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STOCHASTIC PERFORMANCE MEASUREMENT IN TWO-STAGE NETWORK PROCESSES: A DATA ENVELOPMENT ANALYSIS APPROACH

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In classic data envelopment analysis models, two-stage network structures are studied in cases in which the input/output data set are deterministic. In many real applications, however, we face uncertainty. This paper proposes a two-stage network DEA model when the input/output data are stochastic. A stochastic two-stage network DEA model is formulated based on the chance-constrained programming. Linearization techniques and the assumption of single underlying factor of the data are used to construct the equivalent deterministic linear programming model. The relationship between the stochastic efficiency of each stage and stochastic centralized efficiency of the whole process, at different confidence levels, is discussed. To illustrate the real applicability of the proposed approach, a real case on 16 commercial banks in China is given.

Keywords: stochastic DEA, chance-constrained models, two-stage network systems, efficiency

Classification: 90C05, 90B50

1. INTRODUCTION

Since the introduction of Data Envelopment Analysis (DEA) as a nonparametric method for assessing the relative efficiency of decision-making units (DMUs) with multiple inputs and outputs by Charnes et al. [5], many theoretical and empirical studies have applied DEA to diverse fields of science and engineering, in both private and public sectors, such as healthcare, agriculture, retailing, manufacturing, banking, energy and environment, information technology, public policy, operations, and supply chains. For further details, the reader is referred to the studies by Dyson et al. [13], Ray [32], Coelli et al. [7], Zhu and Cook [42], Cooper et al. [11], and Zhu [41].

In many cases, there are DMUs with the two-stage process where the outputs of the first stage are used as inputs to the second stage. To achieve an efficient status, the inputs of the second stage should be reduced while, as the output of the first stage they should be increased. To evaluate such processes, there are two major approaches: cooperative and non-cooperative approaches. Consider a two-stage process in which is considered to be leader and another one is follower. In non-cooperative approaches, we

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first optimize the leader and then, the efficiency of the follower is maximized [23]. In contrast, in the cooperative or centralized approaches, the importance of both stages is the same and the performance of the overall system is maximized.

In the last two decades, many studies have been conducted on the performance analysis of two-stage systems. Centralized and leader-follower models from the perspective of game theory are constructed by Liang et al. [23], where the overall efficiency is decomposed into the product of the efficiencies of the stages [23]. Also, Kao and Hwang [20] considered each stage as an independent system and the efficiency of the whole system is considered as the product of the efficiencies of each stage. The equivalence between the relational model and the frontier model is proved by Cook et al. [8]. Chen et al. [6] proposed an additive relational model, where the overall efficiency is decomposed as a weighted sum of the efficiencies of the stages [25]. From another perspective, Kordrostami and Amirteimoori [22] have studied a multistage system when there are undesirable final products. An extension of their approach to the more general network processes is proposed by Hua and Bian [16]. Wang et al. [35] used additive two-stage DEA in Chen et al. [6] applied the data translation approach to address undesirable outputs [25]. In addition, production possibility sets (PPS) and envelopment models for network system are constructed by Fre et al. [14].

In many situations, such as in a manufacturing system, in a production process, or in a service system, due to volatility and complexity of the processes, measurement of inputs and outputs are difficult in an accurate way. The traditional DEA models could not consider data sensitivity. Thus, some researchers have proposed several models to deal with the data variation in DEA by stochastic models. All of the above-mentioned network models ignore the stochastic variability and uncertainty in the input and output data caused by factors such as measurement errors, sample noise, specification errors, etc. Although in some studies, uncertainty in the data in two-stage DEA models based on the fuzzy theory have been considered (see, for example, [1, 12, 15, 21, 24, 26], but there is little work on two-stage network DEA models considering stochastic data. Zhou et al. [39] proposed the stochastic two-stage network DEA model based on centralized control organization mechanism. In addition, the relationship of their results and the two sub-processes are discussed. Izadikhah and Saen [18] have proposed a new stochastic two-stage DEA model in the presence of undesirable data. Although they claimed that they proposed a linear model to obtain overall efficiency, they did not represent any transformation to linearization of their stochastic model. Wanke et al. [36] proposed an assessment of OECD banks during 2004-2013 in light of relevant accounting and financial indicators to reflect the production process and performance of banking industry. They have used dynamic network DEA and SFA models for accounting and financial indicators with an analysis of super-efficiency in stochastic frontiers. Moheb-Alizadeh et al. [29] proposed a two-stage stochastic formulation with a hybrid solution methodology to identify Efficient and sustainable closed-loop supply chain network design. Mehdizadeh et al. [28] proposed a two-stage network DEA model with stochastic data and formulated it based on the satisficing DEA models of chance-constrained programming and the leader-follower concepts. Moreover, they discussed the relationship between the two-stages as the leader and the follower, respectively, at different confidence levels and under different aspiration levels. Their proposed model was applied to a real case concerning

16 commercial banks in China. Zhou et al. [40] proposed the stochastic DEA for a two-stage process based on the envelopment form of the DEA model. In addition, their model assumed that the stochastic data follow a more generalized distribution function rather than the normal distribution. They also showed that the stochastic efficiency of the whole system can be decomposed into the product of the stochastic efficiencies of the leader and follower.

The aim of this paper is to incorporate chance-constraint DEA and cooperative game theory concepts into the network processes with a two-stage structure. For this purpose, we utilize centralized two-stage model proposed by Liang et al. [23] and stochastic DEA proposed by Huang and Li [17] and Cooper et al. [9] and we propose a stochastic centralized two-stage DEA model. Actually, by using this methodology, we can replace deterministic concepts such as "efficiency" and "inefficiency" with concepts such as "stochastic efficiency" and "stochastic inefficiency". Moreover, we will apply goal programming to linearization of our proposed model.

The paper unfolds as follows: In the next section, we discuss the deterministic centralized model. Next, in Section 3, the stochastic centralized two-stage network DEA model according to the concepts of the cooperative game theory is proposed and its transformation into a deterministic and linear model is explained. In addition, stochastic production possibility set of the two-stage system will be discussed. In Section 4, the proposed model is applied to a case study of 16 commercial banks in China. In the end, Section 5 concludes the paper.

2. PRELIMINARIES

In this section, we first review the deterministic centralized two-stage DEA model and then we present its dual programming.

2.1. Deterministic centralized model

Suppose that there are n independent Decision-Making Units, denoted by DMU_j ($j = 1, 2, \ldots, n$). Each DMU, as depicted in Figure 1, composed of two stages in series. In the process of production, for each DMU_j (j=1,2,,n), the first stage consumes $x_j = (x_{1j}, x_{2j}, \ldots, x_{Mj})^T$ as inputs to produce $z_j = (z_{1j}, z_{2j}, \ldots, z_{Dj})^T$ as outputs. These outputs are consumed by the second stage as inputs to produce $y_j = (y_{1j}, y_{2j}, \ldots, y_{Sj})^T$ as final outputs.

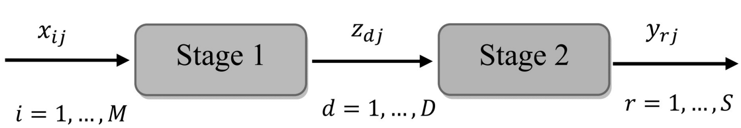

Fig. 1. Two-stage process.

In real-world situations, there are many cases in which the stages of the whole system cooperate with each other to achieve the maximal overall performance of the whole system [27]. For example, marketing and production departments work together to maximize the companys profit [27].

In this section, we present the centralized model based on the concept of cooperative game theory. In our approach, the two-stage process is viewed as one stage where the two stages jointly determine one optimal plan to maximize the total efficiency of the whole system [27]. In order to measure the relative efficiency of DMU_o , the following multiplier model is proposed:

$$
\begin{aligned}
& Max \ \ \frac{Uy_o}{Vx_o} \\
& s.t. \\
& \frac{Uy_j}{Wz_j} \leq 1, \quad j = 1, \ldots, n, \\
& \frac{Wz_j}{Vx_j} \leq 1, \quad j = 1, \ldots, n, \\
& \frac{Uy_j}{Vx_j} \leq 1, \quad j = 1, \ldots, n, \\
& W, U, V \geq 0,
\end{aligned}
\tag{1}
$$

in which, V, W and U are weight vectors associated with input, intermediate measures and outputs, respectively. In the objective function of (1), the weighted sum of the final outputs to the weighted sum of the initial inputs is maximized. The constraints guarantee that the ratio of the weighted sum of the outputs to the weighted sum of the inputs to both stages and the whole system do not exceed unity. Clearly, the mathematical programme (1) is a linear fractional programming problem. By using the Charnes and Cooper [3] transformation, programme (1) can be transformed into the following linear form:

$$
\begin{aligned}
& Max \ \ Uy_o \\
& s.t. \\
& Vx_o = 1, \\
& Uy_j - Wz_j \leq 0, \quad j = 1, \ldots, n, \\
& Wz_j - Vx_j \leq 0, \quad j = 1, \ldots, n, \\
& Uy_j - Vx_j \leq 0, \quad j = 1, \ldots, n, \\
& W, U, V \geq 0.
\end{aligned}
\tag{2}
$$

The dual formulation of (2) is as follows:

$$
\begin{aligned}
& Min \ \theta \\
& s.t. \\
& \Sigma_{j=1}^{n} (\lambda_j + \gamma_j) x_{ij} \leq \theta x_{io} \quad i = 1, \ldots, M, \\
& \Sigma_{j=1}^{n} (\lambda_j + \xi_j) y_{rj} \geq y_{ro} \quad r = 1, \ldots, S, \\
& \Sigma_{j=1}^{n} (\gamma_j - \xi_j) z_{dj} \geq 0 \quad d = 1, \ldots, D, \\
& \lambda_j, \gamma_j, \xi_j \geq 0, (\forall j).
\end{aligned}
\tag{3}
$$

In programme (3), $\lambda_j+\gamma_j, \lambda_j+\xi_j$ and $\gamma_j-\xi_j$ are multipliers related to the inputs, intermediate measures and final outputs, respectively.

3. STOCHASTIC CENTRALIZED MODEL

In this section, we develop a stochastic centralized DEA model to a process with a two-stage structure, which permits the presence of stochastic variability in the data. Let us consider all initial inputs, intermediate products, and final outputs to be jointly normally distributed in the following chance-constrained DEA model, which is the stochastic version of programme (3).

$$\begin{aligned} e^{(\alpha)}_{SCN} =& Min\ \theta \\ & s.t. \\ & \mathbb{P}\Big(\Sigma^n_{j=1}(\lambda_j+\gamma_j)\tilde{x}_{ij} \le \theta\tilde{x}_{io}\Big) \ge 1-\alpha, \quad i=1,\dots,M, \\ & \mathbb{P}\Big(\Sigma^n_{j=1}(\lambda_j+\xi_j)\tilde{y}_{rj} \ge \tilde{y}_{ro}\Big) \ge 1-\alpha, \quad r=1,\dots,S, \\ & \mathbb{P}\Big(\Sigma^n_{j=1}(\gamma_j-\xi_j)\tilde{z}_{dj} \ge 0\Big) \ge 1-\alpha, \quad d=1,\dots,D, \\ & \lambda_j,\gamma_j,\xi_j \ge 0, (\forall j). \end{aligned} \tag{4}$$

In the stochastic programming problem (4), $\tilde{x}_j=(\tilde{x}_{1j},\tilde{x}_{2j},\dots,\tilde{x}_{Mj})^T$, $\tilde{z}_j=(\tilde{z}_{1j},\tilde{z}_{2j},\dots,\tilde{z}_{Dj})^T$ and $\tilde{y}_j=(\tilde{y}_{1j},\tilde{y}_{2j},\dots,\tilde{y}_{Sj})^T$ respectively show the random initial input vector, intermediate product vector, and final output vector, and $x_j=(x_{1j},x_{2j},\dots,x_{Mj})^T, z_j=(z_{1j},z_{2j},\dots,z_{Dj})^T$ and $y_j=(y_{1j},y_{2j},\dots,y_{Sj})^T$ are associated mean vectors. Moreover, here $\mathbb{P}$ means probability and $\alpha\in[0,1]$ is a scalar, specified in advance, which represents the allowable chance (risk) of failing to satisfy the constraints with which it is associated.

Definition 3.1. For a predetermined level α, DMU_o is stochastically efficient if and only if $e^{(\alpha)}_{SCN}=1$. Suppose $SPPS_{stage1}$ and $SPPS_{stage2}$ are stochastic production possibility sets of the first and second stages, respectively, defined as

$$SPPS_{stage1}=\Big\{(\tilde{x}^T,\tilde{z}^T)\in\mathbb{R}^{M+D}_+ | \exists(\tilde{x}^T_j,\tilde{z}^T_j)\in D_j(1-\alpha) \text{ and } \lambda_j,\gamma_j\ge 0, \\ j\in\{1,\dots,n\} \text{ such that } \Sigma^n_{j=1}(\lambda_j+\gamma_j)\tilde{x}_j\le\tilde{x} \text{ and } \Sigma^n_{j=1}(\lambda_j+\gamma_j)\tilde{z}_j\ge\tilde{z}\Big\}$$

$$SPPS_{stage2}=\Big\{(\tilde{z}^T,\tilde{y}^T)\in\mathbb{R}^{D+S}_+ | \exists(\tilde{z}^T_j,\tilde{y}^T_j)\in D'_j(1-\alpha) \text{ and } \lambda_j,\xi_j\ge 0, \\ j\in\{1,\dots,n\} \text{ such that } \Sigma^n_{j=1}(\lambda_j+\xi_j)\tilde{z}_j\le\tilde{z} \text{ and } \Sigma^n_{j=1}(\lambda_j+\xi_j)\tilde{y}_j\ge\tilde{y}\Big\}$$

where $D_j(1-\alpha)$ and $D'_j(1-\alpha)$ are confidence regions for stage 1 and stage 2 to DMU_j, respectively, defined by Olesen and Petersen[30] as follows:

$$D_j(1-\alpha)=\Big\{(\tilde{x}^T,\tilde{z}^T)\in\mathbb{R}^{M+D}_+ | \big[(\tilde{x}_j-x_j)^T,(\tilde{z}_j-z_j)^T\big]\Lambda^{-1}_j\big[(\tilde{x}_j-x_j)^T,(\tilde{z}_j-z_j)^T\big]^T\le c^2_j\Big\},$$

$$D'_j(1-\alpha) = \Big\{(\tilde{z}^T, \tilde{y}^T) \in \mathbb{R}_+^{D+S} | \big[(\tilde{z}_j - z_j)^T, (\tilde{y}_j - y_j)^T\big] \Lambda_j^{'-1} \big[(\tilde{z}_j - z_j)^T, (\tilde{y}_j - y_j)^T\big]^T \leq c_j^2\Big\},$$

in which Λ_j^{-1} and $\Lambda_j^{'-1}$ are the inverse of the variance-covariance matrix of $(\tilde{x}_j, \tilde{z}_j)$ and $(\tilde{z}_j, \tilde{y}_j)$, respectively. Moreover, c_j: $j = 1, \ldots, n$ is determined by $\mathbb{P}(\chi^2_{M+D} \leq c_j^2) = 1 - \alpha_j$, and χ^2_{M+D} is the Chi-square random variable with $M + D$ degrees of freedom.

Olesen and Petersen[30] clarified random realizations of DMU_j that fall within the confidence region $D_j(1 - \alpha_j)$, positioned inside the PPS if $\alpha \leq 0.5$ [37]. Therefore, $SPPS_{stage1}$ and $SPPS_{stage2}$ are envelopment of n confidence regions $D_j(1 - \alpha_j)$ and $D'_j(1 - \alpha_j)$ for $j = 1, , n$, respectively [30]. Then, the stochastic production possibility set of the stochastic centralized network DEA Model (4) can be represented as

$$\begin{aligned} SPPS_{SCN} &= \big\{(\tilde{x}, \tilde{y}) | \exists \tilde{z} \in D_j(1-\alpha_j) \cap D'_j(1-\alpha) : (\tilde{x}, \tilde{z}) \in SPPS_{stage1}, \\ &\qquad (\tilde{z}, \tilde{y}) \in SPPS_{stage2}\big\} \\ &= \big\{(\tilde{x}, \tilde{y}) | \exists \tilde{z} \in D_j(1-\alpha_j) \cap D'_j(1-\alpha) : \Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_j \leq \tilde{x}, \\ &\qquad \Sigma_{j=1}^n (\gamma_j)\tilde{z}_j \geq \Sigma_{j=1}^n \xi_j \tilde{z}_j, \Sigma_{j=1}^n (\lambda_j + \xi_j)\tilde{y}_j \geq \tilde{y}, \quad \lambda_j, \gamma_j, \xi_j \geq 0\big\} \end{aligned}$$

As it can be seen, $SPPS_{SCN}$ is equivalent to the stochastic production possibility set under centralized control organization mechanism that proposed by [39]. Programme (4) calculates the stochastic efficiency of the whole system according to the Farrell radial measure.

3.1. Transformation to deterministic equivalent linear models

Programme (4) is very general and intended mainly for conceptual interpretation. It can also provide guidance for the more specialized developments that we now undertake to achieve "deterministic equivalents" for computation and implementation in applicable circumstances [10]. To evaluate the performance of DMU_o by using programme (4), one can transform the chance-constraint programming problem to a deterministic form, as discussed by Cooper et al. [10]. To this end, consider the ith input constraint in stochastic programming problem (4) as follows:

$$\mathbb{P}\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij} - \theta \tilde{x}_{io} \leq 0\Big) \geq 1 - \alpha. \tag{5}$$

By introducing the slack variable $\epsilon_i \geq 0$, inequality (5) is transformed into the following equality form:

$$\mathbb{P}\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij} - \theta \tilde{x}_{io} \leq 0\Big) = 1 - \alpha + \epsilon_i. \tag{6}$$

Note 1. Suppose X is a random variable and a, b and c are constants. If $a \geq b$ and $\mathbb{P}(X \leq a) = c$, then $\mathbb{P}(X \leq b) = d$, in which $d \leq c$.

Taking Note 1 in to consideration, and by introducing $s_i^- \geq 0$, we have:

$$\mathbb{P}\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij} - \theta \tilde{x}_{io} \leq -S_i^-\Big) = 1 - \alpha. \tag{7}$$

If we consider $\tilde{h}_i = \Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij} - \theta \tilde{x}_{io}$, then $\tilde{h}_i \sim N\Big(h_i, \big(\sigma_i^I(\lambda,\gamma,\theta)\big)^2\Big)$, where,

$$h_i = \mathbb{E}(\tilde{h}_i) = \mathbb{E}\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij} - \theta \tilde{x}_{io}\Big) = \Sigma_{j=1}^n (\lambda_j + \gamma_j)x_{ij} - \theta x_{io}$$

and

$$\begin{aligned}\big(\sigma_i^I(\lambda,\gamma,\theta)\big)^2 &= Var(\tilde{h}_i) = Var\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij} - \theta \tilde{x}_{io}\Big)\\ &= Var\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij}\Big) + Var(\theta\tilde{x}_{io}) - 2Cov\Big(\Sigma_{j=1}^n (\lambda_j + \gamma_j)\tilde{x}_{ij}, \theta\tilde{x}_{io}\Big)\\ &= \Sigma_{j=1}^n \Sigma_{k=1}^n (\lambda_j + \gamma_j)(\lambda_k + \gamma_k)Cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + \theta^2 Var(\tilde{x}_{io})\\ &\quad -2\theta\Sigma_{j=1}^n (\lambda_j + \gamma_j)Cov(\tilde{x}_{ij}, \tilde{x}_{io}\Big).\end{aligned}$$

Now, considering (6), we have

$$\mathbb{P}(\tilde{h}_i \le -s_i^-) = 1-\alpha \Rightarrow \mathbb{P}\left(\frac{\tilde{h}_i - h_i}{\sigma_i^I(\lambda,\gamma,\theta)} \le \frac{-s_i^- - h_i}{\sigma_i^I(\lambda,\gamma,\theta)}\right) = 1-\alpha$$

where $\tilde{Z}_i = \frac{\tilde{h}_i - h_i}{\sigma_i^I(\lambda,\gamma,\theta)} \sim N(0,1)$. Therefore,

$$\mathbb{P}\left(\tilde{Z}_i \le \frac{-s_i^- - h_i}{\sigma_i^I(\lambda,\gamma,\theta)}\right) = 1-\alpha \Rightarrow \mathbb{P}\left(\tilde{Z}_i \le \frac{s_i^- + h_i}{\sigma_i^I(\lambda,\gamma,\theta)}\right) = \alpha \Rightarrow \Phi\left(\frac{s_i^- + h_i}{\sigma_i^I(\lambda,\gamma,\theta)}\right) = \alpha$$

where $\Phi(.)$ is standard normal distribution function. Clearly, based on the invertibility of $\Phi(.)$, we have

$$\begin{aligned}\frac{s_i^- + h_i}{\sigma_i^I(\lambda,\gamma,\theta)} &= \Phi^{-1}(\alpha) \Rightarrow h_i + s_i^- - \Phi^{-1}(\alpha)\sigma_i^I(\lambda,\gamma,\theta) = 0\\ &\Rightarrow \Sigma_{j=1}^n (\lambda_j + \gamma_j)x_{ij} + s_i^- - \Phi^{-1}(\alpha)\sigma_i^I(\lambda,\gamma,\theta) = \theta x_{io}.\end{aligned}$$

Similarly, for outputs constraint, we have

$$\Sigma_{j=1}^n (\lambda_j + \xi_j)y_{rj} - s_i^+ + \Phi^{-1}(\alpha)\sigma_r^o(\lambda,\xi) = y_{ro}$$

in which

$$\begin{aligned}\big(\sigma_r^o(\lambda,\xi)\big)^2 = \Sigma_{j=1}^n \Sigma_{k=1}^n (\lambda_j + \xi_j)(\lambda_k + \xi_k)Cov(\tilde{y}_{rj}, \tilde{y}_{rk}) + Var(\tilde{y}_{ro})\\ -2\Sigma_{j=1}^n (\lambda_j + \xi_j)Cov(\tilde{y}_{rj}, \tilde{y}_{ro}).\end{aligned}$$

Moreover, for the third constraint, we have

$$\Sigma_{j=1}^n (\xi_j - \gamma_j)z_{dj} + s_d^- - \Phi^{-1}(\alpha)\sigma_d^z(\xi,\gamma) = 0$$

in which

$$\big(\sigma_d^z(\xi,\gamma)\big)^2 = Var\Big(\Sigma_{j=1}^n (\xi_j - \gamma_j)z_{dj}\Big) = \Sigma_{j=1}^n \Sigma_{k=1}^n (\xi_j - \gamma_j)(\xi_k - \gamma_k)Cov(\tilde{z}_{dj}, \tilde{z}_{dk}).$$

Consequently, programme (4) is transformed into the following quadratic form:

$$
\begin{aligned}
e_{SCN}^{(\alpha)} = & Min\ \theta \\
& s.t. \\
& \Sigma_{j=1}^{n}(\lambda_j + \gamma_j)x_{ij} + s_i^- - \Phi^{-1}(\alpha)v_i = \theta x_{io}, \qquad i = 1, \ldots, M, \\
& \Sigma_{j=1}^{n}(\lambda_j + \xi_j)y_{rj} - s_i^+ + \Phi^{-1}(\alpha)u_r = y_{ro}, \qquad r = 1, \ldots, S, \\
& \Sigma_{j=1}^{n}(\xi_j - \gamma_j)z_{dj} + s_d^- - \Phi^{-1}(\alpha)w_d = 0, \qquad d = 1, \ldots, D, \\
& v_i^2 = \Sigma_{j=1}^{n}\Sigma_{k=1}^{n}(\lambda_j + \gamma_j)(\lambda_k + \gamma_k)Cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + \theta^2 Var(\tilde{x}_{io}) \\
& - 2\theta\Sigma_{j=1}^{n}(\lambda_j + \gamma_j)Cov(\tilde{x}_{ij}, \tilde{x}_{io})\Big), \qquad i = 1, \ldots, M, \\
& u_r^2 = \Sigma_{j=1}^{n}\Sigma_{k=1}^{n}(\lambda_j + \xi_j)(\lambda_k + \xi_k)Cov(\tilde{y}_{rj}, \tilde{y}_{rk}) + Var(\tilde{y}_{ro}) \\
& - 2\Sigma_{j=1}^{n}(\lambda_j + \xi_j)Cov(\tilde{y}_{rj}, \tilde{y}_{ro})\Big), \qquad r = 1, \ldots, S, \\
& w_z^2 = \Sigma_{j=1}^{n}\Sigma_{k=1}^{n}(\xi_j - \gamma_j)(\xi_k - \gamma_k)Cov(\tilde{z}_{dj}, \tilde{z}_{dk}), \quad d = 1, \ldots, D, \\
& \lambda_j, \gamma_j, \xi_j \geq 0, (\forall j).
\end{aligned} \tag{8}
$$

Theorem 3.2. The quadratic programming problem (8) is feasible, under all confidence levels α.

Proof. It is easy to show that $theta = 1$, $\lambda_o + \gamma_o = 1$, $\lambda_o + \xi_o = 1$, $\lambda_j + \gamma_j = 0$, $\lambda_j + \xi_j = 0$, $\gamma_j - \xi_j = 0$ for all $j \neq o$ and $v = w = u = 0$ is a feasible solution to programme (8). □

Proposition 3.3. Let $\alpha \leq 0.5$. Then $0 \leq e_{SCN}^{(\alpha)} \leq 1$.

Proof. Considering the feasible solution given in Theorem 3.2, and due to the minimization of programme (8), we conclude that $e_{SCN}^{(\alpha)} \leq 1$. Moreover, since $\alpha \leq 0.5$ then $-\Phi^{-1}(\alpha) \geq 0$ and $v \geq 0$. Taking the input constraints of programme (8) into consideration, we have:

$$\Sigma_{j=1}^{n}(\lambda_j + \gamma_j)x_{ij} \leq \theta x_{io} \Rightarrow \frac{\Sigma_{j=1}^{n}(\lambda_j + \gamma_j)x_{ij}}{x_{io}} \leq \theta,$$

hence, $e_{SCN}^{(\alpha)} > 0$. □

Theorem 3.4. If $\alpha = 0.5$, then the results of programmes (8) and (3) are the same.

Proof. Since $\Phi^{-1}(\alpha) = 0$, the proof is complete. □

Programme (8) is a nonlinear programming problem. In the following section, we use the error term structure to convert it in to a linear programming problem. In this sense, the computational efforts will be reduced substantially.

3.2. Error structure

In this subsection, we employ the linearization approach of Cooper et al. [9] regarding the error structure of data to transform chance-constraint problem to an equivalent linear deterministic form. Toward this end, we suppose the inputs, intermediate measures and final outputs of the DMUs are designated by single factors. Sharpe [33], Kahane [19] and Huang and Li [17] applied the assumption of a single underlying factor for several times in economics.

Suppose that the inputs, intermediate measures and final outputs of the jth DMU are as follows:

$$\begin{aligned} \tilde{x}_{ij} &= x_{ij} + a_{ij}\epsilon_{ij}, \quad && i = 1,\dots,M,\\ \tilde{z}_{dj} &= z_{dj} + c_{dj}\tau_{dj}, \quad && d = 1,\dots,D,\\ \tilde{y}_{rj} &= y_{rj} + b_{rj}\eta_{rj}, \quad && r = 1,\dots,S, \end{aligned}$$

where a_{ij}, b_{rj} and c_{dj} are non-negative values. Moreover, ϵ_{ij}, τ_{dj} and η_{rj} are independent normal random variables, such that $\epsilon_{ij} \sim N(0, \bar{\sigma}^2)$, $\eta_{rj} \sim N(0, \bar{\sigma}^2)$ and $\tau_{dj} \sim N(0, \bar{\sigma}^2)$. Then

$$\begin{aligned} \tilde{x}_{ij} &\sim N(x_{ij}, \bar{\sigma}^2 a_{ij}^2), \quad && i = 1,\dots,M,\\ \tilde{z}_{dj} &\sim N(z_{dj}, \bar{\sigma}^2 c_{dj}^2), \quad && d = 1,\dots,D,\\ \tilde{y}_{rj} &\sim N(y_{rj}, \bar{\sigma}^2 b_{rj}^2), \quad && r = 1,\dots,S. \end{aligned}$$

Now, suppose for each $j = 1,\dots,n$, we have $\epsilon_i = \epsilon_{ij}$, $\eta_r = \eta_{rj}$ and $\tau_d = \tau_{dj}$. Thus, programme (4) is transformed into the following form:

$$\begin{aligned} e_{SCN}^{(\alpha)} =& Min\ \theta \qquad (9)\\ & s.t.\\ & \Sigma_{j=1}^{n}(\lambda_j+\gamma_j)x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}|\Sigma_{j=1}^{n}(\lambda_j+\gamma_j)a_{ij} - \theta a_{io}| \le \theta x_{io}, \quad i = 1,\dots,M,\\ & \Sigma_{j=1}^{n}(\lambda_j+\xi_j)y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma}|\Sigma_{j=1}^{n}(\lambda_j+\xi_j)b_{rj} - b_{ro}| \ge y_{ro}, \quad r = 1,\dots,S,\\ & \Sigma_{j=1}^{n}(\xi_j-\gamma_j)z_{dj} - \Phi^{-1}(\alpha)\bar{\sigma}|\Sigma_{j=1}^{n}(\xi_j-\gamma_j)c_{dj}| \le 0, \quad d = 1,\dots,D,\\ & \lambda_j, \gamma_j, \xi_j \ge 0,\ (\forall j). \end{aligned}$$

Due to the existence of the absolute function, the mathematical programming problem (9) is non-linear. There are two different approaches to linearization of this programme. The first method is based on the use of the properties of the absolute function and the second one is based on the goal programming theory developed by Charnes and Cooper [2, 4]. In the last technique, problem (9) is transformed into a quadratic programming problem. Herein, we apply the second approach. To do this, we use the following transformations:

$$
\begin{aligned}
&|\Sigma_{j=1}^{n}(\lambda_j+\gamma_j)a_{ij}-\theta a_{io}| = p_i^+ + p_i^-, && i=1,\dots,M,\\
&\Sigma_{j=1}^{n}(\lambda_j+\gamma_j)a_{ij}-\theta a_{io} = p_i^+ - p_i^-, && i=1,\dots,M,\\
&p_i^+ p_i^- = 0, && i=1,\dots,M,\\
&|\Sigma_{j=1}^{n}(\lambda_j+\xi_j)b_{rj}-b_{ro}| = q_r^+ + q_r^-, && r=1,\dots,S,\\
&\Sigma_{j=1}^{n}(\lambda_j+\xi_j)b_{rj}-b_{ro} = q_r^+ - q_r^-, && r=1,\dots,S,\\
&q_r^+ q_r^- = 0, && r=1,\dots,S,\\
&|\Sigma_{j=1}^{n}(\xi_j-\gamma_j)c_{dj}| = \delta_d^+ + \delta_d^-, && d=1,\dots,D,\\
&\Sigma_{j=1}^{n}(\xi_j-\gamma_j)c_{dj} = \delta_d^+ - \delta_d^-, && d=1,\dots,D,\\
&\delta_d^+ \delta_d^- = 0, && d=1,\dots,D.
\end{aligned}
$$

By replacing the above transformations, problem (9) is converted into the following form:

$$
\begin{aligned}
e_{SCN}^{(\alpha)} =& Min\ \theta \qquad\qquad (10)\\
&s.t.\\
&\Sigma_{j=1}^{n}(\lambda_j+\gamma_j)x_{ij}-\Phi^{-1}(\alpha)\bar{\sigma}(p_i^+ + p_i^-) \ \leq \theta x_{io}, && i=1,\dots,M,\\
&\Sigma_{j=1}^{n}(\lambda_j+\gamma_j)a_{ij}-\theta a_{io} = p_i^+ - p_i^-, && i=1,\dots,M,\\
&\Sigma_{j=1}^{n}(\lambda_j+\xi_j)y_{rj}-s_i^+ + \Phi^{-1}(\alpha)\bar{\sigma}(q_r^+ + q_r^-) \geq y_{ro}, && r=1,\dots,S,\\
&\Sigma_{j=1}^{n}(\lambda_j+\xi_j)b_{rj}-b_{ro} = q_r^+ - q_r^-, && r=1,\dots,S,\\
&\Sigma_{j=1}^{n}(\xi_j-\gamma_j)z_{dj}+s_d^- - \Phi^{-1}(\alpha)\bar{\sigma}(\delta_d^+ - \delta_d^-) \leq 0, && d=1,\dots,D,\\
&\Sigma_{j=1}^{n}(\xi_j-\gamma_j)c_{dj} = \delta_d^+ - \delta_d^-, && d=1,\dots,D,\\
&p_i^+ p_i^- = 0, && i=1,\dots,M,\\
&q_r^+ q_r^- = 0, && r=1,\dots,S,\\
&\delta_d^+ \delta_d^- = 0, && d=1,\dots,D,\\
&\lambda_j,\gamma_j,\xi_j,p_i^+,p_i^-,q_r^+,q_r^-,\delta_d^+,\delta_d^- \geq 0,\ \ (\forall j).
\end{aligned}
$$

Due to the existence of the constraints $p_i^+ p_i^- = 0, q_r^+ q_r^- = 0$ and $\delta_d^+ \delta_d^- = 0$, problem (10) is still a non-linear programming problem. However, as we know, if a linear programming problem has an optimal solution, then, it has an extreme optimal solution and this means that at least one of the variables p_j^+ or p_j^- and q_r^+ or q_r^- and δ_d^+ or δ_d^- are zero. Consequently, if we use the simplex algorithm to solve this linear programming problem, we can find an extreme optimal solution to this nonlinear programming problem and we can avoid the nonlinear constraints $p_i^+ p_i^- = 0$, $q_r^+ q_r^- = 0$ and $\delta_d^+ \delta_d^- = 0$.

The following theorem shows that the efficiency score $e_{SCN}^{(\alpha)}$ is monotone decreasing in α. This means that the error term α plays important role in determining efficiency scores. It states that as the error term increases, the efficiency score decreases.

Theorem 3.5. For each $\alpha' < \alpha < 0.5$ we have $e_{SCN}^{(\alpha')} \geq e_{SCN}^{(\alpha)}$.

Proof. Suppose $(\theta^*, \lambda^*, \gamma^*, \xi^*, p^{+*}, p^{-*}, q^{+*}, q^{-*}, \delta^{+*}, \delta^{-*})$ is an optimal solution to programme (10) when DMU_o is under evaluation at confidence level α. Since $\Phi^{-1}(\alpha)$ is a strictly increasing function, then $\Phi^{-1}(\alpha') < \Phi^{-1}(\alpha)$ and we have

$$\Sigma_{j=1}^n(\lambda_j^* + \gamma_j^*)x_{ij} - \Phi^{-1}(\alpha')\bar{\sigma}(p_i^{+*} + p_i^{-*}) \leq \Sigma_{j=1}^n(\lambda_j^* + \gamma_j^*)x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}(p_i^{+*} + p_i^{-*}) \leq \theta^* x_{io},$$

$$\Sigma_{j=1}^n(\lambda_j^* + \xi_j^*)y_{rj} + \Phi^{-1}(\alpha')\bar{\sigma}(q_r^{+*} + q_r^{-*}) \geq \Sigma_{j=1}^n(\lambda_j^* + \xi_j^*)y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma}(q_r^{+*} + q_r^{-*}) \geq y_{ro},$$

$$\Sigma_{j=1}^n(\xi_j^* - \gamma_j^*)z_{dj} - \Phi^{-1}(\alpha')\bar{\sigma}(\delta_d^{+*} + \delta_d^{-*}) \leq \Sigma_{j=1}^n(\xi_j^* - \gamma_j^*)z_{dj} - \Phi^{-1}(\alpha)\bar{\sigma}(\delta_d^{+*} + \delta_d^{-*}) \leq 0.$$

Hence $(\theta^*, \lambda^*, \gamma^*, \xi^*, p^{+*}, p^{-*}, q^{+*}, q^{-*}, \delta^{+*}, \delta^{-*})$ is a feasible solution to programme (10) when DMU_o is under evaluation at confidence level α'. Due to the minimization of the programme, the proof is completed. □

Corollary 3.6. If DMU_o is efficient under the confidence level α, then it is efficient under the level $\alpha' < \alpha$. Also, at confidence level α', if DMU_o is inefficient, then it is inefficient at each confidence level $\alpha > \alpha'$.

Note that by considering a fixed predetermined value α, the whole system is stochastically efficient if both sub-processes are stochastically efficient. However, the converse is not true in the sense that the sub-processes may be stochastically efficient, while the whole system is inefficient. Theorem 3.7 states this issue.

Theorem 3.7. Let $e_1^{(\alpha)}$ and $e_2^{(\alpha)}$ are the stochastic efficiency scores of the first and second stages in $SPPS_{stage1}$ and $SPPS_{stage2}$, respectively. Then $e_{SCN}^{(\alpha)} \leq e_1^{(\alpha)}$ and $e_{SCN}^{(\alpha)} \leq e_2^{(\alpha)}$.

Proof. Suppose $(\lambda_1^{(1)*} + \gamma_1^{(1)*}, \lambda_2^{(1)*} + \gamma_2^{(1)*}, \ldots, \lambda_n^{(1)*} + \gamma_n^{(1)*}, e_1^{(\alpha)})$ is an optimal solution of the stochastic input-oriented model of the first stage. Let $\lambda_j + \gamma_j = \lambda_j^{(1)*} + \gamma_j^{(1)*}, \lambda_j + \xi_j = \lambda_j^{(1)*} + \gamma_j^{(1)*}$ and $\gamma_j - \xi_j = 0$ for all $j = 1, \ldots, n$. As we can see, $(\lambda_1 + \gamma_1, , \lambda_n + \gamma_n, \lambda_1 + \xi_j, \ldots, \lambda_n + \xi_n, 0, , 0, e_1^{(\alpha)})$ is a feasible solution to programme (4). Suppose the optimal value of programme (4) is $e_{SCN}^{(\alpha)}$. Hence $e_{SCN}^{(\alpha)} \leq e_1^{(\alpha)}$. The proof of the second part is similar. □

It is easy to show that if $e_1^{(\alpha)}$ and $e_2^{(\alpha)}$ are stochastic efficiency scores of the first and second stages in $SPPS_{stage1}$ and $SPPS_{stage2}$, respectively, then $e_{SCN}^{(\alpha)} \leq \frac{(e_1^{(\alpha)} + e_2^{(\alpha)}}{2}$. Moreover, for any predetermined confidence level α, if the whole process is stochastically efficient under the stochastic centralized model, then each sub-process is stochastically efficient.

4. AN EMPIRICAL EXAMPLE

After formulating our theoretical framework, we use a real case on bank branches to illustrate the real applicability of the approach. One of the most frequently studied application of DEA is performance analysis in commercial banks (Seiford, & Zhu[34], Yang et al. [38], Paradi and Zhu [31], Zhou at al. [39]). Although, many DEA-based applications have focused on performance analysis in banking sector, however, most of these studies in this field have used deterministic data. Here, we employ the stochastic dataset on 16 commercial banks taken from [39].

The data set consists of 16 commercial banks in China taken from [39]. The first stage uses Employees, Fixed Assets and Expenses as inputs to produce Deposits and Interbank Deposits as outputs. Then, by consuming the outputs of the first stage as inputs to the second stage, Loan and Profit are produced as final outputs. Furthermore, all inputs and outputs are supposed to follow a normal distribution. As [39] reported, Table 1 (Fig. 2) provides the approximated mean values and standard deviations from annual reports and internal databases for the 16 banks, the ten years period from 2000 to 2010 from the Almanac of China's Finance and Banking.

Table 1. Estimated mean values and standard deviations for 16 banks

DMU	Employees (10^3)	Fixed Assets (RMB 10^8)	Expenses (RMB 10^8)	Deposits (RMB 10^8)	Interbank Deposits (RMB 10^8)	Loan (RMB 10^8)	Profit (RMB 10^8)
1	(21.39,5.39)	(8.65,0.92)	(13.24,3.74)	(935.41,310.39)	(108.61,103.81)	(651.35,259.78)	(13.32,4.97)
2	(249.28,13.38)	(90.05,12.14)	(66.07,9.17)	(4925.35,1058.49)	(603.39,320.35)	(3189.65,1076.21)	(65.07,17.79)
3	(441.88,5.40)	(104.29,23.62)	(94.26,5.81)	(5989.71,1139.97)	(289.77,170.88)	(3014.98,451.41)	(51.45,29.15)
4	(49.29,0.57)	(9.91,0.06)	(8.54,1.07)	(183.29,53.73)	(119.83,56.46)	(1206.90,244.25)	(1.63,0.77)
5	(19.85,3.50)	(3.98,0.53)	(14.80,4.15)	(785.79,238.77)	(120.24,37.66)	(646.48,181.90)	(7.89,3.47)
6	(300.30,1.75)	(64.48,9.76)	(99.19,16.94)	(6375.92,1462.90)	(447.46,230.58)	(3683.58,821.06)	(92.64,26.62)
7	(16.99,2.54)	(8.18,2.66)	(8.30,1.86)	(523.90,115.67)	(121.69,78.56)	(450.52,128.52)	(7.32,2.28)
8	(385.61,17.29)	(80.30,12.33)	(91.51,15.73)	(8223.45,1558.40)	(592.61,236.65)	(4436.01,884.11)	(111.15,35.27)
9	(36.92,7.74)	(11.68,2.66)	(20.34,6.08)	(1250.65,367.00)	(115.79,67.66)	(852.75,269.32)	(20.95,5.99)
10	(19.54,5.89)	(3.39,0.30)	(10.35,2.84)	(558.34,80.84)	(182.91,54.44)	(489.99,161.84)	(11.39,4.12)
11	(10.38,1.63)	(1.68,0.19)	(5.22,1.32)	(254.97,50.46)	(36.06,24.25)	(281.71,78.97)	(0.61,1.95)
12	(17.60,4.39)	(6.26,0.70)	(12.68,2.74)	(805.30,269.91)	(222.44,98.13)	(681.27,147.24)	(12.52,4.96)
13	(77.73,8.60)	(22.74,2.07)	(23.56,2.65)	(1674.17,337.22)	(483.08,178.37)	(1298.78,386.33)	(28.52,8.23)
14	(11.11,1.86)	(3.64,0.20)	(7.29,1.62)	(394.35,72.21)	(87.01,29.12)	(345.67,70.81)	(3.07,1.02)
15	(6.22,1.05)	(12.44,7.93)	(5.50,2.05)	(242.99,108.07)	(115.45,78.89)	(2840.68,734.78)	(20.76,4.79)
16	(14.19,1.04)	(3.29,0.21)	(6.53,1.18)	(337.37,65.30)	(51.05,9.38)	(298.34,71.92)	(2.78,1.81)

* For (a, b), a is the mean value, and b is the standard deviation.

Fig. 2. Table of estimated mean values and standard deviations for the 16 banks from 2000 to 2010

The efficiency scores of overall processes under model (10), at different confidence levels $\alpha = 0.1, 0.2, 0.5$, are shown in Table 2 (Fig. 3). As the results show, DMU15 have the best performance and it is stochastically efficient under all confidence levels. Moreover, DMUs 11, 14 and 16 perform the worst under every confidence level. The last column in Table 2 represents the stochastic efficiency scores at $\alpha = 0.5$, which are equivalent to the deterministic centralized model (3) with only input and output means. The obtained results under different levels, one can see that choosing a confidence level plays an important role in evaluating performance. For a better understanding and explanation of the results, Figure 4 depicts graphically the confidence levels provided in

Table 2 (Fig. 3). As we should expect, by increasing the confidence level α, the efficiency score decreases.

Table 2. Efficiency scores from stochastic centralized model ($\boldsymbol{e_{SCN}}$)

DMU	Confidence levels				
	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
1	0.444	0.399	0.366	0.334	0.304
2	0.329	0.312	0.297	0.280	0.262
3	0.254	0.220	0.194	0.169	0.145
4	0.392	0.353	0.327	0.303	0.279
5	0.370	0.326	0.290	0.256	0.222
6	0.349	0.331	0.313	0.294	0.268
7	0.312	0.297	0.284	0.268	0.251
8	0.441	0.416	0.392	0.364	0.330
9	0.433	0.394	0.366	0.340	0.310
10	0.513	0.463	0.422	0.383	0.342
11	0.202	0.166	0.152	0.139	0.126
12	0.413	0.384	0.359	0.334	0.307
13	0.421	0.395	0.372	0.348	0.323
14	0.172	0.158	0.149	0.140	0.130
15	1.000	1.000	1.000	1.000	1.000
16	0.225	0.197	0.174	0.152	0.127

Fig. 3. Table of efficiency scores from stochastic centralized model.

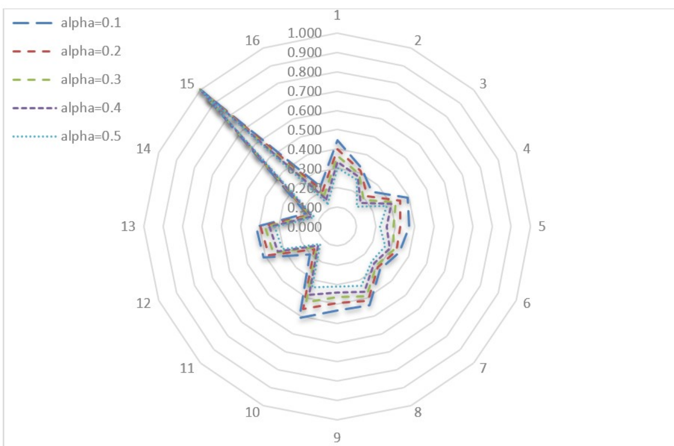

Fig. 4. The efficiency scores at different confidence levels.

Table 3 (Fig. 5) represents the stochastic efficiencies of the first stage (e_1), second stage (e_2) and the mean of e_1 and e_2 under different confidence levels. Looking at

Tables 2 and 3, we see that the stochastic centralized efficiency of each DMU ($e_{SCN}^{(\alpha)}$) at every confidence level, is less than or equal to the mean of e_1 and e_2, as we expected. For example, DMU15 is stochastically efficient under the centralized model that implies both stages are stochastic efficient. However, the converse is not true. For DMUs 1 and 8, at confidence level $\alpha = 0.1$, both stages are stochastically efficient. But, the overall systems of these two DMUs are not stochastically efficient. It should be pointed out that in deterministic centralized model, based on the definitions of the efficiency of the overall system and efficiencies of the stages, we should expect that the efficiencies of the stages must guarantee the efficiency of the overall system. However, in stochastic case, this is not necessarily true.

Table 3. Efficiency scores of the stages and mean efficiency of two stages

DMU	Confidence Level														
	0.1			0.2			0.3			0.4			0.5		
	e_1	e_2	e_{mean}	e_1	e_2	e_{mean}	e_1	e_2	e_{mean}	e_1	e_2	e_{mean}	e_1	e_2	e_{mean}
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.907	0.954	1.000	0.721	0.860	1.000	0.631	0.815
2	0.932	0.589	0.761	0.920	0.584	0.752	0.910	0.576	0.743	0.901	0.567	0.734	0.890	0.558	0.724
3	0.800	1.000	0.900	0.770	1.000	0.885	0.747	1.000	0.874	0.727	0.952	0.839	0.707	0.883	0.795
4	0.739	0.712	0.726	0.721	0.658	0.690	0.708	0.623	0.666	0.696	0.592	0.644	0.682	0.563	0.623
5	1.000	0.509	0.754	1.000	0.461	0.730	1.000	0.422	0.711	1.000	0.385	0.692	1.000	0.346	0.673
6	0.893	1.000	0.946	0.875	1.000	0.937	0.862	1.000	0.931	0.854	1.000	0.927	0.848	1.000	0.924
7	1.000	0.352	0.676	1.000	0.345	0.672	0.982	0.338	0.660	0.948	0.332	0.640	0.919	0.326	0.622
8	1.000	1.000	1.000	1.000	0.992	0.996	1.000	0.965	0.982	1.000	0.939	0.969	1.000	0.910	0.955
9	1.000	0.934	0.967	0.963	0.930	0.946	0.940	0.928	0.934	0.921	0.917	0.919	0.899	0.908	0.904
10	1.000	0.513	0.756	1.000	0.463	0.732	1.000	0.422	0.711	1.000	0.383	0.691	1.000	0.342	0.671
11	0.910	0.355	0.633	0.905	0.324	0.615	0.902	0.322	0.612	0.894	0.320	0.607	0.868	0.318	0.593
12	1.000	0.413	0.707	1.000	0.384	0.692	1.000	0.359	0.679	1.000	0.334	0.667	1.000	0.307	0.654
13	1.000	0.421	0.710	1.000	0.395	0.698	1.000	0.372	0.686	1.000	0.348	0.674	1.000	0.323	0.661
14	1.000	0.262	0.631	1.000	0.242	0.621	0.930	0.225	0.577	0.881	0.208	0.545	0.843	0.190	0.517
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
16	0.889	0.540	0.714	0.872	0.466	0.669	0.836	0.404	0.620	0.805	0.348	0.577	0.778	0.291	0.534

Fig. 5. Table of efficiency scores of the stages and mean efficiency of two stages.

A comparison between the results in deterministic case (when $\alpha = 0.5$) with the results in stochastic case provides interesting insight. In overall sense, it is obvious that the stochastic efficiency score for each bank is changed under different confidence levels. Moreover, as the results show, it is obvious that the efficiency scores of the DMUs under the probability level $\alpha = 0.1$ is higher than the efficiency scores in comparison with other probability levels. This is due to the fact that α is a pre-determined confidence level and it forms an ellipse around the mean value of the inputs and outputs of each firm. As a result, a firm might experience a change in its efficiency score if α is changed. Moreover, Table 5 in [40] listed the corresponding products of stochastic CCR efficiencies of the two stages. Comparing these stochastic efficiency scores with the results obtained from our proposed approach, we see that the products of stochastic CCR efficiency scores of the two stages are greater than or equal to those from stochastic centralized DEA models in Table 2, which means that products of stochastic CCR model may overestimate the efficiencies of the whole system. Again, Table 5 in [40] shows that DMU1 is stochastically efficient under levels $\alpha = 0.1, 0.2, 0.3$ and DMU8 is stochastically efficient under levels $\alpha = 0.1, 0.2$. However, in our proposed approach, only one unit (DMU15) is stochastically efficient in different confidence levels.

5. CONCLUSIONS

Performance evaluation of decision-making units with two-stage structures and stochastic data are studied in this paper. Toward this end, we have presented a cooperative model to calculate the relative performance of a two-stage network system when the input/output data are stochastic. Then, the chance-constrained model was converted to a deterministic equivalent form. In addition, by using techniques of stochastic problems linearization and assumption of the single underlying factor of components of inputs, intermediate measures, and final outputs, the linear form was obtained. The relationship between the stochastic efficiency of each stage and mean of their efficiencies and stochastic centralized efficiency of the whole process, at different confidence levels, was discussed. The results indicated that the efficiency score of the centralized model is less than or equal to the mean efficiencies of the stages. At the end of this paper, a real case on 16 commercial banks in China was discussed to illustrate the applicability of the proposed approach.

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