ROBUST OBSERVER-BASED FINITE-TIME H_∞ CONTROL DESIGNS FOR DISCRETE NONLINEAR SYSTEMS WITH TIME-VARYING DELAY

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This paper investigates the problem of observer-based finite-time H_∞ control for the uncertain discrete-time systems with nonlinear perturbations and time-varying delay. The Luenberger observer is designed to measure the system state. The observer-based controller is constructed. By constructing an appropriated Lyapunov-.Krasovskii functional, sufficient conditions are derived to ensure the resulting closed-loop system is H_∞ finite-time bounded via observer-based control. The observer-based controller for the finite-time H_∞ control problem is developed. Finally, a numerical example illustrates the efficiency of proposed methods.

Keywords: observer-based control, H_∞ finite-time boundedness, Lyapunov–Krasovskii functional, discrete-time systems, time-varying delay

Classification: 93D15, 93D09, 93C10, 93B35, 93B52

1. INTRODUCTION

Feedback control is a long-standing topic and has been one of the key research areas in control theory [7, 8, 16]. State feedback control is developed on the premise that the complete state information of the system is available. But often this is not true in practice, and the current state must be estimated by another dynamical system, a state observer [10, 17, 18, 20]. In recent years, observer-based control has attracted the attention of scholars, and some results have been obtained [3, 10, 15]. In [10], the design of observer-based feedback control was presented for a class of discrete-time nonlinear systems. Ahmad et al. [3] considered the observer-based control for one-sided Lipschitz systems.

In many practical applications, it is necessary to consider the behavior of the system in a fixed finite time interval. Within a given finite time interval, the state of the system will not exceed a bound. Therefore, the concepts of finite-time stability and finite-time boundedness were proposed in [4, 11]. In recent years, finite-time stability, finite-time boundedness and related control problems have attracted the attention of researchers, and many results have been reported in [4, 5, 9, 14, 21]. Zhang et al. [21] investigated finite-time stability and stabilization of uncertain continuous-time delay system. Amato

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et al. [4] considered the finite-time control of linear system. In [5], finite-time H_∞ filtering problem for discrete-time Markovian jump systems was investigated.

On the other hand, in practical systems, the phenomenon of time delay is inevitable. The existence of time delay can deteriorate system performance or lead to system instability. Therefore, the stability and control of time-delay systems have attracted wide attention in academic circles. In recent years, many results about time-delay systems have been presented [6, 7, 9, 10, 12, 13, 19]. In [6], the observer design of neutral neural networks with discrete and distributed time-varying delays was given. Stojanovic [19] investigated the robust finite-time stability for discrete-time systems with interval time-varying delays and nonlinear perturbations. Lin et al. [13] considered the finite-time boundedness and L_2 gain analysis for switched delay systems. However, as far as we know, for uncertain discrete-time systems with nonlinear perturbations and time-varying delays, finite-time boundedness and observer-based finite-time H_∞ stabilization have not been thoroughly studied.

In this paper, we investigate the observer-based finite-time control for discrete uncertain nonlinear systems with time-varying delay. The main contributions of this paper are as follows: First, by constructing an appropriate Lyapunov–Krasovsky functional, utilizing sector-bounded conditions and observer-based control, the sufficient conditions are derived in term of matrices inequality technique which ensure that the closed-loop system obtained is finite-time bounded with H_∞ performance. Then, the observer-based finite-time H_∞ control is designed to guarantee the closed-loop system has desired performance. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, the system description and preliminaries are introduced. In Section 3, based on matrix inequalities and observer-based control, sufficient conditions for H_∞ finite-time stabilization of uncertain discrete-time systems are developed, and H_∞ finite-time observer-based control is given. In Section 4, we give an example to show the performance of our method. Conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

Consider the following uncertain discrete-time system

$$\begin{array}{lcl} x(k+1) & = & (A+\Delta A)x(k) + (A_d+\Delta A_d)x(k-\tau(k)) + Du(k) + Gf(x(k)) \\ & & +Hg(x(k-\tau(k))) + (B+\Delta B)\omega(k), \\ y(t) & = & Cx(k), \\ z(k) & = & N_1x(k) + N_3x(k-\tau(k)) + N_2\omega(k), \\ x(\theta) & = & \varphi(\theta), \quad \theta = -\tau_M,\ -\tau_M+1, \ldots, 0, \end{array} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^p$ is the measured output, $z(k) \in \mathbb{R}^q$ is the control output, $\varphi(\theta)$ is an initial function, $u(k) \in \mathbb{R}^m$ is the control input. $\omega(k) \in \mathbb{R}^l$ is the exogenous disturbance. $A, A_d, B, C, D, N_1, N_2, N_3, G, H$ are appropriate dimension constant matrices. The positive integer $\tau(k)$ is the time -varying delay satisfying

$$0 < \tau_m \le \tau(k) \le \tau_M, \tag{2}$$

where τ_m, τ_M are positive integers. Assume that the admissible uncertainties $\Delta A, \Delta A_d$ and ΔB satisfy

$$[\Delta A,\ \Delta A_d,\ \Delta B] = \Gamma Y(k)[K_a,\ K_d,\ K_b],$$

where Γ, K_a, K_d, K_b are known real constant matrices, $Y(k)$ is unknown time-varying matrix satisfying

$$Y^T(k)Y(k) \leq I, \quad \forall K \in N^+.$$

The nonlinear functions $f(\cdot)$ and $g(\cdot)$ are assumed to be continuous and satisfy $f(0) = 0, g(0) = 0$ and satisfy the following sector-bounded conditions:

$$\begin{array}{l} [f(x) - f(y) - F_1(x-y)]^T[f(x) - f(y) - F_2(x-y)] \leq 0, \quad \forall x, y \in \mathbb{R}^n, \\ [g(x) - g(y) - U_1(x-y)]^T[g(x) - g(y) - U_2(x-y)] \leq 0, \quad \forall x, y \in \mathbb{R}^n, \end{array} \tag{3}$$

where F_1, F_2, U_1, U_2 are real matrices of appropriate dimensions. $\omega(k) \in \mathbb{R}^l$ is the exogenous disturbance which satisfies

$$\sum_{k=0}^{M} \omega^T(k)\omega(k) < d^2. \tag{4}$$

We construct an observer-based controller for system (1) as follows

$$\begin{array}{rcl} \hat{x}(k+1) &=& A\hat{x}(k) + A_d\hat{x}(k-\tau(k)) + Du(k) + L(y - C\hat{x}(k)) + Gf(\hat{x}(k)) \\ && +Hg(\hat{x}(k-\tau(k))), \\ u(k) &=& K\hat{x}(k), \end{array} \tag{5}$$

where $\hat{x}(k)$ is the estimated state, $L \in \mathbb{R}^p$ is the observer gain required to be designed later. Let $e(k) = x(k) - \hat{x}(k)$. The error dynamic system is

$$\begin{array}{rcl} e(k+1) &=& (A - LC)e(k) + A_d e(k-\tau(k)) + \Delta A x(k) + \Delta A_d x(k-\tau(k)) \\ && +G(f(x(k)) - f(\hat{x}(k))) + H(g((k-\tau(k)) - g(\hat{x}(k-\tau(k)))). \end{array} \tag{6}$$

Let $\eta(k) = [x^T(k), e^T(k)]^T$, then by (1),(4)and(5), we can get the following closed-loop system

$$\begin{array}{l} \eta(k+1) = \bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k))), \\ z(k) = \bar{N}_1\eta(k) + \bar{N}_3\eta(k-\tau(k)) + N_2\omega(k), \end{array} \tag{7}$$

where

$$\bar{A} = \breve{A} + \Delta\bar{A}, \quad \breve{A} = \begin{bmatrix} A + DK & -DK \\ 0 & A - LC \end{bmatrix}, \quad \Delta\bar{A} = \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix} = \bar{\Gamma}Y(k)\bar{K}_a,$$

$$\bar{\Gamma} = \begin{bmatrix} \Gamma \\ \Gamma \end{bmatrix}, \quad \breve{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \quad \Delta\bar{A}_d = \begin{bmatrix} \Delta A_d & 0 \\ \Delta A_d & 0 \end{bmatrix} = \bar{\Gamma}Y(k)\bar{K}_d,$$

$$\bar{A}_d = \breve{A}_d + \Delta\bar{A}_d, \quad \bar{K}_a = [K_a \quad 0], \quad \bar{K}_d = [K_d \quad 0], \quad \bar{N}_1 = [N_1 \quad 0],$$

$$\bar{N}_3 = [N_3 \quad 0], \quad \bar{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}, \quad \bar{B} = \breve{B} + \Delta\bar{B},$$

$$\breve{B} = \begin{bmatrix} B \\ B \end{bmatrix}, \quad \Delta\bar{B} = \begin{bmatrix} \Delta B \\ \Delta B \end{bmatrix} = \bar{\Gamma}Y(k)\bar{K}_b, \quad \bar{f}(\eta(k)) = \begin{bmatrix} f(x(k)) \\ f(x(k)) - f(\hat{x}(k)) \end{bmatrix},$$

$$\bar{g}(\bar{x}(k-\tau(k))) = \begin{bmatrix} g(x(k-\tau(k))) \\ g(x(k-\tau(k))) - g(\hat{x}(k-\tau(k))) \end{bmatrix}.$$

Definition 1. The closed-loop system (7) is said to be finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M)$ with $0 < \gamma_1 < \gamma_2, R > 0, M \in Z^+$, if

$$\sup_{\theta \in \{-\tau_M, -\tau_M+1, \ldots, 0\}} \{\eta^T(\theta) R \eta(\theta)\} \le \gamma_1 \Rightarrow \eta^T(k) R \eta(k) < \gamma_2, \ \forall k \in \{1, \ldots, M\},$$

for all disturbance $\omega(k)$ satisfying (4).

Definition 2. The closed-loop system (7) is said to be H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$ with $0 < \gamma_1 < \gamma_2, R > 0, M \in Z^+$, if the system (7) is finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M)$, and under zero-initial condition, for any exogenous disturbance $\omega(k)$ satisfying (4), the output $z(k)$ satisfies

$$\sum_{k=0}^{M} z^T(k) z(k) \le \gamma^2 \sum_{k=0}^{M} \omega^T(k) \omega(k).$$

The main task of this paper is to design an observer-based control to ensure that the discrete systems (7) is H_∞ finite-time bounded.

3. MAIN RESULTS

In this section, we give sufficient conditions which guarantee that the resultant closed-loop system (7) is H_∞ finite-time bounded via observer-based control.

Theorem 1. For given integers $0 < \tau_m \le \tau_M$, the closed-loop system (7) is H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$, if there exist matrices $\bar{P} = diag(P, P) > 0, \bar{R} = diag(R_1, R_2) > 0$, positive scalars $\mu_1, \mu_2, \varepsilon$ such that the following conditions hold:

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_{11} & 0 & 0 & -\mu_1 \bar{F}_2 & 0 & \breve{A}^T \bar{P} & \bar{N}_1^T & 0 & \bar{K}_a^T \\ * & -\bar{R} - \mu_2 \bar{U}_1 & 0 & 0 & -\mu_2 \bar{U}_2 & \breve{A}_d^T \bar{P} & \bar{N}_3^T & 0 & \bar{K}_d^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T \bar{P} & \bar{N}_2^T & 0 & \bar{K}_b^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T \bar{P} & 0 & 0 & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T \bar{P} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{P} & 0 & \bar{P}\bar{\Gamma} & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & 0 & -\varepsilon I & 0 \\ * & * & * & * & * & * & 0 & 0 & -\varepsilon^{-1} I \end{bmatrix} < 0, \tag{9}$$

$$\gamma_1 [\lambda_{\max}(H_1) + (\tau_M + (\tau_M + \tau_m - 1)(\tau_M - \tau_m)/2) \lambda_{\max}(H_2)] + \gamma d^2 < \gamma_2 \lambda_{\min}(H_1), \tag{10}$$

where

$$\breve{A} = \begin{bmatrix} A+DK & -DK \\ 0 & A-LC \end{bmatrix}, \quad \bar{\Gamma} = \begin{bmatrix} \Gamma \\ \Gamma \end{bmatrix}, \quad \breve{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix},$$
$$\bar{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} B \\ B \end{bmatrix}, \quad \bar{K}_a = [K_a \quad 0], \quad \bar{K}_d = [K_d \quad 0], \quad \bar{N}_1 = [N_1 \quad 0],$$
$$\bar{N}_3 = [N_3 \quad 0], \quad \bar{F}_1 = I \otimes [(F_1^T F_2 + F_2^T F_1)/2], \quad \bar{F}_2 = -I \otimes [(F_1 + F_2)/2],$$
$$\bar{U}_1 = I \otimes [(U_1^T U_2 + U_2^T U_1)/2], \quad \bar{U}_2 = -I \otimes [(U_1 + U_2)/2], \quad \bar{\tau} = \tau_M - \tau_m + 1,$$
$$H_1 = R^{-1/2} \bar{P} R^{-1/2}, \quad H_2 = R^{-1/2} \bar{R} R^{-1/2}, \quad \bar{\Sigma}_{11} = \bar{\tau}\bar{R} - \bar{P} - \mu_1 \bar{F}_1.$$

Proof. Construct the Lyapunov–Krasovskii functional candidate

$$V(k) = \eta^T(k)\bar{P}\eta(k) + \sum_{i=k-\tau(k)}^{k-1} \eta^T(i)\bar{R}\eta(i) + \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{i=k-1+j}^{k-1} \eta^T(i)\bar{R}\eta(i), \tag{11}$$

where $\bar{P} = diag(P, P) > 0, \bar{R} = diag(R_1, R_2) > 0$.

The difference of $V(k)$ along the solution of the closed-loop system (7) is given by

$$\begin{aligned}
\Delta V(k) &= V(k+1) - V(k) \\
&= [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))]^T \bar{P} \\
&\quad \times [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))] \\
&\quad -\eta^T(k)\bar{P}\eta(k) + \sum_{i=k+1-\tau(k+1)}^{k} \eta^T(i)\bar{R}\eta(i) - \sum_{i=k-\tau(k)}^{k-1} \eta^T(i)\bar{R}\eta(i) \\
&\quad + \sum_{j=-\tau_M+1}^{-\tau_m} \Big(\sum_{i=k+j}^{k} \eta^T(i)\bar{R}\eta(i) - \sum_{i=k+j-1}^{k-1} \eta^T(i)\bar{R}\eta(i) \Big) \\
&\le [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))]^T \bar{P} \\
&\quad \times [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))] \\
&\quad -\eta^T(k)\bar{P}\eta(k) + \eta^T(k)\bar{R}\eta(k) - \eta^T(k-\tau(k))\bar{R}\eta(k-\tau(k)) \\
&\quad + \sum_{i=k+1-\tau_M}^{k-\tau_m} \eta^T(i)\bar{R}\eta(i) + \sum_{j=-\tau_M+1}^{-\tau_m} (\eta^T(k)\bar{R}\eta(k) - \eta^T(k+j)\bar{R}\eta(k+j)) \\
&= [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))]^T \bar{P} \\
&\quad \times [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))] \\
&\quad -\eta^T(k)\bar{P}\eta(k) - \eta^T(k-\tau(k))\bar{R}\eta(k-\tau(k)) + \sum_{i=k+1-\tau_M}^{k-\tau_m} \eta^T(i)\bar{R}\eta(i) \\
&\quad +(\tau_M - \tau_m + 1)\eta^T(k)\bar{R}\eta(k) - \sum_{i=k+1-\tau_M}^{k-\tau_m} \eta^T(i)\bar{R}\eta(i).
\end{aligned} \tag{12}$$

From (3), it follows that

$$\begin{aligned}
&\begin{bmatrix} \eta(k) \\ \bar{f}(\eta(k)) \end{bmatrix}^T \begin{bmatrix} \bar{F}_1 & \bar{F}_2 \\ \bar{F}_2^T & I \end{bmatrix} \begin{bmatrix} \eta(k) \\ \bar{f}(\eta(k)) \end{bmatrix} \le 0, \\
&\begin{bmatrix} \eta(k-\tau(k)) \\ \bar{g}(\eta(k-\tau(k))) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{bmatrix} \begin{bmatrix} \eta(k-\tau(k)) \\ \bar{g}(\eta(k-\tau(k))) \end{bmatrix} \le 0,
\end{aligned} \tag{13}$$

where

$$\bar{F}_1 = I \otimes [(F_1^T F_2 + F_2^T F_1)/2], \quad \bar{F}_2 = -I \otimes [(F_1 + F_2)/2],$$
$$\bar{U}_1 = I \otimes [(U_1^T U_2 + U_2^T U_1)/2], \quad \bar{U}_2 = -I \otimes [(U_1 + U_2)/2].$$

From (13) and (14), we get

$$\begin{aligned}
\Delta V(k) \leq\ & [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))]^T \bar{P} \\
& \times [\bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k)))] \\
& -\eta^T(k)\bar{P}\eta(k) - \eta^T(k-\tau(k))\bar{R}\eta(k-\tau(k)) + (\tau_M - \tau_m + 1)\eta^T(k)\bar{R}\eta(k) \\
& -\mu_1 \begin{bmatrix} \eta(k) \\ \bar{f}(\eta(k)) \end{bmatrix}^T \begin{bmatrix} \bar{F}_1 & \bar{F}_2 \\ \bar{F}_2^T & I \end{bmatrix} \begin{bmatrix} \eta(k) \\ \bar{f}(\eta(k)) \end{bmatrix} \\
& -\mu_2 \begin{bmatrix} \eta(k-\tau(k)) \\ \bar{g}(\eta(k-\tau(k))) \end{bmatrix}^T \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2^T & I \end{bmatrix} \begin{bmatrix} \eta(k-\tau(k)) \\ \bar{g}(\eta(k-\tau(k))) \end{bmatrix} \\
& = \varsigma^T(k)\Theta\varsigma(k),
\end{aligned} \tag{14}$$

where

$$\varsigma^T(k) = [\eta^T(k) \quad \eta^T(k-\tau(k)) \quad \omega^T(k) \quad \bar{f}^T(\eta(k)) \quad \bar{g}^T(\eta(k-\tau(k)))],$$

$$\Theta = \begin{bmatrix}
\Theta_{11} & \bar{A}^T\bar{P}\bar{A}_d & \bar{A}^T\bar{P}\bar{B} & \bar{A}^T\bar{P}\bar{G} - \mu_1\bar{F}_2 & \bar{A}^T\bar{P}\bar{H} \\
* & \Theta_{22} & \bar{A}_d^T\bar{P}\bar{B} & \bar{A}_d^T\bar{P}\bar{G} & \bar{A}_d^T\bar{P}\bar{H} - \mu_2\bar{U}_2 \\
* & * & \bar{B}^T\bar{P}\bar{B} & \bar{B}^T\bar{P}\bar{G} & \bar{B}^T\bar{P}\bar{H} \\
* & * & * & -\mu_1 I + \bar{G}^T\bar{P}\bar{G} & \bar{G}^T\bar{P}\bar{H} \\
* & * & * & * & -\mu_2 I + \bar{H}^T\bar{P}\bar{H}
\end{bmatrix},$$

$$\Theta_{11} = \bar{\tau}\bar{R} - \bar{P} + \bar{A}^T\bar{P}\bar{A} - \mu_1\bar{F}_1,$$
$$\Theta_{22} = -\bar{R} - \mu_2\bar{U}_1 + \bar{A}_d^T\bar{P}\bar{A}_d,$$
$$\bar{\tau} = \tau_M - \tau_m + 1.$$

Let

$$J(z(k),\omega(k)) = \gamma\omega^T(k)\omega(k) - \gamma^{-1}z^T(k)z(k).$$

We have

$$\Delta V(k) - J(z(k),\omega(k)) \leq \zeta^T(k)\tilde{\Theta}\zeta(k),$$

where

$$\tilde{\Theta} = \begin{bmatrix}
\Pi_{11} & \Pi_{12} & \bar{A}^T\bar{P}\bar{B} + \gamma^{-1}\bar{N}_1^T\bar{N}_2 & \bar{A}^T\bar{P}\bar{G} - \mu_1\bar{F}_2 & \bar{A}^T\bar{P}\bar{H} \\
* & \Pi_{22} & \bar{A}_d^T\bar{P}\bar{B} + \gamma^{-1}\bar{N}_3^T\bar{N}_2 & \bar{A}_d^T\bar{P}\bar{G} & \bar{A}_d^T\bar{P}\bar{H} - \mu_2\bar{U}_2 \\
* & * & \bar{B}^T\bar{P}\bar{B} + \gamma^{-1}\bar{N}_2^T\bar{N}_2 - \gamma I & \bar{B}^T\bar{P}\bar{G} & \bar{B}^T\bar{P}\bar{H} \\
* & * & * & -\mu_1 I + \bar{G}^T\bar{P}\bar{G} & \bar{G}^T\bar{P}\bar{H} \\
* & * & * & * & -\mu_2 I + \bar{H}^T\bar{P}\bar{H}
\end{bmatrix},$$

$$\Pi_{11} = (\tau_M - \tau_m + 1)\bar{R} - \bar{P} + \bar{A}^T\bar{P}\bar{A} - \mu_1\bar{F}_1 + \gamma^{-1}\bar{N}_1^T\bar{N}_1,$$
$$\Pi_{12} = \bar{A}^T\bar{P}\bar{A}_d + \gamma^{-1}\bar{N}_1^T\bar{N}_3,$$
$$\Pi_{22} = -\bar{R} - \mu_2\bar{U}_1 + \bar{A}_d^T\bar{P}\bar{A}_d + \gamma^{-1}\bar{N}_3^T\bar{N}_3.$$

It follows that $\tilde{\Theta} < 0$ imply that

$$\Delta V(k) - J(z(k),\omega(k)) \leq 0.$$

Applying Schur complement, we have that $\tilde{\Theta} < 0$ if and only if

$$\Sigma = \begin{bmatrix} \bar{\tau}\bar{R} - \bar{P} - \mu_1\bar{F}_1 & 0 & 0 & -\mu_1\bar{F}_2 & 0 & \bar{A}^T\bar{P} & \bar{N}_1^T \\ * & -\bar{R} - \mu_2\bar{U}_1 & 0 & 0 & -\mu_2\bar{U}_2 & \bar{A}_d^T\bar{P} & \bar{N}_3^T \\ * & * & -\gamma I & 0 & 0 & \bar{B}^T\bar{P} & \bar{N}_2^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T\bar{P} & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T\bar{P} & 0 \\ * & * & * & * & * & -\bar{P} & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0.$$

Σ can be written as

$$\Sigma = \Sigma_1 + \Sigma_2,$$

where

$$\Sigma_1 = \begin{bmatrix} \bar{\tau}\bar{R} - \bar{P} - \mu_1\bar{F}_1 & 0 & 0 & -\mu_1\bar{F}_2 & 0 & \breve{A}^T\bar{P} & \bar{N}_1^T \\ * & -\bar{R} - \mu_2\bar{U}_1 & 0 & 0 & -\mu_2\bar{U}_2 & \breve{A}_d^T\bar{P} & \bar{N}_3^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T\bar{P} & \bar{N}_2^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T\bar{P} & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T\bar{P} & 0 \\ * & * & * & * & * & -\bar{P} & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \Delta\bar{A}^T\bar{P} & 0 \\ * & 0 & 0 & 0 & 0 & \Delta\bar{A}_d^T\bar{P} & 0 \\ * & * & 0 & 0 & 0 & \Delta\bar{B}^T\bar{P} & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} = \Psi_1 Y(k) O + O^T Y^T(k) \Psi_1^T,$$

$$\Psi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & (\bar{P}\bar{\Gamma})^T & 0 \end{bmatrix}^T,$$
$$O = \begin{bmatrix} \bar{K}_a & \bar{K}_d & \bar{K}_b & 0 & 0 & 0 & 0 \end{bmatrix}.$$

So, one has

$$\Sigma = \Sigma_1 + \Psi_1 Y(k) O + O^T Y^T(k) \Psi_1^T.$$

We have that $\Sigma < 0$ is equivalent to

$$\Xi = \Sigma_1 + \varepsilon^{-1}\Psi_1\Psi_1^T + \varepsilon O^T O < 0. \tag{15}$$

Applying Schur complement, it follows that $\Xi < 0$ if and only if

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_{11} & 0 & 0 & -\mu_1\bar{F}_2 & 0 & \breve{A}^T\bar{P} & \bar{N}_1^T & 0 & \bar{K}_a^T \\ * & -\bar{R} - \mu_2\bar{U}_1 & 0 & 0 & -\mu_2\bar{U}_2 & \breve{A}_d^T\bar{P} & \bar{N}_3^T & 0 & \bar{K}_d^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T\bar{P} & \bar{N}_2^T & 0 & \bar{K}_b^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T\bar{P} & 0 & 0 & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T\bar{P} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{P} & 0 & \bar{P}\bar{\Gamma} & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon^{-1} I \end{bmatrix} < 0, \tag{16}$$

where $\bar{\Sigma}_{11} = \bar{\tau}\bar{R} - \bar{P} - \mu_1\bar{F}_1$.

It follows from (9) that $\Xi < 0$. So, we get that

$$\Delta V(k) - \gamma\omega^T(k)\omega(k) + \gamma^{-1}\bar{z}^T(k)\bar{z}(k) < 0.$$

Then

$$\Delta V(k) - \gamma\omega^T(k)\omega(k) < 0. \tag{17}$$

One has

$$V(k) < V(0) + \sum_{l=0}^{k-1}\gamma\omega^T(l)\omega(l).$$

From (4), we get that

$$V(k) < V(0) + \gamma d^2. \tag{18}$$

From (11), we obtain

$$V(k) \geq \eta^T(k)R^{1/2}R^{-1/2}\bar{P}R^{-1/2}R^{1/2}\eta(k). \tag{19}$$

One has

$$\begin{aligned} V(0) \;\leq\;& \lambda_{\max}(H_1)\eta^T(0)R\eta(0) + \lambda_{\max}(H_2)\sum_{j=-\tau_M}^{-1}\eta^T(j)R\eta(j) \\ &+\lambda_{\max}(H_2)\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=-1+i}^{-1}\eta^T(j)R\eta(j) \\ \leq\;& \gamma_1[\lambda_{\max}(H_1) + (\tau_M + (\tau_M+\tau_m-1)(\tau_M-\tau_m)/2)\lambda_{\max}(H_2)], \end{aligned} \tag{20}$$

where $H_1 = R^{-1/2}\bar{P}R^{-1/2}, H_2 = R^{-1/2}\bar{R}R^{-1/2}$.

From (18)–(20), one obtains

$$\begin{aligned} &\lambda_{\min}(H_1)\eta^T(k)R\eta(k) \leq V(k) \\ &\quad < \gamma_1[\lambda_{\max}(H_1) + (\tau_M + (\tau_M+\tau_m-1)(\tau_M-\tau_m)/2)\lambda_{\max}(H_2)] + \gamma d^2 \\ &\quad < \gamma_2\lambda_{\min}(H_1). \end{aligned}$$

So, from (10), we have

$$\begin{aligned} \eta^T(k)R\eta(k) \;<\;& \tfrac{1}{\lambda_{\min}(H_1)}\{\gamma_1[\lambda_{\max}(H_1) + (\tau_M + (\tau_M+\tau_m-1)(\tau_M-\tau_m)/2) \\ &\times\lambda_{\max}(H_2)] + \gamma d^2\} \\ <\;& \gamma_2. \end{aligned}$$

Thus, the system (7) is finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M)$.

Under the zero-initial condition, one obtains

$$\sum_{k=0}^{M}\Delta V(k) = V(M+1) - V(0) > 0.$$

One has

$$\sum_{l=0}^{M}(\gamma\omega^T(l)\omega(l) - \gamma^{-1}z^T(l)z(l)) \geq 0 \Rightarrow \sum_{l=0}^{M}z^T(l)z(l) \leq \sum_{l=0}^{M}\gamma^2\omega^T(l)\omega(l),$$

which means that the system (7) is H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$. This completes the proof of the theorem. □

Next, sufficient conditions which guarantee time-delay system (7) is H_∞ observer-based finite-time bounded via observer-based control are proposed, and the controller gain and observer gain are given.

Theorem 2. For given integers $0 < \tau_m \le \tau_M$, the closed-loop system (7) is H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$, if there exist matrices $0 < X = diag(X_1, X_2) \in \mathbb{R}^{2n\times 2n}, 0 < Z \in \mathbb{R}^{2n\times 2n}$, and positive scalars $\mu_1, \mu_2, \varepsilon$, and any matrix $\bar{K}, \bar{L}$ such that the following conditions hold:

$$\begin{bmatrix} \Pi_{11} & 0 & 0 & -\mu_1 X\bar{F}_2 & 0 & \Pi_{16} & X\bar{N}_1^T & 0 & X\bar{K}_a^T \\ * & \Pi_{22} & 0 & 0 & -\mu_2 X\bar{U}_2 & X\breve{A}_d^T & X\bar{N}_3^T & 0 & X\bar{K}_d^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T & \bar{N}_2^T & 0 & \bar{K}_b^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T & 0 & 0 & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T & 0 & 0 & 0 \\ * & * & * & * & * & -X & 0 & \bar{\Gamma} & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon^{-1} I \end{bmatrix} < 0, \tag{21}$$

$$\gamma_1[\lambda_{\max}(H_1) + (\tau_M + (\tau_M + \tau_m - 1)(\tau_M - \tau_m)/2)\lambda_{\max}(H_2) + \gamma d^2] < \gamma_2 \lambda_{\min}(H_1), \tag{22}$$

$$X_1 C^T = C^T \breve{X}_1, \tag{23}$$

where

$$\begin{aligned} &\Pi_{11} = \bar{\tau} Z - X + \mu_1(\bar{F}_1^{-T} - 2X), \quad \Pi_{22} = -Z + \mu_2(\bar{U}_1^{-T} - 2X), \\ &\Pi_{16} = \begin{bmatrix} X_1 A^T + \bar{K} D^T & 0 \\ -\bar{K} D^T & X_1 A^T - C^T \bar{L} \end{bmatrix}, \quad \breve{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \\ &\bar{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} B \\ B \end{bmatrix}, \quad \bar{N}_1 = [N_1 \quad 0], \quad \bar{N}_3 = [N_3 \quad 0], \\ &\bar{F}_1 = I \otimes [(F_1^T F_2 + F_2^T F_1)/2], \quad \bar{F}_2 = -I \otimes [(F_1 + F_2)/2], \\ &\bar{U}_1 = I \otimes [(U_1^T U_2 + U_2^T U_1)/2], \quad \bar{U}_2 = -I \otimes [(U_1 + U_2)/2], \\ &\bar{\tau} = \tau_M - \tau_m + 1, \quad \bar{K}_a = [K_a \quad 0], \quad \bar{K}_d = [K_d \quad 0], \\ &H_1 = R^{-1/2} X^{-1} R^{-1/2}, \quad H_2 = R^{-1/2} \bar{R} R^{-1/2}, \quad Z = X\bar{R}X. \end{aligned}$$

Furthermore, if the conditions (21)-(23) have feasible solutions, the controller gain K and observer gain L can be given by

$$K = \bar{K}^T X_1^{-1}, L = \bar{L}^T \breve{X}_1^{-T}.$$

P r o o f. Let $\Phi = diag(\bar{P}^{-1}, \bar{P}^{-1}, I, I, I, \bar{P}^{-1}, I, I, I), X = \bar{P}^{-1}$. Pre- and post-multiplying

(9) by Φ^T and Φ, we have

$$\hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & 0 & 0 & -\mu_1 X\bar{F}_2 & 0 & X\breve{A}^T & X\bar{N}_1^T & 0 & X\bar{K}_a^T \\ * & \hat{\Sigma}_{22} & 0 & 0 & -\mu_2 X\bar{U}_2 & X\breve{A}_d^T & X\bar{N}_3^T & 0 & X\bar{K}_d^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T & \bar{N}_2^T & 0 & \bar{K}_b^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T & 0 & 0 & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T & 0 & 0 & 0 \\ * & * & * & * & * & -X & 0 & \bar{\Gamma} & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon^{-1} I \end{bmatrix} < 0, \tag{24}$$

where

$$\hat{\Sigma}_{11} = \bar{\tau} X\bar{R}X - X - \mu_1 X\bar{F}_1 X,$$
$$\hat{\Sigma}_{22} = -X\bar{R}X - \mu_2 X\bar{U}_1 X.$$

Since

$$\begin{aligned} -\mu_1 X\bar{F}_1 X &\le \mu_1(\bar{F}_1^{-T} - 2X), \\ -\mu_2 X\bar{U}_1 X &\le \mu_2(\bar{U}_1^{-T} - 2X), \end{aligned} \tag{25}$$

then $\hat{\Sigma} < 0$ if

$$\hat{\Sigma}_1 = \begin{bmatrix} \hat{\Sigma}_{11} & 0 & 0 & -\mu_1 X\bar{F}_2 & 0 & X\breve{A}^T & X\bar{N}_1^T & 0 & X\bar{K}_a^T \\ * & \hat{\Sigma}_{22} & 0 & 0 & -\mu_2 X\bar{U}_2 & X\breve{A}_d^T & X\bar{N}_3^T & 0 & X\bar{K}_d^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T & \bar{N}_2^T & 0 & \bar{K}_b^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T & 0 & 0 & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T & 0 & 0 & 0 \\ * & * & * & * & * & -X & 0 & \bar{\Gamma} & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon^{-1} I \end{bmatrix} < 0,$$

where

$$\hat{\Sigma}_{11} = \bar{\tau} Z - X + \mu_1(\bar{F}_1^{-T} - 2X),$$
$$\hat{\Sigma}_{22} = -Z + \mu_2(\bar{U}_1^{-T} - 2X), \quad Z = X\bar{R}X.$$

Let $X_1 = P^{-1}, \bar{K} = X_1 K^T, \bar{L} = \breve{X}_1 L^T$. From (21) and (23), we get that $\bar{\Sigma} < 0$. So, according to Theorem 1, the system (7) is H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$. This completes the proof of the theorem. □

Consider system (1) with $f(x(k)) = 0, g(x(k-\tau(k))) = 0$, then the closed-loop system (7) can be written as

$$\begin{aligned} \eta(k+1) &= \bar{A}\eta(k) + \bar{A}_d\eta(k-\tau(k)) + \bar{B}\omega(k), \\ z(k) &= \bar{N}_1\eta(k) + \bar{N}_3\eta(k-\tau(k)) + N_2\omega(k). \end{aligned} \tag{26}$$

Theorem 3. For given integers $0 < \tau_m \leq \tau_M$, the system (26) is H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$, if there exist matrices $0 < X = diag(X_1, X_1) \in \mathbb{R}^{2n\times 2n}, 0 < Z \in \mathbb{R}^{2n\times 2n}$, and a scalar $\varepsilon > 0$, and any matrix $\bar{K}$ and $\bar{L}$ such that (22),(23) and the following conditions hold:

$$\begin{bmatrix} \bar{\tau}Z - X & 0 & 0 & \Pi_{14} & X\bar{N}_1^T & 0 & X\bar{K}_a^T \\ * & -Z & 0 & X\breve{A}_d^T & X\bar{N}_3^T & 0 & X\bar{K}_d^T \\ * & * & -\gamma I & \breve{B}^T & N_2^T & 0 & \bar{K}_b^T \\ * & * & * & -X & 0 & \bar{\Gamma} & 0 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -\varepsilon^{-1} I \end{bmatrix} < 0, \tag{27}$$

where

$$\begin{aligned} &\Pi_{14} = \begin{bmatrix} X_1A^T + \bar{K}D^T & 0 \\ -\bar{K}D^T & X_1A^T - C^T\bar{L} \end{bmatrix}, \breve{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \breve{B} = \begin{bmatrix} B \\ B \end{bmatrix}, \\ &\bar{N}_1 = [N_1 \quad 0], \quad \bar{N}_3 = [N_3 \quad 0], \\ &\bar{\tau} = \tau_M - \tau_m + 1, \quad \bar{K}_a = [K_a \quad 0], \quad \bar{K}_d = [K_d \quad 0], \\ &H_1 = R^{-1/2}X^{-1}R^{-1/2}, \quad H_2 = R^{-1/2}\bar{R}R^{-1/2}, \quad Z = X\bar{R}X. \end{aligned}$$

Furthermore, if the conditions (22),(23) and (27) have feasible solutions, the controller gain K and observer gain L can be given by

$$K = \bar{K}^T X_1^{-1}, \quad L = \bar{L}^T \breve{X}_1^{-T}.$$

Proof. The proof is similar to that for Theorem 1 and is omitted here. □

Consider the following system

$$\begin{aligned} &x(k+1) = Ax(k) + A_dx(k-\tau(k)) + Du(k) + Gf(x(k)) + Hg(x(k-\tau(k))) + B\omega(k), \\ &y(k) = Cx(k), \\ &z(k) = N_1x(k) + N_3x(k-\tau(k)) + N_2\omega(k), \\ &x(\theta) = \varphi(\theta), \theta = -\tau_M, -\tau_M+1, \dots, 0. \end{aligned} \tag{28}$$

We construct an observer-based controller for system (28) of the following form

$$\begin{aligned} \hat{x}(k+1) = {} & A\hat{x}(k) + A_d\hat{x}(k-\tau(k)) + Du(k) + L(y - C\hat{x}(k)) + Gf(\hat{x}(k)) \\ & + Hg(\hat{x}(k-\tau(k))), \\ u(k) = {} & K\hat{x}(k). \end{aligned} \tag{29}$$

Let $\eta(k) = [x^T(k) \quad e^T(k)]^T$, then by (28) and (29), we get the closed-loop system as follows:

$$\begin{aligned} &\eta(k+1) = \breve{A}\eta(k) + \breve{A}_d\eta(k-\tau(k)) + \breve{B}\omega(k) + \bar{G}\bar{f}(\eta(k)) + \bar{H}\bar{g}(\eta(k-\tau(k))), \\ &z(k) = \bar{N}_1\eta(k) + \bar{N}_3\eta(k-\tau(k)) + N_2\omega(k). \end{aligned} \tag{30}$$

Theorem 4. For given integers $0 < \tau_m \leq \tau_M$, the system (30) is H_∞ finite-time bounded via observer-based control with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$, if there exist scalars $\mu_1 > 0, \mu_2 > 0$, matrices $0 < X = diag(X_1, X_1) \in \mathbb{R}^{2n\times 2n}, 0 < Z \in \mathbb{R}^{2n\times 2n}$, and any matrix $\bar{K}$ and $\bar{L}$ such that (22),(23) and the following conditions hold:

$$\begin{bmatrix} \hat{\Pi}_{11} & 0 & 0 & -\mu_1 X\bar{F}_2 & 0 & \Pi_{16} & X\bar{N}_1^T \\ * & -Z+\mu_2(U_1^{-T}-2X) & 0 & 0 & -\mu_2 X\bar{U}_2 & X\breve{A}_d^T & X\bar{N}_3^T \\ * & * & -\gamma I & 0 & 0 & \breve{B}^T & N_2^T \\ * & * & * & -\mu_1 I & 0 & \bar{G}^T & 0 \\ * & * & * & * & -\mu_2 I & \bar{H}^T & 0 \\ * & * & * & * & * & -X & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0, \tag{31}$$

where

$$\begin{aligned} &\hat{\Pi}_{11} = \bar{\tau}Z - X + \mu_1(\bar{F}_1^{-T} - 2X), \\ &\Pi_{16} = \begin{bmatrix} X_1A^T + \bar{K}D^T & 0 \\ -\bar{K}D^T & X_1A^T - C^T\bar{L} \end{bmatrix}, \quad \breve{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \\ &\bar{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} B \\ B \end{bmatrix}, \quad \bar{N}_1 = [N_1 \quad 0], \quad \bar{N}_3 = [N_3 \quad 0], \\ &\bar{F}_1 = I \otimes [(F_1^T F_2 + F_2^T F_1)/2], \quad \bar{F}_2 = -I \otimes [(F_1 + F_2)/2], \\ &\bar{U}_1 = I \otimes [(U_1^T U_2 + U_2^T U_1)/2], \quad \bar{U}_2 = -I \otimes [(U_1 + U_2)/2], \\ &\bar{\tau} = \tau_M - \tau_m + 1, \quad H_1 = R^{-1/2}X^{-1}R^{-1/2}, \\ &H_2 = R^{-1/2}\bar{R}R^{-1/2}, \quad Z = X\bar{R}X. \end{aligned}$$

Furthermore, if the conditions (22),(23) and (31) have feasible solutions, the controller gain K and observer gain L can be given by

$$K = \bar{K}^T X_1^{-1}, \quad L = \bar{L}^T \breve{X}_1^{-T}.$$

Proof. The proof is similar to that for Theorem 1 and is omitted here. □

4. NUMERICAL EXAMPLE

Consider the system (1) with the following parameters:

$$\begin{aligned} &A = \begin{bmatrix} -0.2 & 0.01 \\ 0.01 & -0.3 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & -0.01 \\ 0.01 & -0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & -0.1 \end{bmatrix}, \\ &C = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad G = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad H = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ &D = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.02 & 0 \\ 0.01 & -0.03 \end{bmatrix}, \\ &N_2 = \begin{bmatrix} 0.1 & 0 \\ 0.01 & -0.02 \end{bmatrix}, \quad N_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ &K_a = \begin{bmatrix} -0.05 & 0 \\ 0.01 & -0.02 \end{bmatrix}, \quad K_d = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0.01 \end{bmatrix}, \quad K_b = \begin{bmatrix} -0.02 & 0.03 \\ 0 & 0.01 \end{bmatrix}, \end{aligned}$$

$\tau_M = 3, \quad \tau_m = 1, \quad \gamma = 0.4, \quad d = 4, \quad \mu_1 = 0.01, \quad \mu_2 = 0.02, \quad \tau(k) = 2 + \sin(\frac{k\pi}{2}),$
$f(x) = \begin{bmatrix} 0.5x_1 + 0.1\sin(x_1) \\ 0.3x_2 + 0.1\sin(x_2) \end{bmatrix}, \quad \omega(k) = \begin{bmatrix} 0.1\sin(k) \\ 0.2\sin(k) \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.4 \end{bmatrix},$
$F_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0.4x_1 + 0.2\sin(x_1) \\ 0.3x_2 + 0.2\sin(x_2) \end{bmatrix},$
$U_1 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}.$

By using Matlab LMI control Toolbox to solve inequalities (21) and (22), we get

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_1 \end{bmatrix} = \begin{bmatrix} 62.3540 & 4.3215 & 0 & 0 \\ 4.3215 & 85.7378 & 0 & 0 \\ 0 & 0 & 62.3540 & 4.3215 \\ 0 & 0 & 4.3215 & 85.7378 \end{bmatrix},$$
$$Z = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} = \begin{bmatrix} 14.6949 & 1.9785 & 0 & 0 \\ 1.9785 & 19.9356 & 0 & 0 \\ 0 & 0 & 14.6751 & 1.3906 \\ 0 & 0 & 1.3906 & 21.0228 \end{bmatrix},$$
$$\bar{K} = \begin{bmatrix} 12.1045 & 28.7151 \\ 28.7151 & -98.1268 \end{bmatrix}, \bar{L} = \begin{bmatrix} -63.0796 & -0.1727 \\ -0.1727 & -130.0344 \end{bmatrix},$$
$$\gamma_2 = 6.41, \varepsilon = 226.0842.$$

The controller gain K and observer gain L are

$$K = \begin{bmatrix} 0.1715 & 0.3263 \\ 0.5417 & -1.1718 \end{bmatrix}, L = \begin{bmatrix} -1.0150 & 0.0491 \\ 0.1027 & -1.5218 \end{bmatrix}.$$

According to Theorem 2, the system (1) is H_∞ finite-time stabilization with respect to $(\gamma_1, \gamma_2, d, R, M, \gamma)$ under the observer-based control (5). Figure 1 and 2 show the states and estimate states of the closed-loop system. From Figure 1 and 2, it is easy to see that the closed-loop system is finite-time bounded.

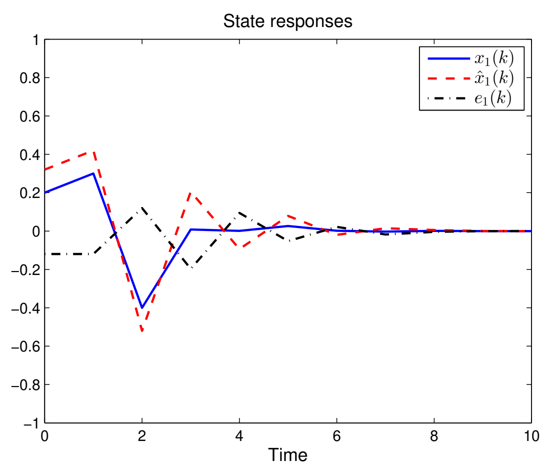

Fig. 1. The trajectories of $x_1(k), \hat{x}_1(k)$ and $e_1(k)$.

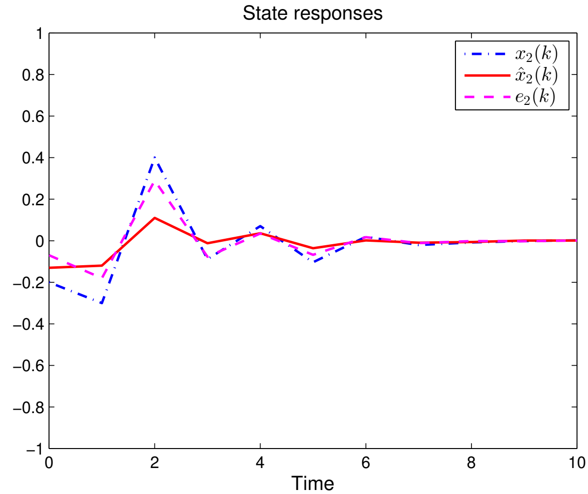

Fig. 2. The trajectories of $x_2(k), \hat{x}_2(k)$ and $e_2(k)$.

Remark 1. In [2], the finite-time stabilization of linear systems was dealt. Amato et al.[1] dealt with the finite-time boundedness (FTB) of linear time-varying systems. However, in [2, 22], the nonlinear system was not considered. In this paper, we investigated the finite-time boundedness of nonlinear system with H_∞ performance.

Remark 2. In [22], the problem of finite-time stability for linear discrete-time systems with time-varying delay was considered and the state feedback control was used. The availability of all the state variables for direct measurement is a rare occasion in practice. In most cases, a reliable estimation of the unmeasurable state variables is needed. In this paper, we given the state observer, and design the observer-based finite-time H_∞ control to guarantee the closed-loop system is H_∞ finite-time bounded. Compared with [22], our design has a wider application range.

5. CONCLUSION

Observer-based finite-time H_∞ control for the uncertain discrete-time systems with nonlinear perturbations and time-varying delay is investigated in this paper. By constructing the Lyapunov–Krasovskii functional and using the matrix inequality technique, we establish the sufficient conditions in terms of matrix inequalities to guarantee that the resulting closed-loop system is H_∞ finite-time bounded via observer-based control. We design the observer-based control to ensure that the resulting closed-loop system is observer-based H_∞ finite-time bounded. Finally, a numerical example is given to demonstrate the validity of the proposed results. How to extend the main results to the discrete nonlinear switched systems with time-varying delay and the singular discrete nonlinear systems with interval time-varying delay, are very meaningful topics that deserves further exploration.

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REFERENCES

[1] F. Amato, R. Ambrosino, M. Ariola, G. De Tommasi, and A. Pironti: On the finite-time boundedness of linear systems. Automatica *107* (2019), 454–466. DOI:10.1016/j.automatica.2019.06.002

[2] F. Amato, M. Darouach, and G. De Tommasi: Finite-time stabilizability, detectability, and dynamic output feedback finite-time stabilization of linear systems. IEEE Trans. Automat. Control *62* (2017), 12, 6521–6528. DOI:10.1109/TAC.2017.2660758

[3] S. Ahmad and M. Rehan: On observer-based control of one-sided Lipschitz systems. J. Frankl. Inst. *353* (2016), 4, 903–916. DOI:10.1016/j.jfranklin.2016.01.010

[4] F. Amato, M. Arioial, and P. Dorato: Finite-time control of linear system subject to parametric uncertainties and disturbances. Automatica *37* (2001), 1459–1463. DOI:10.1016/S0005-1098(01)00087-5

[5] J. Cheng, H. Zhu, S. M. Zhong, Y. Zeng, and L. Y. Hou: Finite-time H_∞ filtering for a class of discrete-time Markovian jump systems with partly unknown transition probabilities. Int. J. Adapt. Control Signal Process. *28* (2014), 1024–1042. DOI:10.1002/acs.2425

[6] Y. Dong, L. Chen, and S. Mei: Observer design for neutral-type neural networks with discrete and distributed time-varying delays. Int. J. Adapt. Control Signal Process. *33* (2019), 1, 527–544. DOI:10.1002/acs.2970

[7] Y. Dong, T. Li, and S. Mei: Exponential stabilization and L_2-gain for uncertain switched nonlinear systems with interval time-varying delay. Math. Meth. Appl. Sci. *39* (2016), 3836–3854. DOI:10.1002/mma.3828

[8] Y. Dong, S. Liang, and H. Wang: Robust stability and H_∞ control for nonlinear discrete-time switched systems with interval time-varying delay. Math. Meth. Appl. Sci. *42* (2019), 1999–2015. DOI:10.1002/mma.5493

[9] Y. Dong, W. Liu, T. Li, and S. Liang: Finite-time boundedness analysis and H_∞ control for switched neutral systems with mixed time-varying delays. J. Frankl. Inst. *354* (2017), 787–811. DOI:10.1016/j.jfranklin.2016.10.037

[10] Y. Dong, Y. Zhang, and X. Zhang: Design of observer-based feedback control for a class of discrete-time nonlinear systems with time-delay. Appl. Comput. Math., *13* (2014), 1, 107–121.

[11] P. Dorato: Short time stability in linear time-varying system. In: Proc. IRE International Convention Record. Part 4, New York 1961, pp. 83–87.

[12] I. Karafyllis: Finite-time global stabilization by means of time-varying distributed delay feedback. SIAM J. Control Optim. *45* (2006), 1, 320–342. DOI:10.1137/040616383

[13] X. Lin, H. Du, and S. Li: Finite-time boundedness and L_2 gain analysis for switched delay systems with norm-bounded disturbance. Appl. Math. Comput. *217* (2011), 12, 5982–5993. DOI:10.1016/j.amc.2010.12.032

[14] Y. C. Ma, L. Fu, Y. H. Jing, and Q. L. Zhang: Finite-time H_∞ control for a class of discrete-time switched singular time-delay systems subject to actuator saturation. Appl. Math. Comput. *261* (2015), 264–283. DOI:10.1016/j.amc.2015.03.111

[15] C. M. Nguyen, P. N. Pathirana, and H. Trinh: Robust observer-based control designs for discrete nonlinear systems with disturbances. Europ. J. Control *44* (2018), 65–72. DOI:10.1016/j.ejcon.2018.09.002

[16] C. M. Nguyen, P. N. Pathirana, and H. Trinh: Robust observer design for uncertain one-sided Lipschitz systems with disturbances. Int. J. Robust Nonlinear Control *28* (2018), 1366–1380. DOI:10.1002/rnc.3960

[17] M. C. Nguyen and H. Trinh: Observer design for one-sided Lipschitz discrete-time systems subject to delays and unknown inputs. SIAM J. Control Optim *54* (2016), 3, 1585–1601. DOI:10.1137/15M1030182

[18] J. Song and S. He: Robust finite-time H_∞ control for one-sided Lipschitz nonlinear systems via state feedback and output feedback. J. Frankl. Inst. *352* (2015), 8, 3250–3266. DOI:10.1016/j.jfranklin.2014.12.010

[19] S. B. Stojanovic: Robust finite-time stability of discrete time systems with interval time-varying delay and nonlinear perturbations. J. Frankl. Inst. *354* (2017), 4549–4572. DOI:10.1016/j.jfranklin.2017.05.009

[20] W. Zhang, H. Su, F. Zhu, and G. Azar: Unknown input observer design for one-sided Lipschitz nonlinear systems. Nonlinear Dyn. *79* (2015), 2, 1469–1479. DOI:10.1007/s11071-014-1754-x

[21] Z. Zhang, Z. Zhang, and H. Zhang: Finite-time stability analysis and stabilization for uncertain continuous-time system with time-varying delay. J. Frankl. Inst. *352* (2015), 1296–1317. DOI:10.1016/j.jfranklin.2014.12.022

[22] Z. Zhang, Z. Zhang, H. Zhang, B. Zheng, and H. R. Karimi: Finite-time stability analysis and stabilization for linear discrete-time system with time-varying delay. J. Frankl. Inst. *351* (2014), 3457–3476. DOI:10.1016/j.jfranklin.2014.02.008

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