - \check{H}^T(\check{Y}(t-\tau(t)))\check{H}(\check{Y}(t-\tau(t)))]\Big\}, \qquad (31)
\end{aligned}
$$

which implies

$$
D^+\boldsymbol{\mathscr{V}}(t) \le \sum_{p=1}^{m} \varphi_p(\chi(t))\Big\{\varsigma^T(t)[\Theta_{i,j}^p + \Pi^T\tau^2\boldsymbol{\mathscr{R}}_1\Pi + \Pi^T\tau\boldsymbol{\mathscr{S}}_{22}\Pi]\varsigma(t)\Big\}, \qquad (32)
$$

where $\varsigma(t) = \big[\check{Y}^T(t),\ \check{Y}^T(t-\tau(t)),\ \check{Y}^T(t-\tau),\ \check{H}^T(\check{Y}(t)),\ \check{H}^T(\check{Y}(t-\tau(t)))\big]^T$, and $\Theta_{i,j}^p$, Π are given in (15).

Furthermore, pre and post multiplication of (14) by $diag\{I, \mathscr{P}^{-1}\}$, we have

$$\begin{aligned}
&\Leftrightarrow \begin{pmatrix} I & 0 \\ 0 & \mathscr{P} \end{pmatrix} \begin{pmatrix} \mathscr{P} & (I-\check{\mathscr{J}}_k)^T\mathscr{P} \\ \star & \mathscr{P} \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & \mathscr{P} \end{pmatrix} \\
&\Leftrightarrow \begin{pmatrix} \mathscr{P} & (I-\check{\mathscr{J}}_k)^T \\ \star & \mathscr{P}^{-1} \end{pmatrix} \geq 0 \\
&\Leftrightarrow \mathscr{P} - (I-\check{\mathscr{J}}_k)^T\mathscr{P}(I-\check{\mathscr{J}}_k) \geq 0. \qquad (33)
\end{aligned}$$

On the other hand, from the NN model (11) it follows that

$$\begin{aligned}
\check{Y}(t_k) - \check{Y}(t_k^-) &= -\check{\mathscr{J}}_k\check{Y}(t_k^-) \\
\check{Y}(t_k) &= (I-\check{\mathscr{J}}_k)\check{Y}(t_k^-). \qquad (34)
\end{aligned}$$

Based on the definition of $\mathscr{V}(t)$, we have

$$\begin{aligned}
\mathscr{V}_1(t_k) &= \check{Y}(t_k)^T\mathscr{P}\check{Y}(t_k) \\
&= \check{Y}(t_k^-)^T(I-\check{\mathscr{J}}_k)^T\mathscr{P}(I-\check{\mathscr{J}}_k)\check{Y}(t_k^-) \\
&\leq \check{Y}(t_k^-)^T\mathscr{P}\check{Y}(t_k^-) \\
\mathscr{V}_1(t_k) &\leq \mathscr{V}_1(t_k^-).
\end{aligned}$$

Moreover, it can be easily verified that $\mathscr{V}_2(t_k) \leq \mathscr{V}_2(t_k^-)$, $\mathscr{V}_3(t_k) \leq \mathscr{V}_3(t_k^-)$, $\mathscr{V}_4(t_k) \leq \mathscr{V}_4(t_k^-)$ and $\mathscr{V}_5(t_k) \leq \mathscr{V}_5(t_k^-)$ which implies that

$$\mathscr{V}(t_k) \leq \mathscr{V}(t_k^-),\ k \in \mathbb{Z}_+. \qquad (35)$$

Using the Schur complement Lemma (3.1) it can be derived from (32) that

$$D^+\mathscr{V}(t) \leq \sum_{p=1}^{m} \varphi_p(\chi(t))\Big\{\varsigma^T(t)\Xi_p\varsigma(t)\Big\},\ t \in [t_{k-1}, t_k),\ k \in \mathbb{Z}_+. \qquad (36)$$

From condition (15), we have

$$D^+\mathscr{V}(t) \leq -\eta\|\check{Y}(t)\|^2 < 0, \qquad (37)$$

for a sufficiently small $\eta > 0$. It can be seen that $D^+\mathscr{V}(t)$ is negative if LMI (15) holds. This implies from Lyapunov stability theory that the equilibrium point of NN (8) or (11) is globally asymptotically stable. This completes the proof of Theorem (3.4).

□

Remark 3.5. When there are no impulses, NN (11) becomes

$$\begin{cases} \dot{\check{Y}}(t) = \sum_{p=1}^{m} \varphi_p(\chi(t))\Big\{ -\check{\mathscr{D}}_p\check{Y}(t) + \check{\mathscr{A}}_p\check{H}(\check{Y}(t)) + \check{\mathscr{B}}_p\check{H}(\check{Y}(t-\tau(t)))\Big\},\ t \geq 0, \\ \check{Y}(t) = \check{\Phi}(t),\ t \in [-\tau, 0]. \end{cases} \qquad (38)$$

The global asymptotic stability of NN (38) without impulsive effect is investigated with corollary (3.6).

Corollary 3.6. Suppose Assumptions **(A1)** – **(A2)** are satisfied, NN model (38) is globally asymptotically stable if there exist positive definite symmetric matrices $\mathscr{P}$, $\mathscr{Q}_1$, $\mathscr{Q}_2$, $\mathscr{R}_1$ and $\begin{pmatrix}\mathscr{S}_{11} & \mathscr{S}_{12}\\ \star & \mathscr{S}_{22}\end{pmatrix} > 0$, any matrix $\mathscr{X}$ and scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$ such that the following LMIs hold for $p = 1, 2, \ldots, m$:

$$\bar{\Xi}_p = \begin{pmatrix}(\bar{\Theta}^p_{i,j})_{5\times 5} & \tau\bar{\Pi}^T & \sqrt{\tau}\bar{\Pi}^T\\ \star & -\mathscr{R}_1 & 0\\ \star & \star & -\mathscr{S}_{22}\end{pmatrix} < 0, \tag{39}$$

where $\bar{\Theta}^p_{1,1} = -2\mathscr{P}\check{\mathscr{D}}_p + \mathscr{Q}_1 - \mathscr{R}_1 + \epsilon_1\check{\mathscr{K}}$, $\bar{\Theta}^p_{1,2} = \mathscr{R}_1 - \mathscr{X} + \mathscr{S}^T_{12}$, $\bar{\Theta}^p_{1,3} = \mathscr{X}$, $\bar{\Theta}^p_{1,4} = \mathscr{P}\check{\mathscr{A}}_p$, $\bar{\Theta}^p_{1,5} = \mathscr{P}\check{\mathscr{B}}_p$, $\bar{\Theta}^p_{2,2} = -(1-\mu)\mathscr{Q}_1 - \mathscr{R}_1 - \mathscr{R}_1 + \mathscr{X} + \mathscr{X}^T + \tau\mathscr{S}_{11} - 2\mathscr{S}^T_{12} + \epsilon_2\check{\mathscr{K}}$, $\bar{\Theta}^p_{2,3} = \mathscr{R}_1 - \mathscr{X}$, $\bar{\Theta}^p_{3,3} = -\mathscr{R}_1$, $\bar{\Theta}^p_{4,4} = \mathscr{Q}_2 - \epsilon_1 I$, $\bar{\Theta}^p_{5,5} = -(1-\mu)\mathscr{Q}_2 - \epsilon_2 I$, $\bar{\Pi} = \begin{bmatrix} -\check{\mathscr{D}}_p & 0 & 0 & \check{\mathscr{A}}_p & \check{\mathscr{B}}_p\end{bmatrix}$.

Remark 3.7. Clifford network models have been successfully applied to solving problems in optimization, neural computing, and image processing. Recent studies have investigated various types of Clifford-valued NN models, including Clifford-valued neutral-type NNs [30], Clifford-valued neutral high-order Hopfield NNs [17], Clifford-valued inertial NNs [19], and Clifford-valued inertial Cohen-Grossberg [19] and so on. However, no work has been published on fuzzy T-S Clifford-valued NNs with time-varying delays and impulsive effects. In order to fill such gaps, we derived for the first time new sufficient conditions that ensure the global asymptotic stability of T-S fuzzy Clifford-valued NNs models with time delays. In the current literature, there are no studies comparing the obtained global asymptotic stability of T-S fuzzy Clifford-valued NN (11). It is noteworthy that our results in this study are new, which is indicative of the efficacy of our work.

Remark 3.8. Clifford algebra is a unital algebra, so research into Clifford-valued networks is difficult due to the noncommutative of Clifford numbers multiplication. Therefore, there are very few studies examining the dynamical behavior of Clifford-valued networks. Meanwhile, the decomposition method is very effective for solving the noncommutativity problem in the multiplication of Clifford numbers. As shown by Theorem (3.4), the original Clifford-valued networks have been examined by dividing them into multidimensional real-valued networks, but the results are only concerning Clifford-valued networks.

Remark 3.9. It is well known that the number of decision variables determines the computation complexity of LMI. When using the augmented LKFs and free-weighting matrix method, the number of decision variables increases. Moreover, when the number of delay subintervals increases, it might increase the computational complexity. Therefore, we used the standard LKFs, and the time derivative has been calculated without using the free-weighting matrix method and delay decomposition methods. As a result, the proposed results may lead to a small computational burden.

4. NUMERICAL EXAMPLES

This section provides a numerical example to demonstrate the usefulness and benefit of the analyses and results.

Example 1: In the case $n = 2$ and $m = 3$, consider the T-S fuzzy Clifford-valued NNs and the plant rule with $p = 1, 2$.

$$\begin{cases} \dot{y}(t) = & \sum_{p=1}^{2} \varphi_p(\chi(t))\Big\{ -\mathscr{D}_p y(t) + \mathscr{A}_p g(y(t)) + \mathscr{B}_p g(y(t-\tau(t)))\Big\},\ t \geq 0,\ t \neq t_k, \\ \triangle y(t_k) = & \mathscr{I}_k y(t_k^-),\ t = t_k,\ k \in \mathbb{Z}_+, \\ y(t) = & \phi(t),\ t \in [-\tau, 0]. \end{cases} \tag{40}$$

Plant Rule 1: IF $\{\chi_1(t)$ is $\varpi_{11}\}$, THEN

$$\begin{cases} \dot{y}(t) = & -\mathscr{D}_1 y(t) + \mathscr{A}_1 g(y(t)) + \mathscr{B}_1 g(y(t-\tau(t))),\ t \geq 0,\ t \neq t_k, \\ \triangle y(t_k) = & \mathscr{I}_k y(t_k^-),\ t = t_k,\ k \in \mathbb{Z}_+, \\ y(t) = & \phi(t),\ t \in [-\tau, 0]. \end{cases}$$

Plant Rule 2: IF $\{\chi_2(t)$ is $\varpi_{22}\}$, THEN

$$\begin{cases} \dot{y}(t) = & -\mathscr{D}_2 y(t) + \mathscr{A}_2 g(y(t)) + \mathscr{B}_2 g(y(t-\tau(t))),\ t \geq 0,\ t \neq t_k, \\ \triangle y(t_k) = & \mathscr{I}_k y(t_k^-),\ t = t_k,\ k \in \mathbb{Z}_+, \\ y(t) = & \phi(t),\ t \in [-\tau, 0], \end{cases}$$

in which the following parameters are used

$$\mathscr{D}_1 = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix},\ \mathscr{D}_2 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix},$$

$$\mathscr{I}_k = \begin{pmatrix} -0.5 & 0 \\ 0 & -0.5 \end{pmatrix},\ \mathscr{K} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix},$$

$$\mathscr{A}_1 = \begin{pmatrix} 0.3e_0 + 2e_1 + 0.1e_{13} & 0.2e_0 + 0.4e_2 - 0.7e_{12} + 0.1e_{13} + 0.1e_{23} + 0.1e_{123} \\ 0.06e_0 - 0.3e_2 + 0.05e_{12} + 0.2e_{13} & 0.2e_0 + 0.2e_1 + 0.1e_3 + 0.06e_{12} + 0.1e_{13} + 0.1e_{123} \end{pmatrix},$$

$$\mathscr{A}_2 = \begin{pmatrix} 0.2e_0 + e_1 + 0.1e_{23} & 0.1e_0 + 0.3e_2 + 0.1e_3 - 0.6e_{12} + 0.1e_{23} + 0.1e_{123} \\ 0.05e_0 - 0.2e_2 + 0.4e_{12} + 0.1e_{123} & 0.1e_0 + 0.1e_1 + 0.1e_2 + 0.05e_{12} + 0.1e_{13} + 0.1e_{123} \end{pmatrix},$$

$$\mathscr{B}_1 = \begin{pmatrix} 0.4e_0 + 0.02e_1 + 0.1e_3 & 0.2e_0 + 0.03e_2 - 0.4e_{12} + 0.1e_{23} + 0.1e_{123} \\ 0.06e_0 - 0.3e_2 + 0.06e_{12} + 0.1e_{13} + 0.1e_{23} & 0.3e_0 + 0.3e_1 + 0.06e_{12} + 0.1e_{13} + 0.1e_{123} \end{pmatrix},$$

$$\mathscr{B}_2 = \begin{pmatrix} 0.3e_0 + 0.01e_1 + 0.1e_3 & 0.1e_0 + 0.02e_2 - 0.3e_{12} + 0.1e_{23} + 0.1e_{123} \\ 0.05e_0 - 0.2e_2 + 0.05e_{12} + 0.1e_{13} + 0.1e_{23} & 0.2e_0 + 0.2e_1 + 0.05e_{12} + 0.1e_{13} + 0.1e_{123} \end{pmatrix}.$$

According to the definitions, we have

$$\mathscr{A}_1^0 = \begin{pmatrix} 0.3 & 0.2 \\ 0.06 & 0.2 \end{pmatrix},\ \mathscr{A}_1^1 = \begin{pmatrix} 2 & 0 \\ 0 & 0.2 \end{pmatrix},$$

$$\mathscr{A}_1^2 = \begin{pmatrix} 0 & 0.4 \\ -0.3 & 0 \end{pmatrix}, \ \mathscr{A}_1^3 = \begin{pmatrix} 0 & 0 \\ 0 & 0.1 \end{pmatrix},$$
$$\mathscr{A}_1^{12} = \begin{pmatrix} 0 & -0.7 \\ 0.5 & 0.06 \end{pmatrix}, \ \mathscr{A}_1^{13} = \begin{pmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{pmatrix},$$
$$\mathscr{A}_1^{23} = \begin{pmatrix} 0 & 0.1 \\ 0 & 0 \end{pmatrix}, \ \mathscr{A}_1^{123} = \begin{pmatrix} 0 & 0.1 \\ 0 & 0.1 \end{pmatrix},$$
$$\mathscr{A}_2^0 = \begin{pmatrix} 0.2 & 0.1 \\ 0.05 & 0.1 \end{pmatrix}, \ \mathscr{A}_2^1 = \begin{pmatrix} 1 & 0 \\ 0 & 0.1 \end{pmatrix},$$
$$\mathscr{A}_2^2 = \begin{pmatrix} 0 & 0.3 \\ -0.2 & 0.1 \end{pmatrix}, \ \mathscr{A}_2^3 = \begin{pmatrix} 0 & 0.1 \\ 0 & 0 \end{pmatrix},$$
$$\mathscr{A}_2^{12} = \begin{pmatrix} 0 & -0.6 \\ 0.4 & 0.05 \end{pmatrix}, \ \mathscr{A}_2^{13} = \begin{pmatrix} 0 & 0 \\ 0 & 0.1 \end{pmatrix},$$
$$\mathscr{A}_2^{23} = \begin{pmatrix} 0.1 & 0.1 \\ 0 & 0 \end{pmatrix}, \ \mathscr{A}_2^{123} = \begin{pmatrix} 0 & 0.1 \\ 0.1 & 0.1 \end{pmatrix},$$
$$\mathscr{B}_1^0 = \begin{pmatrix} 0.4 & 0.2 \\ 0.06 & 0.3 \end{pmatrix}, \ \mathscr{B}_1^1 = \begin{pmatrix} 0.02 & 0 \\ 0 & 0.3 \end{pmatrix},$$
$$\mathscr{B}_1^2 = \begin{pmatrix} 0 & 0.03 \\ -0.3 & 0 \end{pmatrix}, \ \mathscr{B}_1^3 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0 \end{pmatrix},$$
$$\mathscr{B}_1^{12} = \begin{pmatrix} 0 & -0.4 \\ 0.06 & 0.06 \end{pmatrix}, \ \mathscr{B}_1^{13} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.1 \end{pmatrix},$$
$$\mathscr{B}_1^{23} = \begin{pmatrix} 0 & 0.1 \\ 0.1 & 0 \end{pmatrix}, \ \mathscr{B}_1^{123} = \begin{pmatrix} 0 & 0.1 \\ 0.1 & 0 \end{pmatrix},$$
$$\mathscr{B}_2^0 = \begin{pmatrix} 0.3 & 0.1 \\ 0.05 & 0.2 \end{pmatrix}, \ \mathscr{B}_2^1 = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.2 \end{pmatrix},$$
$$\mathscr{B}_2^2 = \begin{pmatrix} 0 & 0.02 \\ -0.2 & 0 \end{pmatrix}, \ \mathscr{B}_2^3 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0 \end{pmatrix},$$
$$\mathscr{B}_2^{12} = \begin{pmatrix} 0 & -0.3 \\ 0.05 & 0.05 \end{pmatrix}, \ \mathscr{B}_2^{13} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.1 \end{pmatrix},$$
$$\mathscr{B}_2^{23} = \begin{pmatrix} 0 & 0.1 \\ 0.1 & 0 \end{pmatrix}, \ \mathscr{B}_2^{123} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.1 \end{pmatrix},$$

and

$$\breve{\mathscr{A}}_1 = \begin{pmatrix}
\mathscr{A}_1^0 & -\mathscr{A}_1^1 & -\mathscr{A}_1^2 & -\mathscr{A}_1^3 & -\mathscr{A}_1^{12} & -\mathscr{A}_1^{13} & -\mathscr{A}_1^{23} & \mathscr{A}_1^{123} \\
\mathscr{A}_1^1 & \mathscr{A}_1^0 & -\mathscr{A}_1^{12} & -\mathscr{A}_1^{13} & \mathscr{A}_1^2 & \mathscr{A}_1^3 & -\mathscr{A}_1^{123} & -\mathscr{A}_1^{23} \\
\mathscr{A}_1^2 & \mathscr{A}_1^{12} & \mathscr{A}_1^0 & -\mathscr{A}_1^{23} & -\mathscr{A}_1^1 & \mathscr{A}_1^{123} & \mathscr{A}_1^3 & \mathscr{A}_1^{13} \\
\mathscr{A}_1^3 & \mathscr{A}_1^{13} & \mathscr{A}_1^{23} & \mathscr{A}_1^0 & -\mathscr{A}_1^{123} & -\mathscr{A}_1^1 & -\mathscr{A}_1^2 & -\mathscr{A}_1^{12} \\
\mathscr{A}_1^{12} & -\mathscr{A}_1^2 & \mathscr{A}_1^1 & -\mathscr{A}_1^{123} & \mathscr{A}_1^0 & -\mathscr{A}_1^{23} & -\mathscr{A}_1^{13} & -\mathscr{A}_1^3 \\
\mathscr{A}_1^{13} & -\mathscr{A}_1^3 & \mathscr{A}_1^{123} & \mathscr{A}_1^1 & \mathscr{A}_1^{23} & \mathscr{A}_1^0 & -\mathscr{A}_1^{12} & \mathscr{A}_1^2 \\
\mathscr{A}_1^{23} & -\mathscr{A}_1^{123} & -\mathscr{A}_1^3 & \mathscr{A}_1^2 & -\mathscr{A}_1^{13} & \mathscr{A}_1^{12} & \mathscr{A}_1^0 & -\mathscr{A}_1^1 \\
\mathscr{A}_1^{123} & \mathscr{A}_1^{23} & -\mathscr{A}_1^{13} & \mathscr{A}_1^{12} & \mathscr{A}_1^3 & -\mathscr{A}_1^2 & \mathscr{A}_1^1 & \mathscr{A}_1^0
\end{pmatrix},$$

$$\check{\mathscr{A}}_2=\begin{pmatrix}
\mathscr{A}_2^0 & -\mathscr{A}_2^1 & -\mathscr{A}_2^2 & -\mathscr{A}_2^3 & -\mathscr{A}_2^{12} & -\mathscr{A}_2^{13} & -\mathscr{A}_2^{23} & \mathscr{A}_2^{123}\\
\mathscr{A}_2^1 & \mathscr{A}_2^0 & -\mathscr{A}_2^{12} & -\mathscr{A}_2^{13} & \mathscr{A}_2^2 & \mathscr{A}_2^3 & -\mathscr{A}_2^{123} & -\mathscr{A}_2^{23}\\
\mathscr{A}_2^2 & \mathscr{A}_2^{12} & \mathscr{A}_2^0 & -\mathscr{A}_2^{23} & -\mathscr{A}_2^1 & \mathscr{A}_2^{123} & \mathscr{A}_2^3 & \mathscr{A}_2^{13}\\
\mathscr{A}_2^3 & \mathscr{A}_2^{13} & \mathscr{A}_2^{23} & \mathscr{A}_2^0 & -\mathscr{A}_2^{123} & -\mathscr{A}_2^1 & -\mathscr{A}_2^2 & -\mathscr{A}_2^{12}\\
\mathscr{A}_2^{12} & -\mathscr{A}_2^2 & \mathscr{A}_2^1 & -\mathscr{A}_2^{123} & \mathscr{A}_2^0 & -\mathscr{A}_2^{23} & -\mathscr{A}_2^{13} & -\mathscr{A}_2^3\\
\mathscr{A}_2^{13} & -\mathscr{A}_2^3 & \mathscr{A}_2^{123} & \mathscr{A}_2^1 & \mathscr{A}_2^{23} & \mathscr{A}_2^0 & -\mathscr{A}_2^{12} & \mathscr{A}_2^2\\
\mathscr{A}_2^{23} & -\mathscr{A}_2^{123} & -\mathscr{A}_2^3 & \mathscr{A}_2^2 & -\mathscr{A}_2^{13} & \mathscr{A}_2^{12} & \mathscr{A}_2^0 & -\mathscr{A}_2^1\\
\mathscr{A}_2^{123} & \mathscr{A}_2^{23} & -\mathscr{A}_2^{13} & \mathscr{A}_2^{12} & \mathscr{A}_2^3 & -\mathscr{A}_2^2 & \mathscr{A}_2^1 & \mathscr{A}_2^0
\end{pmatrix},$$

$$\check{\mathscr{B}}_1=\begin{pmatrix}
\mathscr{B}_1^0 & -\mathscr{B}_1^1 & -\mathscr{B}_1^2 & -\mathscr{B}_1^3 & -\mathscr{B}_1^{12} & -\mathscr{B}_1^{13} & -\mathscr{B}_1^{23} & \mathscr{B}_1^{123}\\
\mathscr{B}_1^1 & \mathscr{B}_1^0 & -\mathscr{B}_1^{12} & -\mathscr{B}_1^{13} & \mathscr{B}_1^2 & \mathscr{B}_1^3 & -\mathscr{B}_1^{123} & -\mathscr{B}_1^{23}\\
\mathscr{B}_1^2 & \mathscr{B}_1^{12} & \mathscr{B}_1^0 & -\mathscr{B}_1^{23} & -\mathscr{B}_1^1 & \mathscr{B}_1^{123} & \mathscr{B}_1^3 & \mathscr{B}_1^{13}\\
\mathscr{B}_1^3 & \mathscr{B}_1^{13} & \mathscr{B}_1^{23} & \mathscr{B}_1^0 & -\mathscr{B}_1^{123} & -\mathscr{B}_1^1 & -\mathscr{B}_1^2 & -\mathscr{B}_1^{12}\\
\mathscr{B}_1^{12} & -\mathscr{B}_1^2 & \mathscr{B}_1^1 & -\mathscr{B}_1^{123} & \mathscr{B}_1^0 & -\mathscr{B}_1^{23} & -\mathscr{B}_1^{13} & -\mathscr{B}_1^3\\
\mathscr{B}_1^{13} & -\mathscr{B}_1^3 & \mathscr{B}_1^{123} & \mathscr{B}_1^1 & \mathscr{B}_1^{23} & \mathscr{B}_1^0 & -\mathscr{B}_1^{12} & \mathscr{B}_1^2\\
\mathscr{B}_1^{23} & -\mathscr{B}_1^{123} & -\mathscr{B}_1^3 & \mathscr{B}_1^2 & -\mathscr{B}_1^{13} & \mathscr{B}_1^{12} & \mathscr{B}_1^0 & -\mathscr{B}_1^1\\
\mathscr{B}_1^{123} & \mathscr{B}_1^{23} & -\mathscr{B}_1^{13} & \mathscr{B}_1^{12} & \mathscr{B}_1^3 & -\mathscr{B}_1^2 & \mathscr{B}_1^1 & \mathscr{B}_1^0
\end{pmatrix},$$

$$\check{\mathscr{B}}_2=\begin{pmatrix}
\mathscr{B}_2^0 & -\mathscr{B}_2^1 & -\mathscr{B}_2^2 & -\mathscr{B}_2^3 & -\mathscr{B}_2^{12} & -\mathscr{B}_2^{13} & -\mathscr{B}_2^{23} & \mathscr{B}_2^{123}\\
\mathscr{B}_2^1 & \mathscr{B}_2^0 & -\mathscr{B}_2^{12} & -\mathscr{B}_2^{13} & \mathscr{B}_2^2 & \mathscr{B}_2^3 & -\mathscr{B}_2^{123} & -\mathscr{B}_2^{23}\\
\mathscr{B}_2^2 & \mathscr{B}_2^{12} & \mathscr{B}_2^0 & -\mathscr{B}_2^{23} & -\mathscr{B}_2^1 & \mathscr{B}_2^{123} & \mathscr{B}_2^3 & \mathscr{B}_2^{13}\\
\mathscr{B}_2^3 & \mathscr{B}_2^{13} & \mathscr{B}_2^{23} & \mathscr{B}_2^0 & -\mathscr{B}_2^{123} & -\mathscr{B}_2^1 & -\mathscr{B}_2^2 & -\mathscr{B}_2^{12}\\
\mathscr{B}_2^{12} & -\mathscr{B}_2^2 & \mathscr{B}_2^1 & -\mathscr{B}_2^{123} & \mathscr{B}_2^0 & -\mathscr{B}_2^{23} & -\mathscr{B}_2^{13} & -\mathscr{B}_2^3\\
\mathscr{B}_2^{13} & -\mathscr{B}_2^3 & \mathscr{B}_2^{123} & \mathscr{B}_2^1 & \mathscr{B}_2^{23} & \mathscr{B}_2^0 & -\mathscr{B}_2^{12} & \mathscr{B}_2^2\\
\mathscr{B}_2^{23} & -\mathscr{B}_2^{123} & -\mathscr{B}_2^3 & \mathscr{B}_2^2 & -\mathscr{B}_2^{13} & \mathscr{B}_2^{12} & \mathscr{B}_2^0 & -\mathscr{B}_2^1\\
\mathscr{B}_2^{123} & \mathscr{B}_2^{23} & -\mathscr{B}_2^{13} & \mathscr{B}_2^{12} & \mathscr{B}_2^3 & -\mathscr{B}_2^2 & \mathscr{B}_2^1 & \mathscr{B}_2^0
\end{pmatrix}.$$

For the simulation purpose, we consider the split activation function, i. e., $g_j(y_j) = 0.5\ tanh(y_j)e_0 + 0.5\ tanh(y_j)e_1 + 0.5\ tanh(y_j)e_2 + 0.5\ tanh(y_j)e_3 + 0.5\ tanh(y_j)e_{12} + 0.5\ tanh(y_j)e_{13} + 0.5\ tanh(y_j)e_{23} + 0.5\ tanh(y_j)e_{123},\ j = 1, 2$. Also, the time-varying delay is fixed as $\tau(t) = 0.4 + 0.3\ sin(t)$, implying that the greatest permissible upper bound is $\tau = 0.7$. It is observable that $0 \le \dot{\tau}(t) \le \mu = 0 \le 0.3\,cos(t) \le 0.3$. Furthermore, we use the following definitions for membership functions $\varphi_1(\chi(t)) = \frac{1}{1+e^{-2t}}$, $\varphi_2(\chi(t)) = 1 - \frac{1}{1+e^{-2t}}$. The LMI conditions of (14) and (15) in Theorem (3.4) are verified by using the MATLAB LMI toolbox. Under the initial values $\phi_1(t) = -0.9e_0 + e_1 - 0.2e_2 - 1.6e_3 + 1.2e_{12} + 1.5e_{13} - 0.9e_{23} - 2e_{123}$ and $\phi_2(t) = -e_0 - e_1 + 1.8e_2 + 2e_3 - 0.6e_{12} - 2e_{13} + 1.2e_{23} + e_{123}$, the time responses of states $y_i^0(t)$, $y_i^1(t)$, $y_i^2(t)$, $y_i^3(t)$, $y_i^{12}(t)$, $y_i^{13}(t)$, $y_i^{23}(t)$, $y_i^{123}(t)$, $i = 1, 2$ are illustrated in Figures (1)- (6).

From the above example, we can conclude that all the conditions associated with Theorem (3.4) are confirmed by this example. As a result of Theorem (3.4), the equilibrium point of NNs (40) is globally asymptotically stable.

5. CONCLUSION

The global asymptotic stability of T-S fuzzy Clifford-valued NN model with time-varying delays and impulsive effects have been examined in this paper. In order to achieve the main results, we design a general network model that includes real-, complex-, and quaternion-valued networks as special cases. First, we decompose the n-dimensional

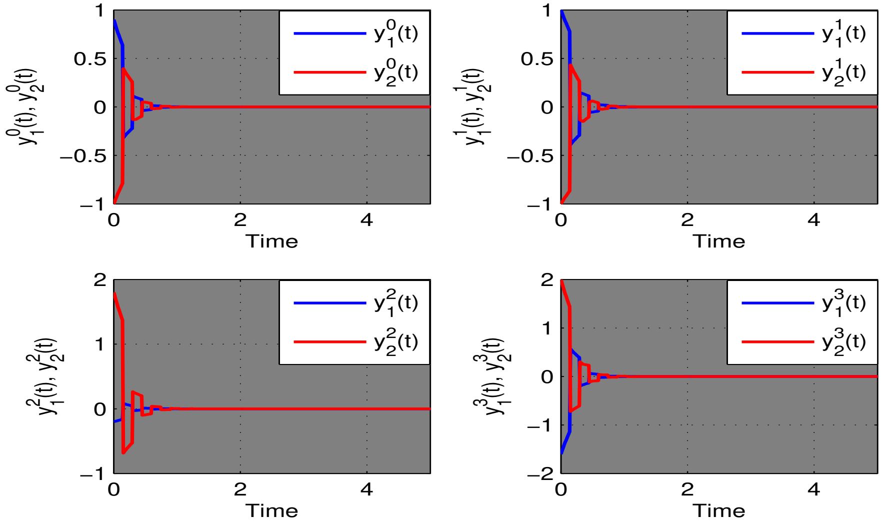

Fig. 1. The trajectories of states $y_i^0(t)$, $y_i^1(t)$, $y_i^2(t)$, $y_i^3(t)$, $i = 1, 2$ of NNs (40) with impulses.

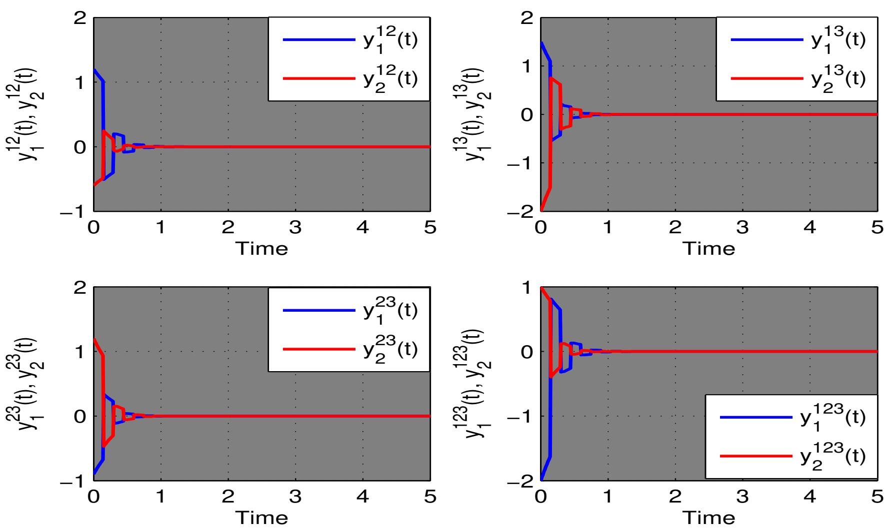

Fig. 2. The trajectories of states $y_i^{12}(t)$, $y_i^{13}(t)$, $y_i^{23}(t)$, $y_i^{123}(t)$, $i = 1, 2$ of NNs (40) with impulses.

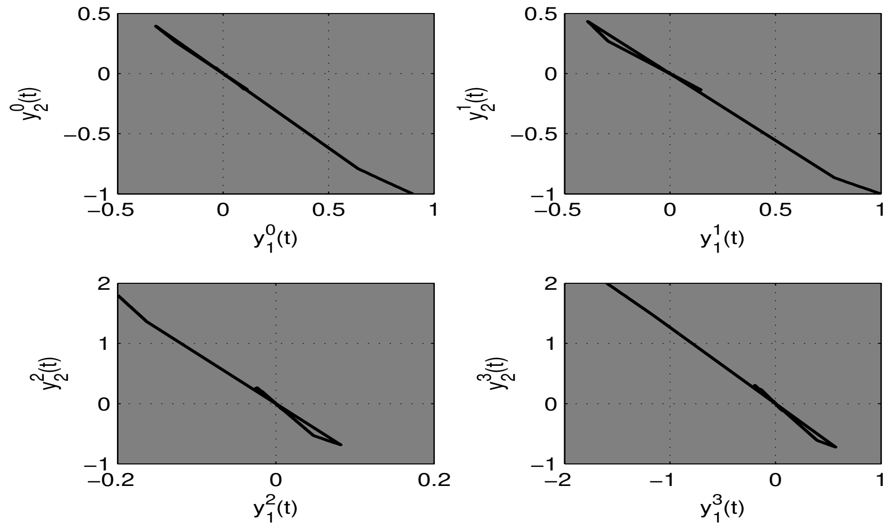

Fig. 3. The trajectories of states $y_i^0(t)$, $y_i^1(t)$, $y_i^2(t)$, $y_i^3(t)$, $i = 1, 2$ of NNs (40) in a 2-dimensional space with impulses.

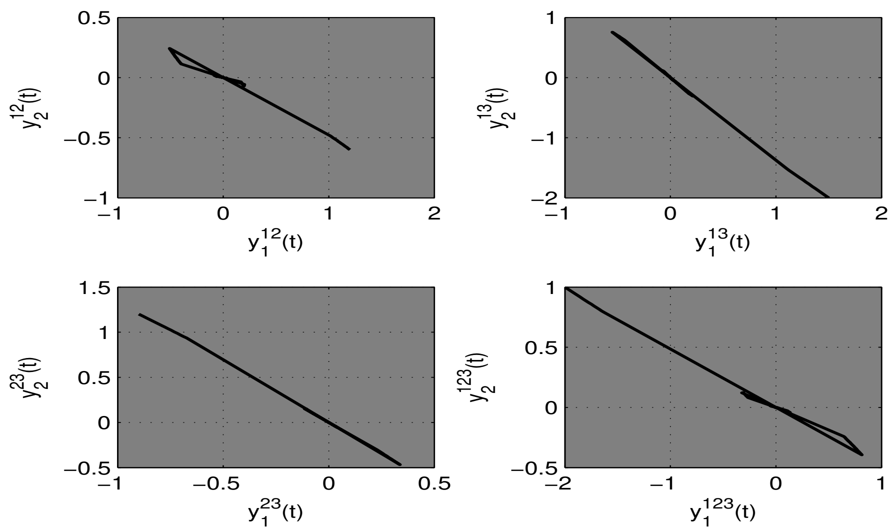

Fig. 4. The trajectories of states $y_i^{12}(t)$, $y_i^{13}(t)$, $y_i^{23}(t)$, $y_i^{123}(t)$, $i = 1, 2$ of NNs (40) in a 2-dimensional space with impulses.

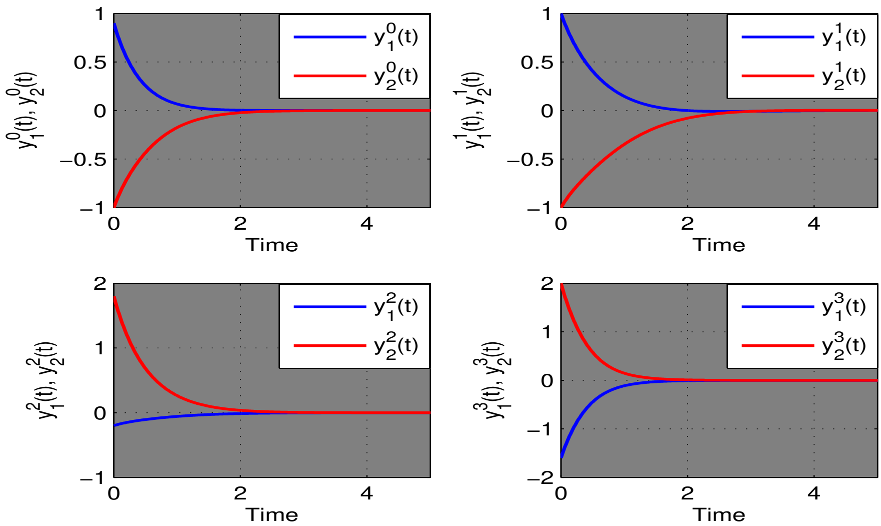

Fig. 5. The trajectories of states $y_i^0(t)$, $y_i^1(t)$, $y_i^2(t)$, $y_i^3(t)$, $i = 1, 2$ of NNs (40) without impulses.

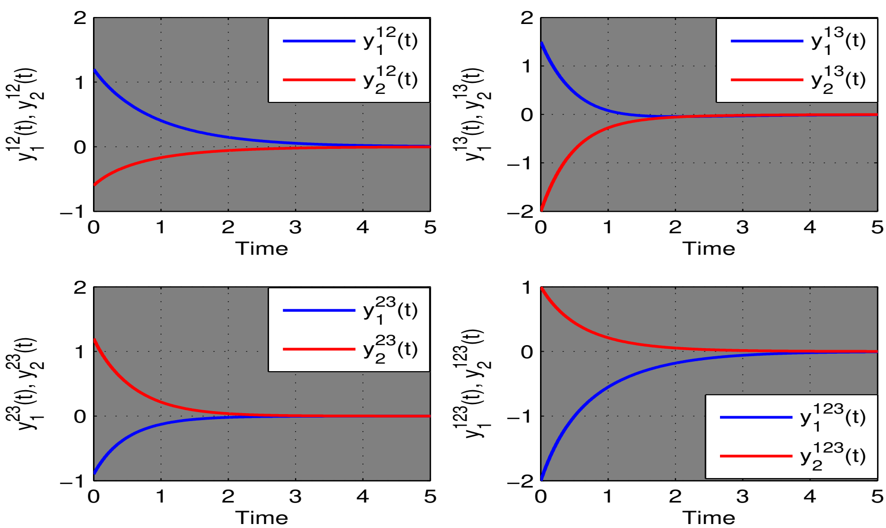

Fig. 6. The trajectories of states $y_i^{12}(t)$, $y_i^{13}(t)$, $y_i^{23}(t)$, $y_i^{123}(t)$, $i = 1, 2$ of NNs (40) without impulses.

Clifford-valued neural network into $2^m n$-dimensional real-valued counterparts in order to solve the non-commutativity of Clifford numbers multiplication. Then, we prove the new LMI-based global asymptotic stability criteria by constructing an appropriate LKFs and employing Jensen's integral inequality together with the reciprocal convex combination method. Finally, we present a numerical example together with their simulation results to illustrate the efficiency of our obtained results.

Undoubtedly, there are certain advancements worth investigating further in this proposed area of research. Shortly, we will attempt to investigate the global stability of T-S fuzzy Clifford-valued bidirectional associative memory NNs with time-varying delays and impulses as shown below.

$$\begin{cases}\dot{x}(t) = & \sum_{p=1}^{m}\varphi_p(\chi(t))\Big\{-\boldsymbol{\mathscr{D}}_p x(t)+\boldsymbol{\mathscr{A}}_p f(y(t))+\boldsymbol{\mathscr{B}}_p f(y(t-\sigma(t)))\Big\},\ t\geq 0,\ t\neq t_k,\\ \triangle x(t_k) = & \boldsymbol{\mathscr{I}}_k x(t_k^-),\ t=t_k,\ k\in\mathbb{Z}_+,\\ x(t) = & \phi_x(t),\ t\in[-\tau,0],\\ \dot{y}(t) = & \sum_{p=1}^{m}\varphi_p(\chi(t))\Big\{-\boldsymbol{\mathscr{C}}_p y(t)+\boldsymbol{\mathscr{E}}_p g(x(t))+\boldsymbol{\mathscr{F}}_p g(x(t-\tau(t)))\Big\},\ t\geq 0,\ t\neq t_k,\\ \triangle y(t_k) = & \boldsymbol{\mathscr{J}}_k y(t_k^-),\ t=t_k,\ k\in\mathbb{Z}_+,\\ y(t) = & \phi_y(t),\ t\in[-\sigma,0].\end{cases}$$

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Ramalingam Sriraman, Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Tamil Nadu-603 203, India and College of Mathematics and Computer Science, Zhejiang Normal University, Jinhua-321004, Zhejiang. P. R. China.
e-mail: sriramar@srmist.edu.in

Asha Nedunchezhiyan, Department of Mathematics, Kalasalingam Academy of Research and Education, Krishnankoil, Tamil Nadu-626 128. India.
e-mail: n.asha@klu.ac.in