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EFFICIENCY EVALUATION OF CLOSED-LOOP SUPPLY CHAINS WITH PROPORTIONAL DUAL-ROLE MEASURES

Monireh Jahani Sayyad Noveiri, Sohrab Kordrostami and Alireza Amirteimoori

Data Envelopment Analysis (DEA) is a beneficial mathematical programming method to measure relative efficiencies. In conventional DEA models, Decision Making Units (DMUs) are usually considered as black boxes. Also, the efficiency of DMUs is evaluated in the presence of the specified inputs and outputs. Nevertheless, in real-world applications, there are situations in which the performance of multi-stage processes like supply chains with forward and reverse flows must be measured such that some of the intervening factors, called proportional dual-role factors, are presented that one part of each proportional dual-role factor plays the input role and the other plays the output role. To address this issue, the current study proposes radial and non-radial DEA models for evaluating the overall and stage efficiencies of the closed-loop supply chains when there are proportional dual-role factors. To illustrate, a proportional dual-role factor is divided into portions of the input of the first stage and the output of the second stage such that the optimal overall and stage efficiency scores of closed-loop supply chain are obtained. A case study is used to illustrate the proposed approach. The experimental results obtained from real world data show the convincing performance of our proposed method.

Keywords: data envelopment analysis (DEA), efficiency, closed-loop supply chain, proportional dual-role factor, input/output

Classification: 90C05, 90B50, 90C90

1. INTRODUCTION

The Data Envelopment Analysis (DEA) technique, initially proposed by Charnes et al. [17], is a non-parametric methodology for evaluating the relative efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. In recent years, there has been an increasing interest in the application of DEA in fields such as banking [30], education [32] and agriculture [7]. In traditional DEA models, each DMU is considered as a black box in which the input/output status of each measure has been specified. However, there are situations in the real world in which DMUs with network structures must be evaluated while some measures that play partially input and output roles, called proportional dual-role measures, are presented. Most of studies on the dual-role factor determine the role of it as either an input or an output at the end. Nevertheless, it seems

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the estimation of the portion of a dual-role factor as input and output is more rational in measuring the relative efficiency of systems. Profit and research funding flows are examples of dual-role factors in networks [38]. The majority of network DEA studies also consider forward flows. However, there are many real world applications that contain forward and reverse flows. For instance, information flows and product returns can be considered as reverse flows. To handle these issues, radial and non-radial DEA models are proposed in this study for measuring the efficiency of two-stage network systems where proportional dual-role factors and reverse flows are present. To illustrate, models are suggested for evaluating the efficiency of the supply chain network structure wherein forward and reverse intermediate flows exist. Also, proportional dual-role factors are handled in this supply chain network system.

A large and growing body of DEA literature has investigated the performance of networks with various structures; see [1, 4, 6, 10, 14, 15, 19, 23, 24, 25, 26, 27, 29, 30, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 47, 50, 55, 56, 57, 58] for more details. The performance measurement of supply chain networks is also addressed by some studies such as [2, 11, 20, 39, 51, 52, 53, 59, 60] due to importance of supply chain management in order to survive in today's competitive world. Balfaqih et al. [11] reviewed some systems, approaches, and criteria for supply chain performance measurement. Several studies have used the DEA technique for measuring the efficiency of supply chain networks [11, 20, 39, 51, 52, 53, 59, 60].

On the other hand, there are published studies describing dual-role factors. At first, Beasley [12, 13] considered research funding as both an input and an output in a study of university departments. Then Cook et al. [21] declared that Beasley's treatment is not an entirely appropriate method and proposed a model for handling factors that can play the roles of both input and output simultaneously. Afterwards, different studies were provided to investigate dual-role factors. For instance, see [3, 18, 22, 37, 54]. Toloo et al. [54] analysed the efficiency of DMUs in the presence of interval inputs, outputs and dual-role measures. Aviles-Sacoto et al. [8] provided a DEA model for handling the occasions in which outputs occur at different stages of time. That is two outputs occur at different points of time that one of them influences the other. Thus, one of them plays a dual role and lastly its role is specified as input or output. Liang et al. [38] presented approaches for evaluating the efficiency of a special form of two-stage systems in which the outputs of the second stage can be fed back as inputs to the first stage. Indeed, feedback variables were deemed as dual-role factors. To illustrate, multiplier forms have been addressed in [38] while reverse intermediate flows have not been incorporated. Aviles-Sacoto et al. [9] behaved with some of intermediate measures as inputs for the second stage and at the same time as final outputs of the second stage. In their study, these intermediate measures play two roles, input and output. Ultimately, the role of dual-role measure is identified as only one of the two. Shabani and Farzipoor Saen [51] proposed a DEA model to determine the prospective benchmarks of green supply chains in the presence of dual-role factors. They used the Program Evaluation and Review Technique/Critical Path Method (PERT/CPM) in which supply chain has been considered as a black box, and the role of a dual-role factor has been finally determined as either an input or an output. However, there have been no systematic investigations to measure the efficiency of closed-loop supply chains in the presence of proportional dual-role measures.

Therefore, the main objective of this paper is to assess the performance of supply chains with forward, reverse flows and proportional dual-role measures. Indeed, the overall and stage efficiency scores of two-stage network (supply chain with forward and reverse flows) systems are estimated when dual-role factors proportionally exist. It means the portion of the proportional dual-role factor is determined as the final output of the second stage and the remainder is specified as the input of the first stage such that the optimal overall efficiency is obtained. In contrast to prior research, a proportional dual-role factor in this study is divided into portions of input and output such that the best performance of closed-loop supply chain is obtained. Overall, our research contributions are as follows:

- We investigate network structures like supply chains when forward, reverse flows, and proportional dual-role factors exist.
- We propose, based on DEA, radial and non-radial models to measure the performance of the closed-loop supply chains with proportional dual-role factors.
- We evaluate the overall and stage efficiency scores of broiler supply chains using the introduced approaches.
- We compare the results obtained from proposed models with the traditional radial and non-radial DEA models.

The paper is organized as follows. In Section 2, a brief overview of fundamental issues, including a basic DEA model, the Enhanced Russell Measure (ERM), and the preliminary on dual-role factors is given. The DEA-based approaches are introduced and developed to estimate the relative efficiency of closed-loop supply chain networks with proportional dual-role factors in Section 3. A case study is applied to illustrate and validate the suggested approaches in Section 4. Conclusions are provided in Section 5.

2. PRELIMINARIES

First, a basic DEA model, the radial CCR (Charnes, Cooper, and Rhodes model) model, is described in the current section. Then, the ERM, which is a non-radial DEA model, is reviewed. Finally, some previous studies on dual-role factors are presented.

2.1. The CCR model and Enhanced Russell Measure (ERM)

Consider n DMUs, DMU_j $(j = 1, \ldots, n)$, with m inputs x_{ij} $(i = 1, \ldots, m)$ and s outputs y_{rj} $(r = 1, \ldots, s)$. For measuring the relative efficiency of DMUs, Charnes et al. [17] proposed the following radial input-oriented model (the CCR model):

$$\begin{aligned} \theta_o^* = &Min \quad \theta_o \\ s.t. \ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}, \ \ i = 1, \ldots, m, \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s, \\ &\lambda_j \geq 0, \qquad \forall j \end{aligned} \tag{1}$$

in which $\lambda_j (j = 1, \ldots, n)$ is the intensity variable. The optimal value of Model (1), θ_o^*, indicates the efficiency of the unit under evaluation, DMU_o.

DMU_o is said to be efficient if and only if $\theta_o^* = 1$. Otherwise, it is inefficient.

By considering non-negative input excesses (s_i^-) and non-negative outputs shortfalls (s_r^+) as $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta_o x_{io}, \forall i$ and $\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \forall r$, we have the following descriptions:

DMU_o is strongly efficient if and only if $\theta_o^* = 1$ and all optimal slacks are obtained equal to zero. DMU_o is weakly efficient iff $\theta_o^* = 1$ and at least an optimal slack is positive.

Pastor et al. [48] defined a non-radial and non-oriented efficiency measure for evaluating the efficiency of DMUs, referred to as ERM, as follows:

$$\begin{aligned} z^*_{ERM_o} = Min \quad & \frac{\frac{1}{m}\sum_{i=1}^{m} \Omega_i}{\frac{1}{s}\sum_{r=1}^{s} \psi_r} \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \Omega_i x_{io}, \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \psi_r y_{ro}, \quad r = 1, \ldots, s, \\ & \lambda_j \geq 0, \forall j, \ \psi_r \geq 1, \forall r, \Omega_i \leq 1, \forall i. \end{aligned} \tag{2}$$

The Russell measure firstly suggested by Fare and Lovell [28] and later revisited by Pastor et al. [48]. In Model (2), the optimal objective value, $z^*_{ERM_o}$, is the efficiency of DMU_o that is the minimization of the ratio, the average efficiency of inputs to the average efficiency of outputs. $\lambda_j (j = 1, \ldots, n)$ is also the intensity variable. DMU_o is called efficient if and only if $z^*_{ERM_o} = 1$, i. e. $\frac{1}{m}\sum_{i=1}^{m} \Omega_i^* = 1$ and $\frac{1}{s}\sum_{r=1}^{s} \psi_r^* = 1$. Model (2) can be linearized by using Charnes–Cooper transformation [16] as follows:

$$\begin{aligned} z^*_{ERM_o} = Min \quad & \frac{1}{m}\sum_{i=1}^{m} \bar{\Omega}_i \\ s.t. \quad & \frac{1}{s}\sum_{r=1}^{s} \bar{\psi}_r = 1, \\ & \sum_{j=1}^{n} \bar{\lambda}_j x_{ij} \leq \bar{\Omega}_i x_{io}, \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \bar{\lambda}_j y_{rj} \geq \bar{\psi}_r y_{ro}, \quad r = 1, \ldots, s, \\ & \bar{\lambda}_j \geq 0, \ \forall j, \ \bar{\psi}_r \geq t, \forall r, \bar{\Omega}_i \leq t, \forall i, \end{aligned} \tag{3}$$

where $t^{-1} = \frac{1}{s}\sum_{r=1}^{s} \psi_r$, $t\lambda_j = \bar{\lambda}_j$, $t\psi_r = \bar{\psi}_r$, and $t\Omega_i = \bar{\Omega}_i$.

Notice that the aforesaid definition of the CCR-efficient DMUs is not equal to it of the ERM-efficient DMUs. Also, without loss of generality, the input-oriented CCR model is investigated. However, the output-oriented form can be easily written.

2.2. Prior studies on dual-role factors

Factors are called dual-role if they can play the roles of input and output simultaneously. According to Cook et al. [21], trainees in organizations, awards to university departments and revenue in banks can be considered as examples of dual-role factors. Also, deposits in banks can be treated as another dual-role measure. Cook et al. [21] proposed a model, the modified version of Beasly's model [13], to evaluate the efficiency of DMUs in the presence of dual-role factors.

Assume $x_{ij}(i = 1, \dots m; j = 1, \dots, n)$, $y_{rj}(r = 1, \dots, s; j = 1, \dots, n)$, and $w_{kj}(k = 1, \dots, K; j = 1, \dots, n)$ denote inputs, outputs, and dual-role factors for $DMU_j(j = 1, \dots, n)$.

The dual of model proposed by Cook et al. [21] in the presence of multiple dual-role factors is as follows:

$$
\begin{aligned}
\theta_o^* = {} & Min \quad \theta_o \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}, \quad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_j w_{kj} \geq w_{ko}, \quad k = 1, \dots, K, \\
& \sum_{j=1}^{n} \lambda_j w_{kj} \leq w_{ko}, \quad k = 1, \dots, K, \\
& \lambda_j \geq 0, \ \forall j.
\end{aligned}
\tag{4}
$$

As Chen [18] mentioned, a dual-role factor is treated as exogenously fixed, or non-discretionary in this case. Dual-role factors are finally interpreted as either inputs or outputs in most models with dual-role factors. Amirteimoori et al. [3] incorporated recyclable outputs in production process. They investigated the situation wherein some portion of the produced outputs may be considered as inputs in a system exhibited as a black box. Nevertheless, there are occasions in the real world wherein the efficiency of network systems should be evaluated while proportional dual-role factors and reverse flows exist. For instance, in evaluating the efficiency of seller-buyer supply chains, products flow and demand forecast flow can be considered as forward and reverse intermediate measures as mentioned in [46] while the profit factor can play the roles of the output of buyer stage and the input of seller stage. To address these situations, the proposed method by Zhu [60] to handle the supply chain performance is extended in this paper to analyze the efficiency of closed-loop supply chains when proportional dual-role factors are present. Also, against of the majority of the existing models that deal with dual-role factors, these factors are split into inputs and outputs proportionally in this study. DEA models proposed to deal with the issue are provided in the next section.

3. CLOSED-LOOP SUPPLY CHAIN WITH PROPORTIONAL DUAL-ROLE FACTORS

In this section, radial and non-radial models are proposed for evaluating the relative efficiency of supply chains with two stages.

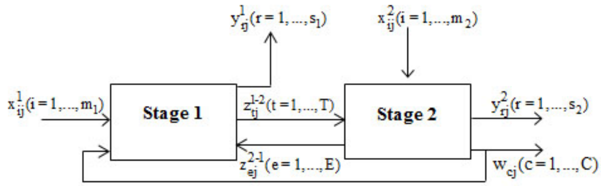

Fig. 1. A structure of supply chain.

Consider n supply chains, SC_j $(j = 1, \dots, n)$, with two components k $(k = 1, 2)$ to be evaluated. The structure of each supply chain can be seen in Figure 1. Each component of the supply chain has external inputs and outputs. External inputs and outputs of component k are denoted with x^k_{ij} $(i = 1, \dots, m_k)$ and y^k_{rj} $(r = 1, \dots, s_k)$, respectively. Also, there are proportional dual-role factors that portions of them are outputs of the second stage and other portions are inputs of the first stage as shown in Figure 1. Proportional dual-role factors are indicated by $w_{cj}(c = 1, \dots, C)$. Furthermore, forward and reverse flows exist as intermediate measures. Intermediate measures from stage one to stage two are denoted by z^{1-2}_{tj} $(t = 1, \dots, T)$ and intermediate measures from stage 2 to stage 1 are shown by z^{2-1}_{ej} $(e = 1, \dots, E)$.

In overall, the following notations are used in models proposed:

n: the number of supply chains,

$j = 1, \dots, n$: the set of supply chains,

m_k: the number of inputs of stage k,

$i = 1, \dots, m_k$: the set of inputs of stage k,

s_k: the number of outputs of stage k,

$r = 1, \dots, s_k$: the set of outputs of stage k,

C: the number of proportional dual-role measures,

$c = 1, \dots, C$: the set of proportional dual-role measures,

x^k_{ij} $(i = 1, \dots, m_k)$: ith external input of component k for jth SC,

y^k_{rj} $(r = 1, \dots, s_k)$: rth external output of component k for jth SC,

$w_{cj}(c = 1, \dots, C)$: cth proportional dual-role factor for jth SC,

z^{1-2}_{tj} $(t = 1, \dots, T)$: tth intermediate measure from stage 1 to stage 2 for jth SC,

T: the number of intermediate measures from stage 1 to stage 2,

$t = 1, \dots, T$: the set of intermediate measures from stage 1 to stage 2,

z_{ej}^{2-1} $(e = 1, \dots, E)$: eth intermediate measure from stage 2 to stage 1 for jth SC,

E: the number of intermediate measures from stage 2 to stage 1,

$e = 1, \dots, E$: the set of intermediate measures from stage 2 to stage 1,

λ_j: intensity variables corresponded to component 1,

β_j: intensity variables corresponded to component 2,

$\tilde{z}_{to}^{1-2}$: variables for the intermediate measures from stage 1 to stage 2 that should be determined,

$\tilde{z}_{eo}^{2-1}$: variables for the intermediate measures from stage 2 to stage 1 that should be determined,

$a_k(k = 1, 2)$: normalized weights (i.e. $\sum_k a_k = 1, a_k \geq 0$) defined by managers and decision makers that show the preference of stages,

α^c: the variable to determine the portion of the proportional dual-role factor c as the input of stage 1,

$(1 - \alpha^c)$: the variable to determine the portion of the proportional dual-role factor c as the output of stage 2,

SC_o: the supply chain under evaluation.

Ω_k: proportional reductions of inputs,

Ω_{ik}: non-proportional reductions of inputs,

ψ_{rk}: non-proportional augmentations of outputs,

Due to the aforementioned notations, the following subsection provides a radial DEA model to evaluate the performance of supply chains in the presence of forward, reverse flows and proportional dual-role factors.

3.1. A radial model for evaluating the closed-loop supply chain efficiency with proportional dual-role factors

Considering the supply chain network structure depicted in Figure 1 and notations mentioned, the following radial input-oriented DEA model is proposed to estimate the efficiency scores of the whole supply chain o and its components:

$$ER_o^* = \underset{\Omega_k, \lambda_j, \beta_j, \alpha^c, \tilde{z}}{Min} \sum_{k=1}^{2} a_k \Omega_k \tag{5}$$

$$\begin{aligned}
&s.t. \quad (\text{Stage}\, 1) \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \Omega_1 x_{io}^1, \quad i = 1, \dots, m_1, \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq y_{ro}^1, \quad r = 1, \dots, s_1,
\end{aligned}$$

$$\alpha^c \left(\sum_{j=1}^{n} \lambda_j w_{cj} \leq w_{co} \right), \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \lambda_j z_{ej}^{2-1} \leq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \lambda_j z_{tj}^{1-2} \geq \tilde{z}_{to}^{1-2}, \ \ t = 1, \ldots, T,$$
(Stage 2)
$$\sum_{j=1}^{n} \beta_j x_{ij}^{2} \leq \Omega_2 x_{io}^{2}, \ \ i = 1, \ldots, m_2,$$
$$\sum_{j=1}^{n} \beta_j y_{rj}^{2} \geq y_{ro}^{2}, \quad r = 1, \ldots, s_2,$$
$$(1-\alpha^c) \left(\sum_{j=1}^{n} \beta_j w_{cj} \geq w_{co} \right), \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \beta_j z_{ej}^{2-1} \geq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \beta_j z_{tj}^{1-2} \leq \tilde{z}_{to}^{1-2}, \ \ t = 1, \ldots, T,$$
$$\lambda_j, \beta_j \geq 0, \ 0 \leq \alpha^c \leq 1, \forall j, \forall c.$$

In Model (5), the variable α^c indicates the portion of the proportional dual-role factor c as the input of stage 1 and $(1-\alpha^c)$ shows it as the output of stage 2. The optimal value $\Omega_k^*(k=1,2)$ shows the efficiency of each component and ER_o^* is the efficiency of the whole supply chain o. The predefined weight $a_k(k=1,2)$ where $\sum_{k=1}^{2} a_k = 1$ and $a_k \geq 0$ indicates the preference over the performance of supply chain's stages.

Note that we have the following constraints in Model (5):

$$\alpha^c \left(\sum_{j=1}^{n} \lambda_j w_{cj} \leq w_{co} \right) \quad \Rightarrow \quad \sum_{j=1}^{n} \alpha^c \lambda_j w_{cj} \leq \alpha^c \, w_{co},$$
$$(1-\alpha^c) \left(\sum_{j=1}^{n} \beta_j w_{cj} \geq w_{co} \right) \quad \Rightarrow \quad \sum_{j=1}^{n} \beta_j w_{cj} - \sum_{j=1}^{n} \alpha^c \beta_j w_{cj} \geq (1-\alpha^c) w_{co},$$

As can be seen, α^c and $(1-\alpha^c)$ can be ignored from the above-mentioned constraints, but they are maintained because of their determining role as the portions of proportional dual-role factors.

Clearly, Model (5) is non-linear. For linearizing Model (5), we use the following the change of variables

$$\alpha^c \, \beta_j = \rho_j^c,$$

due to

$$0 \leq \alpha^c \leq 1,$$

it is resulted

$$0 \leq \rho_j^c \leq \beta_j,$$

(To explain in more details, if $\alpha^c = 0$, then $\rho_j^c = 0$ is obtained from $\alpha^c \beta_j = \rho_j^c$. Also, if $\alpha^c = 1$, we have $\beta_j = \rho_j^c$ due to $\alpha^c \beta_j = \rho_j^c$. Thus we can conclude $0 \leq \rho_j^c \leq \beta_j$).

Also, we apply

$$\alpha^c \lambda_j = \upsilon_j^c,$$

by reason of

$$0 \leq \alpha^c \leq 1,$$

the following expression is satisfied:

$$0 \leq \upsilon_j^c \leq \lambda_j.$$

Thus, Model (5) can be substituted with the following linear programming:

$$ER_o^* = \underset{\Omega_k, \lambda_j, \beta_j, \alpha^c, \rho_j^c, \upsilon_j^c, \tilde{z}}{Min} \sum_{k=1}^{2} a_k \Omega_k \tag{6}$$

$$\begin{aligned}
& s.t. \quad (\text{Stage } 1) \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \Omega_1 x_{io}^1, \ \ i = 1, \ldots, m_1, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq y_{ro}^1, \quad r = 1, \ldots, s_1 \\
& \sum_{j=1}^{n} \upsilon_j^c w_{cj} \leq \alpha^c w_{co}, \quad c = 1, \ldots, C, \\
& \sum_{j=1}^{n} \lambda_j z_{ej}^{2-1} \leq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E, \\
& \sum_{j=1}^{n} \lambda_j z_{tj}^{1-2} \geq \tilde{z}_{to}^{1-2}, \ \ t = 1, \ldots, T, \\
& (\text{Stage } 2) \\
& \sum_{j=1}^{n} \beta_j x_{ij}^2 \leq \Omega_2 x_{io}^2, \ \ i = 1, \ldots, m_2, \\
& \sum_{j=1}^{n} \beta_j y_{rj}^2 \geq y_{ro}^2, \quad r = 1, \ldots, s_2, \\
& \sum_{j=1}^{n} \beta_j w_{cj} - \sum_{j=1}^{n} \rho_j^c w_{cj} \geq (1 - \alpha^c) w_{co}, \quad c = 1, \ldots, C,
\end{aligned}$$

$$\sum_{j=1}^{n} \beta_j z_{ej}^{2-1} \geq \tilde{z}_{eo}^{2-1}, \quad e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \beta_j z_{tj}^{1-2} \leq \tilde{z}_{to}^{1-2}, \quad t = 1, \ldots, T,$$
$$\lambda_j, \beta_j \geq 0,\ 0 \leq \alpha^c \leq 1,\ 0 \leq \rho_j^c \leq \beta_j,\ 0 \leq \upsilon_j^c \leq \lambda_j, \forall j, \forall c.$$

According to the stated change of variables, Model (5) is equivalent to Model (6). Also, as aforementioned, the optimal solution α^{c*} indicates the portion of the proportional dual-role factor c as the input of stage one and $1 - \alpha^{c*}$ shows the portion of the proportional dual-role factor as the output of stage two. α^{c*} is not necessarily unique. For SC_o, we use the following problem to identify the minimum α^{c*}:

$$\begin{aligned} & Min \sum_{c=1}^{C} \alpha^c \\ & s.t. \text{ constriants of model (6) and} \\ & \quad ER_o^* = \sum_{k=1}^{2} a_k \Omega_k. \end{aligned} \tag{7}$$

Actually, the minimum α^{c*} $(c = 1, \ldots, C)$ is found while the efficiency of the whole supply chain stays constant and optimal. Notice that α^c is a continuous variable between zero and one and we find the minimum α^{c*}. Thus, the optimal efficiency of the whole supply chain stays constant and optimal for an interval between α^{c*} and 1. Model (7) can be solved considering the objective function $Max\ \sum_{c=1}^{C} \alpha^c$ for finding the maximum α^{c*} $(c = 1, \ldots, C)$. However, they are obtained equal to one.

Definition 3.1. SC_o, the supply chain under evaluation, is said to be efficient if and only if the optimal value of Model (6) is equal to 1, i. e. $ER_o^* = 1$.

Theorem 3.2. $ER_o^* \leq \sum_{k=1}^{2} a_k \theta_{ok}^*$ that $\theta_{ok}^* (k = 1, 2)$ is the efficiency score of SC_o calculated by the CCR model (i. e. Model (1)) for each stage separately.

Proof. Suppose $(\theta_{o1}^*, \lambda_j^*)$ and $(\theta_{o2}^*, \beta_j^*)$ are optimal solutions of Model (1) for stages 1 and 2, respectively. It is clear that for stage 1:

$$\sum_{j=1}^{n} \lambda_j^* x_{ij}^1 \leq \theta_{o1}^* x_{io}^1,\ i = 1, \ldots, m_1, \quad \sum_{j=1}^{n} \lambda_j^* y_{rj}^1 \geq y_{ro}^1,\ r = 1, \ldots, s_1,$$
$$\sum_{j=1}^{n} \lambda_j^* w_{cj} \leq \theta_{o1}^* w_{co},\ c = 1, \ldots, C, \quad \sum_{j=1}^{n} \lambda_j^* z_{ej}^{2-1} \leq \theta_{o1}^* z_{eo}^{2-1},\ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \lambda_j^* z_{tj}^{1-2} \geq z_{to}^{1-2},\ t = 1, \ldots, T.$$

Note that $\sum_{j=1}^{n} \lambda_j^* w_{cj} \leq \theta_{o1}^* w_{co} \leq w_{co} \overset{0 \leq \alpha^c \leq 1}{\Rightarrow} \alpha^c \left(\sum_{j=1}^{n} \lambda_j^* w_{cj} \leq w_{co} \right),\ c = 1, \ldots, C,$ and for stage 2

$$\sum_{j=1}^{n} \beta_j^* x_{ij}^2 \leq \theta_{o2}^* x_{io}^2,\ i = 1, \ldots, m_2,\quad \sum_{j=1}^{n} \beta_j^* y_{rj}^2 \geq y_{ro}^2,\ r = 1, \ldots, s_2,$$
$$\sum_{j=1}^{n} \beta_j^* w_{cj} \geq w_{co},\quad c = 1, \ldots, C,\quad \sum_{j=1}^{n} \beta_j^* z_{ej}^{2-1} \geq z_{eo}^{2-1},\ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \beta_j^* z_{tj}^{1-2} \leq \theta_{o2}^* z_{to}^{1-2},\quad t = 1, \ldots, T,$$

by which it can be concluded that $(1-\alpha^c)\left(\sum_{j=1}^{n} \beta_j^* w_{cj} \geq w_{co}\right),\ c = 1, \ldots, C$, due to $\sum_{j=1}^{n} \beta_j^* w_{cj} \geq w_{co}$ and $0 \leq 1-\alpha^c \leq 1$. Therefore, by taking $z_{to}^{1-2} = \tilde{z}_{to}^{1-2}$ and $z_{eo}^{2-1} = \tilde{z}_{eo}^{2-1}$, these optimal solutions obtained from Model (1) are a feasible solution for Model (5) that results in $ER_o^* \leq \sum_{k=1}^{2} a_k \theta_{ok}^*$. □

It should be noted that the proposed radial input-oriented model can be conveniently reformulated for the output-oriented version.

3.2. Non-radial models for evaluating the closed-loop supply chain efficiency with proportional dual-role factors

In the previous subsection, a radial input-oriented DEA model was proposed for measuring the efficiency of the supply chain wherein the inputs of each component are reduced in a certain proportion. Here, an extended Russell measure is introduced for measuring the efficiency of supply chains with reverse flows and proportional dual-role factors in which inputs of each component are contracted non-proportionally and outputs of each component are augmented non-proportionally. In the following proposed model, $\Omega_{ik} (i = 1, \ldots, m_k; k = 1, 2)$ and $\psi_{rk}\ (r = 1, \ldots, s_k; k = 1, 2)$ indicate non-proportional reductions of inputs and non-proportional augmentations of outputs, respectively.

$$NR_o^* = Min\ \frac{\sum_{k=1}^{2} a_k \frac{1}{m_k} \sum_{i=1}^{m_k} \Omega_{ik}}{\sum_{k=1}^{2} a_k \frac{1}{s_k} \sum_{r=1}^{s_k} \psi_{rk}} \tag{8}$$

$$s.t.\quad \text{(Stage 1)}$$
$$\sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \Omega_{i1} x_{io}^1,\ i = 1, \ldots, m_1,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq \psi_{r1} y_{ro}^1,\quad r = 1, \ldots, s_1,$$
$$\alpha^c\left(\sum_{j=1}^{n} \lambda_j w_{cj} \leq w_{co}\right),\quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \lambda_j z_{ej}^{2-1} \leq \tilde{z}_{eo}^{2-1},\ e = 1, \ldots, E,$$

$$\sum_{j=1}^{n} \lambda_j z_{tj}^{1-2} \geq \tilde{z}_{to}^{1-2}, \quad t = 1, \ldots, T,$$
(Stage 2)
$$\sum_{j=1}^{n} \beta_j x_{ij}^2 \leq \Omega_{i2} x_{io}^2, \quad i = 1, \ldots, m_2,$$
$$\sum_{j=1}^{n} \beta_j y_{rj}^2 \geq \psi_{r2} y_{ro}^2, \quad r = 1, \ldots, s_2,$$
$$(1-\alpha^c)\left(\sum_{j=1}^{n} \beta_j w_{cj} \geq w_{co}\right), \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \beta_j z_{ej}^{2-1} \geq \tilde{z}_{eo}^{2-1}, \quad e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \beta_j z_{tj}^{1-2} \leq \tilde{z}_{to}^{1-2}, \quad t = 1, \ldots, T,$$
$$\lambda_j, \beta_j \geq 0,\ 0 \leq \alpha^c \leq 1,\ \Omega_{ik} \leq 1, \psi_{rk} \geq 1, \forall j, \forall c, \forall i, \forall r, \forall k.$$

Also, $a_k (k = 1, 2)$ is normalized weights determined by decision makers to show the importance of each stage. It is clear that Model (8) is the fractional non-linear programming problem. At first, we transform it into a fractional linear programming by using the following change of variables:

$$\alpha^c \left(\sum_{j=1}^{n} \lambda_j w_{cj} \leq w_{co}\right) \Rightarrow \sum_{j=1}^{n} \alpha^c \lambda_j w_{cj} \leq \alpha^c w_{co},$$
$$(1-\alpha^c)\left(\sum_{j=1}^{n} \beta_j w_{cj} \geq w_{co}\right) \Rightarrow \sum_{j=1}^{n} \beta_j w_{cj} - \sum_{j=1}^{n} \alpha^c \beta_j w_{cj} \geq (1-\alpha^c) w_{co},$$
$$\alpha^c \beta_j = \rho_j^c, 0 \leq \alpha^c \leq 1 \Rightarrow 0 \leq \rho_j^c \leq \beta_j,$$
$$\alpha^c \lambda_j = \upsilon_j^c, 0 \leq \alpha^c \leq 1 \Rightarrow 0 \leq \upsilon_j^c \leq \lambda_j$$

Thus, with considering the aforementioned change of variables, Model (8) is substituted with the following problem:

$$NR_o^* = Min \ \frac{\sum_{k=1}^{2} a_k \frac{1}{m_k} \sum_{i=1}^{m_k} \Omega_{ik}}{\sum_{k=1}^{2} a_k \frac{1}{s_k} \sum_{r=1}^{s_k} \psi_{rk}} \tag{9}$$

$s.t.$ (Stage 1)
$$\sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \Omega_{i1} x_{io}^1, \quad i = 1, \ldots, m_1,$$

$$
\begin{aligned}
&\sum_{j=1}^{n} \lambda_{_j} y_{rj}^{1} \geq \psi_{r1} y_{ro}^{1}, \quad r=1,\ldots,s_1,\\
&\sum_{j=1}^{n} \upsilon_j^c w_{cj} \leq \alpha^c w_{co}, \quad c=1,\ldots,C,\\
&\sum_{j=1}^{n} \lambda_{_j} z_{ej}^{2-1} \leq \tilde{z}_{eo}^{2-1}, \;\; e=1,\ldots,E,\\
&\sum_{j=1}^{n} \lambda_{_j} z_{tj}^{1-2} \geq \tilde{z}_{to}^{1-2}, \quad t=1,\ldots,T,\\
&\text{(Stage 2)}\\
&\sum_{j=1}^{n} \beta_j x_{ij}^{2} \leq \Omega_{i2} x_{io}^{2}, \;\; i=1,\ldots,m_2,\\
&\sum_{j=1}^{n} \beta_j y_{rj}^{2} \geq \psi_{r1} y_{ro}^{2}, \quad r=1,\ldots,s_2,\\
&\sum_{j=1}^{n} \beta_j w_{cj} - \sum_{j=1}^{n} \rho_j^c w_{cj} \geq (1-\alpha^c) w_{co}, \quad c=1,\ldots,C,\\
&\sum_{j=1}^{n} \beta_j z_{ej}^{2-1} \geq \tilde{z}_{eo}^{2-1}, \;\; e=1,\ldots,E,\\
&\sum_{j=1}^{n} \beta_j z_{tj}^{1-2} \leq \tilde{z}_{to}^{1-2}, \quad t=1,\ldots,T,\\
&\lambda_{_j}, \beta_{_j} \geq 0,\; 0 \leq \alpha^c \leq 1\,, 0 \leq \rho_j^c \leq \beta_j, 0 \leq \upsilon_j^c \leq \; \lambda_j,\\
&\Omega_{ik} \leq 1, \psi_{rk} \geq 1, \forall j, \forall c, \forall i, \forall r, \forall k.
\end{aligned}
$$

Afterwards, we use the Charnes–Cooper transformation [16] for linearizing Model (9) as follows:

$$
\begin{aligned}
&\frac{1}{\sum_{k=1}^{2} a_k \frac{1}{s_k} \sum_{r=1}^{s_k} \psi_{rk}} = t, t\lambda_{_j} = \bar{\lambda}_{_j}, t\beta_j = \bar{\beta}_j, t\Omega_{ik} = \bar{\Omega}_{ik}, t\psi_{rk} = \bar{\psi}_{rk}, t\alpha^c = \bar{\alpha}^c,\\
&t\rho_j^c = \bar{\rho}_j^c, t\upsilon_j^c = \bar{\upsilon}_j^c,\; t\tilde{z}_{eo}^{2-1} = \bar{z}_{eo}^{2-1}, t\tilde{z}_{to}^{1-2} = \bar{z}_{tjo}^{1-2}.
\end{aligned}
$$

Therefore, Model (9) is reformulated as the following linear programming problem:

$$
NR_o^* = Min \sum_{k=1}^{2} a_k \frac{1}{m_k} \sum_{i=1}^{m_k} \bar{\Omega}_{ik} \tag{10}
$$

$$
\begin{aligned}
&s.t. \quad \text{(Stage 1)}\\
&\sum_{k=1}^{2} a_k \frac{1}{s_k} \sum_{r=1}^{s_k} \bar{\psi}_{rk} = 1,
\end{aligned}
$$

$$\sum_{j=1}^{n} \bar{\lambda}_j x_{ij}^1 \leq \bar{\Omega}_{i1} x_{io}^1, \ \ i = 1, \ldots, m_1,$$
$$\sum_{j=1}^{n} \bar{\lambda}_j y_{rj}^1 \geq \bar{\psi}_{r1} y_{ro}^1, \quad r = 1, \ldots, s_1,$$
$$\sum_{j=1}^{n} \bar{\upsilon}_j^c w_{cj} \leq \bar{\alpha}^c w_{co}, \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \bar{\lambda}_j z_{ej}^{2-1} \leq \bar{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \bar{\lambda}_j z_{tj}^{1-2} \geq \bar{z}_{to}^{1-2}, \ \ t = 1, \ldots, T,$$

(Stage 2)

$$\sum_{j=1}^{n} \bar{\beta}_j x_{ij}^2 \leq \bar{\Omega}_{i2} x_{io}^2, \ \ i = 1, \ldots, m_2,$$
$$\sum_{j=1}^{n} \bar{\beta}_j y_{rj}^2 \geq \bar{\psi}_{r1} y_{ro}^2, \quad r = 1, \ldots, s_2,$$
$$\sum_{j=1}^{n} \bar{\beta}_j w_{cj} - \sum_{j=1}^{n} \bar{\rho}_j^c w_{cj} \geq (t - \bar{\alpha}^c) w_{co}, \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \bar{\beta}_j z_{ej}^{2-1} \geq \bar{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \bar{\beta}_j z_{tj}^{1-2} \leq \bar{z}_{to}^{1-2}, \ \ t = 1, \ldots, T,$$
$$\bar{\lambda}_j, \bar{\beta}_j \geq 0, \ 0 \leq \bar{\alpha}^c \leq t, 0 \leq \bar{\rho}_j^c \leq \bar{\beta}_j, 0 \leq \bar{\upsilon}_j^c \leq \bar{\lambda}_j,$$
$$\bar{\Omega}_{ik} \leq t, \bar{\psi}_{rk} \geq t, \forall j, \forall c, \forall i, \forall r, \forall k.$$

Definition 3.3. A supply chain o is called efficient under Model (10) if and only if the optimal value of Model (10) is equal to 1, that is $NR_o^* = 1$.

It is obvious that all components of the supply chain will be efficient when the whole supply chain is efficient. Also, the whole supply chain is inefficient if and only if $NR_o^* < 1$. In the similar way of the proposed radial model, we can calculate the minimum $\bar{\alpha}^c, (c = 1, \ldots, C)$ for SC_o by the following model:

$$\begin{aligned} &Min \sum_{c=1}^{C} \bar{\alpha}^c \\ &s.t. \ \text{constriants of model (10) and} \\ &\quad NR_o^* = \sum_{k=1}^{2} a_k \frac{1}{m_k} \sum_{i=1}^{m_k} \bar{\Omega}_{ik} \ . \end{aligned} \tag{11}$$

The maximum $\bar{\alpha}^c, (c = 1, \ldots, C)$ can be computed by substituting the objective function (11) with $Max\ \sum_{c=1}^{C} \bar{\alpha}^c$. Nevertheless, the upper bound t is found for $\bar{\alpha}^c, (c = 1, \ldots, C)$ that results in the upper bound one for α^c.

Theorem 3.4. $NR_o^* \leq \sum_{k=1}^{2} a_k z_{ERMo}^{*k}$ that z_{ERMo}^{*k} is the efficiency score of SC_o obtained by Model (2) (i.e. enhanced Russell measure) for each stage $k\,(k = 1, 2)$ separately.

Proof. Similar to Theorem 3.2., it can be proved. □

Theorem 3.5. The optimal value of Model (10) is less or equal to the optimal value of Model (6), i.e. $NR_o^* \leq ER_o^*$ for SC_o.

Proof. Optimal solutions of Model (6) are feasible solutions for Model (10). Thus, the optimal value of Model (10) will not be more than Model (6). It means $NR_o^* \leq ER_o^*$ for each SC_o. □

The aforementioned non-radial models are non-oriented. Conveniently, the non-radial input-oriented (output-oriented) model can be defined. The following model shows the input-oriented extended Russell measure for measuring the efficiency of closed-loop supply chains in the presence of proportional dual-role measures:

$$IR_o^* = Min\ \sum_{k=1}^{2} a_k \left(\sum_{i=1}^{m_k} \Omega_{ik}/m_k \right) \tag{12}$$

$$\begin{aligned}
& s.t. \quad \text{(Stage 1)} \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \Omega_{i1} x_{io}^1, \ \ i = 1, \ldots, m_1, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq y_{ro}^1, \quad r = 1, \ldots, s_1, \\
& \sum_{j=1}^{n} \upsilon_j^c w_{cj} \leq \alpha^c w_{co}, \quad c = 1, \ldots, C, \\
& \sum_{j=1}^{n} \lambda_j z_{ej}^{2-1} \leq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E, \\
& \sum_{j=1}^{n} \lambda_j z_{tj}^{1-2} \geq \tilde{z}_{to}^{1-2}, \quad t = 1, \ldots, T, \\
& \text{(Stage 2)} \\
& \sum_{j=1}^{n} \beta_j x_{ij}^2 \leq \Omega_{i2} x_{io}^2, \ \ i = 1, \ldots, m_2, \\
& \sum_{j=1}^{n} \beta_j y_{rj}^2 \geq y_{ro}^2, \quad r = 1, \ldots, s_2,
\end{aligned}$$

$$\sum_{j=1}^{n} \beta_j w_{cj} - \sum_{j=1}^{n} \rho_j^c w_{cj} \geq (1-\alpha^c) w_{co}, \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \beta_j z_{ej}^{2-1} \geq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \beta_j z_{tj}^{1-2} \leq \tilde{z}_{to}^{1-2}, \ \ t = 1, \ldots, T,$$
$$\lambda_j, \beta_j \geq 0, \ 0 \leq \alpha^c \leq 1, 0 \leq \rho_j^c \leq \beta_j,$$
$$0 \leq \upsilon_j^c \leq \lambda_j, \Omega_{ik} \leq 1, \forall j, \forall c, \forall i, \forall k.$$

And the output-oriented extended Russell measure is formulated as follows:

$$OR_o^* = Max \ \sum_{k=1}^{2} a_k \left(\sum_{r=1}^{s_k} \psi_{rk} / s_k \right) \tag{13}$$

$$s.t. \quad (\text{Stage}\,1)$$
$$\sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq x_{io}^1, \ \ i = 1, \ldots, m_1,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq \psi_{r1} y_{ro}^1, \quad r = 1, \ldots, s_1,$$
$$\sum_{j=1}^{n} \upsilon_j^c w_{cj} \leq \alpha^c w_{co}, \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \lambda_j z_{ej}^{2-1} \leq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$
$$\sum_{j=1}^{n} \lambda_j z_{tj}^{1-2} \geq \tilde{z}_{to}^{1-2}, \ \ t = 1, \ldots, T,$$
$$(\text{Stage} \ \ 2)$$
$$\sum_{j=1}^{n} \beta_j x_{ij}^2 \leq x_{io}^2, \ \ i = 1, \ldots, m_2,$$
$$\sum_{j=1}^{n} \beta_j y_{rj}^2 \geq \psi_{r2} y_{ro}^2, \quad r = 1, \ldots, s_2,$$
$$\sum_{j=1}^{n} \beta_j w_{cj} - \sum_{j=1}^{n} \rho_j^c w_{cj} \geq (1-\alpha^c) w_{co}, \quad c = 1, \ldots, C,$$
$$\sum_{j=1}^{n} \beta_j z_{ej}^{2-1} \geq \tilde{z}_{eo}^{2-1}, \ \ e = 1, \ldots, E,$$

$$\sum_{j=1}^{n} \beta_j z_{tj}^{1-2} \leq \tilde{z}_{to}^{1-2}, \quad t = 1, \ldots, T,$$
$$\lambda_j, \beta_j \geq 0, \; 0 \leq \alpha^c \leq 1, \; 0 \leq \rho_j^c \leq \beta_j,$$
$$0 \leq v_j^c \leq \lambda_j, \psi_{rk} \geq 1, \forall j, \forall c, \forall r, \forall k.$$

Notice that the introduced models were investigated under CRS assumption. However, they can be extended under Variable Returns to Scale (VRS) assumption too.

4. AN APPLICATION

In this section, our suggested approaches are used to analyze the performance in the poultry industry. In the DEA literature, some authors [5, 31, 49] measured the performance of broiler production farms. The purpose of this case study is to assess the efficiency of 13 broiler supply chains from Iran over one period of six months when reverse flows and proportional dual-role factors are present. As can be seen in Figure 2, the broiler supply chain under consideration has been constituted from two components, broiler chicken farm and also Chicken Slaughterhouse and Waste Management (CSWM).

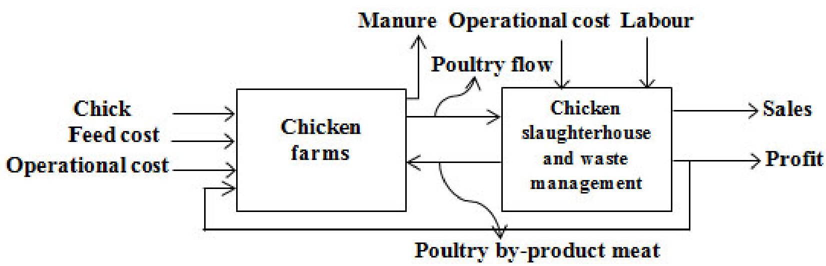

Fig. 2. The structure of a case study.

Inputs, outputs, intermediate measures and proportional dual-role factors in this study were chosen according to data availability and literature review. Factors used are described as follows:

Component 1: Broiler chicken farm

- Inputs (I): New born chicken, feed cost and operational cost.
  - ∗ New born chicken: The number of broilers bred to produce meat.
  - ∗ Feed cost (1000 Rials): Chicken feed cost consumed for each poultry farm.
  - ∗ Operational cost (1000 Rials): It contains costs of labourer, vehicles, energy and drugs.
- Outputs (O): Manure.
  - ∗ Manure (Kg): Poultry manure generated by chickens.

- Proportional dual-role factor: Profit.
  - Profit (1000 Rials): The resulting profits in the CSWM component are divided into two portions as both final outputs of the CSWM component and capital to finance the ongoing operations that are consumed by the broiler chicken farm component.

Component 2: Chicken slaughterhouse and waste management

- Inputs (I): Operational cost and labour.
  - Operating cost (1000 Rials): It contains operating expenses and labourer and products costs.
  - Labour: The number of labourers.
- Outputs (O): Sales.
  - Sales: The number of products sold.
- Proportional dual-role factor: Profit.
  - Profit (1000 Rials): The resulting profits in the CSWM component are divided into two portions as both final outputs of the CSWM component and capital to finance the ongoing operations that are consumed by the broiler chicken farm component.

Intermediate measures

- Intermediate measures from component 1 to component 2 (1-2): Product flow.
  - Product flow: The number of poultry shipped from broiler chicken farm to chicken slaughterhouse.
- Intermediate measure from component 2 to component 1 (2-1): Poultry by-product meat,
  - Poultry by-product meat (Kg): Volume of poultry by-product meat used in the broiler diet that is sent from the CSWM component to broiler chicken farm.

Data of supply chains' components are shown in Table 1. Without a loss of generality, we firstly focus upon the introduced radial input-oriented model that is under CRS. Thus, Models (6) and (7) are initially calculated for evaluating the efficiency of the whole broiler supply chain and components, and determining the portions of the proportional dual-role factor. The results are presented in Table 2. We assume the equal preference between two stages, broiler chicken farm and CSWM, that is $a_1 = a_2 = 1/2$. Columns 2-4 of Table 2 show the overall efficiency of the whole supply chain, farm and CSWM efficiencies, respectively. α^* in the fifth column shows the portion of the proportional dual-role factor as the input of broiler chicken farm component. The range of α^* has been found by calculating the minimum value of α^* via Model (7) and the upper bound

SC	Chicken farm (1)				CSWM (2)			Dual-role	Intermediate	
	New born chicken (I)	Feed cost (I)	Operational cost (I)	Manure (O)	Operational Cost (I)	Labour (I)	Sales (O)	Profit	Product flow(1-2)	Poultry by-product meat flow (2-1)
1	12700	148500	573700	3500	402000	1046	45786	4284	6691	6500
2	14670	171740	639000	3600	624000	2534	44643	3824	7871	8740
3	13300	154930	632200	3400	541000	2262	43845	6732	6921	6532
4	15000	182880	665900	4200	672000	2847	46320	5164	8280	7240
5	12000	147490	570300	3400	720000	3124	44757	3745	6340	7420
6	14000	165080	636400	3900	684000	2964	47648	5633	7134	7460
7	13000	168930	620200	4600	704000	3011	46514	6476	7202	7100
8	14900	175430	716800	4800	462000	1067	48737	5884	7475	7605
9	13500	169520	623000	5750	523000	2141	48243	6512	7399	7423
10	12800	144130	609300	3600	721000	3402	32844	7420	6359	6457
11	19800	235970	809600	5150	462000	1824	48246	7134	10373	10400
12	11000	133540	513400	3200	485000	1942	31465	6182	5933	5812
13	12000	148870	572100	3800	634000	2589	58243	5814	6521	6342

Tab. 1. Data of a case study.

one. One can show the upper bound one by solving Model (7) where the minimization objective function is converted into maximization objective function. $1-\alpha^*$ in the sixth column indicates the portion of the proportional dual-role factor as the output of CSWM component. Interval $1-\alpha^*$ can be conveniently determined due to the range of α^*. As can be seen in column 2, no broiler supply chain is overall efficient. However, broiler supply chain 1 has better performance in comparison to other supply chains. Furthermore, the portion of the proportional dual-role factor as input and/or output in SCs 1, 2, 5, 8 and 13 does not have any influence on the efficiency results. It means managers can decide arbitrary. Comparing the efficiency scores of two stages, it can be found that SC 9 is efficient in the broiler farm stage while SCs 1, 8 and 13 are efficient in the CSWM stage.

To illustrate the results presented in Table 2, we consider SC 7 as an instance. Its overall efficiency is 0.758 and the stages efficiencies, broiler chicken farm and CSWM, are 0.907 and 0.609, respectively. For identifying the portion of profit as the input of the farm stage and the output of the CSWM stage, the decision maker can choose from intervals. That is the interval [0.319, 1] can be used to specify the profit portion as the input while the interval [0, 0.681] can be applied to obtain it as the output. For example, managers can select 0.580 of profit as the input, that is, $0.580 \times 6476 = 3756.08$ of profit is considered as the input of the farm stage and $0.42 \times 6476 = 2719.92$ of profit is taken as the output of the CSWM stage. In fact, managers can select each value of interval [0.319, 1] to estimate the portion of profit as the input of the farm stage and according to it the portion of profit as the output of the CSWM stage is attained. To show further details, the best performance values of supply chains are found by considering these ranges. In other words, falling out of these ranges causes worse performance of supply chains. Managers can also apply this information for future planning and reallocation. And more rational efficiency results are achieved by incorporating proportional dual-role factors.

At the next stage, we use Models (10) and (11) for measuring the efficiency of SCs. The results are shown in Table 3. The overall efficiency scores are indicated in the second column. SC 1 with score equal to 0.959 has the best overall efficiency in contrast to other SCs. The efficiency scores of farm and CSWM stages are presented in the third and fourth columns, respectively. Broiler supply chain 9 is efficient in the farm stage and two SCs 1 and 8 are efficient in the CSWM stage. Also, the portion of the proportional

SC	Efficiency			α^*	$1-\alpha^*$
	Overall	Farm	CSWM		
1	0.998	0.996	1	[0,1]	[0,1]
2	0.721	0.668	0.774	[0,1]	[0,1]
3	0.8	0.718	0.882	[0.35,1]	[0,0.65]
4	0.711	0.738	0.685	[0.13,1]	[0,0.87]
5	0.722	0.768	0.677	[0,1]	[0,1]
6	0.749	0.753	0.746	[0.16,1]	[0,0.84]
7	0.758	0.907	0.609	[0.319,1]	[0,0.681]
8	0.979	0.957	1	[0,1]	[0,1]
9	0.927	1	0.855	[0.16,1]	[0,0.84]
10	0.59	0.77	0.409	[0.551,1]	[0,0.449]
11	0.83	0.732	0.928	[0.34,1]	[0,0.66]
12	0.674	0.766	0.582	[0.521,1]	[0,0.479]
13	0.993	0.985	1	[0,1]	[0,1]

Tab. 2. Results of Models (6) and (7).

dual-role factor as the input of the farm stage can be seen in the fifth column while it as the output of the CSWM stage can be found in the sixth column. Similar to the previous approach, we can interpret the results of Models (10) and (11). For instance, consider SC 10. The overall efficiency of SC 10 is equal to 0.498. Also, farm and CSWM efficiency scores of SC 10 are 0.679 and 0.318, respectively. Furthermore, the interval [0.371, 1] in the fifth column shows the interval that it is used to determine the portion of profit as the input of the farm stage. Indeed, each amount of this interval can be chosen to specify the profit portion as the input. For instance, regarding the value 0.80 that belongs to this interval, $0.80 \times 7420 = 5936$ of profit is considered as the input and $0.20 \times 7420 = 1484$ of profit is deemed as the output. Notice that for SCs 1, 2, 4, 5, 8 and 13, each amount between zero and one can be selected to calculate the portions of profit as the input of the farm stage and the output of the CSWM stage, as shown in columns 5 and 6. This implies that decision-making about profit will be arbitrary in these SCs.

To summarize, comparing the results of Models (6) and (10) reflects the following results:

- The overall efficiency of SCs obtained by Model (10) is less than or equal to it of Model (6).
- The efficiency scores of the farm stage obtained by Model (10) are less than or equal to them by Model (6).
- The efficiency scores of the CSWM stage obtained by Model (10) are less than or equal to them by Model (6).
- SC 1 has the best overall performance in both Models (6) and (10).

SC	Efficiency			α^*	$1-\alpha^*$
	Overall	Farm	CSWM		
1	0.959	0.919	1	[0,1]	[0,1]
2	0.564	0.602	0.527	[0,1]	[0,1]
3	0.6	0.61	0.59	[0.255,1]	[0,0.745]
4	0.586	0.673	0.5	[0,1]	[0,1]
5	0.577	0.664	0.491	[0,1]	[0,1]
6	0.586	0.672	0.501	[0.051,1]	[0,0.949]
7	0.645	0.812	0.477	[0.184,1]	[0,0.816]
8	0.881	0.763	1	[0,1]	[0,1]
9	0.839	1	0.677	[0.157,1]	[0,0.843]
10	0.498	0.679	0.318	[0.371,1]	[0,0.629]
11	0.712	0.648	0.777	[0.216,1]	[0,0.784]
12	0.584	0.688	0.48	[0.3,1]	[0,0.7]
13	0.897	0.798	0.996	[0,1]	[0,1]

Tab. 3. Results of Models (10) and (11).

Finally, to compare the results obtained of the proposed models with the conventional DEA models, we solve the CCR model (Model (1)) and the ERM model (Model (3)) for the farm and CSWM stages separately. To illustrate in more details, individual solutions of the CCR and ERM models were provided for each stage. Then, the average of stages efficiencies is calculated for obtaining the overall efficiency of SCs. The results are indicated in Table 4. Columns 2 and 3 show the results obtained from calculating the CCR model for each stage separately. Then, the average efficiency is computed that it is displayed in the fourth column. As can be seen the overall efficiency of Model (6) is less than or equal to that of the CCR model. Indeed, investigating the intermediate measures and proportional dual-role factors in the proposed approach causes that different results are obtained. Columns 5 and 6 of Table 4 show the stages efficiency scores resulted via the enhanced Russell model individually. The average efficiency is present in the seventh column of Table 4.

Similarly, the resulting efficiency scores of Model (10) are less than or equal to the average efficiency scores obtained from the ERM model. Interestingly, the portions of profit (the proportional dual-role factor) is also determined as input and output in the proposed approach such that optimal efficiency scores are resulted.

To more explain the results presented in Table 4, we consider SC 5 for instance. Farm and CSWM efficiencies obtained from Model (1) are 0.99 and 1, respectively. The average of the stages' efficiency is taken as the overall efficiency of SC 5. It means that the whole efficiency of SC 5 is equal to 0.995. Furthermore, by calculating the enhanced Russell model for SC 5, farm and CSWM efficiencies are 0.95 and 1 while the resulting overall efficiency is 0.975.

Figure 3 shows an overview of the overall efficiency scores of broiler supply chains that have been obtained from the proposed approaches and the traditional DEA models. As shown in Figure 3, the overall efficiency scores resulted from Models (6) and (10) are

SC	CCR Efficiency			ERM Efficiency		
	Farm	CSWM	Average	Farm	CSWM	Average
1	1	1	1	0.96	1	0.98
2	1	1	1	1	1	1
3	0.98	1	0.99	0.82	1	0.91
4	1	0.83	0.915	1	0.61	0.805
5	0.99	1	0.995	0.95	1	0.975
6	0.96	0.98	0.97	0.85	0.83	0.84
7	1	0.96	0.98	1	0.79	0.895
8	0.96	1	0.98	0.91	1	0.955
9	1	1	1	1	1	1
10	0.98	1	0.99	0.79	1	0.895
11	1	1	1	1	1	1
12	0.98	1	0.99	0.84	1	0.92
13	0.99	1	0.995	0.87	1	0.935

Tab. 4. Results of CCR and ERM models.

SC	α				
	0	0.03	0.1	[0.551,1]	[0.521,1]
10	0.948	0.921	0.857	0.59	
12	0.992	0.969	0.914		0.674

Tab. 5. Results of Model (6) for different values α.

more distinguished relative to CCR and ERM models. Also, broiler supply chain 10 has the least overall efficiency in both Models (6) and (10). Looking at Tables 2 and 3, it is apparent that SC 10 has weaker performance in the CSWM component as compared to the broiler farm component. It can thus be suggested that the management reviews and reconsiders its operations and practices in the CSWM component.

4.1. Sensitivity analysis

Here, sensitivity analysis of findings gained from proposed models is appraised. We consider supply chains 10 and 12 as instances. Three cases $\alpha = 0$, 0.03 and 0.1 are addressed. Therefore, the introduced radial input-oriented and non-radial non-oriented models are calculated by taking different cases $\alpha = 0$, 0.03 and 0.1. Results of the overall efficiency values obtained from radial and non-radial approaches are shown in Tables 5 and 6, respectively. As can be seen in Tables 5 and 6, solutions might change for different values α. The last two columns of Tables 5 and 6 show the overall efficiency scores obtained from the offered models for credible intervals. According to findings, different overall efficiency scores could be derived for various points α. These detections demonstrate the sensitivity of appeared models to the changes of values α. Also, the range of intervals might change due to different data sets.

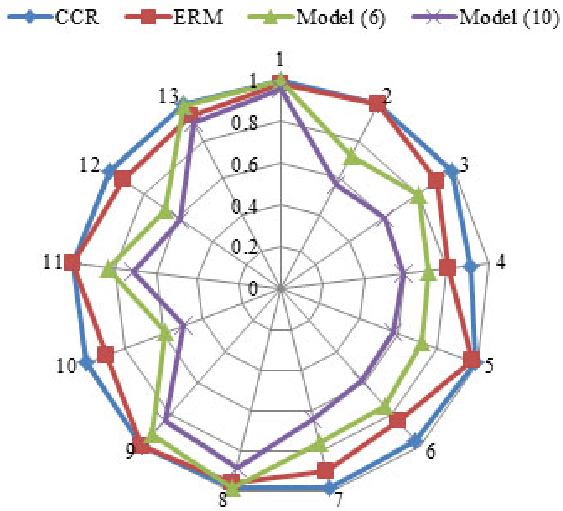

Fig. 3. Overall efficiencies of broiler supply chains.

SC	α				
	0	0.03	0.1	[0.371,1]	[0.3,1]
10	0.859	0.787	0.702	0.498	
12	0.898	0.823	0.744		0.584

Tab. 6. Results of Model (10) for different values α .

5. CONCLUSIONS

Supply chain is one of the most important multi-stage systems whose efficiency identification is significant for managers in order to make effective decisions. Data envelopment analysis (DEA) is one of the useful approaches for evaluating the efficiency of supply chains and their components. The efficiency of supply chains with forward flows and specified status of inputs and outputs are usually measured via the DEA methodology. Nevertheless, there are occasions in real-world applications in which the supply chain performance with forward, reverse flows and proportional dual-role (partial input and output roles) factors must be estimated.

The current paper has been designed to determine the efficiency of supply chains and components in the presence of reverse flows and proportional dual-role factors. Radial and non-radial DEA models have been proposed to investigate these closed-loop supply chains with proportional dual-role factors. The efficiency scores of the whole supply chains and their components have been measured at the same time. Furthermore, the portion of the proportional dual-role factor has been split into input and output while the optimal efficiency has been obtained. The application of broiler supply chain has been provided to illustrate and analyze the approaches. This is a primary study to evaluate the performance of Iranian broiler closed-loop supply chains while the proportional dual-role factor is included. This study can be also discussed in a series of directions. Firstly, the case of small data set is a limitation of this research. Further investigation should be conducted to analyze the performance of more broiler supply chains. Moreover, more discussion can be performed on slack variables and multiple optima. Models have been also based on performance assessment in a special period. More detailed analysis is needed to examine the performance of dynamic closed-loop supply chains with proportional dual-role measures.

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Monireh Jahani Sayyad Noveiri, Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan. Iran.
e-mail: monirehjahani@yahoo.com

Sohrab Kordrostami, Corresponding author. Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan. Iran.
e-mail: sohrabkordrostami@gmail.com

Alireza Amirteimoori, Department of Applied Mathematics, Rasht Branch, Islamic Azad University, Rasht. Iran.
e-mail: aamirteimoori@gmail.com