KYBERNETIKA — VOLUME 57 (2021), NUMBER 3, PAGES 513–533

EVENT-TRIGGERED H_∞ STATIC OUTPUT FEEDBACK CONTROL OF DISCRETE TIME PIECEWISE-AFFINE SYSTEMS

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This paper is concerned with the problem of H_∞ event-triggered output feedback control of discrete time piecewise-affine systems. Relying on system outputs, a piecewise-affine triggering condition is constructed to release communication burden. Resorting to piecewise Lyapunov functional and robust control techniques, sufficient conditions are built to ensure the closed-loop systems to be asymptotically stable with the prescribed H_∞ performance. By utilizing a separation strategy, the static output feedback controller is solved by means of linear matrix inequalities. The validity of the proposed method are demonstrated by numerical examples.

Keywords: event-triggered control, output feedback control, linear matrix inequality

Classification: 93D09, 65Z99

1. INTRODUCTION

As a special class of switched systems, the piecewise-affine (PWA) paradigm has been widely applied in computer vision, systems biology, electromechanical and automotive systems since it has the ability of describing the hybrid and nonlinear phenomena that are frequent in practical situations[1]. For example, this paradigm is employed by [20] to approximate the nonlinearity of tire force and also utilized by [25] to describe the nonlinear characteristics embedded in hydraulic wind power transfer systems. Regarding to the theory study on PWAs, fruitful results have been obtained. To be specific, a so-called S-arbitrary switching approach is developed in [35] to discuss the stability and stabilization of PWA systems via a relaxed piecewise quadratic Lyapunov function technique. By prescribed the admissible control performance, [22] proposes a PWA controller design method to treat the fault recoverability evaluation problem via the the piecewise linear quadratic control performance bound. Combining recursive multi-model least-squares and linear multi-category discrimination techniques, [5] gives a two-stage PWA regression for nonlinear systems to overcome the deficiencies of simple model and over-parametrized model. In [14], the stability of uncertain PWA systems is tested by two manners, namely, one is based on a linear matrix inequality with conservativeness and another is based on robust simulation. A discontinuous Lyapunov function ap-

DOI: 10.14736/kyb-2021-3-0513

proach is adopted by [11] to discuss the stability of planar PWA systems. [13] presents a cone-copositive piecewise quadratic Lyapunov function approach for the stability of continuous PWA systems via cone-copositivity.

Note that control signals in the afore mentioned results are continuously transmitted to actuators, which is also called the time-triggered approach. Actually, there is a tiny difference among transmissions when systems are running in normal mode. As a result, this approach could render the control channel be occupied by redundant transmissions, which may cause time-delay, packet dropout, and congestion. To alleviate the channel overload, the event-based triggering manner has stimulated the interesting of the researchers since the transmission is determined by the predefined condition [2, 9, 21]. Around this topic, fruit results have been reported in [10, 12, 15, 26, 34]. [12] employs perturbed linear and piecewise linear systems to study the even generators embeded between sensor-to-controller and the controller-to-actuator, respectively. An output-based triggering solution is given in [15] for discrete-time systems with Gaussian process and measurement noises. Moreover, this work also discusses the connections the existing absolute and relative threshold schemes. The event-triggered sliding control of discrete two-dimensional systems is provided in [32] via the constructed horizontal and vertical linear sliding surface.[26] treats a distributed optimal consensus under the event-triggered environment by means of a novel gradient-based algorithm and takes into account the global convergence performance and the Zeno phenomenon. In [34], two observer-based event triggering tracking strategies are proposed for a class of leader-follower multi-agent systems under the connected communication graph. Based on on distributed observers, [10] explores new dynamic triggering mechanisms for heterogeneous multiagent systems with nonuniform communication delays. However, only few are available to PWAs [16, 17, 18]. To be specific, in [16], the fault detection of a PWA T-S system with output saturation is set up in the event-triggered framework which is related to the fault. Unfortunately, an implicit assumption is that the concerned system should be stable. To relax this assumption, an event-triggered state feedback control of PWAs with guaranteed cost performance is discussed in [17] where a relative triggering manner is established on the control signal. This result is further extended by the same authors to the input saturation scenario [18]. It should be pointed out that these two results related to controller design are based on the availability of the system state. As it is known, getting the exact system state is costly or even impossible [3]. Although interesting results on output feedback for linear systems have been reported in [6, 19, 23, 30, 33], almost no output-based product has been established for the event-triggered PWAs, especially when the triggering parameter is co-designed with the output controller by means of linear matrix inequalities (LMIs).

Motivated by the above discussions, this paper is dedicated to the event-triggered H_∞ static output feedback control of PWA system. According to the property of PWAs, a piecewise static output feedback controller with an affine part is constructed firstly. Then, a relative triggering rule is fixed by the constructed controller to alleviate the unnecessary control signal transmission. With the help of the piecewise Lyapunov method, conditions for the closed-loop systems to be asymptotically stable with the prescribed H_∞ performance are built by the partition of system outputs. Resorting to Finsler lemma, an effective decoupling strategy is put forward to linearize the nonlinearity incurred by Lyapunov matrix, controller gain, system inputs and outputs. Owing to this treatment, the resultant output controller calculation containing the triggering threshold falls in the scope of LMIs. At last, two examples are provided to demonstrate the effectiveness of the proposed method.

The subsequent content is organized as follows: System descriptions, Definitions, and Lemmas are given in Section 2. Under the designed triggering condition discussed in Section 2, Section 3 covers two sufficient conditions on system stability analysis and controller synthesis with H_∞ performance, respectively. Two numerical examples are given in Section 4 to show the effectiveness of the proposed H_∞ approach. Some conclusions are given in Section 5.

2. SYSTEM DESCRIPTION AND PREPARATION

Consider an event-triggered discrete-time PWA system as

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k_l) + a_i + D_i \omega(k) \\ y(k) = C_{1i} x(k) \\ z(k) = C_{2i} x(k) + E_i u(k_l), y(k) \in \mathcal{Y}_i, \ i \in \wp \end{cases} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $y(k) \in \mathcal{R}^p$ means the system output, $\omega(k) \in \mathcal{R}^q$ belongs to $\mathcal{L}_2$ is the disturbance input, and $z(t) \in \mathcal{R}^r$ is the regulated output. $u(k_l) \in \mathcal{R}^s$ is the event-triggered control input, where $k_l (l = 1, 2, \dots)$ represent the event-triggered time sequence. The system matrix $A_i \in \mathcal{R}^{n\times n}$, $B_i \in \mathcal{R}^{n\times m}$, $C_{1i} \in \mathcal{R}^{p\times n}$, $C_{2i} \in \mathcal{R}^{r\times n}$, $D_i \in \mathcal{R}^{n\times 1}$, $E_i \in \mathcal{R}^{r\times m}$, $a_i \in \mathcal{R}^{n\times 1}$. $\mathcal{Y}_i \subseteq \mathcal{R}^p$ is the partition of the output space into a number of polyhedral regions. $\wp := \{1, 2, 3 \dots, M\}$ is the index set of these regions. If the output partition contains the origin of output space, the corresponding indices belong to $\wp_0$, otherwise, they belong to $\wp_1$. Therefore, $\wp = \wp_0 \cup \wp_1$.

Considering that the system output may transfer from one region to another, here let Ω represent the possible transitions of the output trajectories, that is,

$$\Omega = \{(i,j) | y(k) \in \mathcal{Y}_i, y(k+1) \in \mathcal{Y}_j, \ i,j \in \wp\}. \tag{2}$$

When the output of the system transmits from region $\mathcal{Y}_i$ to $\mathcal{Y}_j$ at kth instance, the dynamics of the system is determined by the dynamics of the local model of $\mathcal{Y}_i$.

As stated in [28], PWA described by ellipsoids could be often approximated by the polyhedral cells since it requires less parameters and can be handled by the LMIs solved by the existing Matlab toolbox. To introduce the ellipsoid, it is further assumed that there exist vector $Q_i \in R^{n\times 1}$ and scalar q_i meeting $\mathcal{Y}_i \subseteq S_i$ where

$$S_i = \{x | \|Q_i x + q_i\| \leqslant 1\}. \tag{3}$$

Specially, $\mathcal{Y}_i = \{y(k) | d_1^i < \theta_i y(k) < d_2^i\}$, where $\theta_i \in \mathcal{R}^{1\times p}$ is a scalar, d_1^i and d_2^i are the boundaries of the each subsystem. Then, Q_i, q_i are given by $Q_i = 2\theta_i C_{1i}/(d_2^i - d_1^i)$ and $q_i = -(d_2^i + d_1^i)/(d_2^i - d_1^i)$.

From (3), one has

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -Q_i^T Q_i & * \\ -q_i^T Q_i & 1 - {q_i}^2 \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} \geqslant 0, i \in \wp. \tag{4}$$

Now, the static output feedback controller with affine term for system (1) is designed as:

$$u(k) = K_i y(k) + m_i. \tag{5}$$

Where $K_i \in \mathcal{R}^{m \times p}$, $m_i \in \mathcal{R}^n$. With (5), select $e(k)$ to denote the difference between the last time triggered control signal $u(k_l)$ and the current time control signal $u(k)$, namely,

$$e(k) = u(k_l) - u(k). \tag{6}$$

Therefore, $u(k_l)$ is updated by the following triggering condition

$$k_{l+1} = min\{k > k_l | e^T(k)e(k) > \sigma^2 u(k_l)^T u(k_l)\}. \tag{7}$$

As a result, the closed-loop system composed of (1) and (2) is

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k_l) + a_i + D_i \omega(k) \\ &= A_i x(k) + B_i [u(k) + e(k)] + a_i + D_i \omega(k) \\ &= \bar{A}_i x(k) + \bar{a}_i + B_i e(k) + D_i \omega(k) \\ z(k) &= C_{2i} x(k) + E_i u(k_l) \\ &= C_{2i} x(k) + E_i (u(k) + e(k)) \\ &= (C_{2i} + E_i K_i C_{1i}) x(k) + E_i m_i + E_i e(k), \end{aligned} \tag{8}$$

where

$$\begin{aligned} \bar{A}_i &= A_i + B_i K_i C_{1i}, \\ \bar{a}_i &= B_i m_i + a_i. \end{aligned}$$

Remark 1. Compared with [17] and [18] related to system states, the proposed triggering condition (7) just relies on system outputs. As a result, this could render new challenge of the co-design the affine term m_i and static output feedback controller gain K_i with the required disturbance attenuation performance.

Remark 2. From (2), it is seen that the larger σ is, the lower the frequency of signal transmission is. In particular, when $\sigma = 0$, the triggering rule (2) is reduced to a time-triggered rule.

Before ending this section, a definition and two lemmas are given firstly as below:

Definition 1. (Yan et al. [31]) With a scalar $\gamma > 0$, system(1) is said to have an H_∞ control performance index γ, if it satisfies

$$\sum_{k=0}^{\infty} z^T(k)z(k) < \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k), \tag{9}$$

for all nonzero $\omega(t) \in \mathcal{L}_2$ under zero initial conditions.

Lemma 1. (Boyd et al. [4]) (Finsler lemma) Let $v \in \mathcal{R}^{n_v}, \mathcal{P} = \mathcal{P}^T$,and $\mathcal{H} \in \mathcal{R}^{m\times n_v}$, such that $rank(\mathcal{H}) = r < n_v$. Then the following statements are equivalent:

1) $v^T\mathcal{P}v < 0$, for all $v \neq 0, \mathcal{H}v = 0$;

2) $\mathcal{H}^{\perp T}\mathcal{P}\mathcal{H}^{\perp} < 0$;

3) There exists $\mathcal{S} \in \mathcal{R}^{n_v\times m}$ such that $\mathcal{P} + He(\mathcal{S}\mathcal{H}) < 0$.

Lemma 2. (Boyd et al. [4]) (S-Procedure) Let $W_0(x), W_1(x), \ldots, W_1(x)$ is quadratic functions, i. e., $W_i(x) = x^T F_i x$ with $F_i = F_i^T (i = 0, 1, ..., \rho)$. then the following implication:

$$W_1(x) \leq 0, W_2(x) \leq 0, ..., W_\rho(x) \leq 0 \Longrightarrow W_0(x) < 0 \tag{10}$$

is true if there exist scalars $\tau_1 \geq 0, \cdots, \tau_\rho \geq 0$ such that

$$F_0 - \sum_{i=1}^{\rho} \tau_i F_i < 0. \tag{11}$$

3. MAIN RESULTS

In this section, sufficient conditions are provided to guarantee the closed-loop system (8) to be asymptotically stable with the required H_∞ performance firstly. Then, an LMI based H_∞ static output feedback controller synthesis method is built by means of decoupling the nonlinearity among Lyapunov matrix, controller gain, system input and output matrices.

Theorem 1. For given scalars $\sigma \in (0, 1]$, if there exist scaler $\eta_i > 0, \xi_i > 0 (i \in \wp)$ and matrices $P_i \in \mathcal{R}^{n\times n} > 0$, $P_j \in \mathcal{R}^{n\times n} > 0, (i, j \in \wp)$ as follow, the closed-loop PWA system (1) is stochastically stable with the prescribed H_∞ performance index γ

$$\begin{bmatrix} \Phi_{(1,1)} & * & * & * & * & * & * \\ 0 & -\xi_i I_m & * & * & * & * & * \\ \Phi_{(3,1)} & 0 & \Phi_{(3,3)} & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 & * & * & * \\ P_j\bar{A}_i & P_jB_i & P_j\bar{a}_i & P_jD_i & -P_j & * & * \\ \Phi_{(6,1)} & E_i & E_im_i & 0 & 0 & -I_r & * \\ \xi_iK_iC_{1i} & 0 & \xi_im_i & 0 & 0 & 0 & -\xi_i\sigma^{-2}I_m \end{bmatrix} < 0, (i \in \wp_1) \tag{12}$$

$$\begin{bmatrix} \Phi_{(1,1)} & * & * & * & * & * \\ 0 & -\xi_i I_m & * & * & * & * \\ 0 & 0 & -\gamma^2 & * & * & * \\ P_j\bar{A}_i & P_jB_i & P_jD_i & -P_j & * & * \\ \Phi_{(6,1)} & E_i & 0 & 0 & -I_r & * \\ \xi_iK_iC_{1i} & 0 & 0 & 0 & 0 & -\xi_i\sigma^{-2}I_m \end{bmatrix} < 0, (i \in \wp_0). \tag{13}$$

Where $\Phi_{(1,1)} = -P_i - \eta_i Q_i^T Q_i$, $\Phi_{(3,1)} = -\eta_i q_i^T Q_i$, $\Phi_{(3,3)} = -\eta_i({q_i}^2 - 1)$,$\Phi_{(6,1)} = C_{2i} + E_iK_iC_{1i}$.

Proof. The candidate PWA Lyapunov function is chosen as:

$$V(x(k)) = x^T(k)P_i x(k), P_i > 0. \tag{14}$$

Assuming that the output $y(k)$ transits from $\mathcal{Y}_i$ to $\mathcal{Y}_j$ $(i, j \in \wp)$, the difference of (14) is

$$\begin{aligned} \Delta V &= V(x(k+1)) - V(x(k)) \\ &= x^T(k+1)P_j x(k+1) - x^T(k)P_i x(k). \end{aligned} \tag{15}$$

According to Definition 1, the closed-loop system (8) is asymptotically stable with the prescribed H_∞ performance index γ once the following inequality holds

$$J = \Delta V + z(k)^T z(k) - \gamma^2 \omega^T(k)\omega(k) < 0. \tag{16}$$

To make (16) hold, two cases are considered as below.

Case 1: $i \in \wp_1$, namely $a_i \neq 0$, $m_i \neq 0$.
Setting $\delta^T = \begin{bmatrix} x^T(k) & e^T(k) & 1 & \omega^T(k) \end{bmatrix}$, (16) is rewritten

$$J = \delta^T \Xi_2 \delta < 0 \tag{17}$$

where

$$\Xi_2 = \begin{bmatrix} \Upsilon_{(1,1)} & * & * & * \\ \Upsilon_{(2,1)} & \Upsilon_{(2,2)} & * & * \\ \Upsilon_{(3,1)} & \Upsilon_{(3,2)} & \Upsilon_{(3,3)} & * \\ D_i^T P_j \bar{A}_i & D_i^T P_j B_i & D_i^T P_j \bar{a}_i & \Upsilon_{(4,4)} \end{bmatrix}$$

$$\begin{aligned}
\Upsilon_{(1,1)} &= \bar{A}_i^T P_j \bar{A}_i - P_i + (C_{2i} + E_i K_i C_{1i})^T (C_{2i} + E_i K_i C_{1i}) \\
\Upsilon_{(2,1)} &= B_i^T P_j \bar{A}_i + E_i^T (C_{2i} + E_i K_i C_{1i}) \\
\Upsilon_{(2,2)} &= B_i^T P_j B_i + E_i^T E_i \\
\Upsilon_{(3,1)} &= \bar{a}_i^T P_j \bar{A}_i + (E_i m_i)^T (C_{2i} + E_i K_i C_{1i}) \\
\Upsilon_{(3,2)} &= \bar{a}_i^T P_j B_i + (E_i m_i)^T E_i \\
\Upsilon_{(3,3)} &= \bar{a}_i^T P_j \bar{a}_i + (E_i m_i)^T (E_i m_i) \\
\Upsilon_{(4,4)} &= D_i^T P_j D_i - \gamma^2.
\end{aligned}$$

To make $J < 0$, the main task is to ensure $\Xi_2 < 0$.
On the other hand, the triggered condition (2) is rewritten as

$$\delta^T \Xi_1 \delta \geqslant 0, \tag{18}$$

where

$$\Xi_1 = \begin{bmatrix} \sigma^2 (K_i C_{1i})^T (K_i C_{1i}) & * & * & * \\ 0 & -I & * & * \\ \sigma^2 m_i^T K_i C_{1i} & 0 & \sigma^2 m_i^T m_i & * \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Meanwhile, (4) is also equivalent to

$$\delta^T \Xi_3 \delta \geqslant 0, \tag{19}$$

in which

$$\Xi_3 = \begin{bmatrix} -Q_i^T Q_i & * & * & * \\ 0 & 0 & * & * \\ -q_i^T Q_i & 0 & 1-{q_i}^2 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Applying Lemma 2 to (17) with (18) and (19), (17) could be ensured once the following inequality holds

$$\begin{bmatrix} \bar{\Upsilon}_{(1,1)} & * & * & * \\ \Upsilon_{(2,1)} & \Upsilon_{(2,2)} - \xi_i I & * & * \\ \bar{\Upsilon}_{(3,1)} & \Upsilon_{(3,2)} & \bar{\Upsilon}_{(3,3)} & * \\ D_i^T P_j \bar{A}_i & D_i^T P_j B_i & D_i^T P_j \bar{a}_i & \Upsilon_{(4,4)} \end{bmatrix}$$

in which

$$\begin{aligned} \bar{\Upsilon}_{(1,1)} &= \Upsilon_{(1,1)} + \xi_i \sigma^2 (K_i C_{1i})^T (K_i C_{1i}) - \eta_i Q_i^T Q_i \\ \bar{\Upsilon}_{(3,1)} &= \Upsilon_{(3,1)} + \xi_i \sigma^2 m_i^T K_i C_{1i} - \eta_i q_i^T Q_i \\ \bar{\Upsilon}_{(3,3)} &= \Upsilon_{(3,3)} + \xi_i \sigma^2 m_i^T m_i - \eta_i ({q_i}^2 - 1). \end{aligned}$$

Applying Schur complement once again, (19) could be ensured by

$$\begin{bmatrix} \Phi_{(1,1)} & * & * & * & * & * & * \\ 0 & -\xi_i I_m & * & * & * & * & * \\ \Phi_{(3,1)} & 0 & \Phi_{(3,3)} & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 & * & * & * \\ \bar{A}_i & B_i & \bar{a} & D_i & -P_j^{-1} & * & * \\ \Phi_{(6,1)} & E_i & E_i m_i & 0 & 0 & -I_r & * \\ K_i C_{1i} & 0 & m_i & 0 & 0 & 0 & -\xi_i^{-1}\sigma^{-2} I_m \end{bmatrix} < 0 \tag{20}$$

which is just (12) via Pre-/post-multiplying (20) with $diag(I, I, I, I, P_j, I, \xi_i I)$. Therefore, once (12) holds, then one has

$$\Delta V(k) + z(k)^T z(k) - \gamma^2 \omega^T(k)\omega(k) < 0. \tag{21}$$

Namely,

$$V(k+1) - V(k) + z(k)^T z(k) - \gamma^2 \omega^T(k)\omega(k) < 0. \tag{22}$$

Then summing up (3) from $k = 0$ to $k = \infty$ yields

$$\sum_{k=0}^{\infty} V(k+1) - \sum_{k=0}^{\infty} V(k) + \sum_{k=0}^{\infty} z^T(k)z(k) - \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k) < 0. \tag{23}$$

Resorting to zero initial assumption ($V(0) = 0$), (3) is rewritten as

$$\sum_{k=0}^{\infty} z^T(k)z(k) - \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k) < 0. \tag{24}$$

According to Definition 1, the required H_∞ performance index γ is guaranteed.

Case 2: $i \in \wp_0$, namely $a_i = 0$, $m_i = 0$.
Taking the similar steps as Case 1, one has (12) which could ensure the closed-loop system 8 is asymptotically stable with the prescribed H_∞ performance index γ. □

Remark 3. Due to the coupling of P_j and the controller gains K_i, m_i, the conditions 12 and 13 in Theorem 1 are non-convex. As a result, they cannot be directly solved by the existing Matlab LMI TOOLBOX. To overcome this difficulty, a decoupling approach based on the Lemma 1 is adopted to deal with the coupling terms and the resulted synthesis conditions are given in Theorem 2 as below.

Theorem 2. For given scalars $\sigma \in (0, 1], b_1, b_2, b_3, b_4$, the closed-loop PWA system 1 is asymptotically stable with the prescribed H_∞ performance index γ if there exist scalars $\eta_i > 0, \xi_i > 0 (i \in \wp), f_1 \in \mathcal{R}^{m\times m}$, and matrices $P_i \in \mathcal{R}^{n\times n} > 0,\ P_j \in \mathcal{R}^{n\times n} > 0, L_i \in \mathcal{R}^{m\times p}, M_i \in \mathcal{R}^{m\times 1}, (i, j \in \wp)$ satisfying

$$\begin{bmatrix} \Lambda_1 & * \\ \Lambda_2 & \Lambda_3 \end{bmatrix} < 0, (i \in \wp_1) \tag{25}$$

$$\begin{bmatrix} \Lambda_4 & * \\ \Lambda_5 & \Lambda_3 \end{bmatrix} < 0, (i \in \wp_0) \tag{26}$$

$$\Lambda_1 = \begin{bmatrix} \Phi_{(1,1)} & * & * & * \\ 0 & -\xi_i I_m & * & * \\ \Phi_{(3,1)} & 0 & \Phi_{(3,3)} & * \\ 0 & 0 & 0 & -\gamma^2 \end{bmatrix}, \Lambda_2 = \begin{bmatrix} \Gamma_{(5,1)} & P_j B_i & \Gamma_{(5,3)} & P_j D_i \\ \Gamma_{(6,1)} & E_i & b_2 E_i M_i & 0 \\ \Gamma_{(7,1)} & 0 & b_3 M_i & 0 \\ \Gamma_{(8,1)} & 0 & b_4 M_i & 0 \end{bmatrix},$$

$$\Lambda_3 = \begin{bmatrix} -P_j & * & * & * \\ 0 & -I_r & * & * \\ 0 & 0 & -\xi_i \sigma^{-2} I_m & * \\ \Gamma_{(8,5)} & \Gamma_{(8,6)} & \Gamma_{(8,7)} & \Gamma_{(8,8)} \end{bmatrix}, \Lambda_4 = \begin{bmatrix} \Phi_{(1,1)} & * & * \\ 0 & -\xi_i I_m & * \\ 0 & 0 & -\gamma^2 \end{bmatrix},$$

$$\Lambda_5 = \begin{bmatrix} \Gamma_{(5,1)} & P_j B_i & P_j D_i \\ \Gamma_{(6,1)} & E_i & 0 \\ \Gamma_{(7,1)} & 0 & 0 \\ \Gamma_{(8,1)} & 0 & 0 \end{bmatrix},$$

where $\Gamma_{(5,1)} = P_j A_i + b_1 B_i L_i C_{1i}, \Gamma_{(5,3)} = P_j a_i + b_1 B_i M_i, \Gamma_{(6,1)} = C_{2i} + b_2 E_i L_i C_{1i}$, $\Gamma_{(7,1)} = b_3 L_i C_{1i}, \Gamma_{(8,1)} = b_4 L_i C_{1i}, \Gamma_{(8,5)} = (P_j B_i - b_1 B_i f_1)^T, \Gamma_{(8,6)} = (E_i - b_2 E_i f_1)^T$, $\Gamma_{(8,7)} = (\xi_i I - b_3 f_1)^T, \Gamma_{(8,8)} = -b_4(f_1 + f_1^T)$.

Moreover, $K_i = f_1^{-1} L_i$, $m_i = f_1^{-1} M_i$.

Proof. **Case 1:** $i \in \wp_1$. Reformulate (12) in Theorem 1 as

$$\mathcal{H}_1^{\perp^T} \mathcal{P}_1 \mathcal{H}_1^{\perp} < 0, \tag{27}$$

where

$$\mathcal{P}_1 = \begin{bmatrix} \Phi_{(1,1)} & * & * & * & * & * & * & * \\ 0 & -\xi_i I_m & * & * & * & * & * & * \\ \Phi_{(3,1)} & 0 & \Phi_{(3,3)} & * & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 & * & * & * & * \\ P_j \bar{A}_i & P_j B_i & P_j \bar{a}_i & P_j D_i & -P_j & * & * & * \\ \Phi_{(6,1)} & E_i & E_i m_i & 0 & 0 & -I_r & * & * \\ \xi_i K_i C_{1i} & 0 & \xi_i m_i & 0 & 0 & 0 & -\xi_i \sigma^{-2} I_m & * \\ 0_{m\times n} & 0_m & 0_{m\times 1} & 0_{m\times 1} & 0_{m\times n} & 0_{m\times r} & 0_m & 0_m \end{bmatrix}$$
$$\mathcal{H}_1^{\perp} = \begin{bmatrix} I_n & * & * & * & * & * & * \\ 0 & I_m & * & * & * & * & * \\ 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & I_n & * & * \\ 0 & 0 & 0 & 0 & 0 & I_r & * \\ 0 & 0 & 0 & 0 & 0 & 0 & I_m \\ K_i C_{1i} & 0_{m\times m} & m_i & 0 & 0_{m\times n} & 0_{m\times r} & 0_m. \end{bmatrix}. \tag{28}$$

Applying Lemma 1 to (27) yields

$$\mathcal{P}_1 + He(\mathcal{S}_1 \mathcal{H}_1) < 0, \tag{29}$$

where

$$\begin{aligned} \mathcal{S}_1 &= \begin{bmatrix} 0_{m\times 1} & 0_{m\times 1} & 0_{m\times n} & 0_{m\times n} & \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 \end{bmatrix}^T \\ \mathcal{H}_1 &= \begin{bmatrix} K_i C_{1i} & 0_{m\times n} & m_i & 0_m & 0_{m\times r} & 0_m & 0_{m\times 1} & -I_m \end{bmatrix} \end{aligned} \tag{30}$$

with

$$\begin{aligned} \Gamma_1 &= (-P_j B_i + b_1 B_i f_1)^T \\ \Gamma_2 &= (-E_i + b_2 E_i f_1)^T \\ \Gamma_3 &= (-\xi_i + b_3 f_1)^T I_m \\ \Gamma_4 &= (b_4 f_1)^T I_m \end{aligned}$$

which is just (25).

Case 2: $i \in \wp_0$. As the same process as above, rewrite (13) as

$$\mathcal{H}_2^{\perp^T} \mathcal{P}_2 \mathcal{H}_2^{\perp} < 0, \tag{31}$$

where

$$\mathcal{P}_2 = \begin{bmatrix} \Phi_{(1,1)} & * & * & * & * & * & * \\ 0 & -\xi_i I_m & * & * & * & * & * \\ 0 & 0 & -\gamma^2 & * & * & * & * \\ P_j\bar{A}_i & P_jB_i & P_jD_i & -P_j & * & * & * \\ \Phi_{(6,1)} & E_i & 0 & 0 & -I_r & * & * \\ \xi_iK_iC_{1i} & 0 & 0 & 0 & 0 & -\xi_i\sigma^{-2}I_m & * \\ 0_{m\times n} & 0_m & 0_{m\times 1} & 0_{m\times n} & 0_{m\times r} & 0_m & 0_m \end{bmatrix}$$
$$\mathcal{H}_2^{\perp} = \begin{bmatrix} I_n & * & * & * & * & * \\ 0 & I_m & * & * & * & * \\ 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & I_n & * & * \\ 0 & 0 & 0 & 0 & I_r & * \\ 0 & 0 & 0 & 0 & 0 & I_m \\ K_iC_{1i} & 0_m & 0 & 0_{m\times n} & 0_{m\times r} & 0_m \end{bmatrix}. \quad (32)$$

Resorting to Lemma 1, (31) is equivalent to:

$$\mathcal{P}_2 + He(\mathcal{S}_2\mathcal{H}_2) < 0, \quad (33)$$

where

$$\begin{aligned} \mathcal{S}_2 &= \begin{bmatrix} 0_{m\times 1} & 0_{m\times n} & 0_{m\times n} & \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4 \end{bmatrix}^T \\ \mathcal{H}_2 &= \begin{bmatrix} K_iC_{1i} & 0_{m\times n} & 0_m & 0_{m\times r} & 0_m & 0_{m\times 1} & -I_m \end{bmatrix} \end{aligned} \quad (34)$$

which is just 26. □

Remark 4. Compared with 27 where the coupling terms are solved by the cone complement linearization with multiple iterations, conditions given in Theorem 2 are in terms of linear matrix inequalities, which can be solved by MATLAB LMI Toolbox directly.

4. EXAMPLE

In this section, two examples are supplied to illustrate the effectiveness of the proposed method.

Example 1: Consider a nonlinear system with the following evolution

$$\begin{cases} \dot{x_1} = x_2 - 10x_1 + 1.2, \\ \dot{x_2} = -0.1x_2 + sin(x_1) + u, \\ y_1 = x_1, \\ y_2 = -0.04x_1 + 0.9x_2. \end{cases} \quad (35)$$

The nonlinear term $sin(y_1)$ is approximated by the PWA method as

$$sin(x_1)=\begin{cases}-0.85y_1-2.6,\ -4<y_1<-2\\ -0.9,\ -2<y_1<-1\\ 0.9y_1,\ -1<y_1<1\\ 0.9,\ 1<y_1<2\\ 2.6-0.85y_1,\ 2<y_1<4\end{cases}$$

which is also indicated in the following Figure 1.

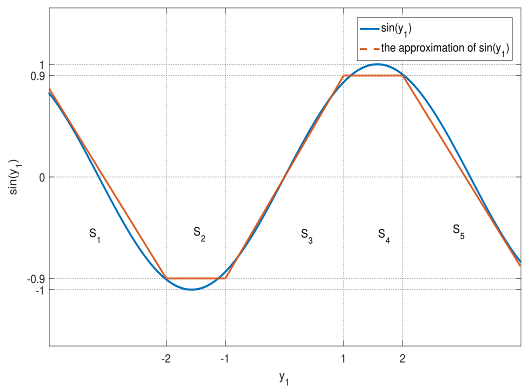

Fig. 1. The PWA of $sin(y_1)$.

It is obvious that the output space is separated into five regions: $S_1=\{y\in\mathcal{R}^2|-d_3<y_1<-d_2\}$, $S_2=\{y\in\mathcal{R}^2|-d_2<y_1<-d_1\}$, $S_3=\{y\in\mathcal{R}^2|-d_1<y_1<d_1\}$, $S_4=\{y\in\mathcal{R}^2|d_1<y_1<d_2\}$ and $S_5=\{y\in\mathcal{R}^2|d_2<y_1<d_3\}$.And the corresponding parameters are $Q1=Q3=Q5=[10]$, $Q2=Q4=[20]$, $q1=q2=3$, $q3=0$, $q4=q5=-3$. Meanwhile, the system 4 is changed to the PWA system 1 with the following parameters

$$\begin{cases}A_1=A_5=\begin{bmatrix}0.9 & 0.01\\ -0.0085 & 0.999\end{bmatrix}, A_2=\begin{bmatrix}0.9 & 0.01\\ 0 & 0.999\end{bmatrix},\\ A_3=\begin{bmatrix}0.9 & 0.01\\ 0.009 & 0.999\end{bmatrix}, A_4=\begin{bmatrix}0.9 & 0.01\\ 0 & 0.019\end{bmatrix},\\ a_1=\begin{bmatrix}0.012\\ -0.026\end{bmatrix}, a_2=\begin{bmatrix}0.012\\ -0.009\end{bmatrix}, a_3=\begin{bmatrix}0\\ 0\end{bmatrix},\\ a_4=\begin{bmatrix}0.012\\ 0.009\end{bmatrix}, a_5=\begin{bmatrix}0.012\\ 0.026\end{bmatrix},\\ B_i=\begin{bmatrix}0\\ 0.01\end{bmatrix}, C_{1i}=\begin{bmatrix}1 & 0\\ -0.04 & 0.9\end{bmatrix}, C_{2i}=\begin{bmatrix}0.9 & 0.09\\ -0.4 & 0.1\end{bmatrix},\\ E_i=\begin{bmatrix}0.6\\ 0.6\end{bmatrix}, D_i=\begin{bmatrix}0.124\\ 0\end{bmatrix}, (i=1,2,3,4,5).\end{cases}$$

Considering the initial state $x(0) = [0; 0]$. Applying Theorem 2 with $\sigma = 0.1$, and $\omega(k) = e^{-0.8kh\times sin(kh)}$ when $k \geq 2$, one can get the H_∞ performance index $\gamma = 5.1962$, and the corresponding controller gain can be obtained by the calculation of Theorem 2, so that the system under consideration obtains reliable H_∞ performance.

Solving the conditions proposed in Theorem 2 with $\sigma = 0.1$ by means of Matlab LMI Toolbox, the H_∞ performance index γ is 5.0990, and the corresponding controller gains are

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.0104 & -0.1768 \end{bmatrix}, m_1 = 0.0032, \\ K_2 &= \begin{bmatrix} -0.0098 & -0.1839 \end{bmatrix}, m_2 = 0.0021, \\ K_3 &= \begin{bmatrix} -0.1755 & -0.3337 \end{bmatrix}, m_3 = 0, \\ K_4 &= \begin{bmatrix} -0.1380 & -0.3991 \end{bmatrix}, m_4 = 0.0003, \\ K_5 &= \begin{bmatrix} 1.1171 & -0.3358 \end{bmatrix}, m_5 = 0.0003. \end{aligned} \tag{36}$$

Under the zero initial state $x(0) = [00]^T$ and $\omega(k) = e^{-0.8kh\times sin(kh)}$ $(k \geq 2)$, simulation curves are obtained for H_∞ SOF control and SOF control without disturbances in Figures 2 and 3, respectively.

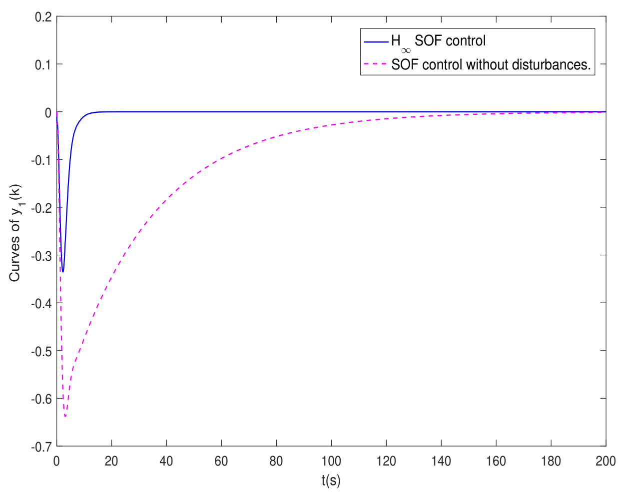

Fig. 2. Curves of $y_1(k)$ under the H_∞ SOF control and the SOF control without disturbances.

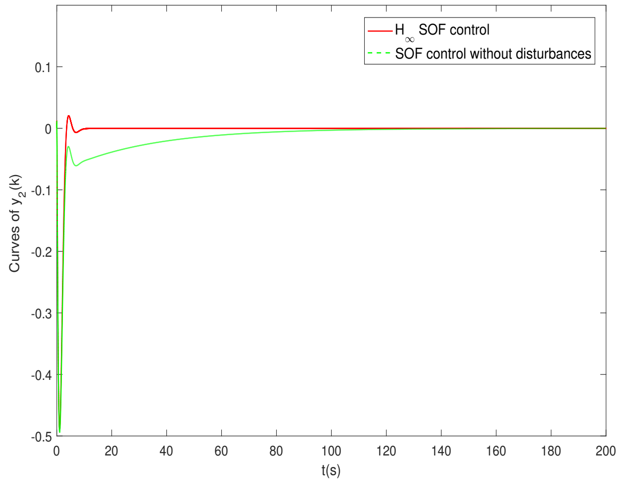

Fig. 3. Curves of $y_2(k)$ under the H_∞ SOF control and the SOF control without disturbances.

From the comparisons of these figures, it is seen that the outputs produced by the H_∞ approach are much faster and smoother. Additionally, release intervals under three cases of σ ($\sigma = 0.01$, 0.05 and 0.1) are depicted in Figure 4.

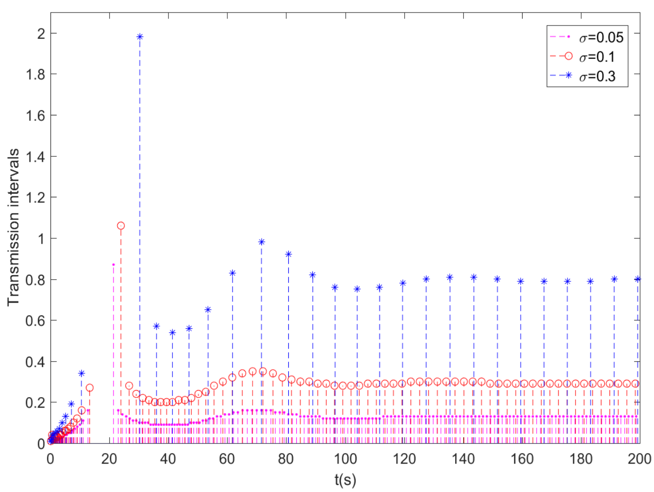

Fig. 4. Transmission intervals for different σ.

This figure indicates that the bigger the σ is, the longer the release intervals is. Thus, the selection of σ is important to reduce the unnecessary transmission and save the limited channel. Furthermore, to deliver the influence of σ to the system output $y(k)$, the outputs $y_1(k)$ and $y_2(k)$ under $\sigma = 0.05, 0.1, 0.3$ are depicted in Figures 5 − 6, respectively.

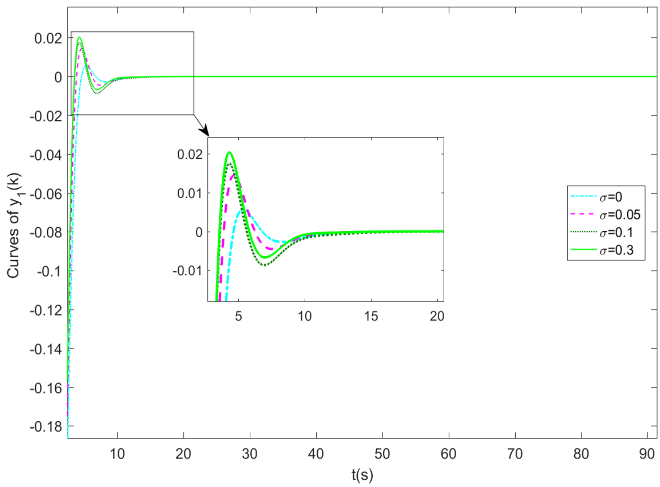

Fig. 5. Curves of $y_1(k)$ under $\sigma = 0.05, 0.1, 0.3$.

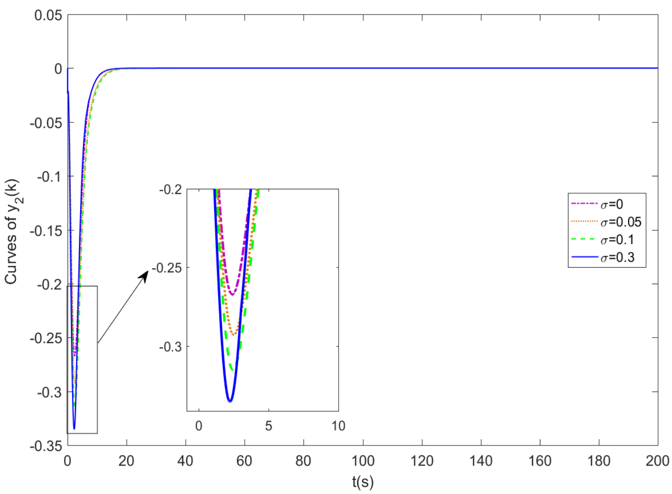

Fig. 6. Curves of $y_2(k)$ under $\sigma = 0.05, 0.1, 0.3$.

It is seen in Figures 5 and 6 that the value of σ has an impact on the outputs y_1 and y_1, namely, the smaller σ is, the more the transmission is, the better performance $y(k)$ is.

Example 2: A simplified autonomous land vehicle is given in Figure 7

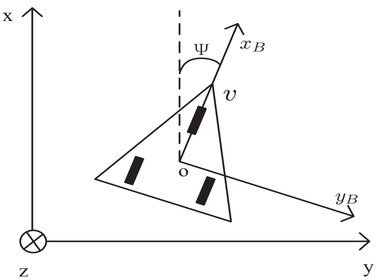

Fig. 7. The autonomous land vehicle.

where xyz is a rectangular coordinate system, x_Boy_B is rectangular coordinate system of the vehicle, ψ means the angle between the autonomous vehicle's forward direction and the x-axis direction, l indicates the displacement of y-axis.

Supposing that the autonomous vehicle has a constant speed v, the control aim is to keep $l = 0$ and $\psi = 0$ when the vehicle maintains a constant speed in the forward direction. Moreover, the dynamic equations of the system are described by

$$\begin{bmatrix} \dot{\psi} \\ \dot{\omega} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{\beta}{\alpha} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ \omega \\ l \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ vsin(\psi) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\alpha} \\ 0 \end{bmatrix} u \tag{37}$$

where ω means direction angular velocity, u stands for the control torque, α is the moment of inertia of the vehicle with respect to the center of mass, β is the damping coefficient.

Setting the state vector $(x_1^T, x_2^T, x_3^T)^T = (\psi^T, \omega^T, l^T)^T$ with $\alpha = 1Kgm^2$, $\beta = 0.01$, $v = 1m/s$, (37) is rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -0.01x_2 + u, \\ \dot{x}_3 = \sin(x_1), \\ y_1 = x_1, \\ y_2 = x_2, \\ y_3 = x_3. \end{cases} \tag{38}$$

Assuming $\psi \in [-3\pi/5, 3\pi/5]$, it is divided as five parts: $(-3\pi/5, -\pi/5)$, $(-\pi/5, -\pi/15)$, $(-\pi/15, \pi/15)$, $(\pi/15, \pi/5)$, and $(\pi/5, 3\pi/5)$. Then, $sin(y_1)$ is linearized to PWA form as below:

$$sin(y_1) = \begin{cases} -0.309y_1 - 0.637 \ (-3\pi/5 \le y_1 < -\pi/5), \\ 0.914y_1 - 0.016 \ (-\pi/5 \le y_1 < -\pi/15), \\ y_1 \ (-\pi/15 \le y_1 < \pi/15), \\ 0.914y_1 + 0.016 \ (\pi/15 \le y_1 < \pi/5), \\ 0.309y_1 + 0.637 \ (\pi/5 \le y_1 \le 3\pi/5). \end{cases}$$

As a result, five regions are obtained: $S_1 = \{y| -3\pi/5 \leq y_1 \leq -\pi/5\}$, $S_2 = \{y| -\pi/5 < y_1 < -\pi/15\}$, $S_3 = \{y| -\pi/15 \leq y_1 \leq \pi/15\}$, $S_4 = \{y|\pi/15 \leq y_1 \leq \pi/5\}$ and $S_5 = \{y|\pi/5 \leq y_1 \leq 3\pi/5\}$.

Discretizing the resulted PWA models with the sampling period $h = 0.02s$ leads to

$$\begin{cases} A_1 = A_5 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.01 & 0 \\ 0.309 & 0 & 0 \end{bmatrix}, A_2 = A_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.01 & 0 \\ 0.914 & 0 & 0 \end{bmatrix}, \\ A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.01 & 0 \\ 1 & 0 & 0 \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, a_1 = \begin{bmatrix} 0 \\ 0 \\ -0.637 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 0 \\ -0.016 \end{bmatrix}, \\ a_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, a_4 = \begin{bmatrix} 0 \\ 0 \\ 0.016 \end{bmatrix}, a_5 = \begin{bmatrix} 0 \\ 0 \\ 0.637 \end{bmatrix}, \\ C_{1i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, C_{2i} = \begin{bmatrix} 0.9 & 0.9 & 0 \\ -0.4 & 0.1 & 0 \\ -0.4 & 0.1 & 0 \end{bmatrix}, D_i = \begin{bmatrix} 1.24 \\ 0 \\ 0 \end{bmatrix}, E_i = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.2 \end{bmatrix}. \end{cases} \tag{39}$$

According to (3), the corresponding parameters are

$$\begin{aligned} &Q_1 = Q_5 = \begin{bmatrix} 1.5915 & 0 & 0 \end{bmatrix}, \\ &Q_2 = Q_3 = Q_4 = \begin{bmatrix} 4.7746 & 0 & 0 \end{bmatrix}, \\ &q_1 = q_2 = 2, q_3 = 0, q_4 = q_5 = -2. \end{aligned}$$

Taking the above parameters to (25) and (26) in Theorem 2 with $\sigma = 0.1$ via the Matlab LMI Toolbox, the H_∞ performance index γ is 5.1962 and the event-triggered controller gains are

$$\begin{aligned} &K_1 = K_5 = \begin{bmatrix} 0.05081 & -9.6947 & 2.0441 \times 10^{-8} \end{bmatrix}, \\ &K_2 = K_4 = \begin{bmatrix} -0.0461 & -0.1939 & -6.4233 \times 10^{-10} \end{bmatrix}, \\ &K_3 = \begin{bmatrix} 0.0239 & -0.1262 & -4.0079 \times 10^{-10} \end{bmatrix}, \\ &m_1 = m_5 = 1.7299 \times 10^{-6}, m_2 = m_4 = 4.3359 \times 10^{-8}, m_3 = 0. \end{aligned}$$

Applying the above obtained controller gains with the zero initial condition $x(0) = [0 \;\; 0 \;\; 0]^T$ and $\omega(k) = e^{-0.8kh \times -0.2sin(-0.1kh)}$ $(k \geq 2)$, simulation curves are given in Figures 8−12. To be specific, the comparison of system outputs between the proposed H_∞ SOF control and the SOF control without disturbances are provided in Figures 8−10, respectively. The transmission interval for the proposed event-triggered H_∞ SOF control is given in Figure 11 and Curves of $u(k)$ is supplied in Figure 12.

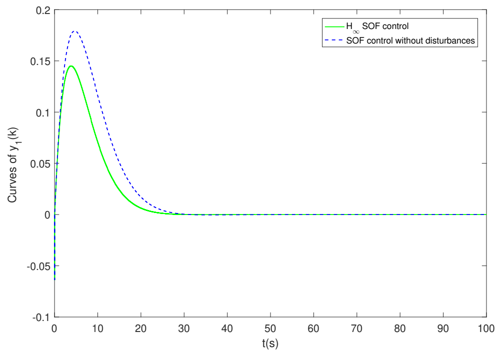

Fig. 8. Curves of $y_1(k)$ under the H_∞ SOF control and the SOF control without disturbances.

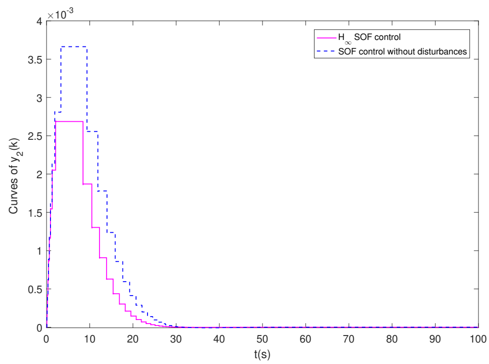

Fig. 9. Curves of $y_2(k)$ under the H_∞ SOF control and the SOF control without disturbances.

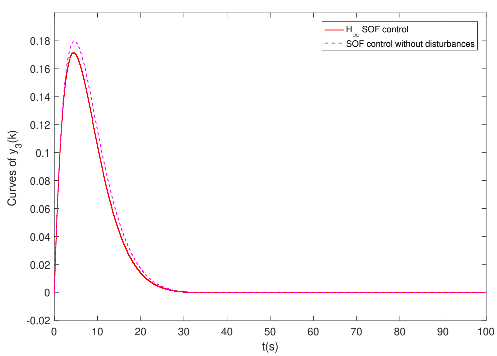

Fig. 10. Curves of $y_3(k)$ under the H_∞ SOF control and the SOF control without disturbances.

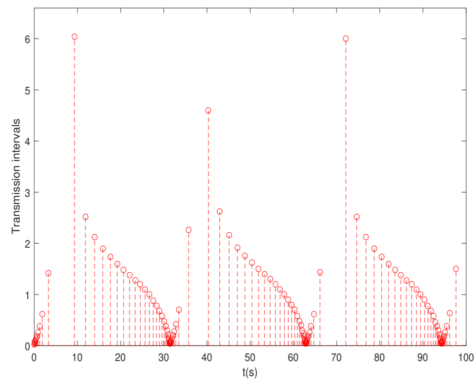

Fig. 11. Transmission interval.

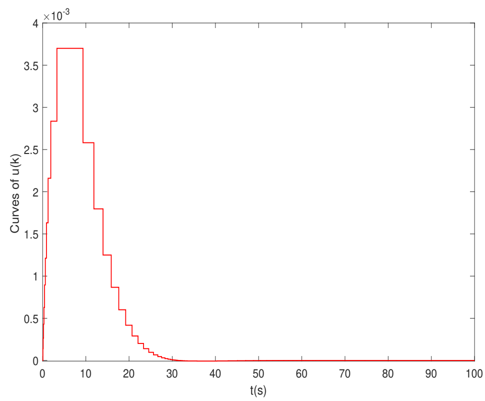

Fig. 12. Curves of $u(k)$.

From these figures, it is seen that the proposed event-triggered H_∞ SOF controller with affine term could keep $l = 0$ and $\psi = 0$ when the vehicle maintains a constant speed in the forward direction.

5. CONCLUSIONS

This paper investigates the event-triggered H_∞ SOF control of PWA systems. By means of piece-wise Lyapunov approach, sufficient conditions for the resultant closed-loop system to be asymptotically stable with prescribed H_∞ performance is established. A separation strategy is adopted to get an LMI based H_∞ controller gain. The validity of the proposed approach is verified by two examples.

6. ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China under Grants (61773200, 61403189), the peak of six talents in Jiangsu Province under Grant 2015XXRJ-011, the Doctoral Foundation of Ministry of Education of China under Grant 20133221120012, the Natural Science Foundation of Jiangsu Province of China under Grant BK20130949.

(Received August 3, 2019)

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