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CONSENSUS OF HETEROGENEOUS MULTI-AGENT SYSTEMS WITH UNCERTAIN DOS ATTACK: APPLICATION TO MOBILE STAGE VEHICLES

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In this paper, the consensus of heterogeneous multi-agent systems (MASs) with uncertain Deny-of-Service (DoS) attack strategies is studied. In our system, all agents are time synchronized and they communicate with each other with a constant sampling period normally. When the system is under attack, all agents use the hold-input mechanism to update the control protocol. By assuming that the attack duration is upper bounded and the occurrence of the attack follows a Markovian jumping process, the closed-loop system in presence of such a kind of random DoS attack is modeled as a Markovian jumping system, and the attack probabilities are allowed to be partially unknown and uncertain. By means of Lyapunov stability theory and Markovian jumping system approach, sufficient conditions are proposed such that the output consensus can be achieved, and the controller gains are determined by solving some matrix inequalities. Finally, a simulation study on the mobile stage vehicles is performed, showing the effectiveness of main results.

Keywords: heterogeneous multi-agent systems (MASs), Markovian jumping system, Deny-of-Service (DoS) attack, output feedback control

Classification: 93D05, 93C57, 60J05

1. INTRODUCTION

In recent years, cooperation of MASs has received increasing attention due to its wide application in various areas such as coordination of intelligent transportation systems, multi-spacecraft, mobile stages and smart grid, distributed target tracking of sensor networks [1, 2, 3, 4, 5, 6], and so on. The consensus problem is the basis of cooperation and coordination control of MASs, which includes the leader-following consensus and the leader-less one. In [7], a necessary and sufficient condition for the output consensus of heterogeneous linear MASs based on the internal model principle was proposed. A neural network-based adaptive leader-following consensus control method was proposed for a class of nonlinear state-delay MASs in [8]. Hu et al. [9] proposed a distributed dynamic event trigger mechanism to ensure the consensus of linear MASs. In [10], the consensus problem of MASs with Markovian network topologies and external interference was solved by introducing a new network topology mode regulator which consists

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of a randomly overlapping decomposer and a high-level decision maker. More work on MASs can be found in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] and references therein, where the consensus of MASs was studied by means of adaptive control, reinforcement learning and event-triggered control based on the matrix method and graph theory.

In reality, data transmission between each agent is usually realized through the wireless network. Therefore, how to ensure the consensus of MASs when the occurrence of various network attacks has become a common concern. In [22], the distributed coordinated control problem for a class of linear MASs affected by two types of network attacks on the edges was addressed. A sufficient condition for secure consensus tracking was given by using the so called average dwell time-based multiple Lyapunov function approach. Leader-following consensus of heterogeneous linear MASs under DoS attack was investigated by using a switched system approach in [23], where different attack intensities were considered. In [24], Zhi et al. studied the event-triggered secure cooperative control of linear MASs subject to DoS attacks. The frequency and duration of DoS attacks were analyzed for the problem of secure average consensus. Due to the existence of random DoS attack, the communication links would be broken and each agent uses hold-input mechanism to update the state and therefore resulting in the non-periodic sampling phenomena. Note that some ideas have been presented in the literature [25, 26, 27] for the non-periodic sampling problem of continuous systems whereas most of the above studies assume that the sampling process is deterministic. As pointed out in [28], random sampling schemes are preferable to deterministic sampling schemes. [29, 30, 31, 32, 33, 34, 35, 36] considered the consensus problem of MASs during random sampling of continuous systems for the purpose of reducing energy consumption or in the case of network attacks during data transmission, and the sampling probabilities are preciously known. However, it is worth mentioning that the above results were obtained based on the accurate statistical information of DoS attack process, which has great limitations in practical applications as the attack behavior is usually hard to capture. To the best of the author's knowledge, up to now, the consensus of heterogeneous MASs with uncertain or unknown DoS attack behaviors has not been investigated yet, which motivates our work.

Inspired by the recent works on the random sampling mechanism in existing works, the consensus of heterogeneous MASs with uncertain DoS attacks is studied in this paper, where the statistical information of DoS attack process is uncertain or even unknown. Normally, the agent sampling process is time synchronized and the sampling period is constant. Under the mild assumption that the attack is randomly triggered and follows the Markovian process, we formulate the closed-loop MASs in presence of DoS attack as a stochastic sampling system, where a Markovian jumping system approach is adopted. In our framework, the uncertain or unknown DoS attack strategies are modeled as uncertain and unknown elements of Markovian switching probabilities, where the number of subsystem depends on the attack duration directly. A sufficient condition for guaranteeing the consensus of heterogeneous MASs subject to uncertain DoS attack is obtained by using the decomposition technique, Lyapunov stability theory and matrix transformation method. An algorithm that calculating the gain of output feedback controller is obtained by solving a set of matrix inequalities. Finally, the effectiveness of main results is verified by a simulation study on the mobile stage vehicles.

Notations: R^n denotes the n-dimensional Euclidean space. $X > 0$ denotes a positive-definite matrix X. The superscript W^T means the transpose of a real matrix W, $he(X) = X + X^T$. $\mathbb{E}\{\bullet\}$ and $\Pr\{\bullet\}$ are the mathematical expectation and probability of the event "$\bullet$", respectively. $\| \bullet \|$ denotes the two-norm of matrix, "$*$" stands for the symmetry of a matrix. "$\otimes$" denote the Kronecker product. $diag\{\cdots\}$ is used to describe the block-diagonal matrix. $\lambda_{\min}\{\Omega\}$ is the minimal eigenvalue of matrix Ω. I and 0 represent the identity matrix and zero matrix with appropriate dimensions.

2. PROBLEM FORMULATION

First of all, we introduce some basic knowledge of graph theory. An undirected graphs $\mathcal{G}$ is formed by set $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}=\{\nu_1,\nu_2,\ldots,\nu_n\}$ represents a set of n nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges formed by ordered pairs of nodes. If there is a path from ν_1 to ν_n, then ν_1 and ν_n are connected. We called the graph $\mathcal{G}$ is a connected graph if for any two nodes in $\mathcal{G}$ are connected. Let the adjacency matrix be $\mathcal{A} = [a_{ij}]$, where $a_{ij} > 0$ when $(\nu_i,\nu_j) \in \mathcal{E}$, it represents the case that node i can receive the information from node j, otherwise, $a_{ij} = 0$. Defining the set of neighbors of node i as $\mathcal{N}_i = \{j : a_{ij} > 0\}$, and the matrix $\mathcal{D} = diag\{d_i\}$ is called as the in-degree matrix, where $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ is in-degree weights of node i. The Laplacian matrix is $\mathcal{L} = \mathcal{D} - \mathcal{A}$. The pining matrix $G = diag\{g_1, g_{2,\ldots,}g_n\}$ is used to show the interaction between the leader and followers. It is defined that $g_i = 1$ if the ith follower can receive information from the leader, otherwise, $g_i = 0$.

Assumption 2.1. The communication graph is connected(connected graph indicates that any two nodes in graph $\mathcal{G}$ are connected) and there is no isolated agent(nodes with zero degree are called isolated nodes).

Assumption 2.2. The DoS attack duration is upper bounded, and the occurrence of attack follows a Markov chain.

Remark 2.3. The DoS attack duration is generally upper bounded, see, e. g., the attack detection problem studied in the connected vehicles, where a sliding mode observer was designed in [37] to estimate the attack bound. In addition, the Markovian jumping system approach was adopted in many works to model the DoS attack phenomenon, see, e. g., [38]. In this work, we follow this method but extend it to the uncertain and unknown case.

The state space model of N followers and one leader of the continuous-time heterogeneous MASs are described as follows.
Follower:

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + D_i \omega_i \\ y_i = C_i x_i \quad i = 1, \ldots, N \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}, u_i(t) \in \mathbb{R}^{p_i}, \omega_i(t) \in \mathbb{R}^{m_i}$ and $y_i(t) \in \mathbb{R}^q$ are the state, control input, external disturbance and output of agent i, respectively. $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times p_i}$, $C_i \in \mathbb{R}^{q\times n_i}$, $D_i \in \mathbb{R}^{n_i\times m_i}$ are some constant matrices and they are generally different

in heterogeneous MASs.
Leader:

$$\begin{cases} \dot{x}_0 = Mx_0 \\ y_0 = \overline{R}x_0 \end{cases} \tag{2}$$

where $x_0(t) \in \mathbb{R}^m, y_0(t) \in \mathbb{R}^q$ are the state and measure output of the leader, respectively. $M \in \mathbb{R}^{m\times m}$ and $\bar{R} \in \mathbb{R}^{q\times m}$ are two constant matrices.

Traditionally, the following output feedback controller was designed [39]:

$$\begin{cases} u_i = K_i\left(y_i - C_i\Pi_i\zeta_i\right) + \Gamma_i\zeta_i \\ \dot{\zeta}_i = M\zeta_i + F\left(\sum\limits_{j\in\mathcal{N}_i} a_{ij}\left(\zeta_j - \zeta_i\right) + g_i\left(x_0 - \zeta_i\right)\right) \end{cases} \tag{3}$$

where $K_i \in \mathbb{R}^{p_i\times q}$ and $F \in \mathbb{R}^{m\times m}$ are the gains of the output feedback controller. In addition, matrices $\Pi_i \in \mathbb{R}^{n_i\times \mathrm{m}}$ and $\Gamma_i \in \mathbb{R}^{p_i\times \mathrm{m}}$ satisfy the following relationship:

$$A_i\Pi_i + B_i\Gamma_i = \Pi_iM; C_i\Pi_i = \bar{R} \qquad i = 1, \ldots, N. \tag{4}$$

Due to the fact that the control system is usually designed in a digital way, we assume that each agent can trigger the sampling process periodically and synchronously, and then send the real-time sampling data to the neighbor agent who needs to communicate. So the sampling time instants are $t_0^{'},\ t_1^{'}\ldots t_k^{'}\ldots$ and normally the sampling period is $h_k^{'} = t_{k+1}^{'} - t_k^{'} = T_0$. Due to the existence of adversaries, the communication links would be broken and each agent uses hold-input mechanism to update the state. For instance, the actual sampling period $h_k = t_{k+1} - t_k$ may become $2T_0,\ 3T_0,\ 4T_0\ldots$ when the DoS attack happens, where $\{t_0,\ t_1\ldots t_k\ldots\} \subseteq \left\{t_0^{'},\ t_1^{'}\ldots t_k^{'}\ldots\right\}$. Under the mild assumption that the attack is randomly triggered and follows the Markovian process, the sampling period h_k is thus taken from the set $\Re = \{\delta_1T_0, \delta_2T_0, \ldots, \delta_nT_0\}$, where T_0 is a fixed sampling time, $\delta_j, j = 1, 2, \ldots, n$ is a positive integer. For the simplicity, t_k will be shortly denoted as k. Denote $\rho(k) \in \phi \triangleq \{1, 2, \ldots, n\}$, then we have the following discrete-time system:
Follower:

$$\begin{cases} x_i(k+1) = A_{i\rho(k)}x_i(k) + B_{i\rho(k)}u_i(k) + D_{i\rho(k)}\omega_i(k) \\ y_i(k) = C_ix_i(k) \qquad\qquad\qquad i = 1, \ldots, N. \end{cases} \tag{5}$$

Leader:

$$\begin{cases} x_0(k+1) = M_{\rho(k)}x_0(k) \\ y_0(k) = \overline{R}x_0(k) \end{cases} \tag{6}$$

where $A_{i\rho(k)} = (A_{i0})^{\delta_{\rho(k)}}$, $B_{i\rho(k)} = \sum_{t=1}^{\delta_{\rho(k)}}(A_{i0})^{t-1}B_{i0}$, $D_{i\rho(k)} = \sum_{t=1}^{\delta_{\rho(k)}}(A_{i0})^{t-1}D_{i0}$, $M_{\rho(k)} = (M_0)^{\delta_{\rho(k)}}$ with $A_{i0} = e^{A_iT_0}$, $B_{i0} = B_i\int_0^{T_0}e^{A_i\tau}\mathrm{d}\tau$, $D_{i0} = D_i\int_0^{T_0}e^{A_i\tau}\mathrm{d}\tau$, $M_0 = e^{MT_0}$. In this paper, the Markov chain $\{\rho(k), k \in \mathbb{N}^+\}$ is used to describe the DoS attack process. Denote $\Pr(\rho(k+1) = t|\rho(k) = s) = \pi_{st}$ and for any $s, t \in \phi$, we have $\pi_{st} > 0$ and $\sum_{t=1}^{n}\pi_{st} = 1$.

The Markovian transition probability matrix is:

$$\Lambda = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1n} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2n} \\ & & \ddots & \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn} \end{bmatrix}. \tag{7}$$

In this paper, we propose the following control protocol

$$\begin{cases} \zeta_i(k+1) = M_{\rho(k)}\zeta_i(k) + F_{\rho(k)}\left(\sum\limits_{j\in\mathcal{N}_i} a_{ij}\left(\zeta_j(k) - \zeta_i(k)\right) + g_i\left(x_0(k) - \zeta_i(k)\right)\right) \\ u_i(k) = K_i\left(y_i(k) - C_i\Pi_i\zeta_i(k)\right) + \Gamma_i\zeta_i(k) \end{cases} \tag{8}$$

where $F_{\rho(k)} = \sum_{t=1}^{\delta_{\rho(k)}} (M_0)^{t-1} F_0$, $F_0 = F\int_0^{T_0} e^{M\tau}\mathrm{d}\tau$.

In the actual attack process, the attacker or hacker will deliberately hide the attack behavior. Therefore, some elements of the Markovian transition probability matrix is uncertain or even completely unknown. Inspired by [40], we assume that the transition probability matrix $\Lambda = [\pi_{st}]_{n\times n}$ is affected by a polytopes $\mathbf{P}_\Lambda$, where $\mathbf{P}_\Lambda \triangleq \left\{\Lambda | \Lambda = \sum_{r=1}^{\mathbb{Z}} \alpha_r\Lambda_r; \alpha_r \geqslant 0, \sum_{r=1}^{\mathbb{Z}} \alpha_r = 1\right\}$ with $\Lambda_r = [\pi_{st}]_{n\times n}, s, t \in \phi$, and $r = 1, \ldots, \mathbb{Z}$ is a transition probability matrix containing uncertain elements. In addition, for the sake of simplicity, we define $\phi = \phi_{\mathcal{K}}^{(s)} \cup \phi_{\mathcal{UC}}^{(s)} \cup \phi_{\mathcal{UK}}^{(s)}$.

$$\phi_{\mathcal{K}}^{(s)} \triangleq \{t : \pi_{st}\ is\ \text{know}\},\ \phi_{\mathcal{UC}}^{(s)} \triangleq \{t : \tilde{\pi}_{st}\ \text{is}\quad \text{uncertain}\},\ \phi_{\mathcal{UK}}^{(s)} \triangleq \{t : \hat{\pi}_{st}\ is\ unknow\}$$

$$\pi_{\mathcal{UC}}^{(s)} \triangleq \textstyle\sum_{t\in\phi_{\mathcal{UC}}^{(s)}} \tilde{\pi}_{st}^{r}, \forall r = 1, \ldots, \mathbb{Z} \text{ and } \pi_{\mathcal{K}}^{(s)} \triangleq \sum_{t\in\phi_{\mathcal{K}}^{(s)}} \pi_{st}$$

where uncertain and unknown elements are indicated by the superscripts "$\sim$" and "$\wedge$", respectively.

Remark 2.4. It is noted that the number of subsystems of (5) depends on the attack duration directly. The sampling period can be T_0 and $2T_0$ when the maximum attack duration is T_0, and the number of subsystems is 2. When the maximum attack duration is $2T_0$, the sampling period can be T_0, $2T_0$ and $3T_0$, so there are three subsystems. Thus when the maximum attack duration is nT_0, we may get $n+1$ subsystems.

Remark 2.5. It is assumed that the maximum attack duration is known in our work, when the bound of attack duration is unknown, one can use the sliding mode observer to estimate it, see [37] for more details.

Remark 2.6. The Markovian jumping system method was proposed in the paper [38] to deal with the DoS attack. Due to the fact that the precious attack behavior is hard to know, and the precise modeling method in [38] can not be applied. Although the behavior is not completely known, it is possible to have the bound of attack probability and the transition probability of different attack duration. Then we can transform those bound to the polytope-type uncertainty. Therefore, our modeling method is much more flexible than that in [38].

Define $e_i(k) = y_i(k) - y_0(k)$ as the output tracking error signal and also define the local tracking error and local reference synchronization error as follows

$$\begin{cases} \varepsilon_i(k) = x_i(k) - \Pi_i \zeta_i(k) \\ \eta_i(k) = \zeta_i(k) - x_0(k) \end{cases} \tag{9}$$

where $\varepsilon_i(k) \in \mathbb{R}^{n_i}$ and $\eta_i(k) \in \mathbb{R}^m$. At the same time, we need to define the following notations for the purpose of expression

$$\begin{cases} e(k) = \begin{bmatrix} e_1^T(k), & \dots & , e_N^T(k) \end{bmatrix}^T \\ \varepsilon(k) = \begin{bmatrix} \varepsilon_1^T(k), & \dots & , \varepsilon_N^T(k) \end{bmatrix}^T \\ \eta(k) = \begin{bmatrix} \eta_1^T(k), & \dots & , \eta_N^T(k) \end{bmatrix}^T \\ x_c(k) = \begin{bmatrix} \varepsilon^T(k), & \eta^T(k) \end{bmatrix}^T \\ \omega(k) = \begin{bmatrix} \omega_1^T(k), & \dots & , \omega_N^T(k) \end{bmatrix}^T \\ A_{\rho(k)} = diag(A_{i\rho(k)}), B_{\rho(k)} = diag(B_{i\rho(k)}), D_{\rho(k)} = diag(D_{i\rho(k)}) \\ C = diag(C_i), K = diag(K_i), \Pi = diag(\Pi_i) \end{cases} \tag{10}$$

where $A_{\rho(k)}, B_{\rho(k)}, D_{\rho(k)}, C, K, \Pi$ represent the system matrix of N-dimensional system composed of low-dimensioal $A_{i\rho(k)}, B_{i\rho(k)}, D_{i\rho(k)}, C_i, K_i, \Pi_i$. Then we can obtain the following closed-loop tracking error system

$$\begin{cases} x_c(k+1) = A_c x_c(k) + D_c \omega(k) \\ e(k) = C_c x_c(k) \end{cases} \tag{11}$$

where

$$A_c = \begin{bmatrix} A_{\rho(k)} + B_{\rho(k)} KC & \Pi(\mathcal{L} + G) \otimes F_{\rho(k)} \\ 0 & I_N \otimes M_{\rho(k)} - (\mathcal{L} + G) \otimes F_{\rho(k)} \end{bmatrix}$$

$$C_c = \begin{bmatrix} C & \bar{R}_c \end{bmatrix}, \quad D_c = \begin{bmatrix} D_{\rho(k)}^T & 0 \end{bmatrix}^T, \quad \bar{R}_c = I_N \otimes \bar{R}.$$

The consensus problem of heterogeneous MASs with uncertain DoS attack could be solved if we design the control protocol (8) such that:

1) for each initial condition $\varepsilon_i(0)$, $\eta_i(0)$ and $\rho(0) \in \phi$, the following inequalities

$$\mathbb{E}\left\{ \sum_{k=0}^{\infty} \| \eta_i(k) \|^2 | \chi(0) \right\} < \infty \tag{12}$$

$$\mathbb{E}\left\{ \sum_{k=0}^{\infty} \| \varepsilon_i(k) \|^2 | \chi(0) \right\} < \infty \tag{13}$$

are true, where $\chi(0) = \{\eta_i(0), \varepsilon_i(0), \rho(0)\}$ is initial condition.

2) for all non-zero $\omega_i(k) \in \mathcal{L}\,[0, \infty)$ and the zero-initial condition,

$$\mathbb{E}\left\{ \sum_{k=0}^{\infty} \| e_i\,(k)\,\|^2 \right\} \leqslant \gamma^2 \sum_{k=0}^{\infty} \| \omega_i(k) \|^2 \tag{14}$$

holds, where $\gamma > 0$ refers to the perturbation attenuation rate.

The following lemmas are the requirements to derive the main results.

Lemma 2.7. (Ni et al. [30]) For matrices with appropriate dimensions $\mathbb{T}$, $\mathbb{M}$, $\mathbb{U}$ and $\mathbb{W}$, the sufficient condition of $\mathbb{T} + he(\mathbb{M}\mathbb{W}) < 0$ is

$$\begin{bmatrix} \mathbb{T} & * \\ \mathbb{M}^T + \mathbb{U}\mathbb{W} & -\mathbb{U} - \mathbb{U}^T \end{bmatrix} < 0.$$

Lemma 2.8. There exists a positiive definite matrix $\mathcal{Q}$ such that inequality

$$-\mathcal{Q}^T \mathbb{I}^{-1} \mathcal{Q} \leqslant -\mathcal{Q} - \mathcal{Q}^T + \mathbb{I}$$

holds for any real matrix $\mathbb{I} \geqslant 0$.

Proof. For any $\mathbb{I} \geqslant 0$, we have $\mathbb{I} - \mathcal{Q} - \mathcal{Q}^T + \mathcal{Q}^T \mathbb{I}^{-1} \mathcal{Q} = \left(\mathbb{I} - \mathcal{Q}^T\right) \mathbb{I}^{-1} \left(\mathbb{I} - \mathcal{Q}\right) \geqslant 0.$ so we can obtain that $-\mathcal{Q}^T \mathbb{I}^{-1} \mathcal{Q} \leqslant -\mathcal{Q} - \mathcal{Q}^T + \mathbb{I}$. □

Lemma 2.9. (Schur complement): Given a symmetric matrix:

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12}^T & \Delta_{22} \end{bmatrix}$$

then the following inequalities

$$\begin{aligned} &(i)\, \Delta < 0; \\ &(ii)\, \Delta_{11} < 0, \Delta_{22} - \Delta_{12}^T \Delta_{11}^{-1} \Delta_{12} < 0; \\ &(iii) \Delta_{22} < 0, \Delta_{11} - \Delta_{12} \Delta_{22}^{-1} \Delta_{12}^T < 0. \end{aligned}$$

are equivalent.

3. MAIN RESULTS

Theorem 3.1. The consensus problem of heterogeneous MASs is solvable if the following low-dimensional closed-loop systems

$$\begin{cases} \hat{\varepsilon}_i(k+1) = \left(A_{i\rho(k)} + B_{i\rho(k)} K_i C_i\right) \hat{\varepsilon}_i(k) + D_{i\rho(k)} \omega_i(k) \\ e_i(k) = C_i \hat{\varepsilon}_i(k) \qquad\qquad i = 1, \ldots, N \end{cases} \tag{15}$$

are simultaneously asymptotically stable in the mean-square sense with a prescribed attenuation level $\gamma > 0$, and the following low-dimensional closed-loop systems

$$\hat{\eta}_i(k+1) = \left(M_{\rho(k)} - \lambda_i F_{\rho(k)}\right) \hat{\eta}_i(k) \quad i = 1, \ldots, N \tag{16}$$

are simultaneously asymptotically stable in the mean-square sense, where λ_i is non-zero eigenvalue of topology matrix $(\mathcal{L} + G)$.

Proof. We can easily find a transformation matrix $\mathcal{T}$ such that

$$\mathcal{T}(\mathcal{L}+G)\mathcal{T}^{-1}=\Lambda=\begin{pmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_N\end{pmatrix}. \tag{17}$$

Define $\hat{x}_c(k)=\begin{bmatrix}\hat{\varepsilon}(k)\\ \hat{\eta}(k)\end{bmatrix}=\begin{bmatrix}I & 0\\ 0 & \mathcal{T}\otimes I\end{bmatrix}x_c(k)$, then we have

$$\begin{cases}\hat{x}_c(k+1)=A_c^{'}\hat{x}_c(k)+D_c^{'}\omega(k)\\ e(k)=C_c^{'}\hat{x}_c(k)\end{cases} \tag{18}$$

where

$$\begin{aligned}A_c^{'}&=\begin{bmatrix}A_{\rho(k)}+B_{\rho(k)}KC & \Pi(\mathcal{L}+G)\mathcal{T}^{-1}\otimes F_{\rho(k)}\\ 0 & I_N\otimes M_{\rho(k)}-\mathcal{T}(\mathcal{L}+G)\mathcal{T}^{-1}\otimes F_{\rho(k)}\end{bmatrix}\\ C_c^{'}&=\begin{bmatrix}C & \mathcal{T}^{-1}\otimes\bar{R}_c\end{bmatrix},\quad D_c^{'}=D_c.\end{aligned}$$

It remains to show that the following low-dimensional systems

$$\begin{cases}\hat{x}_{c_i}(k+1)=\begin{bmatrix}\hat{\varepsilon}_i(k)\\ \hat{\eta}_i(k)\end{bmatrix}=A_{c_i}^{'}\hat{x}_{c_i}(k)+D_{c_i}^{'}\omega_i(k)\\ e_i(k)=C_{c_i}^{'}\hat{x}_{c_i}(k)\end{cases} \tag{19}$$

are simultaneously asymptotically stable, where

$$\begin{aligned}A_{c_i}^{'}&=\begin{bmatrix}A_{i\rho(k)}+B_{i\rho(k)}KC & \aleph\\ 0 & M_{\rho(k)}-\lambda_i F_{\rho(k)}\end{bmatrix}\\ C_{c_i}^{'}&=\begin{bmatrix}C_i & \Im\end{bmatrix},\quad D_{c_i}^{'}=\begin{bmatrix}D_{i\rho(k)}\\ 0\end{bmatrix}.\end{aligned}$$

$\aleph$ and $\Im$ are two terms do not affect our analysis. It is easy to know that systems (19) are a set of cascaded systems, where the input $\omega_i(k)$ does not affect $\hat{\eta}_i(k)$, then if we design suitable $F_{\rho(k)}$ such that systems (16) are simultaneously asymptotically stable, then $\aleph$ and $\Im$ block does not appear in the transfer function. Therefore, if systems (15) are simultaneously asymptotically stable with a prescribed attenuation level $\gamma>0$, the consensus problem is solvable and the proof is completed. □

Theorem 3.2. For given controller gains F_s, K_i, if there exist a set of matrices $P_s>0$ such that the following matrix inequalities

$$\begin{bmatrix}\psi_{is}^T\mathcal{P}_\Omega^{(s)}\psi_{is}-P_s+C_i^TC_i & \psi_{is}^T\mathcal{P}_\Omega^{(s)}D_{is}\\ * & D_{is}^T\mathcal{P}_\Omega^{(s)}D_{is}-\gamma^2 I\end{bmatrix}<0 \tag{20}$$

$$\begin{bmatrix}-P_s & (M_s-\lambda_i F_s)^T\\ * & -\left(\mathcal{P}_\Omega^{(s)}\right)^{-1}\end{bmatrix}<0 \tag{21}$$

are satisfied for all $i \in \{1, 2, \ldots, N\}$, $s \in \phi$, then for a given $\gamma > 0$, there exists a control protocol in form of (8) which guarantees the consensus of heterogeneous MASs subject to uncertain DoS attacks, where

$$\begin{cases} \mathcal{P}_{\mathcal{K}}^{(s)} \triangleq \sum_{t\in\phi_{\mathcal{K}}^{(s)}} \pi_{st}P_t, \mathcal{P}_{\mathcal{UC}}^{(s)} \triangleq \sum_{t\in\phi_{\mathcal{UC}}^{(s)}} \tilde{\pi}_{st}^{r}P_t, \\ \mathcal{P}_{\mathcal{UK}}^{(s)} \triangleq \sum_{t\in\phi_{\mathcal{UK}}^{(s)}} \hat{\pi}_{st}P_t, \\ \mathcal{P}^{s} \triangleq \mathcal{P}_{\mathcal{K}}^{(s)} + \sum_{t\in\phi_{\mathcal{UC}}^{(s)}} (\sum_{r=1}^{\mathbb{Z}} \alpha_r\tilde{\pi}_{st}^{r})P_t + \mathcal{P}_{\mathcal{UK}}^{(s)}, \\ \mathcal{P}_{\Omega}^{(s)} \triangleq \mathcal{P}_{\mathcal{K}}^{(s)} + \mathcal{P}_{\mathcal{UC}}^{(s)} + (1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)})P_t, \forall t \in \phi_{\mathcal{UK}}^{(s)} \end{cases} \tag{22}$$

and $\psi_{is} = A_{is} + B_{is}K_iC_i$.

P r o o f. First for the system (16): $\hat{\eta}_i(k+1) = \left(M_{\rho(k)} - \lambda_i F_{\rho(k)}\right)\hat{\eta}_i(k) \quad i = 1, \ldots, N$. Define the Lyapunov function as $V(\hat{\eta}_i(k), k, \rho(k)) = \hat{\eta}_i^T(k)P_{\rho(k)}\hat{\eta}_i(k)$ and let $\rho(k) = s$, $\rho(k+1) = t$. Then we have

$$\begin{aligned} &\mathbb{E}\{\Delta V(\hat{\eta}_i(k), k)\} \\ &\quad = \mathbb{E}\{V(\hat{\eta}_i(k+1), k+1, \rho(k+1)|\hat{\eta}_i(k), \rho(k) = s) - V(\hat{\eta}_i(k), k, \rho(k))\} \\ &\quad = \hat{\eta}_i^T(k+1)\mathcal{P}^{s}\hat{\eta}_i(k+1) - \hat{\eta}_i^T(k)P_s\hat{\eta}_i(k). \end{aligned} \tag{23}$$

In addition, we define

$$\begin{cases} \Phi_i = \hat{\eta}_i(k+1) \\ \Omega_i = -\hat{\eta}_i^T(k)P_s\hat{\eta}_i(k). \end{cases} \tag{24}$$

Then

$$\mathbb{E}\{\Delta V(\hat{\eta}_i(k), k)\} = \Phi_i^T\mathcal{P}^{s}\Phi_i + \Omega_i. \tag{25}$$

It is noted that $\mathcal{P}^{s} = \sum_{t=1}^{n} \pi_{st}P_t = \mathcal{P}_{\mathcal{K}}^{(s)} + \sum_{t\in\phi_{\mathcal{UC}}^{(s)}} (\sum_{r=1}^{\mathbb{Z}} \alpha_r\tilde{\pi}_{st}^{r})P_t + \mathcal{P}_{\mathcal{UK}}^{(s)}$, where $\sum_{r=1}^{\mathbb{Z}} \alpha_r\tilde{\pi}_{st}^{r}$, $\forall t \in \phi_{\mathcal{UC}}^{(s)}$ represent an uncertain element in the polytope uncertainty description, $\sum_{r=1}^{\mathbb{Z}} \alpha_r = 1$, $\alpha_r \in [0, 1]$, then we can acquire that

$$\begin{aligned} &\mathbb{E}\{\Delta V(\hat{\eta}_i(k), k)\} \\ &= \sum_{r=1}^{\mathbb{Z}} \alpha_r \left(\Phi_i^T\left(\mathcal{P}_{\mathcal{K}}^{(s)} + \sum_{t\in\phi_{\mathcal{UC}}^{(s)}} \tilde{\pi}_{st}^{r}P_t + \mathcal{P}_{\mathcal{UK}}^{(s)}\right)\Phi_i + \Omega_i\right) \\ &= \Phi_i^T\left(\mathcal{P}_{\mathcal{K}}^{(s)} + \sum_{t\in\phi_{\mathcal{UC}}^{(s)}} \tilde{\pi}_{st}^{r}P_t + \mathcal{P}_{\mathcal{UK}}^{(s)}\right)\Phi_i + \Omega_i \\ &= \Phi_i^T\left(\mathcal{P}_{\mathcal{K}}^{(s)} + \mathcal{P}_{\mathcal{UC}}^{(s)} + (1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)}) \times \sum_{t\in\phi_{\mathcal{UK}}^{s}} \frac{\hat{\pi}_{st}}{1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)}}P_t\right)\Phi_i + \Omega_i. \end{aligned}$$

It is easy to know the fact that $0 \leqslant \frac{\hat{\pi}_{st}}{1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)}} \leqslant 1$ and $\sum_{t\in\phi_{\mathcal{UK}}^{s}} \frac{\hat{\pi}_{st}}{1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)}} = 1$, we obtain

$$\begin{aligned} &\mathbb{E}\{\Delta V(\hat{\eta}_i(k), k)\} \\ &= \sum_{t\in\phi_{\mathcal{UK}}^{s}} \frac{\hat{\pi}_{st}}{1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)}} \left(\Phi_i^T\left(\mathcal{P}_{\mathcal{K}}^{(s)} + \mathcal{P}_{\mathcal{UC}}^{(s)} + (1-\pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)})P_t\right)\Phi_i + \Omega_i\right). \end{aligned}$$

Therefore, for $0 \leqslant \hat{\pi}_{st} \leqslant 1 - \pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)}$, the above formula is equivalent to

$$\mathbb{E}\{\Delta V(\hat{\eta}_i(k),k)\} = \Phi_i^T \left(\mathcal{P}_{\mathcal{K}}^{(s)} + \mathcal{P}_{\mathcal{UC}}^{(s)} + (1 - \pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)})P_t\right) \Phi_i + \Omega_i \tag{26}$$

for $\forall t \in \phi_{\mathcal{UK}}^{(s)}$. Let $\mathcal{P}_{\Omega}^{(s)} \triangleq \mathcal{P}_{\mathcal{K}}^{(s)} + \mathcal{P}_{\mathcal{UC}}^{(s)} + (1 - \pi_{\mathcal{K}}^{(s)} - \pi_{\mathcal{UC}}^{(s)})P_t, \forall t \in \phi_{\mathcal{UK}}^{(s)}$, we have

$$\begin{aligned}\mathbb{E}\{\Delta V(\hat{\eta}_i(k),k)\} &= \hat{\eta}_i^T(k) \left[(M_s - \lambda_i F_s)^T \mathcal{P}_{\Omega}^{(s)} (M_s - \lambda_i F_s) - P_s\right] \hat{\eta}_i(k) \\ &= \hat{\eta}_i^T(k)\Theta\hat{\eta}_i(k)\end{aligned} \tag{27}$$

where $\Theta \triangleq (M_s - \lambda_i F_s)^T \mathcal{P}_{\Omega}^{(s)} (M_s - \lambda_i F_s) - P_s$. By using Lemma 2.9, it is easy to show that $\Theta < 0$ from (21). Then

$$\mathbb{E}\{\Delta V(\hat{\eta}_i(k),k)\} \leqslant -\lambda_{\min}\left\{\bar{\Theta}\right\} \hat{\eta}_i^T(k)\hat{\eta}_i(k)$$

where $\bar{\Theta} = -\Theta$, thus for any $T \geqslant 1$, we have

$$\begin{aligned}\mathbb{E}\left\{\sum_{k=0}^{T} \| \hat{\eta}_i(k)\|^2\right\} \leqslant &-\frac{1}{\lambda_{\min}\left\{\bar{\Theta}\right\}} \left\{\mathbb{E}\left(V\left(\hat{\eta}_i(T+1), T+1\right)\right)\right\} \\ &+\frac{1}{\lambda_{\min}\left\{\bar{\Theta}\right\}} \left\{\mathbb{E}\left(V\left(\hat{\eta}_i(0), 0\right)\right)\right\}.\end{aligned} \tag{28}$$

Since the Lyapunov function is non-negative, that is $V\left(\hat{\eta}_i(T+1), T+1\right) \geqslant 0$, then we can obtain

$$\mathbb{E}\left\{\sum_{k=0}^{T} \| \hat{\eta}_i(k)\|^2\right\} \leqslant \frac{1}{\lambda_{\min}\left\{\bar{\Theta}\right\}} \left\{\mathbb{E}\left(V\left(\hat{\eta}_i(0), 0\right)\right)\right\}. \tag{29}$$

Let $\beta = \lambda_{\min}\left\{\bar{\Theta}\right\}$, it is easy to derive

$$\mathbb{E}\left\{\sum_{k=0}^{T} \| \hat{\eta}_i(k)\|^2\right\} \leqslant \frac{1}{\beta} \left\{\mathbb{E}\left(V\left(\hat{\eta}_i(0), 0\right)\right)\right\} < \infty$$

It can be seen that the closed-loop system (16) is asymptotically stable.

Now we consider the stability of system (15). To do so, we let the external disturbance be zero, and system (15) becomes

$$\hat{\varepsilon}_i(k+1) = \left(A_{i\rho(k)} + B_{i\rho(k)}K_iC_i\right)\hat{\varepsilon}_i(k).$$

Define the Lyapunov function as $V(\hat{\varepsilon}_i(k), k, \rho(k)) = \hat{\varepsilon}_i^T(k)P_s\hat{\varepsilon}_i(k)$, then

$$\begin{aligned}\mathbb{E}\{\Delta V(\hat{\varepsilon}_i(k),k)\} &= \mathbb{E}\{V(\hat{\varepsilon}_i(k+1), k+1, \rho(k+1)|\hat{\varepsilon}_i(k), \rho(k) = s) - V(\hat{\varepsilon}_i(k), k, \rho(k))\} \\ &= (A_{is} + B_{is}K_iC_i)^T\hat{\varepsilon}_i(k)^T\mathcal{P}^s(A_{is} + B_{is}K_iC_i)\hat{\varepsilon}_i(k) - \hat{\varepsilon}_i^T(k)P_s\hat{\varepsilon}_i(k) \\ &= \hat{\varepsilon}_i^T(k)(\psi_{is}^T\mathcal{P}_{\Omega}^{(s)}\psi_{is} - P_s)\hat{\varepsilon}_i(k)\end{aligned} \tag{30}$$

where $\psi_{is} = A_{is} + B_{is}K_iC_i$. By using Lemma 2.9, it is known from (20) that $\psi_{is}^T \mathcal{P}_\Omega^{(s)} \psi_{is} - P_s + C_i^T C_i < 0$, by the fact that $C_i^T C_i > 0$, then $\psi_{is}^T \mathcal{P}_\Omega^{(s)} \psi_{is} - P_s < 0$. Thus, the system (15) is asymptotically stable in the mean-square sense when $\omega_i(k) = 0$.

Now we consider the case when the external disturbance is $\omega_i(k) \neq 0$, we also define the Lyapunov function as $V(\hat{\varepsilon}_i(k), k, \rho(k)) = \hat{\varepsilon}_i^T(k) P_s \hat{\varepsilon}_i(k)$, then

$$\begin{aligned}
&\mathbb{E}\{\Delta V(\hat{\varepsilon}_i(k), k)\} \\
&= \mathbb{E}\{V(\hat{\varepsilon}_i(k+1), k+1, \rho(k+1)|\hat{\varepsilon}_i(k), \rho(k) = s) - V(\hat{\varepsilon}_i(k), k, \rho(k))\} \\
&= \hat{\varepsilon}_i^T(k)(\psi_{is}^T \mathcal{P}_\Omega^{(s)} \psi_{is} - P_s)\hat{\varepsilon}_i(k) + \omega_i^T(k) D_{is}^T \mathcal{P}_\Omega^{(s)} D_{is}\omega_i(k) \\
&\quad + he((\psi_{is}\hat{\varepsilon}_i(k))^T \mathcal{P}_\Omega^{(s)} D_{is}\omega_i(k)).
\end{aligned}$$

Define the Hamiltonian function as follows:

$$H = \mathbb{E}\{\Delta V(\hat{\varepsilon}_i(k), k)\} + e_i^T(k)e_i(k) - \gamma^2 \omega_i^T(k)\omega_i(k). \tag{31}$$

Let $\xi_k = \begin{bmatrix} \hat{\varepsilon}_i^T(k) & \omega_i^T(k) \end{bmatrix}$, it follows from (20) that

$$H = \xi_k \begin{bmatrix} \psi_{is}^T \mathcal{P}_\Omega^{(s)} \psi_{is} - P_s + C_i^T C_i & \psi_{is}^T \mathcal{P}_\Omega^{(s)} D_{is} \\ * & D_{is}^T \mathcal{P}_\Omega^{(s)} D_{is} - \gamma^2 I \end{bmatrix} \xi_k^T < 0.$$

Accumulating from $k = 0$ to ∞ on both sides of the Hamiltonian function (31), we have

$$\mathbb{E}\{V(\infty) - V(0)\} + \mathbb{E}\left\{\sum_{k=0}^{\infty} \| e_i(k) \|^2\right\} - \gamma^2 \sum_{k=0}^{\infty} \| \omega_i(k)\|^2 < 0. \tag{32}$$

According to the zero initial conditions $V(0) = 0$ and by the fact that the Lyapunov function $V(\infty) > 0$, it is easy to see that

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \| e_i(k) \|^2\right\} - \gamma^2 \sum_{k=0}^{\infty} \| \omega_i(k)\|^2 < 0$$

that is

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \| e_i(k) \|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} \| \omega_i(k)\|^2.$$

Thus, it can be seen that the robust performance is guaranteed. In summary, the consensus of heterogeneous MASs under uncertain DoS attacks is guaranteed and the proof is completed. □

Based on Theorem 3.2, the method for solving the controller gain is given as below.

Theorem 3.3. If there exist positive definite matrix Q, H and a set of matrices $P_s > 0$, as well as matrices $L_i, V_i, \Xi_i, \Upsilon, \bar{H}_s$ with appropriate dimensions and a positive integer $\gamma > 0$ such that the following matrix inequalities

$$\begin{bmatrix} -Q - Q^T + \mathcal{P}_\Omega^{(s)} & * & * & * \\ A_{is}^T Q + C_i^T L_i \Upsilon & C_i^T C_i - P_s & * & * \\ D_{is}^T Q & 0 & -\gamma^2 I & * \\ B_{is}^T - V_i \Upsilon & \Xi_i^T L_i^T C_i & 0 & -V_i \Xi_i - \Xi_i^T V_i^T \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} -P_s & M_s^T H - \lambda_i \bar{H}_s \\ * & -H^T - H + \mathcal{P}_\Omega^{(s)} \end{bmatrix} < 0 \tag{34}$$

have feasible solutions for all $i \in \{1, 2, \ldots, N\}$, $s \in \phi$, then the consensus performance of MASs is guaranteed, where

$$\begin{cases} \mathcal{P}_\mathcal{K}^{(s)} \triangleq \sum_{t \in \phi_\mathcal{K}^{(s)}} \pi_{st} P_t, \mathcal{P}_{\mathcal{UC}}^{(s)} \triangleq \sum_{t \in \phi_{\mathcal{UC}}^{(s)}} \tilde{\pi}_{st}^r P_t, \\ \mathcal{P}_{\mathcal{UK}}^{(s)} \triangleq \sum_{t \in \phi_{\mathcal{UK}}^{(s)}} \hat{\pi}_{st} P_t, \\ \mathcal{P}^s \triangleq \mathcal{P}_\mathcal{K}^{(s)} + \sum_{t \in \phi_{\mathcal{UC}}^{(s)}} (\sum_{r=1}^{\mathbb{Z}} \alpha_r \tilde{\pi}_{st}^r) P_t + \mathcal{P}_{\mathcal{UK}}^{(s)}, \\ \mathcal{P}_\Omega^{(s)} \triangleq \mathcal{P}_\mathcal{K}^{(s)} + \mathcal{P}_{\mathcal{UC}}^{(s)} + (1 - \pi_\mathcal{K}^{(s)} - \pi_{\mathcal{UC}}^{(s)}) P_t, \forall t \in \phi_{\mathcal{UK}}^{(s)} \end{cases}$$

Ξ_i and Υ are given in advance and $K_i = \left(L_i V_i^{-1}\right)^T$, $F_s = \left(\bar{H}_s H^{-1}\right)^T$.

P r o o f. First of all, left and right multiplying (21) by $diag\{\ I,\quad H^T\ \}$ and its transpose, respectively, we obtain

$$\begin{bmatrix} -P_s & (M_s - \lambda_i F_s)^T H \\ * & -H^T \left(\mathcal{P}_\Omega^{(s)}\right)^{-1} H \end{bmatrix} < 0. \tag{35}$$

Let $\bar{H}_s = F_s^T H$, and using Lemma 2.8, we have that if (36) is true that (21) must be true

$$\begin{bmatrix} -P_s & M_s^T H - \lambda_i \bar{H}_s \\ * & -H^T - H + \mathcal{P}_\Omega^{(s)} \end{bmatrix} < 0. \tag{36}$$

Now, applying Lemma 2.9, it follows from (20) that

$$\begin{bmatrix} -(\mathcal{P}_\Omega^{(s)})^{-1} & \psi_{is} & D_{is} \\ * & -P_s + C_i^T C_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{37}$$

Using the matrix $diag\{\ Q^T,\quad I,\quad I\ \}$ and its transpose to pre- and post-multiply the inequalities given in (37), respectively, then we have

$$\begin{bmatrix} -Q^T(\mathcal{P}_\Omega^{(s)})^{-1} Q & * & * \\ \psi_{is}^T Q & C_i^T C_i - P_s & * \\ D_{is}^T Q & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{38}$$

By applying Lemma 2.8, we obtain that

$$\begin{bmatrix} -Q^T - Q + \mathcal{P}_\Omega^{(s)} & * & * \\ \psi_{is}^T Q & C_i^T C_i - P_s & * \\ D_{is}^T Q & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{39}$$

Due to the fact that $\psi_{is} = A_{is} + B_{is} K_i C_i$, we can write the above formula as follows:

$$\begin{bmatrix} -Q^T - Q + \mathcal{P}_\Omega^{(s)} & * & * \\ A_{is}^T Q & C_i^T C_i - P_s & * \\ D_{is}^T Q & 0 & -\gamma^2 I \end{bmatrix} + he\left(\begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} C_i^T K_i^T B_{is}^T Q \begin{bmatrix} I & 0 & 0 \end{bmatrix} \right) < 0. \tag{40}$$

Let $K_i^T = L_i V_i^{-1}$, we have

$$\begin{aligned} &\begin{bmatrix} -Q^T - Q + \mathcal{P}_\Omega^{(s)} & * & * \\ A_{is}^T Q & C_i^T C_i - P_s & * \\ D_{is}^T Q & 0 & -\gamma^2 I \end{bmatrix} \\ &+he\left(\begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} C_i^T L_i V_i^{-1} [(B_{is}^T Q - V_i \Upsilon) + V_i \Upsilon] \begin{bmatrix} I & 0 & 0 \end{bmatrix} \right) < 0 \end{aligned} \tag{41}$$

which is equivalent to

$$\begin{aligned} &\begin{bmatrix} -Q^T - Q + \mathcal{P}_\Omega^{(s)} & * & * \\ A_{is}^T Q + C_i^T L_i \Upsilon & C_i^T C_i - P_s & * \\ D_{is}^T Q & 0 & -\gamma^2 I \end{bmatrix} \\ &+he\left(\begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} C_i^T L_i \Xi_i \Xi_i^{-1} V_i^{-1} (B_{is}^T Q - V_i \Upsilon) \begin{bmatrix} I & 0 & 0 \end{bmatrix} \right) < 0. \end{aligned} \tag{42}$$

Let:

$$\begin{cases} \mathbb{T} = \begin{bmatrix} -Q^T - Q + \mathcal{P}_\Omega^{(s)} & * & * \\ A_{is}^T Q + C_i^T L_i \Upsilon & C_i^T C_i - P_s & * \\ D_{is}^T Q & 0 & -\gamma^2 I \end{bmatrix} \\ \mathbb{M} = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} C_i^T L_i \Xi_i \\ \mathbb{W} = \Xi_i^{-1} V_i^{-1} (B_{is}^T Q - V_i \Upsilon) \begin{bmatrix} I & 0 & 0 \end{bmatrix} \\ \mathbb{U} = V_i \Xi_i. \end{cases}$$

By using Lemma 2.7, it is easy to know that

$$\begin{bmatrix} -Q^T - Q + \mathcal{P}_\Omega^{(s)} & * & * & * \\ A_{is}^T Q + C_i^T L_i \Upsilon & C_i^T C_i - P_s & * & * \\ D_{is}^T Q & 0 & -\gamma^2 I & * \\ B_{is}^T Q - V_i \Upsilon & \Xi_i^T L_i^T C_i & 0 & -V_i \Xi_i - \Xi_i^T V_i^T \end{bmatrix} < 0. \tag{43}$$

Thus, the proof is completed. □

4. SIMULATION EXAMPLES

In this section, a simulation study on the mobile stage vehicles is performed, showing the effectiveness of main results. The mobile stage vehicle is basically a mobile robot, where many mathematical models are proposed to study the cooperative control problem.

In this example, a three-order LTI model in [39] is adopted to describe the dynamic model of the mobile stage vehicles, where $x_i(t) = \begin{bmatrix} x_{1i}(t) & x_{2i}(t) & x_{3i}(t) \end{bmatrix}^T$, and $x_{1i}(t)$, $x_{2i}(t)$, $x_{3i}(t)$ are the position state, the velocity state, the acceleration state, respectively.

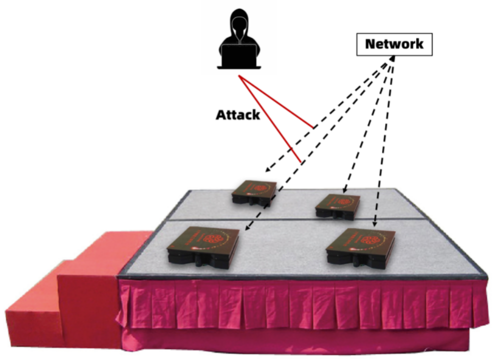

Fig. 1. An illustration of mobile stage vehicle tracking system.

The dynamics of each vehicle is modeled by

$$\dot{x}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & -a_i \end{bmatrix} x_i + \begin{pmatrix} 0 \\ 0 \\ b_i \end{pmatrix} u_i + \begin{pmatrix} 0 \\ 0 \\ e_i \end{pmatrix} \omega_i$$
$$y_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x_i \quad i = 1,2,3$$

where $\{a_i, b_i, c_i, d_i, e_i\}$, $i = 1,2,3$ for three vehicles are chosen as $\{2,1,1,10,1\}$, $\{2,1,1,3,1\}$, $\{2,2,1,10,1\}$, respectively. The leading vehicle dynamics is modeled as:

$$\dot{x}_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_0$$
$$y_0 = \begin{pmatrix} 1 & 0 \end{pmatrix} x_0$$

Our main task is to design an output feedback controller so that the positions of following stage vehicles can track the position of the leading stage vehicle in presence of adversaries. Due to the network connection between mobile stage vehicles, adversaries may randomly launch attacks on the network. Figure 1 illustrates the system structure. In our system, the attack behavior is partially uncertain and unknown to the defender. In simulation, we assume that the heterogeneous mobile stage vehicle tracking system has one leader and three followers, and its topology is shown in Figure 2.

Based on equation (4), we can compute that $\Pi_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\Gamma_i = \begin{pmatrix} 0 & d_i/b_i \end{pmatrix}$. In our system, the disturbances are set as $0.5\sin(k)$, $\sin(k)$, $-\sin(k)$. Normally, the system sampling period is set to be $T_0 = 0.01$, and the maximal attack duration is set to be $2T_0$. The transition probability matrix of attack takes the following case:

$$\begin{bmatrix} 0.5 & 0.2 & 0.3 \\ ? & \begin{bmatrix} 0.5 & 0.6 \end{bmatrix} & ? \\ 0.4 & 0.1 & 0.5 \end{bmatrix}. \tag{44}$$

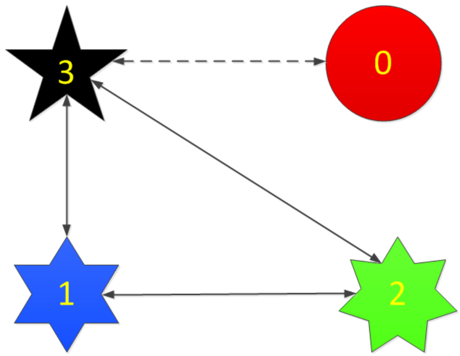

Fig. 2. Communication topology.

According to communication topology, we can obtain that $\lambda_i = 0.2679,\ 3.0000,\ 3.7321$ for $i = 1,\ 2,\ 3$, respectively, we choose $\Upsilon = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ and $\Xi_i = 0.6$ for $i = 1,\ 2,\ 3$, respectively, such that matrix inequalities (33), (34) have feasible solutions, and the controller gains are listed as follows:

$$\begin{cases} K_1 = -1.6456,\ K_2 = -1.6051,\ K_3 = -1.6342 \\ F_1 = \begin{bmatrix} 0.3043 & 0.0030 \\ 0.0000 & 0.3043 \end{bmatrix},\ F_2 = \begin{bmatrix} 0.3043 & 0.0061 \\ 0.0000 & 0.3043 \end{bmatrix},\ F_3 = \begin{bmatrix} 0.3043 & 0.0091 \\ 0.0000 & 0.3043 \end{bmatrix} \end{cases}$$

Choosing the initial conditions as $x_0(0) = \begin{bmatrix} 10 & 1 \end{bmatrix}^T$, $x_1(0) = \begin{bmatrix} 15 & 1 & 1 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 18 & 1 & 1 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} -5 & 1 & 1 \end{bmatrix}^T$ and $\zeta_1(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, $\zeta_2(0) = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$, $\zeta_3(0) = \begin{bmatrix} 2 & 1 \end{bmatrix}^T$, and the attack process is assumed to be triggered as in Figure 3. The consensus performance of the heterogenous mobile stage vehicles are shown in Figure 4 and Figure 5. The performance is generally satisfactory.

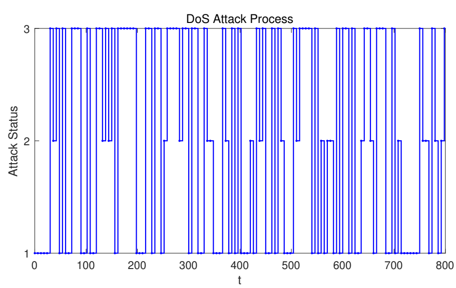

Fig. 3. A possible DoS attack process.

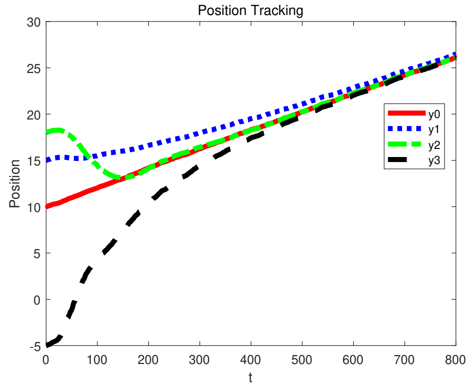

Fig. 4. Tracking performance of position.

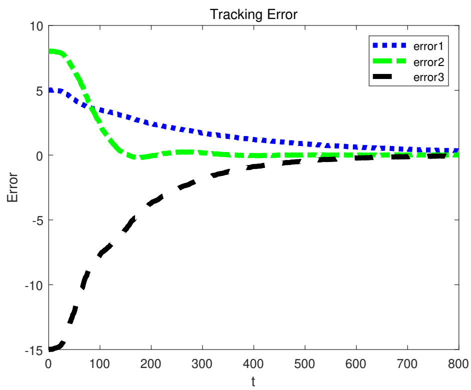

Fig. 5. Tracking error performance.

5. CONCLUSION

This paper has been concerned with the consensus of heterogeneous MASs with uncertain DoS attack, where the attack is randomly triggered and assumed to satisfy the Markovian process. The major feature is that the DoS attack strategy of MASs is allowed to be partly uncertain or even unknown, which is more realistic in practice. A sufficient condition for guaranteeing the output consensus of heterogeneous MASs subject to uncertain DoS attack is obtained by using the decomposition technique, Lyapunov stability theory and matrix transformation method. In addition, a matrix inequality-based controller gain design method has been proposed. Finally, the simulation study on the mobile stage vehicles is performed, showing the effectiveness of main results. In our future work, we will pay our attention to event-based communication mechanism [41, 42, 43].

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