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FIXED-TIME TRACKING CONTROL FOR NONHOLONOMIC MOBILE ROBOT

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This paper investigates the fixed-time trajectory tracking control problem for a nonholonomic mobile robot. Firstly, the tracking error system is derived for the mobile robot by the aid of a global invertible transformation. Then, based on the unified error dynamics and by using the fixed-time control method, continuous fixed-time tracking controllers are developed for the mobile robot such that the robot can track the desired trajectory in a fixed time. Moreover, the settling time is independent of the system initial conditions and only determined by the controller parameters. Finally, numerical simulations are provided to demonstrate the effectiveness of the theoretical results.

Keywords: nonholonomic mobile robot systems, fixed-time control, trajectory tracking

Classification: 93A14, 93D15, 93D21

1. INTRODUCTION

In recent years, the tracking control of nonholonomic mobile robot system has been highly valued and favored by scholars and researchers all over the world, and has become a research hotspot. Although according to Brocket theory, a nonholonomic system is not able to be asymptotically stable using the smooth and time invariant control laws [3]. With the development of mathematical theory and control theory, many advanced theories have been deeply discussed and widely used in the field of trajectory tracking control for mobile robot system [1, 5, 7, 11, 17, 28]

It is worth noting that, for these above works, the tracking control can only be achieved in an asymptotic manner, namely, the settling time is infinite. In reality, it is of particular interest to realise the control system in a finite time to meet specifc system requirement. Therefore, finite-time control problems draw some researchers' attention [2, 6, 12, 18, 26]. At present, many meaningful finite-time tracking control results for nonholonomic mobile robot systems have been reported in the literature [14, 20, 24, 27] and the references therein. In [14], two continuous finite-time tracking control laws were developed for two different cases of a nonholonomic mobile robot in a kinematic model, and the global finite time stability was guaranteed by using the cascaded system results. The authors of [27] proposed finite-time tracking controller for the nonholonomic

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systems with extended chained form. The authors of [20] studied finite-time tracking control problem of a nonholonomic wheeled mobile robot in dynamic model with external disturbances, finite-time disturbance observers and finite-time tracking control laws were designed for the mobile robot. In [24], an adaptive finite-time neural control was designed for robotic manipulators.

Although the previously listed finite-time control algorithms can guarantee that the closed-loop system convergence in a finite time, the settling time is difficult to estimate or is dependent on the initial condition. So it would be useful if the settling time could be predetermined no matter whether the initial conditions are known or not. Recently, a new concept, called fixed-time stability, has been proposed in [21]. Fixed-time control is more preferable than finite-time control in practical applications since the fixed-time approach can generate a control law prescribing a transition time which is independent of the operation domain [13]. Based on the fixed-time stability notion, some new results are reported. For example, the fixed-time control problem for second-order and high-order systems has been investigated in [15, 16, 25, 30]. The fixed-time stabilisation for a kind of uncertain nonholonomic systems subject to perturbations was considered in paper [29], and a globally fixed-time stabilisation strategy was proposed by taking advantage of adding a power integrator technique and switching ideal. The authors of paper [9] discussed fixed-time tracking control problem for nonholonomic mobile robot, and fixed time control algorithm was designed by proposing a new integral terminal sliding mode surface. In [23], the fixed-time attitude tracking control problem for rigid spacecraft with input quantization and external disturbances was investigated, fixed-time disturbance observer was designed to estimate unknown disturbances and fixed-time controller was constructed for the rigid spacecraft system.

Motivated by the above works, the main purpose of this paper is to tackle the fixed-time tracking control problem of a nonholonomic mobile robot, which is more challenging because of mobile robots' nonlinear dynamics and nonholonomic constraints. We first introduce the unified tracking error system for the mobile robot, which consists of two subsystems, i. e., a first-order subsystem and a second-order subsystem. Then, based on fixed-time control theory and adding a power integrator technique, the two subsystems are discussed respectively, and fixed-time control laws are proposed such that the states of the mobile robot converge to the desired reference trajectory in a fixed time. Since the resulting error system consists of two subsystems, we will give two stages to design the fixed-time control laws for the mobile robot. In the first stage, the first-order subsystem is discussed, fixed-time angular controller is design for the mobile robot based on fixed-time stability theory. In the second stage, the second-order subsystem is investigated and the translational velocity is given based on fixed-time control theory and adding a power integrator technique.

The rest of this paper is organized as follows. In the next Section, some preliminaries are first introduced. Then the model description and problem formulation are presented. The main results are given in Section 3. Numerical simulations are shown in Section 4. Conclusions are given in Section 5.

2. PRELIMINARIES

2.1. Problem formulation

As we known, Campion et al [4] have divided nonholonomic wheeled mobile robots into four types: $(2,0),(2,1),(1,1)$ and $(1,2)$. In this paper, we will consider fixed-time tracking control problem for the type (2,0) nonholonomic mobile robot system, as shown in Figure 1, which consists of a front castor wheel and two rear wheels. The

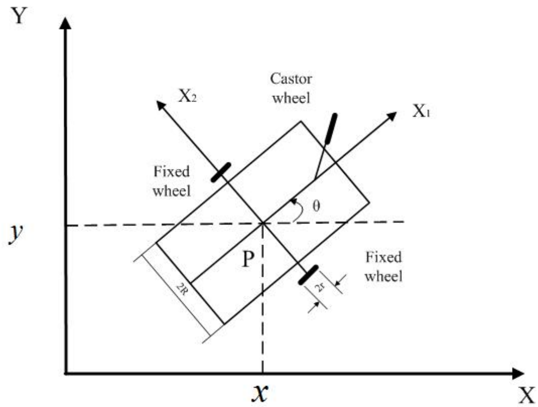

Fig. 1. Type (2,0) wheeled mobile robot.

two rear wheels of the robot are controlled independently by motors, and a front castor wheel prevents the robot from tipping over as it moves on a plane. Assume that the geometric center point and the mass center point of the robot are the same. Then, the nonholonomic constraint can be written as

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0, \tag{1}$$

where (x, y) denotes the position P of the center of mass, θ is the angle between X axis and X_1 axis with a positive anticlockwise direction. By this formula, the kinematics of the mobile robot can be described by the following equation in global coordinates:

$$\dot{x} = v\cos\theta, \tag{2a}$$
$$\dot{y} = v\sin\theta, \tag{2b}$$
$$\dot{\theta} = \omega, \tag{2c}$$

where v and ω are the linear velocity and the angular velocity of the mobile robot, respectively.

The dynamics of the reference trajectory is described by

$$\dot{x}_r = v_r\cos\theta_r, \tag{3a}$$
$$\dot{y}_r = v_r\sin\theta_r, \tag{3b}$$
$$\dot{\theta}_r = \omega_r, \tag{3c}$$

where (x_r, y_r) is the desired path of the mass center (x, y) in the image frame, θ_r is the desired direction, v_r and ω_r are the linear velocity and the angular velocity of the reference mobile robot, respectively.

2.2. Related lemmas

In this subsection, some important lemmas in obtaining the fixed-time controller are presented.

Lemma 2.1. (Polyakov [21]) Considering the following system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \tag{4}$$

suppose that there exists a continuous, positive definite function $V(x) : R^n \to R$ such that

$$\dot{V}(x) \leq -\alpha V^p(x) - \beta V^q(x), \quad x \in U_0, \tag{5}$$

where $\alpha > 0, \beta > 0$, $0 < p < 1, q > 1$, then the origin is a fixed-time stable equilibrium of system (4) and the finite settling time T satisfies $T \leq \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)}$.

Lemma 2.2. (Hardy et al. [8]) For $x_1, x_2 \in R$, $0 < p \leq 1$ is a real number, then the following inequality holds:

$$(|x_1| + |x_2|)^p \leq |x_1|^p + |x_2|^p. \tag{6}$$

Lemma 2.3. (Zuo and Tie [31]) For $x_i \in R$, $i = 1, 2, \cdots, n$ and $p > 1$, then

$$n^{1-p}(\sum_{i=1}^{n} |x_i|)^p \leq \sum_{i=1}^{n} |x_i|^p. \tag{7}$$

Lemma 2.4. (Hardy et al. [8]) For any real numbers a and b, if $0 < p = \frac{p_1}{p_2} \leq 1$, and $p_1 > 0, p_2 > 0$ are positive odd integers, then

$$|a^p - b^p| \leq 2^{1-p}|a - b|^p. \tag{8}$$

Lemma 2.5. (Qian and Lin [22]) Let $c, d > 0$, for any $\gamma > 0$, the following inequality holds for any $x, y \in R$

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}|y|^{c+d}. \tag{9}$$

3. MAIN RESULT

In order to deduce the main result, we first convert the global coordinates representation to Cartesian coordinates by the following global transformation [10]:

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix}, \tag{10}$$

i. e.,

$$
\begin{aligned}
x_e = & \quad (x_r - x)\cos\theta + (y_r - y)\sin\theta, \\
y_e = & \quad -(x_r - x)\sin\theta + (y_r - y)\cos\theta, \\
\theta_e = & \quad \theta_r - \theta.
\end{aligned}
\tag{11}
$$

Taking the time derivative of x_e, y_e, θ_e along system (2) and (3), the error dynamics equations can be obtained as

$$
\begin{aligned}
\dot{x}_e &= (\dot{x}_r - \dot{x})\cos\theta - (x_r - x)\dot{\theta}\sin\theta + (\dot{y}_r - \dot{y})\sin\theta + (y_r - y)\dot{\theta}\cos\theta \\
&= (v_r\cos\theta_r - v\cos\theta)\cos\theta - \omega(x_r - x)\sin\theta \\
&\quad +(v_r\sin\theta_r - v\sin\theta)\sin\theta + \omega(y_r - y)\cos\theta \\
&= \omega\big((y_r - y)\cos\theta - (x_r - x)\sin\theta\big) - v(\cos^2\theta + \sin^2\theta) \\
&\quad +v_r(\cos\theta_r\cos\theta + \sin\theta_r\sin\theta) \\
&= \omega y_e - v + v_r cos\theta_e, \qquad (12a)\\
\dot{y}_e &= -(\dot{x}_r - \dot{x})\sin\theta - (x_r - x)\dot{\theta}\cos\theta + (\dot{y}_r - \dot{y})\cos\theta - (y_r - y)\dot{\theta}\sin\theta \\
&= -(v_r\cos\theta_r - v\cos\theta)\sin\theta - \omega(x_r - x)\cos\theta \\
&\quad +(v_r\sin\theta_r - v\sin\theta)\cos\theta - \omega(y_r - y)\sin\theta \\
&= -\omega\big((x_r - x)\cos\theta + (y_r - y)\sin\theta\big) + v(\cos\theta\sin\theta - \sin\theta\cos\theta) \\
&\quad +v_r(\sin\theta_r\cos\theta - \cos\theta_r\sin\theta) \\
&= -\omega x_e + v_r sin\theta_e, \qquad (12b)\\
\dot{\theta}_e &= \dot{\theta}_r - \dot{\theta} = \omega_r - \omega. \qquad (12c)
\end{aligned}
$$

The objective of this paper is to design appropriate control laws v and ω such that system (2) can track the reference system (3) in a fixed time, i. e., the error system (12) is fixed-time stable.

Based on the structure of error dynamics equations (12), we will give two steps to design the controllers. In the first step, we design ω such that θ_e is forced to converge to zero in a fixed time. In the second step, we design v such that x_e, y_e can converge to zero in a fixed time.

3.1. Angular velocity design

Theorem 3.1. Consider system (2), if the angular controller is chosen as follows

$$\omega = \omega_r + k_1 sig^{\beta_1}\theta_e + k_2 sig^{\beta_2}\theta_e, \tag{13}$$

where $k_1, k_2 > 0$, $0 < \beta_1 < 1, \beta_2 > 1$, then the desired angular velocity trajectory can be tracked in a fixed time.

P r o o f. Choose a Lyapunov function as

$$V(\theta) = \frac{1}{2}\theta_e^2. \tag{14}$$

Computing the derivative of $V(\theta)$ along system (13), we obtain

$$\begin{aligned} \dot{V}(\theta) = &\ \theta_e\dot{\theta}_e = -k_1\theta_e sig^{\beta_1}\theta_e - k_2\theta_e sig^{\beta_2}\theta_e \\ = &\ -k_1|\theta_e|^{1+\beta_1} - k_2|\theta_e|^{1+\beta_2} \\ = &\ -k_1 2^{\frac{1+\beta_1}{2}}(\tfrac{1}{2}\theta_e^2)^{\frac{1+\beta_1}{2}} - k_2 2^{\frac{1+\beta_2}{2}}(\tfrac{1}{2}\theta_e^2)^{\frac{1+\beta_2}{2}} \\ \leq &\ -k_1 2^{\frac{1+\beta_1}{2}}(V(\theta))^{\frac{1+\beta_1}{2}} - k_2 2^{\frac{1+\beta_2}{2}}(V(\theta))^{\frac{1+\beta_2}{2}}. \end{aligned} \tag{15}$$

Noticing that $0 < \beta_1 < 1, \beta_2 > 1$, it can calculate that $0 < \frac{1+\beta_1}{2} < 1, \frac{1+\beta_2}{2} > 1$. By virtue of Lemma 2.1, we can obtain that $V(\theta)$ reaches zero in a fixed time

$$T_\theta \leq \frac{1}{T_1} + \frac{1}{T_2}, \tag{16}$$

where $T_1 = k_1 2^{\frac{1+\beta_1}{2}}\frac{1-\beta_1}{2}$, $T_2 = k_2 2^{\frac{1+\beta_2}{2}}\frac{1-\beta_2}{2}$. On the other hand, if $V(\theta) = 0$, then $\theta_e = 0$. Therefore system (12c) is fixed-time stable, i. e., the desired angular velocity trajectory can be tracked in a fixed time. This completes the proof. □

3.2. Velocity control law design

In this subsection, systems (12a) – (12b) will be discussed and fixed-time controller v for the mobile robot will be developed via adding a power integrator technique.

Theorem 3.2. Consider system (2), if the controller v is chosen as

$$\begin{aligned} v = &\ v_r - \tfrac{1}{\omega_r}\big(\eta_3\bar{\sigma}^{1+m_1}(y_e) + \bar{\sigma}_3(x_e, y_e) + 1\big)\xi^{r_1+m_1-1} \\ &-\tfrac{1}{\omega_r}\big(2^{1-r_1}\bar{\sigma}(y_e) + \eta_2\bar{\sigma}^{1+r_1}(y_e) + \eta_1 + \bar{\sigma}_4(x_e, y_e) + \bar{\sigma}_1(y_e) + 1\big)\xi^{2r_1-1}, \end{aligned} \tag{17}$$

where

$$\eta_1 = \frac{r_1 2^{\frac{3}{r_1}-r_1}}{(1+r_1)^{1+\frac{1}{r_1}}}, \quad \eta_2 = \frac{r_1^{r_1}2^{2+3r_1-r_1^2}}{(1+r_1)^{1+r_1}}, \quad \eta_3 = \frac{m_1^{m_1}2^{2-r_1-r_1m_1+4m_1}}{(1+m_1)^{1+m_1}},$$

$$\bar{\sigma}(y_e) = 2^{\frac{1}{r_1}}(2-r_1)(1+y_e^{m_1-r_1})^{\frac{1}{r_1}} + 2^{\frac{1}{r_1}}(2-r_1)\frac{m_1-r_1}{r_1}(1+y_e^{m_1-r_1})^{\frac{1}{r_1}-1}y_e^{m_1-r_1},$$

$$\bar{\sigma}_1(y_e) = \frac{2-r_1}{1+r_1}\Big(\frac{8r_1-4}{1+r_1}\Big)^{\frac{2r_1-1}{2-r_1}}\omega_r^{\frac{2(1+r_1)}{2-r_1}}|y_e|^{\frac{2(1-r_1^2)}{2-r_1}},$$

$$\bar{\sigma}_2(x_e, y_e) = \frac{2^{1+r_1}}{1+r_1}\Big(\frac{4r_1}{1+r_1}\Big)^{r_1}\Big|\frac{\dot{\omega}_r}{\omega_r}\Big|^{1+r_1}|\xi|^{1-r_1^2},$$

$$\bar{\sigma}_3(x_e, y_e) = \frac{2^{1+m_1}}{1+m_1}\Big(\frac{2m_1}{1+m_1}\Big)^{m_1}\Big|\frac{\dot{\omega}_r}{\omega_r}\Big|^{1+m_1}|\xi|^{1+m_1-r_1-r_1m_1},$$

$$\bar{\sigma}_4(x_e, y_e) = \Big|\frac{\dot{\omega}_r}{\omega_r}\Big|\xi^{1-r_1} + \bar{\sigma}_2(x_e, y_e), \quad \xi = (-\omega_r x_e)^{\frac{1}{r_1}} - (-2y_e^{r_1} - 2y_e^{m_1})^{\frac{1}{r_1}},$$

in addition, $r_1 = 1+\tau_1$, $m_1 = 1+\tau_2$ and $-\frac{1}{2} < \tau_1 < 0$, $\tau_2 > 0$ which are the ratio of positive even integer and positive odd integer, then the state x_e and y_e of systems (12a) – (12b) will be stabilized to zero in a fixed time.

Proof. Define the following transformation

$$e_1 = y_e, \quad e_2 = -\omega_r x_e, \quad \theta_e = \theta_e. \tag{18}$$

Differentiating (18) and substituting (12) and control law (17) into it, one obtains

$$\begin{aligned}
\dot{e}_1 &= \frac{\omega}{\omega_r} e_2 + v_r \sin\theta_e, && \text{(19a)}\\
\dot{e}_2 &= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r \omega e_1 + \omega_r v - \omega_r v_r \cos\theta_e \\
&= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r \omega e_1 - \omega_r v_r \cos\theta_e + \omega_r v_r \\
&\quad -\big(\eta_3 \sigma^{1+m_1}(e_1) + \sigma_3(e_1, e_2) + 1\big)\xi^{r_1+m_1-1} \\
&\quad -\big(2^{1-r_1}\sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1 + \sigma_4(e_1, e_2) + \sigma_1(e_1) + 1\big)\xi^{2r_1-1}, && \text{(19b)}\\
\dot{\theta}_e &= -k_1 sig^{\beta_1}\theta_e - k_2 sig^{\beta_2}\theta_e, && \text{(19c)}
\end{aligned}$$

where $\eta_1, \eta_2, \eta_3, m_1, r_1$ are defined as above and

$$\sigma(e_1) = 2^{\frac{1}{r_1}}(2 - r_1)(1 + e_1^{m_1-r_1})^{\frac{1}{r_1}} + 2^{\frac{1}{r_1}}(2 - r_1)\frac{m_1 - r_1}{r_1}(1 + e_1^{m_1-r_1})^{\frac{1}{r_1}-1} e_1^{m_1-r_1},$$

$$\sigma_1(e_1) = \frac{2 - r_1}{1 + r_1}\left(\frac{8r_1 - 4}{1 + r_1}\right)^{\frac{2r_1-1}{2-r_1}} \omega_r^{\frac{2(1+r_1)}{2-r_1}} |e_1|^{\frac{2(1-r_1^2)}{2-r_1}},$$

$$\sigma_2(e_1, e_2) = \frac{2^{1+r_1}}{1 + r_1}\left(\frac{4r_1}{1 + r_1}\right)^{r_1} \left|\frac{\dot{\omega}_r}{\omega_r}\right|^{1+r_1} |\xi|^{1-r_1^2},$$

$$\sigma_3(e_1, e_2) = \frac{2^{1+m_1}}{1 + m_1}\left(\frac{2m_1}{1 + m_1}\right)^{m_1} \left|\frac{\dot{\omega}_r}{\omega_r}\right|^{1+m_1} |\xi|^{1+m_1-r_1-r_1 m_1},$$

$$\sigma_4(e_1, e_2) = \left|\frac{\dot{\omega}_r}{\omega_r}\right| \xi^{1-r_1} + \sigma_2(e_1, e_2), \quad \xi = e_2^{\frac{1}{r_1}} - (-2e_1^{r_1} - 2e_1^{m_1})^{\frac{1}{r_1}}.$$

According to transformation (18), we only need to prove that $e_j = 0 (j = 1, 2)$ in a fixed time. Based on Theorem 3.1, we can obtain that $\theta_e(t) = 0$ in a fixed time T_θ. Thus, for any $t > T_\theta$, $\omega_r = \omega$ and the closed-loop system (19) can be rewritten as follows

$$\begin{aligned}
\dot{e}_1 &= e_2, && \text{(20a)}\\
\dot{e}_2 &= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r \omega e_1 + \omega_r v - \omega_r v_r \\
&= \frac{\dot{\omega}_r}{\omega_r} e_2 - \omega_r^2 e_1 - \big(\eta_3 \sigma^{1+m_1}(e_1) + \sigma_3(e_1) + 1\big)\xi^{r_1+m_1-1} \\
&\quad -\big(2^{1-r_1}\sigma(e_1) + \eta_2 \sigma^{1+r_1}(e_1) + \eta_1 + \sigma_4(e_1, e_2) + \sigma_1(e_1) + 1\big)\xi^{2r_1-1}. && \text{(20b)}
\end{aligned}$$

Two steps will be given in this part and adding a power integrator technique is employed.

Step 1 Choose the following Lyapunov function candidate

$$V_1(e_1) = \frac{1}{2}e_1^2, \tag{21}$$

whose derivative along system (20) is

$$\dot{V}_1(e_1) = e_1 e_2 = e_1 e_2^* + e_1(e_2 - e_2^*). \tag{22}$$

With the help of the backstepping design idea, a virtual control law is designed as

$$e_2^* = -2e_1^{r_1} - 2e_1^{m_1}, \tag{23}$$

which leads to

$$\dot{V}_1(e_1) \leq -2e_1^{1+r_1} - 2e_1^{1+m_1} + e_1(e_2 - e_2^*). \tag{24}$$

Step 2 The Lyapunov function is constructed as

$$V_2(e_1, e_2) = V_1(e_1) + \int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}})^{2-r_1}\,\mathrm{d}s. \tag{25}$$

According to the results in paper [22], we can obtain that $\int_{e_2^*}^{e_2}(s^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}})^{2-r_1}\,\mathrm{d}s$ is differentiable, positive definite and proper. For brevity, denote $\xi = e_2^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}}$. The derivative of $V_2(e_1, e_2)$ along systems (20) and (24) is

$$\begin{aligned} \dot{V}_2(e_1, e_2) \ &\leq -2e_1^{1+r_1} - 2e_1^{1+m_1} + e_1(e_2 - e_2^*) + \xi^{2-r_1}\dot{e}_2 \\ &\quad +(2 - r_1)\tfrac{d(-e_2^{*\frac{1}{r_1}})}{dt} \int_{e_2^*}^{e_2} (s^{\frac{1}{r_1}} - e_2^{*\frac{1}{r_1}})^{1-r_1}\,\mathrm{d}s. \end{aligned} \tag{26}$$

Using Lemmas 2.4 and 2.5, one obtains

$$e_1(e_2 - e_2^*) \leq |e_1||(e_2^{\frac{1}{r_1}})^{r_1} - (e_2^{*\frac{1}{r_1}})^{r_1}| \leq 2^{1-r_1}|e_1||\xi|^{r_1} \leq \tfrac{1}{4}|e_1|^{1+r_1} + \eta_1|\xi|^{1+r_1}. \tag{27}$$

Noticing that

$$-e_2^{*\frac{1}{r_1}} = \left(2e_1^{r_1}(1 + e_1^{m_1-r_1})\right)^{\frac{1}{r_1}} = 2^{\frac{1}{r_1}} e_1(1 + e_1^{m_1-r_1})^{\frac{1}{r_1}}, \tag{28}$$

which leads to

$$\begin{aligned} (2 - r_1)\tfrac{d(-e_2^{*\frac{1}{r_1}})}{de_1} \ &= \ 2^{\frac{1}{r_1}}(2 - r_1)(1 + e_1^{m_1-r_1})^{\frac{1}{r_1}} \\ &\quad +2^{\frac{1}{r_1}}(2 - r_1)\tfrac{m_1-r_1}{r_1}(1 + e_1^{m_1-r_1})^{\frac{1}{r_1}-1}e_1^{m_1-r_1} \\ &\triangleq \ \sigma(e_1). \end{aligned} \tag{29}$$

In addition, based on Lemma 2.2, from (23) and the definition of ξ, we have

$$|e_2| = |\xi + e_2^{*\frac{1}{r_1}}|^{r_1} \leq |\xi|^{r_1} + |e_2^*| \leq |\xi|^{r_1} + 2|e_1|^{r_1} + 2|e_1|^{m_1}. \tag{30}$$

By Lemma 2.4, we can also obtain that

$$\int_{e_2^*}^{e_2}(s^{\frac{1}{r_1}}-e_2^{*\frac{1}{r_1}})^{1-r_1}\,\mathrm{d}s \le |\xi|^{1-r_1}|e_2-e_2^*| = |\xi|^{1-r_1}|(e_2^{\frac{1}{r_1}})^{r_1}-(e_2^{*\frac{1}{r_1}})^{r_1}| \le 2^{1-r_1}|\xi|. \quad (31)$$

From (29), (30), (31) and Lemma 2.5, one obtains

$$\begin{aligned} & (2-r_1)\frac{\mathrm{d}(-e_2^{*\frac{1}{r_1}})}{\mathrm{d}t}\int_{e_2^*}^{e_2}(s^{\frac{1}{r_1}}-e_2^{*\frac{1}{r_1}})^{1-r_1}\,\mathrm{d}s \\ = \; & (2-r_1)\frac{\mathrm{d}(-e_2^{*\frac{1}{r_1}})}{de_1}\frac{de_1}{\mathrm{d}t}\int_{e_2^*}^{e_2}(s^{\frac{1}{r_1}}-e_2^{*\frac{1}{r_1}})^{1-r_1}\,\mathrm{d}s \\ \le \; & \sigma(e_1)(|\xi|^{r_1}+2|e_1|^{r_1}+2|e_1|^{m_1})2^{1-r_1}|\xi| \\ \le \; & \tfrac{1}{4}|e_1|^{1+r_1}+\tfrac{1}{2}|e_1|^{1+m_1}+\big(2^{1-r_1}\sigma(e_1) \\ & +\eta_2\sigma^{1+r_1}(e_1)\big)|\xi|^{1+r_1}+\eta_3\sigma^{1+m_1}(e_1)|\xi|^{1+m_1}. \end{aligned} \quad (32)$$

Substituting (27) and (32) into (26), we have

$$\begin{aligned} \dot V_2(e_1,e_2) \le \; & -\tfrac{3}{2}|e_1|^{1+r_1}-\tfrac{3}{2}|e_1|^{1+m_1}+\eta_3\sigma^{1+m_1}(e_1)|\xi|^{1+m_1} \\ & +\big(2^{1-r_1}\sigma(e_1)+\eta_2\sigma^{1+r_1}(e_1)+\eta_1\big)|\xi|^{1+r_1}+\xi^{2-r_1}\dot e_2 \\ \le \; & -\tfrac{3}{2}|e_1|^{1+r_1}-\tfrac{3}{2}|e_1|^{1+m_1}+\eta_3\sigma^{1+m_1}(e_1)|\xi|^{1+m_1} \\ & +\big(2^{1-r_1}\sigma(e_1)+\eta_2\sigma^{1+r_1}(e_1)+\eta_1\big)|\xi|^{1+r_1}+\xi^{2-r_1}\dot e_2. \end{aligned} \quad (33)$$

Combining (20b) with (33), yields

$$\begin{aligned} \dot V_2(e_1,e_2) \le \; & -\tfrac{3}{2}|e_1|^{1+r_1}-\tfrac{3}{2}|e_1|^{1+m_1}-|\xi|^{1+m_1}-|\xi|^{1+r_1}+|\xi|^{2-r_1}|\tfrac{\dot\omega_r}{\omega_r}||e_2| \\ & +|\xi|^{2-r_1}\omega_r^2|e_1|-\sigma_3(e_1,e_2)|\xi|^{1+m_1}-\sigma_4(e_1,e_2)|\xi|^{1+r_1} \\ & -\sigma_1(e_1)|\xi|^{1+r_1}. \end{aligned} \quad (34)$$

By Lemma 2.5, we obtain

$$|\xi|^{2-r_1}|\omega_r^2||e_1| \le |\omega_r^{\frac{2}{2-r_1}}e_1^{\frac{2-2r_1}{2-r_1}}\xi|^{2-r_1}|e_1|^{2r_1-1} \le \sigma_1(e_1)|\xi|^{1+r_1}+\frac{1}{4}|e_1|^{1+r_1}. \quad (35)$$

Note that $\xi=e_2^{\frac{1}{r_1}}-e_2^{*\frac{1}{r_1}}$, $e_2^*=-2e_1^{r_1}-2e_1^{m_1}$, using Lemmas 2.2 and 2.5, we have

$$\begin{aligned} |\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|e_2| \le \; & |\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|\xi+e_2^{*\frac{1}{r_1}}|^{r_1} \le |\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}(|\xi|^{r_1}+|e_2^*|) \\ = \; & |\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|\xi|^{r_1}+|\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|2e_1^{r_1}+2e_1^{m_1}| \\ \le \; & |\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|\xi|^{r_1}+2|\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|e_1|^{r_1}+2|\tfrac{\dot\omega_r}{\omega_r}||\xi|^{2-r_1}|e_1|^{m_1} \\ \le \; & |\tfrac{\dot\omega_r}{\omega_r}||\xi|^{1-r_1}|\xi|^{1+r_1}+\tfrac{1}{4}|e_1|^{1+r_1}+\sigma_2(e_1,e_2)|\xi|^{1+r_1} \\ & +\tfrac{1}{2}|e_1|^{1+m_1}+\sigma_3(e_1,e_2)|\xi|^{1+m_1} \\ = \; & \sigma_4(e_1,e_2)|\xi|^{1+r_1}+\tfrac{1}{4}|e_1|^{1+r_1}+\tfrac{1}{2}|e_1|^{1+m_1}+\sigma_3(e_1,e_2)|\xi|^{1+m_1}. \end{aligned} \quad (36)$$

Substituting (35) and (36) into (34), one obtains

$$\dot V_2(e_1,e_2) \le -|e_1|^{1+r_1}-|e_1|^{1+m_1}-|\xi|^{1+m_1}-|\xi|^{1+r_1}. \quad (37)$$

On the other hand, with the definition of $V_2(e_1, e_2)$ in (25), it follows from Lemma 2.4 that

$$V_2(e_1,e_2) \le \frac{1}{2}e_1^2 + 2^{1-r_1}\xi^2 \le \lambda(e_1^2+\xi^2), \tag{38}$$

where $\lambda = \max\{\frac{1}{2}, 2^{1-r_1}\}$. By Lemmas 2.2-2.3, it can be concluded that

$$(e_1^2+\xi^2)^{\frac{1+r_1}{2}} \le |e_1|^{1+r_1} + |\xi|^{1+r_1}, \tag{39}$$

and

$$(e_1^2+\xi^2)^{\frac{1+m_1}{2}} \le 2^{\frac{m_1-1}{2}}(|e_1|^{1+m_1} + |\xi|^{1+m_1}). \tag{40}$$

With the help of these two inequalities, it follows from (37) and (38) that

$$\dot{V}_2(e_1,e_2) \le -\lambda^{-\frac{1+r_1}{2}} V_2^{\frac{1+r_1}{2}}(e_1,e_2) - 2^{-\frac{m_1-1}{2}}\lambda^{-\frac{1+m_1}{2}} V_2^{\frac{1+m_1}{2}}(e_1,e_2). \tag{41}$$

Based on Lemma 2.1, we conclude that $V_2(e_1, e_2)$ reaches zero in a fixed time. In other words, there exists a time constant $T_0 = \lambda^{\frac{1+m_1}{2}}\frac{2}{1-r_1} + 2^{\frac{m_1-1}{2}}\lambda^{\frac{1+m_1}{2}}\frac{2}{m_1-1} < \infty$, such that $V_2(e_1, e_2) = 0$, $\forall t \ge T_0$. It means that $e_1 = 0$ and $e_2 = 0$ in fixed time. Therefore, one can concludes that system (19a) and (19b) with the controller (17) is globally fixed-time stable. □

Remark 1. It is worth mentioning that we do not prove that the control law (17) can guarantee the boundedness of states $e_j = 0 (j = 1, 2)$ in the interval $[0,\ T_\theta]$, it is mainly because the analysis of the dynamics of the closed-loop system is a difficult task due to the complex nonlinear items. In simulation section, we have done a great number of simulations for the nonholonomic mobile robot systems (2) and (3) under the control laws (13) and (17). We do not observe any divergence phenomenon. Actually, in practice, to guarantee the boundedness of system states, we can employ a bounded control law in the interval $[0,\ T_\theta]$.

By virtue of Theorems 3.1–3.2 and Remark 1, we have the following main result.

Theorem 3.3. For the nonholonomic mobile robot systems (2), if the control laws ω and v are designed as (13) and (17), then system (2) can globally track the desired reference trajectory (3) in a fixed time, where the control parameters used in (13) and (17) are chosen as those in above Theorems 3.1 and 3.2.

Proof. Firstly, based on Theorems 3.1 and 3.2, we get that states θ_e, e_1 and e_2 in system (19) can reach zero in fixed time under control laws (13) and (17). Secondly, combining the state transformation equations (10) and (18), it is shown that θ_e, e_1 and e_2 reach zero implies that $x_r = x$, $y_r = y$, $\theta_r = \theta$. Thus, nonholonomic mobile robot system (2) can globally track the desired reference trajectory (3) in a fixed time. □

Remark 2. The authors of paper [19] has discussed finite-time tracking control problem for systems (2) and (3), and distributed finite-time tracking control laws have been given as follows

$$\omega = \omega_r + \mathcal{K}_1 sig^{\beta}\theta_e, \tag{42a}$$

$$v = v_r - \frac{1}{\omega_r}\big(\mathcal{K}_3 + \rho_1(y_e) + \rho_2(x_e,y_e)\big)(\mathcal{K}_2^p y_e - \omega_r^p x_e^p)^{\frac{2}{p}-1} \tag{42b}$$

where $0 < \beta < 1$, $\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3 > 0$ are appropriate constants, $1 < p = \frac{p_1}{p_2} < 2$, p_1, p_2 are positive odd integers, $\rho_1(y_e) = \frac{2p-1}{1+p}\omega_r^{\frac{2(p+1)}{2p-1}} y_e^{\frac{2(p^2-1)}{p(2p-1)}}$, $\rho_2(x_e, y_e) = |\frac{\dot{\omega}_r}{\omega_r}|(\mathcal{K}_2^p y_e - \omega_r^p x_e^p)^{1-\frac{1}{p}} + \frac{2p-1}{1+p}(\frac{\dot{\omega}_r}{\omega_r}\mathcal{K}_2)^{\frac{1+p}{2p-1}} y_e^{\frac{p^2-1}{p(2p-1)}}$. Compared with the finite-time control laws (42), the main advantage of the proposed fixed-time control result lies in the convergence time can be pre-determined without considering the initial condition. The simulations will illustrate this statement in the next section.

4. SIMULATION RESULTS

In this section, a numerical example is provided to illustrate our theoretical results derived in the previous section, and two cases will be considered. In the first case, simulation results will be given to show the effectiveness of the proposed fixed-time control laws (13) and (17). In the second case, under different initial condition, we will compare the convergent performance of two kinds of control laws, i. e. fixed-time control laws (13) and (17), and finite-time control law (42).

Case 1: For system (3), the desired reference velocities are chosen as $v_r = 1.5 - \frac{1.5t}{t+10} m/s$, $\omega_r = 1 + \frac{2t}{t+10} rad/s$. Let the initial value $[x_r(0), y_r(0), \theta_r(0)] = (2, 1.5, 0)$, $[x(0), y(0), \theta(0)] = (-0.4, 1, 0.1)$. The control gains of fixed-time control laws (13) and (17) are selected $k_1 = k_2 = 2$, the value of the fraction power are taken as $\tau_1 = -\frac{2}{87}$, $\tau_2 = \frac{2}{87}$, $\beta_2 = \frac{9}{7}$ and $\beta_1 = \frac{7}{9}$. The simulation results are shown in Figures 2–5.

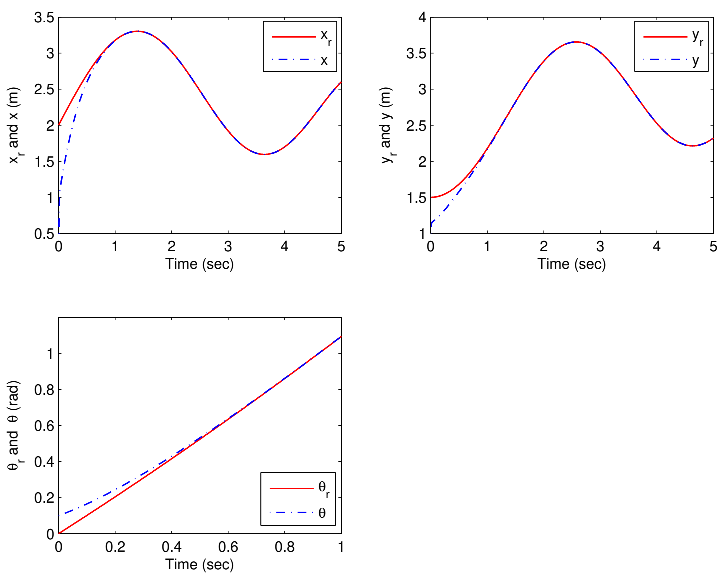

Fig. 2. Response state curves for x_r, y_r, θ_r and x, y, θ.

Figure 2 shows response state curves for x_r, y_r, θ_r and x, y, θ. Figure 3 shows the tracking errors x_e, y_e and θ_e respect to time for the robot. Figure 4 shows response desired trajectory and tracking curves. Figure 5 shows the control outputs of v and ω, respectively. According to Figures 2 − 4, it is easy to observe that fixed-time control laws (13) and (17) can make the system states converge to the desired trajectory in the fixed time.

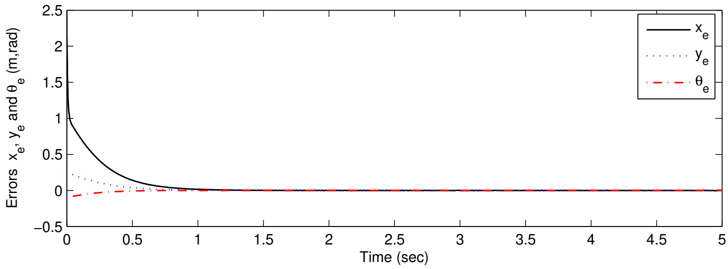

Fig. 3. Response tracking errors curves for x_e, y_e and θ_e.

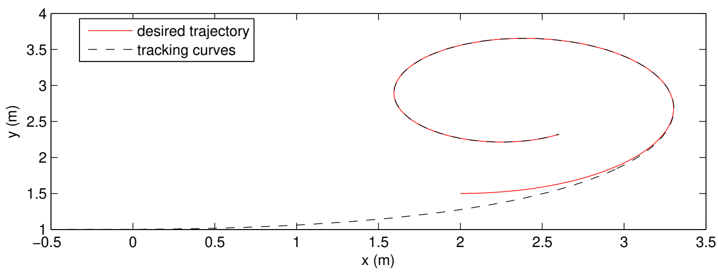

Fig. 4. Response desired trajectory and tracking curves.

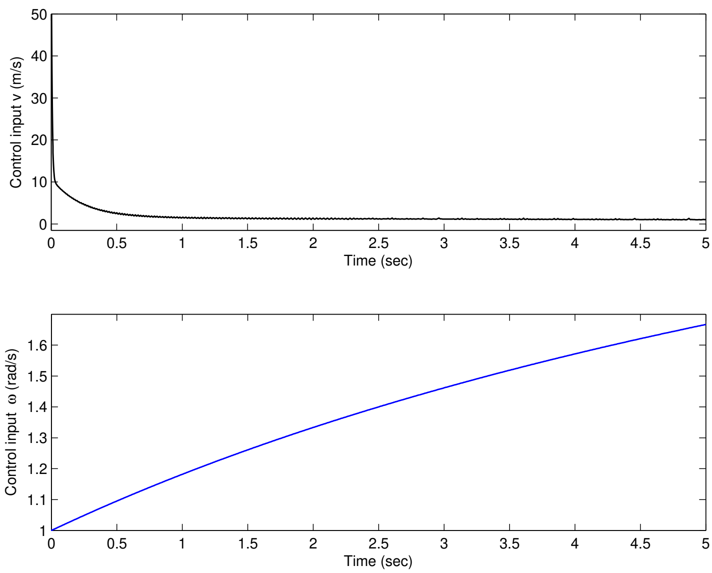

Fig. 5. Response curves of control outputs v and ω.

Case 2: In this case, based on remark 2, under different initial conditions for fixed-time control laws (13) and (17), and finite-time control law (42), we will compare the convergent performance of these two kinds of control laws. Simulation result is shown in Figure 6, where $\delta(0) = \sqrt{(x_r(0) - x(0))^2 + (y_r(0) - y(0))^2}$, it can be seen that the statement that the convergence time is independent of initial state for fixed-time control laws.

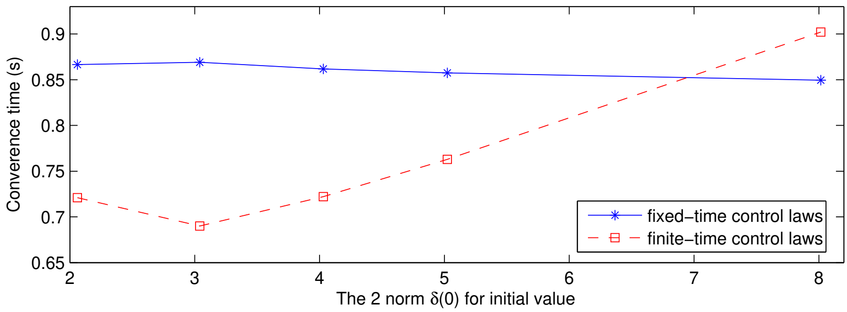

Fig. 6. The convergence time for the different initial conditions.

5. CONCLUSIONS

In this paper, we have investigated the problem of fixed-time tracking control for a nonholonomic mobile robot system. Rigorous theoretic analysis shows that the proposed fixed-time controllers can make the mobile robot track the desired reference trajectory in a fixed time. Simulation results been presented to support the theoretical results.

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