EVENT-TRIGGERED OUTPUT CONSENSUS FOR LINEAR MULTI-AGENT SYSTEMS VIA ADAPTIVE DISTRIBUTED OBSERVER

Limin Zhang, Jian Sun and Qingkai Yang

This paper investigates the distributed event-triggered cooperative output regulation problem for heterogeneous linear continuous-time multi-agent systems (MASs). To eliminate the requirement of continuous communication among interacting following agents, an event-triggered adaptive distributed observer is skillfully devised. Furthermore, a class of closed-loop estimators is constructed and implemented on each agent such that the triggering times on each agent can be significantly reduced while at the same time the desired control performance can be preserved. Compared with the existing open-loop estimators, the proposed estimators can provide more accurate state estimates during each triggering period. It is further shown that the concerned cooperative output regulation problem can be effectively resolved under the proposed control scheme and the undesirable Zeno behavior can be excluded. Finally, the effectiveness of the proposed results is verified by numerical simulations.

Keywords: event-triggered communication (ETC), output regulation, cooperative control, multi-agent systems (MASs)

Classification: 93C02

1. INTRODUCTION

Over the past decade, distributed coordination and cooperation of multiple interacting agents (or nodes) over some communication networks have received intensive attention due to its wide applications in various engineering fields, such as formation of spacecrafts [1, 2], vehicle platoon control [3], sensor network-based monitoring and detection [4, 5, 6], and power system control [7, 8, 9]. As a fundamental problem of multi-agent systems (MASs), the cooperative output regulation has been widely studied [10, 11, 12, 13, 14, 15, 16, 17]. For example, in [10], a distributed observer approach was originally proposed in the case of static communication networks and followed by a case of switching communication networks in [11] and a case of Markovian communication networks in [12]. Later, the distributed observer approach was employed to solve containment problem in [13]. In [14, 15, 16, 17] the estimation of the system matrix was realized via a class of adaptive observers for reaching output regulation.

DOI: 10.14736/kyb-2020-2-0217

Note that some distributed observer approaches proposed in the literature are only valid under the assumption of continuous communication between neighboring agents. By taking the limited on-board energy resources and communication bandwidth into consideration [18, 19, 20], event-triggered control strategy is more favorable. This is because a well-designed event-triggered mechanism can significantly reduce the inter-agent communication frequency, and thus saving certain energy and bandwidth resources that are dedicated for performing data transmissions and exchanges. Some pioneering works on distributed event-triggered strategies can be found in [21, 22, 23, 24, 25, 26]. For more latest results in this regard, we refer interested readers to the surveys [20, 27].

In the context of multi-agent output regulation, not surprisingly, event-triggered strategies have also attracted significant attention [28, 29, 30, 31, 32, 33, 34, 35, 36]. More specifically, [28, 29, 30] addressed the event-triggered output regulation problem for linear MASs under an undirected communication graph; [31, 32] considered the situation where the dynamics of each agent are time-varying and topology is switching; [33, 34] extended the results to the situation of nonlinear MASs; while, in [35, 36], a predictive event-triggered communication strategy was proposed for MASs under directed communication topologies. However, the above mentioned results expose a common limitation, i. e., the control design highly depended on the global information of the network topology, more specifically, the eigenvalues of the resulting Laplacian matrix of the graph. To address this issue, some adaptive control techniques were employed in recent studies [18, 37]. It should be also pointed out that in the existing leader-following MASs, the knowledge of the leader's system matrix is often required to for each follower in such a way to estimate or observe the leader's states, which imposes some extra difficulty in real-world leader-following MAS applications. Apart from this, in an event-triggered setting, the estimation of an internal reference model's state during each triggering interval is normally presented in an open-looped manner, which might lead to some conservatism. Therefore, in this paper, we aim to tackle these challenging issues by developing an event-triggered adaptive distributed observer so as to remove the requirement on the system matrix in the process of leader's state estimation. Besides, a closed-looped estimator is also presented with higher estimation accuracy. Compared with some existing results on event-triggered distributed observer approach to the cooperative output regulation problem, the main contributions of this paper are summarized as follows. First, a new event-triggered adaptive distributed observer, which provides an estimation of the system matrix in an energy-efficient way, is proposed. Second, a novel predictive event-triggered mechanism is developed. Third, different from the existing open-loop estimator used in [18], the estimation of the internal reference model's input is skillfully used as a feedback on the communication intervals, which produces a smaller estimate error and thus fewer triggering times.

The rest of paper is organized as follows. Section 2 introduces some preliminaries and problem formulation. The main results are presented in Section 3. A numerical example is given to demonstrate the effectiveness of our method in Section 4. Section 5 concludes this paper.

Notations: $A \otimes B$ denotes the Keonecker product of matrix A and B. For a matrix $A \in R^{m\times n}$, $vec(A) = col(A_1, \dots, A_n)$ where $A_i \in R^m$ is the ith cloumn of A. For any cloumn vector $C \in R^{nq}$ with positive integers n and q, $\digamma_n^q(C) = [C_1, \dots, C_q]$, where, $C_i \in R^n$ and $C = col(C_1, \dots, C_q)$.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Graph theory

In this section, we first briefly review the graph theory used in this paper. The communication topology among the leader and N followers can be represented by a directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$, where $\bar{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$ is the set of agents with agent v_0 as the leader to be tracked, $\bar{\mathcal{E}} = \{(v_i, v_j), v_i, v_j \in \bar{\mathcal{V}}, i \neq j\}$ is the set of edges among the agents, and $\bar{\mathcal{A}} = [a_{ij}]_{(N+1)\times(N+1)}$ is the adjacent matrix with $a_{ij} = 1$ if agent v_i can receive the state information from agent v_j, i.e., $(v_i, v_j) \in \bar{\mathcal{E}}$, otherwise $a_{ij} = 0$. Furthermore, we assume $a_{ii} = 0$. The neighbor set of agent v_i is denoted by $\bar{\mathcal{N}}_i = \{v_j \in \bar{\mathcal{V}} : (v_i, v_j) \in \bar{\mathcal{E}}\}$. Agent v_p is reachable from agent v_q if there is a sequence of edges $\{(v_p, v_{p+1}), (v_{p+1}, v_{p+2}), \ldots (v_{q-1}, v_q)\}$ among the agents $\{v_p, v_{p+1}, \ldots, v_{q-1}, v_q\}$. If each agent is reachable form node v_q, the directed graph contains a spanning tree with agent v_q as the root.

The topology among the N follower agents is denoted by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, which is a subgraph of $\bar{\mathcal{G}}$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, $\mathcal{E} = \{(v_i, v_j), v_i, v_j \in \mathcal{V}, i \neq j\}$ and $\mathcal{A} = [a_{ij}]_{N\times N}$ refers to the set of follower agents, edges, and adjacent matrix, respectively. The unique feature of the adjacent matrix in undirected graph is that $a_{ij} = a_{ji}$ for any $i \neq j$. The Laplacian matrix $\mathcal{L} = [l_{ij}]_{N\times N}$ is defined by $l_{ij} = -a_{ij}$ for $v_i \neq v_j$ and $l_{ii} = \sum_{i=1, j\neq i}^{N} a_{ij}$. It is easy to check that $\sum_{j=1}^{N} l_{ij} = 0$ for all $i = 1, 2, \ldots, N$.

The matrix H is defined as $H = \mathcal{L} + diag\{a_{10}, \ldots, a_{N0}\}$ and has the following property.

Lemma 2.1. (Hu et al. [38]) If the graph $\bar{\mathcal{G}}$ has a directed spanning tree with agent v_0 being the root, then all the eigenvalues of H have positive real parts.

2.2. Problem formulation

Consider heterogeneous linear MASs consisting of one leader agent and N follwer agents. The dynamics of the leader agent is given by

$$\dot{v} = S_0 v, \tag{1}$$

where $v \in R^q$ is the state of the leader system representing the reference input to be tracked and $S_0 \in R^{q\times q}$ is a constant matrix.

The dynamics of follower agent v_i is given by

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i \\ y_{mi} = C_{mi} x_i + D_{mi} u_i \\ \tilde{e}_i = C_i x_i + D_i u_i + E_i v, \end{cases} \tag{2}$$

where $x_i \in R^{ni}$,$u_i \in R^{mi}$, $y_{mi} \in R^{p_{mi}}$ and $\tilde{e}_i \in R^{p_i}$ represent the state, control input, output, and regulated output, respectively. A_i,B_i,C_{mi},D_{mi},C_i,D_i, and E_i are constant matrices with compatible dimensions.

The problem to be solved in this paper can be formulated as follows.

Problem 2.2. Given systems (1) − (2), under communication graph $\bar{\mathcal{G}}$, the objective of this paper is to design an event-triggered adaptive distributed observer and a dynamic output feedback control law such that the following two statements are true.

1. When v is bounded, the trajectory of the closed-loop system is bounded.
2. For any initial condition, the regulated output satisfies $\lim_{t\to\infty} \tilde{e}_i(t) = 0$ for $i = 1, \ldots, N$.

In order to solve Problem 2.2, we need the following assumptions.

Assumption 2.3. (A_i, B_i) are stabilizable, $i = 1, \ldots, N$.

Assumption 2.4. (C_{mi}, A_i) are detectable, $i = 1, \ldots, N$.

Assumption 2.5. The following linear matrix equations

$$\begin{aligned} X_i S_0 =& A_i X_i + B_i U_i \\ 0 =& C_i X_i + D_i U_i + E_i \end{aligned} \tag{3}$$

have solution pairs (X_i, U_i) for $i = 1, \ldots, N$.

Assumption 2.6. The graph $\bar{\mathcal{G}}$ contains a spanning tree with agent v_0 being the root.

3. MAIN RESULTS

In this section, we will design an event-triggered adaptive distributed observer and the corresponding dynamic output feedback control law. In addition, we will present theoretical analysis on the stability of the resulting closed-loop system as well as the capability of excluding Zeno behavior in the triggering mechanism.

For each agent, the event-triggered adaptive distributed observer is designed as follows

$$\dot{\eta}_i = S_i \eta_i + K_i \bar{\vartheta}_i [\sum_{j=1}^{N} a_{ij}(\hat{\eta}_j - \hat{\eta}_i) + a_{i0}(v - \hat{\eta}_i)], \tag{4a}$$

$$\dot{\hat{S}}_i = \mu_1 [\sum_{j=1}^{N} a_{ij}(\hat{S}_j - \hat{S}_i) + a_{i0}(S_0 - \hat{S}_i)], \tag{4b}$$

$$\dot{\vartheta}_i = \gamma_i [\sum_{j=1}^{N} a_{ij}(\hat{\eta}_j - \hat{\eta}_i) + a_{i0}(v - \hat{\eta}_i)]^T M_i [\sum_{j=1}^{N} a_{ij}(\hat{\eta}_j - \hat{\eta}_i) + a_{i0}(v - \hat{\eta}_i)], \tag{4c}$$

where $\eta_i \in R^q$ is the state of the dynamic compensator. K_i, and M_i are gain matrices to be determined. $\bar{\vartheta}_i \in R$ is a piecewise constant coupling gain. When ϑ_i reaches previous $\bar{\vartheta}_i + c_i$, $\bar{\vartheta}_i$ is updated to ϑ_i. $\bar{\vartheta}_i(0) = \vartheta_i(0) > 0$, $c_i > 0$ and $\gamma_i > 0$ are positive constant scalars. We denote the updating time instant of $\bar{\vartheta}_i$ as $t = t^i_l$. $S_i \in R^{q\times q}$ is the observed system matrix of S_0 by agent v_i. We denote the triggering time instant of S_i as $t = t_m^i$. $\hat{S}_i$ is the estimate of S_i during $t \in [t_m^i, t_{m+1}^i)$, which is updated by

$$\begin{cases} \hat{S}_i(t) = \hat{S}_i(t_m^i), t \in [t_m^i, t_{m+1}^i) \\ \hat{S}_i(t_m^i) = S_i(t_m^i), t = t_m^i. \end{cases} \tag{5}$$

$\hat{\eta}_i \in R^q$ in (4a) and (4c) is the estimate of η_i during $t \in [t_n^i, t_{n+1}^i)$, and $\hat{\eta}_j \in R^q$ in (4a) and (4c) is the estimate of η_j during $t \in [t_n^j, t_{n+1}^j)$, where $t = t_n^i$ and $t = t_n^j$ are the triggering time instants of η_i and η_j, respectively. To get a more accurate state estimate, the closed-loop estimators of $\hat{\eta}_i$ and $\hat{\eta}_j$ in agent v_i are designed as follows

$$\begin{cases} \dot{\hat{\eta}}_i(t) = \hat{S}_i\hat{\eta}_i(t) + K_i\bar{\vartheta}_i\hat{\omega}_i, t \in [t_n^i, t_{n+1}^i) \\ \hat{\eta}_i(t) = \eta_i(t), t = t_n^i \\ \dot{\hat{\hat{\eta}}}_j(t) = \hat{S}_j\hat{\hat{\eta}}_j(t), t \in [t_n^i, t_{n+1}^i) \\ \hat{\hat{\eta}}_j(t) = \hat{\eta}_j(t), t = t_n^i \\ \hat{\omega}_i(t) = \sum_{j=1}^{N} a_{ij}(\hat{\hat{\eta}}_j(t) - \hat{\eta}_i(t)) + a_{i0}(v - \hat{\eta}_i(t)) \end{cases} \tag{6}$$

$$\begin{cases} \dot{\hat{\eta}}_j(t) = \hat{S}_j\hat{\eta}_j(t) + K_j\bar{\vartheta}_j\hat{\omega}_j, t \in [t_n^j, t_{n+1}^j) \\ \hat{\eta}_j(t) = \eta_j(t), t = t_n^j \\ \dot{\hat{\hat{\eta}}}_k(t) = \hat{S}_k\hat{\hat{\eta}}_k(t), t \in [t_n^j, t_{n+1}^j) \\ \hat{\hat{\eta}}_k(t) = \hat{\eta}_k(t), t = t_n^j \\ \hat{\omega}_j(t) = \sum_{j=1}^{N} a_{jk}(\hat{\hat{\eta}}_k(t) - \hat{\eta}_j(t)) + a_{j0}(v - \hat{\eta}_j(t)), \end{cases} \tag{7}$$

where agent v_i is the neighbor of agent v_j, agent v_j is the neighbor of agent v_k. $\hat{\hat{\eta}}_j(t)$ and $\hat{\hat{\eta}}_k(t)$ denote the two-hop estimate of $\eta_j(t)$ and $\eta_k(t)$, respectively, whose updating laws depend on the estimates $\hat{\eta}_j(t_n^i)$ and $\hat{\eta}_k(t_n^j)$. The triggering time instants t_m^i and t_n^i are given by

$$t_m^i = inf\{t > t_{m-1}^i | f_{S_i} \geq 0\} \tag{8}$$

$$t_n^i = inf\{t > t_{n-1}^i | f_{\eta_i} \geq 0\}, \tag{9}$$

where f_{η_i} and f_{S_i} are the triggering functions of η_i and S_i, respectively. The triggering functions are given by

$$f_{S_i} = \|\varepsilon_i\|^2 - \beta_i e^{-\sigma_i t} \tag{10}$$

$$f_{\eta_i} = \|e_i\|^2 - \frac{1}{\iota_i(1+\bar{\vartheta}_i)}(\alpha_i\|\varpi_i\|^2 + \beta_i e^{-\sigma_i t}), \tag{11}$$

where ε_i is used to denote the estimate error of S_i, i.e. $\varepsilon_i = \hat{S}_i - S_i$ and e_i is the estimate error of η_i, i.e. $e_i = \hat{\eta}_i - \eta_i$. α_i, β_i and σ_i are the constant scalars to be determined. ϖ_i is defined as $\varpi_i = \sum_{j=1}^{N} a_{ij}(\hat{\eta}_j - \hat{\eta}_i) + a_{i0}(v - \hat{\eta}_i)$. ι_i is a time-varying variable updated by

$$\dot{\iota}_i = \delta_i(1 + \bar{\vartheta}_i)\|e_i\|^2. \tag{12}$$

The illustrative diagram to describe the whole multi-agent framework for agent v_i is shown in Figure 1.

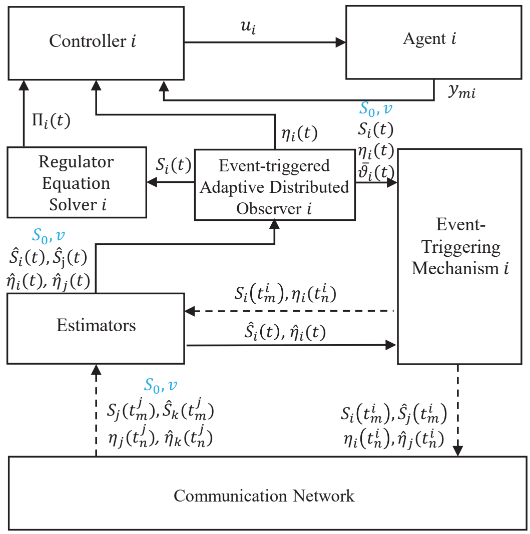


Fig. 1. Event-triggered control schematic. The solid lines indicate the continuous communication and dashed lines indicate the intermittent communication.

Remark 3.1. In this paper, the time delay and package dropouts of the communication network are not taken into account. We assume that the triggered data can be transmitted without error and delay. It should be mentioned that not every follower agent can receive information from the leader, while only the agent who is connected with the leader can receive the information v and S_0. For those agents that are not informed by the leader, they estimate the S_0 and v by (4a) and (4b), respectively.

Remark 3.2. A two-hop estimation mechanism is used to estimate the virtual control input $\hat{\omega}_i(t)$ and $\hat{\omega}_j(t)$ according to (6) and (7) such that the estimation process of $\hat{\eta}_i$ and $\hat{\eta}_j$ are closed-looped. Compared with the open-loop estimator in [18], the closed-loop estimators designed in this paper consider the response of virtual control input $\hat{\omega}_i(t)$, which can lead to some more accurate state estimates, thus can reduce the triggering times on a given time interval. The simulation results in Table. 1 verify this point.

Remark 3.3. Note that the triggering conditions (8) and (9) are checked in parallel, while the transmissions of S_i and η_i are performed separately. When the triggering conditions (8) and (9) hold at the same time, S_i and η_i will be transmitted simultaneously.

However, from (4a), it can be seen that the update of S_i will affect η_i, which indicates that the convergence property of η_i is dependent on S_i.

Now, we give the main results as follows.

Lemma 3.4. Given systems (1)–(2) and communication graph $\bar{\mathcal{G}}$, under Assumptions 2.5–2.6, each follower agent observes the leader's system mateix S_0 with the event-triggered adaptive observer designed as (4b),(5) and (10). Further let $\tilde{S}_i = S_i - S_0$. Then, for any initial condition $\tilde{S}_i(0)$ and any $\mu_1 > 0$, we have $\lim_{t\to\infty} \tilde{S}_i(t) = 0$ exponentially and Zeno behavior is excluded on any finite time interval.

Proof. Taking the time derivative along with $\tilde{S}_i$ during $t \in [t^i_{m-1}, t^i_m)$, we obtain

$$\begin{aligned}\dot{\tilde{S}}_i =& \dot{S}_i - \dot{S}_0\\ =&\mu_1[\sum_{j=1}^{N} a_{ij}(\hat{S}_j - \hat{S}_i) + a_{i0}(S_0 - \hat{S}_i)]\\ =&\mu_1[\sum_{j=1}^{N} a_{ij}(\hat{S}_j - S_j + S_j - S_0 - \hat{S}_i + S_i - S_i + S_0)\\ &+ a_{i0}(S_0 - \hat{S}_i + S_i - S_i)]\\ =&\mu_1[\sum_{j=1}^{N} a_{ij}(\varepsilon_j + \tilde{S}_j - \varepsilon_i - \tilde{S}_i) + a_{i0}(-\tilde{S}_i - \varepsilon_i)].\end{aligned} \tag{13}$$

We rewrite (13) in a compact form as follow

$$\dot{\tilde{S}} = -\mu_1(H \otimes I_q)\tilde{S} - \mu_1(H \otimes I_q)\varepsilon, \tag{14}$$

where $\tilde{S} = [\tilde{S}_1^T, \tilde{S}_2^T, \ldots, \tilde{S}_N^T]^T$ and $\varepsilon = [\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T]^T$. From the triggering function (10), we know that $\|\varepsilon_i\| < \beta_i e^{-\sigma_i t}$ is satisfied during each triggering interval $t \in [t_{m-1}, t_m)$. Due to the fact that $\lim_{t\to\infty} \beta_i e^{-\sigma_i t} = 0$, we have $\lim_{t\to\infty} \varepsilon_i = 0$ and $\mu_1(H \otimes I_q)\varepsilon$ will decay to zero exponentially. Furthermore, for $\mu_1 > 0$, the matrix $-\mu_1(H \otimes I_q)$ is Hurwitz. Thus, $\tilde{S}$ converges to zero exponentially. Next, we prove that triggering interval $\tau_i = t_m - t_{m-1}$ is strictly positive at any finite time interval.

Note that $\frac{\mathrm{d}\|\varepsilon_i\|}{\mathrm{d}t}$ can be upper bounded by $\|\dot{\varepsilon}_i\|$ as follows

$$\frac{\mathrm{d}\|\varepsilon_i\|}{\mathrm{d}t} \leq \|\dot{\varepsilon}_i\| \leq \|\mu_1[\sum_{j=1}^{N} a_{ij}(\hat{S}_j - \hat{S}_i) + a_{i0}(S_0 - \hat{S}_i)]\|. \tag{15}$$

Since S_i is bounded over $[0, \infty)$, we know that $\|\mu_1[\sum_{j=1}^{N} a_{ij}(\hat{S}_j - \hat{S}_i) + a_{i0}(S_0 - \hat{S}_i)]\|$ is also bounded. We use $\bar{S}$ to denote the upper bound of $\|\mu_1[\sum_{j=1}^{N} a_{ij}(\hat{S}_j - \hat{S}_i) + a_{i0}(S_0 - \hat{S}_i)]\|$. Integrating both sides of (15) from t_{m-1} to t_m, we get

$$\begin{aligned}\|\varepsilon_i\| \leq& \int_{t_{m-1}}^{t_m} (\|\mu_1[\sum_{j=1}^{N} a_{ij}(\hat{S}_j - \hat{S}_i) + a_{i0}(S_0 - \hat{S}_i)]\|)\,\mathrm{d}t\\ \leq&\tau_i \bar{S}.\end{aligned} \tag{16}$$

Not that $\|\varepsilon_i\| \geq \beta_i e^{-\sigma_i t}$ at $t = t_m$. Thus, we have

$$\tau_i \bar{S} \geq \beta_i e^{-\sigma_i t}. \tag{17}$$

The triggering interval τ_i thus satisfies

$$\tau_i \geq \frac{\beta_1 e^{-\sigma_i t}}{\bar{S}}. \tag{18}$$

From (18), we know τ_i is strictly positive on any finite time interval. Therefore, Zeno behavior is excluded. The exclusion of Zeno behavior can also be proved by contradiction (see [39] for an example). This completes the proof. □

Lemma 3.5. Given systems (1) − (2) and communication graph $\bar{\mathcal{G}}$, under Assumptions 2.5 − 2.6, the event-triggerd adaptive distributed observer for the state of leader is designed as (4b), (5) and (10). Each solution pair (X_i^*, U_i^*) of linear matrix equations

$$\begin{aligned} X_i S_i =& A_i X_i + B_i U_i \\ 0 =& C_i X_i + D_i U_i + E_i \end{aligned} \tag{19}$$

can be solved by the following equations dynamically

$$\mathcal{Q}_i(t) = S_i(t)^T \otimes \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix} - I_q \otimes \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \tag{20}$$

$$\dot{\Xi}_i(t) = -\mu_2 \mathcal{Q}_i(t)^T (\mathcal{Q}_i(t) \Xi_i(t) - b_i) \tag{21}$$

$$\Pi_i(t) = \digamma^q_{(n_i+m_i)}(\Xi_i(t)), \tag{22}$$

where $b_i = vec\left(\begin{bmatrix} 0 \\ E_i \end{bmatrix}\right)$,$\Pi_i(t) = vec\left(\begin{bmatrix} \mathcal{X}_i(t) \\ \mathcal{U}_i(t) \end{bmatrix}\right)$ and $\mu_2 > 0$ is set to be sufficiently large.

Proof. Let (X_i^*, U_i^*) be some solution pair of the regulator equations (3). By Lemma 3.4, we know that $\lim_{t\to\infty} S_i - S_0 = 0$ exponentially with the designed event-triggerd adaptive distributed observer. Then combining with [16, Lemma 3] and [16, Lemma 4], we have

$$\lim_{t\to\infty} \left(\Pi_i(t) - \begin{bmatrix} X_i^* \\ U_i^* \end{bmatrix} \right) = 0. \tag{23}$$

□

Lemma 3.6. Given systems (1) − (2), under the communication graph $\bar{\mathcal{G}}$, suppose Assumptions 2.3 − 2.6 hold. The event-triggerd adaptive distributed observer for the state of leader is designed as (4) − (12). Choose $K_i = P_i$ and $M_i = P_i^2$, where P_i is symmetric positive-definited and satisfies the following Riccati equation

$$P_i S_i + S_i^T P_i - P_i^2 + I_q = -\dot{P}_i. \tag{24}$$

Let $\xi_i = \eta_i - v$. Then, we have $\lim_{t\to\infty} \xi_i = 0$ exponentially and Zeno behavior is excluded on any finite time interval.

P r o o f. We rewrite ϖ_i in the form of

$$\begin{aligned}\varpi_i =& \sum_{j=1}^{N} a_{ij}(\hat{\eta}_j - \hat{\eta}_i) + a_{i0}(v - \hat{\eta}_i) \\ =& \sum_{j=1}^{N} a_{ij}(\xi_j - \xi_i + e_j - e_i) + a_{i0}(-\xi_i - e_i).\end{aligned} \tag{25}$$

Accordingly, the compact form of ϖ_i is given by

$$\varpi = -(H \otimes I_q)(\xi + e), \tag{26}$$

where $\xi = [\xi_1^T, \xi_2^T, \dots, \xi_N^T]^T$, $e = [e_1^T, e_2^T, \dots, e_N^T]^T$. Then, ξ can be written in terms of ϖ and e as follows

$$\xi = -(H^{-1} \otimes I_q)\varpi - e. \tag{27}$$

Taking the time derivative along ξ_i, we obtain

$$\begin{aligned}\dot{\xi}_i =& \dot{\eta}_i - \dot{v} \\ =& S_i \eta_i + \bar{\vartheta}_i K_i \varpi_i - S_0 v \\ =& S_i \eta_i - S_i v + S_i v - S_0 v + \bar{\vartheta}_i K_i \varpi_i \\ =& S_i \xi_i + \tilde{S}_i v + \bar{\vartheta}_i K_i \varpi_i.\end{aligned} \tag{28}$$

Consider the following Lyapunov function candidate

$$V_1 = \sum_{i=1}^{N} \xi_i^T P_i \xi_i + \sum_{i=1}^{N} \frac{\beta_2}{2\gamma_i}(\vartheta_i - \vartheta_0)^2 + \sum_{i=1}^{N} \frac{1}{2\delta_i}(\iota_i - \iota_0)^2, \tag{29}$$

where ϑ_0 and ι_0 are initial values of ϑ_i and ι_i respectively. Let $t_k, k = 1, 2, \dots$ denote the triggering time instants for the whole MASs. In light of (28) and taking the time derivative of the Lyapunov function candidate (29) during each triggering interval $[t_k, t_{k+1})$, we obtain

$$\begin{aligned}\dot{V}_1 =& \sum_{i=1}^{N} \xi_i^T (S_i^T P_i + P_i S_i)\xi_i + 2\sum_{i=1}^{N} \xi_i^T P_i \tilde{S}_i v + 2\sum_{i=1}^{N} \xi_i^T P_i \bar{\vartheta}_i k_i \varpi_i \\ &+ \sum_{i=1}^{N} \xi_i^T \dot{P}_i \xi_i + \sum_{i=1}^{N} \beta_2(\vartheta_i - \vartheta_0)\varpi_i^T P_i^2 \varpi_i + \sum_{i=1}^{N} (\iota_i - \iota_0)(1 + \bar{\vartheta}_i)\|e_i\|^2.\end{aligned} \tag{30}$$

By substituting the Riccati equation (24) into (30), it follows that

$$\begin{aligned}\dot{V}_1 =& \sum_{i=1}^{N} \xi_i^T (P_i^2)\xi_i - \sum_{i=1}^{N} \xi_i^T I_q \xi_i + 2\sum_{i=1}^{N} \xi_i^T P_i \bar{\vartheta}_i k_i \varpi_i + 2\sum_{i=1}^{N} \xi_i^T P_i \tilde{S}_i v \\ &+ \sum_{i=1}^{N} \beta_2(\vartheta_i - \vartheta_0)\varpi_i^T P_i^2 \varpi_i + \sum_{i=1}^{N} (\iota_i - \iota_0)(1 + \bar{\vartheta}_i)\|e_i\|^2.\end{aligned} \tag{31}$$

The compact form of (31) is given by

$$\begin{aligned}\dot{V}_1 =&\xi^T P^2\xi - \xi^T\xi + 2\xi^T P^2\bar{\vartheta}\varpi + 2\xi^T P\tilde{S}\tilde{v}\\ &+ \beta_2(\vartheta - \vartheta_0 I_{N\times n})\varpi^T P\varpi + \sum_{i=1}^{N}(\iota_i - \iota_0)(1+\bar{\vartheta}_i)\|e_i\|^2,\end{aligned} \tag{32}$$

where $P = diag\{P_1, P_2, \ldots, P_N\}$, $P^2 = diag\{P_1^2, P_2^2, \ldots, P_N^2\}$, $\varpi = [\varpi_1^T, \varpi_2^T, \ldots, \varpi_N^T]^T$, $\tilde{v} = I_N \otimes v$, $\vartheta = diag\{\vartheta_1 I_q, \vartheta_2 I_q, \ldots, \vartheta_N I_q\}$, $\bar{\vartheta} = diag\{\bar{\vartheta}_1 I_q, \bar{\vartheta}_2 I_q, \ldots, \bar{\vartheta}_N I_q\}$. In view of (27), (32) can be rewritten as

$$\begin{aligned}\dot{V}_1 =&\varpi^T(H^{-1}\otimes I_q)P^2(H^{-1}\otimes I_q)\varpi + 2e^T P^2(H^{-1}\otimes I_q)\varpi\\ &+ e^T P^2 e - \xi^T\xi - 2e^T P^2\bar{\vartheta}\varpi - 2\varpi^T(H^{-1}\otimes I_q)P^2\bar{\vartheta}\varpi\\ &+ 2\xi^T P\tilde{S}\tilde{v} + \beta_2(\vartheta - \vartheta_0 I_{N\times n})\varpi^T P\varpi\\ &+ \sum_{i=1}^{N}(\iota_i - \iota_0)(1+\bar{\vartheta}_i)\|e_i\|^2.\end{aligned} \tag{33}$$

Using the Young's inequality, one has

$$\begin{aligned}2\xi^T P\tilde{S}\tilde{v} \le&\xi^T\xi + (\tilde{S}\tilde{v})^T P^2(\tilde{S}\tilde{v})\\ 2e^T P^2(H^{-1}\otimes I_q)\varpi \le&e^T P^2 e + \varpi^T(H^{-1}\otimes I_q)P^2(H^{-1}\otimes I_q)\varpi\\ -2e^T P^2\bar{\vartheta}\varpi \le&\frac{1}{\lambda_2}e^T P^2\bar{\vartheta}e + \lambda_2\varpi^T P^2\bar{\vartheta}\varpi.\end{aligned} \tag{34}$$

Then combining (33) with (34) yields

$$\begin{aligned}\dot{V}_1 \le&2\varpi^T(H^{-1}\otimes I_q)P^2(H^{-1}\otimes I_q)\varpi + 2e^T P^2 e + (\tilde{S}\tilde{v})^T P^2(\tilde{S}\tilde{v})\\ &+ \frac{1}{\lambda_2}e^T P^2\bar{\vartheta}e + \lambda_2\varpi^T P^2\bar{\vartheta}\varpi - 2\varpi^T(H^{-1}\otimes I_q)P^2\bar{\vartheta}\varpi\\ &+ \lambda_2(\vartheta - \vartheta_0 I_{N\times n})\varpi^T P\varpi + \sum_{i=1}^{N}(\iota_i - \iota_0)(1+\bar{\vartheta}_i)\|e_i\|^2.\end{aligned} \tag{35}$$

Defining $\lambda_1 = \lambda_{\max}(H^{-1})$ and $\lambda_2 = \lambda_{\min}(H^{-1})$, one has

$$\begin{aligned}\dot{V}_1 \le&2\lambda_1^2\varpi^T P^2\varpi + 2e^T P^2 e + (\tilde{S}\tilde{v})^T P^2(\tilde{S}\tilde{v}) + \frac{1}{\lambda_2}e^T P^2\bar{\vartheta}e + \lambda_2\varpi^T P^2\bar{\vartheta}\varpi\\ &- 2\lambda_2\varpi^T P^2\bar{\vartheta}\varpi + \lambda_2\varpi^T P^2\bar{\vartheta}\varpi + \lambda_2(\vartheta - \bar{\vartheta} - \vartheta_0 I_{N\times n})\varpi^T P^2\varpi\\ &+ \sum_{i=1}^{N}(\iota_i - \iota_0)(1+\bar{\vartheta}_i)\|e_i\|^2\end{aligned} \tag{36}$$

$$\begin{aligned}\dot{V}_1 \le&(2\lambda_1^2 + \lambda_2 c_{\max} - \vartheta_0)\varpi^T P^2\varpi + \sum_{i=1}^{N}(2 + \frac{1}{\lambda_2}\bar{\vartheta}_i)\|P_i^2\|\|e_i\|^2\\ &- \sum_{i=1}^{N}(\iota_0 + \iota_0\bar{\vartheta}_i)\|e_i\|^2 + (\tilde{S}\tilde{v})^T P^2(\tilde{S}\tilde{v}) + \sum_{i=1}^{N}\iota_i(1+\bar{\vartheta}_i)\|e_i\|^2,\end{aligned} \tag{37}$$

where $c_{\max} = \max_{i=1,2,\ldots,N} c_i$. Assume that ϑ_0 and ι_0 are chosen sufficiently large, such that $2\lambda_1^2 + \lambda_2 c_{\max} \le \vartheta_0$ and $max\{2\lambda_{\max}(\|P\|^2), \frac{1}{\lambda_2}\lambda_{\max}(\|P\|^2)\} \le \iota_0$, then we have

$$\dot{V}_1 \le -\|\varpi\|^2 + (\tilde{S}\tilde{v})^T P^2 (\tilde{S}\tilde{v}) + \sum_{i=1}^{N} \iota_i (1 + \bar{\vartheta}_i)\|e_i\|^2. \tag{38}$$

Since the triggering functions is not satisfied during the triggering intervals, according to (11), e_i satisfies

$$\|e_i\|^2 < \frac{1}{\iota_i(1+\bar{\vartheta}_i)}(\alpha_i\|\varpi_i\|^2 + \beta_i e^{-\sigma_i t}). \tag{39}$$

Substituting (39) into (38), one has

$$\dot{V}_1 \le \sum_{i=1}^{N}[-(1-\alpha_i)\|\varpi_i\|^2 + \beta_i e^{-\sigma_i t}] + (\tilde{S}\tilde{v})^T P^2 (\tilde{S}\tilde{v}). \tag{40}$$

We define a new variable W as follows

$$W = V_1 + \sum_{i=1}^{N} \frac{\rho_i}{\sigma_i} e^{-\sigma_i t}. \tag{41}$$

Then, it follows from (40) and (41) that

$$\dot{W} \le -(1-\alpha_{\max})\|\varpi\|^2 - \sum_{i=1}^{N}(\rho_i - \beta_i)e^{-\sigma_i t} + (\tilde{S}\tilde{v})^T P^2 (\tilde{S}\tilde{v}). \tag{42}$$

Note that $\lim_{t\to\infty} \tilde{S} = 0$ exponentially from Lemma 3.4. It can be ensured that $-(\rho_i - \beta_i)e^{-\sigma_i t} + (\tilde{S}\tilde{v})^T P^2 (\tilde{S}\tilde{v}) < 0$ and $\dot{W} < 0$, under the condition that $\rho_i > \beta_i$ and σ_i are sufficiently small.

When $t = t_k$, we have

$$W(t_l^+) - W(t_l^-) = 0 \tag{43}$$

$$W(t_n^+) - W(t_n^-) = 0 \tag{44}$$

$$W(t_m^+) - W(t_m^-) = \sum_{i=1}^{N} \xi_i^T (P_i(t_m^+) - P_i(t_m^-))\xi_i. \tag{45}$$

Considering the fact that $\lim_{t\to\infty} \tilde{S}_i \to 0$ and $\lim_{t\to\infty} P_i \to P^*$, we get $\lim_{t\to\infty} W(t_m^+) - W(t_m^-) = 0$. As a consequence, W is nonegative and nonincreasing over $[0, \infty)$. We can conclude that $W, \xi, \vartheta_i, \iota_i$ are bounded over $[0, \infty)$. From (4c) and (12), we know that $\dot{\vartheta}_i \ge 0$ and $i_i \ge 0$. Thus, by choosing $\vartheta_i(0) > 0$ and $\iota_i(0) > 0$, it can be concluded that ϑ_i and ι_i converge to some positive constant. According to Cauchy's convergence criterion, for any $\bar{\varrho} > 0$, there exists $T_1 < T_2$ such that $W(T_1^+) - W(T_2^-) < \bar{\varrho}$. From

(42), we obtain

$$
\begin{aligned}
&(1-\alpha_{\max})\int_{T_1}^{T_2}\|\varpi(t)\|^2 \\
&\le -\int_{T_1}^{T_2}(\dot{W})uret \\
&= -\int_{T_1}^{T_{t1}}(\dot{W})uret - \int_{T_{t1}}^{T_{t2}}(\dot{W})uret - \ldots, -\int_{T_{tn}}^{T_2}(\dot{W})uret \\
&= W(T_1^+) - W(T_{t1}^-) + W(T_{t1}^+) - W(T_{t2}^-) + \ldots + W(T_{tn}^-) - W(T_2^-) \\
&\le W(T_1^+) - W(T_2^-) < \bar{\varrho},
\end{aligned}
\tag{46}
$$

where $T_1 < T_{t1} < \ldots < T_{tn} < T_2$.

In light of (46) and Cauchy's convergence criterion, we know $\lim_{t\to\infty}\int_0^t\|\varpi(t)\|^2 ure\tau$ exists. From (31), $\dot{\varpi}(t)$ exists for $t\in[t_n,t_{n+1})$. This implies that $\int_0^t\|\varpi(t)\|ure\tau$ is twice differentiable over each triggering interval $[t_k,t_{k+1})$. Since $\varpi(t)$ and $\dot{\varpi}(t)$ are bounded over $[0,\infty)$, there exists a scalar $Z>0$ such that

$$
\sup_{t\in[t_n,t_{n+1}),n=1,2,\ldots,N}|\varpi(t)^T\dot{\varpi}(t)| < Z.
$$

From the generalized Barbalat's [40, Lemma 1], one can conclude that $\lim_{t\to\infty}\|\varpi(t)\|^2=0$, which implies $\lim_{t\to\infty}\varpi_i(t)=0$ for $i=1,2\ldots,N$. In view of (39), we can obtain $\lim_{t\to\infty}\|e_i\|=0$. Then according to (32), it follows $\lim_{t\to\infty}\xi_i=0$. In the following sequence, we show that Zeno behaviour can be excluded on any finite time interval.

Taking time derivative of e_i, we obtain

$$
\begin{aligned}
\dot{e}_i(t) =&\dot{\hat{\eta}}_i(t) - \dot{\eta}_i(t) \\
=&\hat{S}_i(t)\hat{\eta}_i(t) + \bar{\vartheta}_i(t)k_i(t)\hat{\omega}_i(t) - \hat{S}_i(t)\eta_i(t) \\
&-\bar{\vartheta}_i(t)k_i(t)\varpi_i(t) \\
=&\hat{S}_i(t)e_i(t) + \bar{\vartheta}_i(t)k_i(t)\sum_{i=1}^{N}a_{ij}[\hat{\hat{\eta}}_j(t)-\hat{\eta}_i(t)] \\
&-\bar{\vartheta}_i(t)k_i(t)\sum_{i=1}^{N}a_{ij}[\hat{\eta}_j(t)-\hat{\eta}_i(t)] \\
=&\hat{S}_i(t)e_i(t) + \bar{\vartheta}_i(t)k_i(t)\sum_{i=1}^{N}a_{ij}[\hat{\hat{\eta}}_j(t)-\hat{\eta}_j(t)].
\end{aligned}
\tag{47}
$$

Setting $\Gamma(\eta_j(t)) = \hat{\hat{\eta}}_j(t) - \hat{\eta}_j(t)$, then $\frac{\mathrm{d}\|e_i(t)\|}{\mathrm{d}t}$ can be bounded by $\|\dot{e}_i(t)\|$ as follows

$$
\begin{aligned}
\frac{\mathrm{d}\|e_i(t)\|}{\mathrm{d}t} \le&\|\dot{e}_i(t)\| \\
\le&\|\hat{S}_i(t)\|\|e_i(t)\| + \|\bar{\vartheta}_i(t)\|\|k_i(t)\|\|d_i\bar{\Gamma}(\eta_j(t))\|,
\end{aligned}
\tag{48}
$$

where d_i is the degree of agent v_i and $\bar{\Gamma}(\eta_j(t))$ is the upper bound of $\Gamma(\eta_j(t))$.

Consider the following non-negative function

$$\begin{aligned} \dot{y} =& \|\hat{S}_i(t)\|\|y\| + \|\bar{\vartheta}_i(t)\|\|k_i(t)\|\|d_i\bar{\Gamma}(\eta_j(t))\| \\ y(0) =& \|e_i(t_n^i)\| = 0, n = 1, 2, \dots. \end{aligned} \tag{49}$$

The general solution to the above equation is given by

$$y(t) = \frac{\|\bar{\vartheta}_i(t)\|\|k_i(t)\|\|d_i\bar{\Gamma}(\eta_j(t))}{\|\hat{S}_i(t)\|}(e^{\|\hat{S}_i(t)\|t} - 1). \tag{50}$$

Then, we have $\|e_i(t)\| \leq y(t - t_n^i)$, for $t \in [t_n^i, t_{n+1}^i)$. It is clear that the triggering function satisfies $E_i(t) \geq 0$ at $t = t_n^i$. Denote $\Delta_i = \sqrt{\frac{\alpha_i\|\varpi_i\|^2 + \beta_i e^{-\sigma_i t}}{\iota_i(1+\bar{\vartheta}_i)}}$, and then we get

$$\Delta_i \geq \frac{\|\bar{\vartheta}_i(t)\|\|k_i(t)\|\|d_i\bar{\Gamma}(\eta_j(t))}{\|\hat{S}_i(t)\|}(e^{\|\hat{S}_i(t)\|\tau_i} - 1) \tag{51}$$

$$\tau_i \geq \frac{1}{\|\hat{S}_i(t)\|} ln(1 + \frac{\|\hat{S}_i(t)\|}{\|\bar{\vartheta}_i(t)\|\|k_i(t)\|\|d_i\bar{\Gamma}(\eta_j(t))}\Delta_i). \tag{52}$$

From (52), we know τ_i is strictly positive on any finite time interval, which implies that Zeno behaviour is excluded on any finite time interval. If exclusion of Zeno behavior is still needed when t tends to infinity, one can add a small positive constant in the triggering function. A disadvantage of this triggering function is that the system can only achieve a bounded consensus. This completes the proof. □

Theorem 3.7. Considering systems (1) – (2) and a communication graph $\bar{\mathcal{G}}$, under Assumptions 2.3 – 2.6, for sufficiently large μ_1, μ_2, Problem 2.2 can be solved using the event-triggerd adaptive distributed observer (4) and the following output feedback control law

$$u_i = K_{1i}\zeta_i + K_{2i}(t)\eta_i, \tag{53}$$

$$\dot{\zeta}_i = A_i\zeta_i + B_iu_i + J_i(C_{mi}\zeta_i + D_{mi}u_i - y_{mi}), \tag{54}$$

where $\zeta_i \in R^{n_i}$ is the state of the Luenberger observer; K_{1i}, $K_{2i}(t)$ and J_i are the control gain matrices satisfying the condition that $A_i + B_iK_{1i}$ and $A_i + J_iC_{mi}$ are Hurwitz; $K_{2i}(t) = \mathcal{X}_i(t) - K_{1i}\mathcal{U}_i(t)$; and $(\mathcal{X}_i(t), \mathcal{U}_i(t))$ is the solution of (19) and can be determined by (20) – (22).

Proof. Let $\hat{x}_i = \zeta_i - x_i$, $\tilde{x}_i = x_i - X_i^*v$, $\tilde{u}_i = u_i - U_i^*v$, $K_{2i}^* = U_i^* - K_{1i}X_i^*$, $\tilde{K}_{2i} = K_{2i}(t) - K_{2i}^*$, we have

$$\dot{\hat{x}}_i = (A_i + J_iC_{mi})\hat{x}_i \tag{55}$$

$$\dot{\tilde{x}}_i = A_i\tilde{x}_i + B_i\tilde{u}_i \tag{56}$$

$$e_i = C_i\tilde{x}_i + D_i\tilde{u}_i \tag{57}$$

$$\tilde{u}_i = K_{1i}\tilde{x}_i + K_{1i}\hat{x}_i + K_{2i}(t)\xi_i + \tilde{K}_{2i}v. \tag{58}$$

Substituting (58) into (56) gives

$$\dot{\tilde{x}}_i = (A_i + B_i K_{1i})\tilde{x}_i + B_i K_{1i}\hat{x}_i + B_i K_{2i}(t)\xi_i + B_i \tilde{K}_{2i} v. \tag{59}$$

Taking into consideration the fact that $A_i + J_i C_{mi}$ is Hurwitz, we know $\lim_{t\to\infty} \hat{x}_i = 0$. In addition, as $A_i + B_i K_{1i}$ is Hurwitz and follows $\lim_{t\to\infty} \tilde{x}_i = 0$,$\lim_{t\to\infty} \xi_i = 0$ and $\lim_{t\to\infty} \tilde{K}_{2i} = 0$. We can conclude $\lim_{t\to\infty} \tilde{x}_i = 0$. Thus, we have $\lim_{t\to\infty} \tilde{u}_i = 0$ and $\lim_{t\to\infty} \tilde{e} = 0$. This completes the proof. □

4. NUMERICAL SIMULATIONS

Consider a multi-agent system consisting of 5 heterogeneous follower agents and a leader. The leader to be tracked is given by

$$\dot{v} = \begin{bmatrix} 0 & \varsigma \\ 0 & 0 \end{bmatrix} v, v(0) = \begin{bmatrix} 5.230 \\ 1.270 \end{bmatrix}. \tag{60}$$

The dynamics of each follower agent is given by

$$\begin{aligned} \dot{x}_i &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & s_i \\ 0 & -r_i & -a_i \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix} u_i \\ y_{mi} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_i \\ \tilde{e}_i &= \begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix} x_i + F_i v, i = 1, \ldots, 5. \end{aligned} \tag{61}$$

The parameters and initial states are chosen as $\varsigma = 2$, $s_i = \{1, 2, 1, 2, 1\}$, $r_i = \{10, 6, 6, 8, 8\}$, $a_i = \{10, 2, 1, 10, 2\}$, $b_i = \{6, 5, 8, 5, 6\}$, $i = 1, \ldots, 5$, $E_1 = [-0.5, 0], E_2 = [-1, 0], E_3 = [-1.5, 0], E_4 = [-2, 0], E_5 = [-2.5, 0]$, $J_i = [-10; -5; ; 15]$, $K_{1i} = [-15, -20, -15]$, $x_1(0) = [2; 0; 0]$, $x_2(0) = [1.5; 0; 0]$, $x_3(0) = [0; 0; 0]$, $x_4(0) = [-1.5; 0; 0]$, $x_5(0) = [-2; 0; 0]$, $\eta_1(0) = [1; 6]$, $\eta_2(0) = [9; 2]$, $\eta_3(0) = [2; 3]$, $\eta_4(0) = [10; 8]$, $\eta_5(0) = [4; 8]$, $\zeta_i = [0; 0; 0; 0; 0; 0; 0; 0]$, $\beta_i = 0.01$, $\sigma_i = 0.1$, $\mu_1 = 10$, $\mu_2 = 140$. The communication topology is shown as Figure 2.

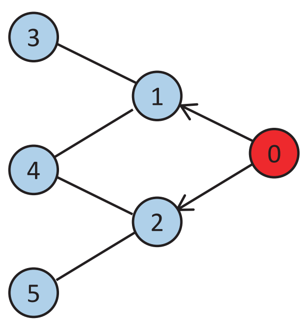


Fig. 2. Communication topology.

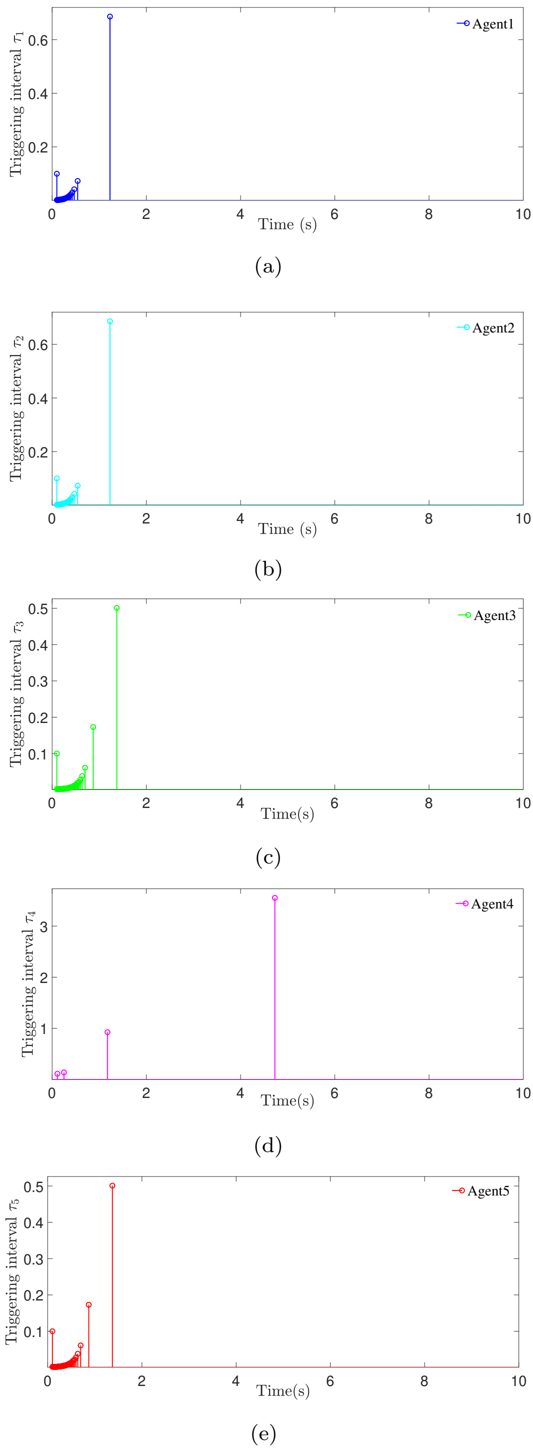
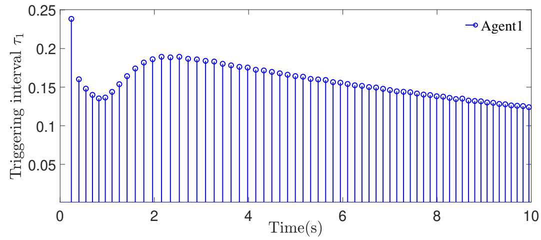
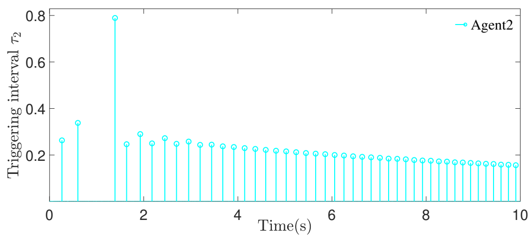


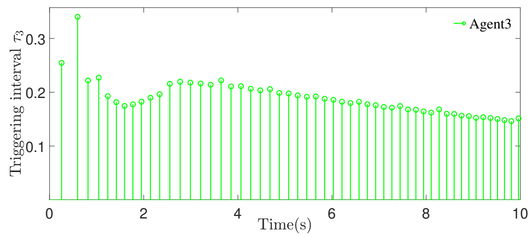
Fig. 3. The triggering intervals for each agent with the triggering function (10) designed for S_i.



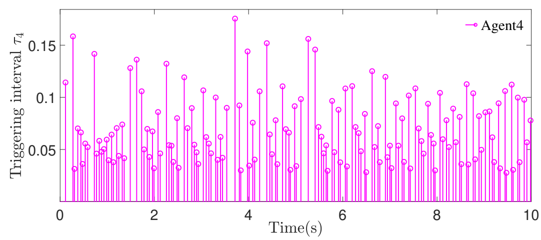
(a)



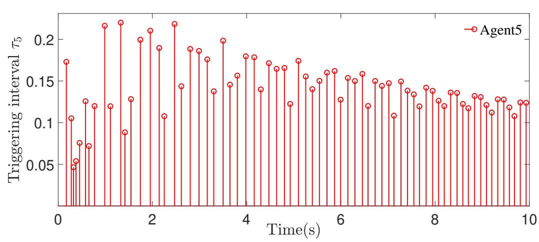
(b)



(c)



(d)



(e)

Fig. 4. The triggering intervals for each agent with the triggering function (11) designed for η_i.

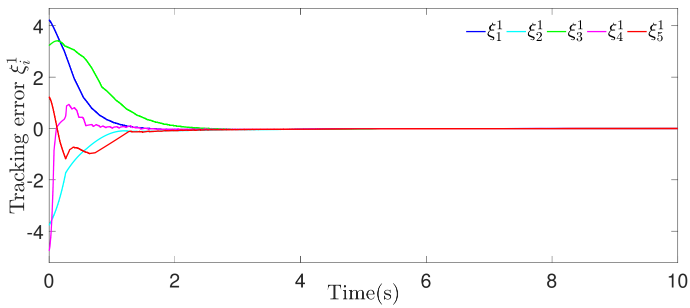
Figure 3 is the triggering time intervals of agent $i, i = 1, \ldots, 5$ associated with the triggering function (10) designed for S_i. Analogously, Figure 4 shows the triggering intervals for the triggering function defined in (11). From Figure 3, we can see that triggering actions only occur during first few seconds. It means that S_i has converged to S_0. Combining Figure 3 and Figure 4, we can observe that the intervals are all positive.

Figure 5 shows the tracking errors between the compensators and the leader using the designed event-triggered adaptive distributed observer. It can be seen that the tracking errors converge to zero.

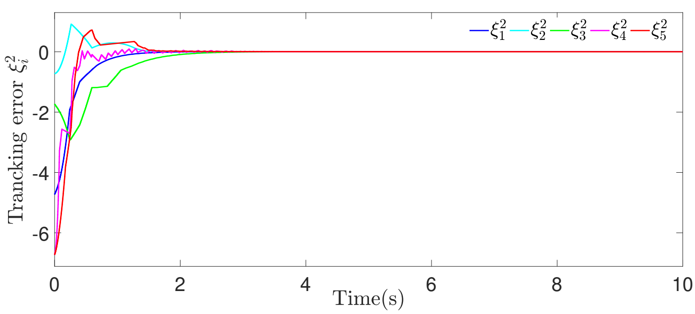
The evolutions of the parameters c_i and ι_i are shown in Figure 6 and Figure 7, from which we can see that both of them converge to some bounded constant scalars.

Figure 8 presents the regulated outputs of each agent. It can be seen that the regulated outputs converge to zero using our proposed control laws.

To show the superiority of the closed-loop estimator, we present the triggering times through the system evolution. Compared with the open-loop estimator used in [18], the triggering times can be largely decreased by employing the closed-loop estimator proposed in our paper.



(a)



(b)

Fig. 5. The tracking errors between the compensators and leader/exosystem. (a) The tracking errors ξ_i^1. (b) The tracking errors ξ_i^2.

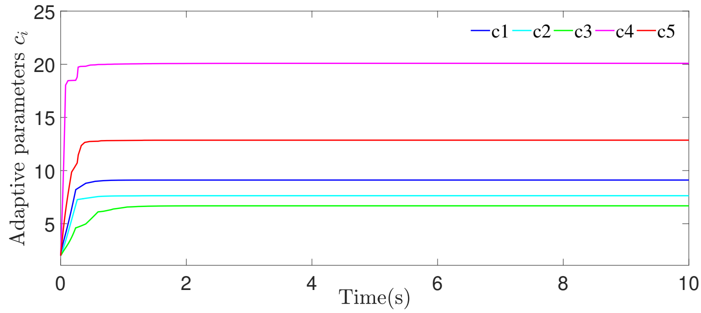


Fig. 6. Adaptive parameters c_i.

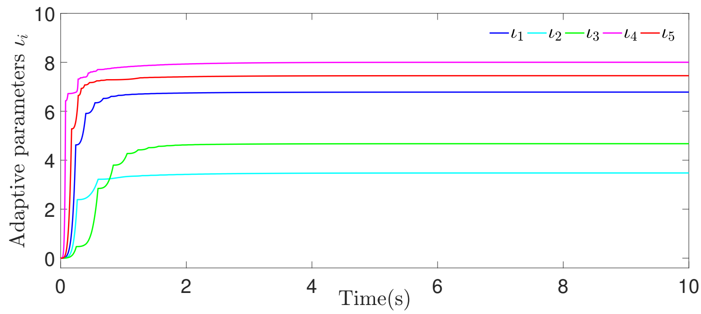


Fig. 7. Adaptive parameters ι_i.

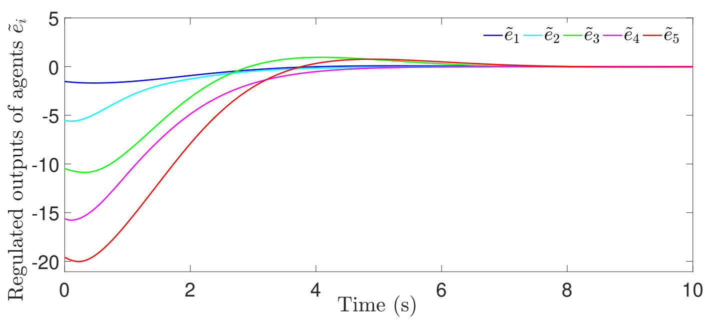


Fig. 8. Regulated outputs of the agents.

	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5
Open-loop estimator[18]:	338	338	392	360	223
Closed-loop estimator:	210	221	231	201	149

Tab. 1. The triggering times of each follower agent.

5. CONCLUSION

The cooperative output regulation problem for heterogeneous linear MASs has been studied in this paper. An event-triggered adaptive distributed observer and a closed-looped estimator have been proposed, each of which allows us to remove the assumptions that all the follower agents need to know the system matrix of the leader system. Moreover, each follower agent can observe the information of the leader without continuous communication. Rigorous theoretical analysis has been presented to show the effectiveness of our proposed method.

Note that the proposed event-triggered control scheme in this paper needs to check triggering conditions continuously, which may be difficult to implement for practical operation[7]. Hence, our future research work will focus on designing sampled-data-based event-triggered communication mechanism to deal with the output regulation probem and extending the results to nonlinear MASs and switching topology.

ACKNOWLEDGEMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 61925303, Grant 61621063, and Grant 61903035, in part by the Program for Changjiang Scholars and Innovative Research Team in University under Grant IRT1208, in part by the ChangJiang Scholars Program, and in part by the National Outstanding Youth Talents Support Program, Beijing Institute of Technology Research Fund Program for Young Scholars.

(Received September 15, 2019)

REFERENCES

[1] W. Ren and N. Sorensen: Distributed coordination architecture for multi-robot formation control. Robotics Autonomous Systems *56* (2008), 324–333. DOI:10.1016/j.robot.2007.08.005

[2] L. Vig, J.A. Adams: Multi-robot coalition formation. IEEE Trans. Robotics *22* (2006), 637–649. DOI:10.1109/tro.2006.878948

[3] T. Arai, E. Pagello, L.E. Parker: Guest editorial Advances in multi-robot systems. IEEE Trans. Robotics Automat. *18* (2002), 655–661. DOI:10.1109/tra.2002.806024

[4] X. Ge, Q.-L. Han and Z. Wang: A threshold-parameter-dependent approach to designing distributed event-triggered H_∞ consensus filters over sensor networks. IEEE Trans. Cybernetics *49* (2019), 1148–1159. DOI:10.1109/tcyb.2017.2789296

[5] Y. Guan and X. Ge: Distributed attack detection and secure estimation of networked cyber-physical systems against false data injection attacks and jamming attacks. IEEE Trans. Signal Inform. Process. Networks *4* (2018), 48–59. DOI:10.1109/tcyb.2017.2789296

[6] X. Ge, Q.-L. Han, M. Zhong and X.-M. Zhang: Distributed Krein space-based attack detection over sensor networks under deception attacks. Automatica *109* (2019), 108557. DOI:10.1016/j.automatica.2019.108557

[7] L. Ding, Q.-L. Han and X.-M. Zhang: Distributed secondary control for active power sharing and frequency regulation in islanded microgrids using an event-triggered communication mechanism. IEEE Trans. Industr. Inform. *15* (2019), 3910–3922. DOI:10.1109/tii.2018.2884494

[8] D. Ding, Q.-L. Han, Z. Wang and X. Ge: A survey on model-based distributed control and filtering for industrial cyber-physical systems. IEEE Trans. Industr. Informatics *15* (2019), 2483–2499. DOI:10.1109/tii.2019.2905295

[9] H. Yang, Q.-L. Han, X. Ge, et al.: Fault tolerant cooperative control of multi-agent systems: A survey of trends and methodologies. IEEE Transactions on Industrial Inform. *16* (2020), 4–17. DOI:10.1109/tii.2019.2945004

[10] Y. Su and J. Huang: Cooperative output regulation of linear multi-agent systems. IEEE Trans. Automat. Control *57* (2011), 1062–1066. DOI:10.1109/tac.2011.2169618

[11] Y. Su and J. Huang: Cooperative output regulation with application to multi-agent consensus under switching network. IEEE Trans. Cybernetics *42* (2012), 864–875. DOI:10.1109/tsmcb.2011.2179981

[12] X. Ge and Q.-L. Han: Consensus of multiagent systems subject to partially accessible and overlapping Markovian network topologies. IEEE Trans. Cybernetics *47* (2017), 1807–1819. DOI:10.1109/tcyb.2016.2570860

[13] H. Haghshenas, M. A. Badamchizadeh, and M. Baradarannia: Containment control of heterogeneous linear multi-agent systems. Automatica *54* (2015), 210–216. DOI:10.1016/j.automatica.2015.02.002

[14] Z. Li, M. Z. Chen, and Z. Ding: Fully distributed adaptive controllers for cooperative output regulation of heterogeneous linear multi-agent systems with directed graphs. arXiv preprint arXiv:1411.7125 *54* (2014).

[15] H. Cai and J. Huang: Cooperative output regulation with application to multi-agent consensus under switching network. IEEE Trans. Automat. Control *61* (2017), 3152–3157. DOI:10.1109/tac.2014.2368231

[16] H. Cai, F. L. Lewis, G. Hu and J. Huang: The adaptive distributed observer approach to the cooperative output regulation of linear multi-agent systems. Automatica *75* (2017), 299–305. DOI:10.1016/j.automatica.2016.09.038

[17] W. Liu and J. Huang: Adaptive leader-following consensus for a class of higher-order nonlinear multi-agent systems with directed switching networks. Automatica *75* (2017), 84–92. DOI:10.1016/j.automatica.2017.02.010

[18] Y. Qian, L. Liu, and G. Feng: Distributed Dynamic Event-Triggered Control for Cooperative Output Regulation of Linear Multiagent Systems. IEEE Trans. Cybernetics (2019), 1–10.

[19] X.-M. Zhang, Q.-L. Han, X. Ge, et al.: Networked control systems: A survey of trends and techniques. IEEE/CAA J. Automat. Sinica *7* (2020), 1–17. DOI:10.1109/jas.2019.1911651

[20] L. Ding, Q.-L. Han, X. Ge and X.-M. Zhang: An overview of recent advances in event-triggered consensus of multiagent systems. IEEE transactions on cybernetics *48* (2017), 1110–1123. DOI:10.1109/tcyb.2017.2771560

[21] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson: Distributed event-triggered control for multi-agent systems. IEEE Trans. Automat. Control *57* (2011), 1291–1297. DOI:10.1109/tac.2011.2174666

[22] Y. Fan, G. Feng, Y. Wang and C.Song: Distributed event-triggered control of multi-agent systems with combinational measurements. Automatica *49* (2013), 671–675. DOI:10.1016/j.automatica.2012.11.010

[23] G. Guo, L. Ding, and Q.-L. Han: A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems. Automatica *50* (2014), 1489–1496. DOI:10.1016/j.automatica.2014.03.017

[24] X. Ge, Q.-L. Han, and F. Yang: Event-based set-membership leader-following consensus of networked multi-agent systems subject to limited communication resources and unknown-but-bounded noise. IEEE Trans. Industr. Electron. *64* (2016), 5045–5054. DOI:10.1109/tie.2016.2613929

[25] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson: Event-based broadcasting for multi-agent average consensus. Automatica *49* (2013), 245–252. DOI:10.1016/j.automatica.2012.08.042

[26] X. Meng and T. Chen: Event based agreement protocols for multi-agent networks. Automatica *49* (2013), 2125–2132. DOI:10.1016/j.automatica.2013.03.002

[27] X. Ge, Q.-L. Han, X.-M. Zhang, L. Ding and F.Yang: Distributed event-triggered estimation over sensor networks: A survey. IEEE Trans. Cybernetics *50* (2019), 3, 1306–1320. DOI:10.1109/tcyb.2019.2917179

[28] W. Hu and L. Liu: Cooperative output regulation of heterogeneous linear multi-agent systems by event-triggered control. IEEE Trans. Cybernet. *47* (2016), 105–116. DOI:10.1109/tcyb.2015.2508561

[29] X. Wang, W. Ni, and Z. Ma: Distributed event-triggered output regulation of multi-agent systems. Int. J. Control *88* (2015), 640–652. DOI:10.1080/00207179.2014.971432

[30] W. Zou and Z. Xiang: Event-triggered distributed containment control of heterogeneous linear multi-agent systems by an output regulation approach. Int. J. Systems Sci. *48* (2017), 2041–2054. DOI:10.1080/00207721.2017.1309595

[31] Y. Ma, Yajing and J. Zhao: Distributed integral-based event-triggered scheme for cooperative output regulation of switched multi-agent systems. Inform. Sci. *457* (2018), 208–221. DOI:10.1016/j.ins.2018.02.021

[32] R. Yang, H. Zhang, G. Feng and H. Yan: Distributed event-triggered adaptive control for cooperative output regulation of heterogeneous multiagent systems under switching topology. IEEE Trans. Neural Networks Learning Systems *29* (2017), 4347–4358. DOI:10.1109/tnnls.2017.2762343

[33] W. Liu and J. Huang: Event-triggered global robust output regulation for a class of nonlinear systems. IEEE Trans. Automat. Control *62* (2017), 5923–5930. DOI:10.1109/tac.2017.2700384

[34] W. Liu and J. Huang: Cooperative global robust output regulation for a class of nonlinear multi-agent systems by distributed event-triggered control. Automatica *93* (2018), 138–148. DOI:10.1016/j.automatica.2018.03.062

[35] Q. Yang, J. Sun, and J. Chen: Output Consensus for Heterogeneous Linear Multiagent Systems With a Predictive Event-Triggered Mechanism. IEEE Trans. Cybernet. (2019), 1–13. DOI:10.1109/tcyb.2019.2895044

[36] X. Meng, L. Xie, and Y.C. Soh: Event-triggered output regulation of heterogeneous multiagent networks. IEEE Trans. Automat. Control *63* (2018), 4429–4434. DOI:10.1109/tac.2018.2823085

[37] R. Yang, H. Zhang, G. Feng et al.: Robust cooperative output regulation of multi-agent systems via adaptive event-triggered control. Automatica *102* (2019), 129–136. DOI:10.1016/j.automatica.2019.01.001

[38] W. Hu, L. Liu, and G. Feng: Cooperative output regulation of linear multi-agent systems by intermittent communication: A unified framework of time-and event-triggering strategies. IEEE Trans. Automat. Control *63* (2017), 548–555. DOI:10.1109/tac.2017.2727821

[39] X. Yi and, T. Yang and J. Wu: Distributed event-triggered control for global consensus of multi-agent systems with input saturation. Automatica *100* (2019), 1–9. DOI:10.1016/j.automatica.2018.10.032

[40] Y. Su and J. Huang: Stability of a class of linear switching systems with applications to two consensus problems. IEEE Trans. Automat. Control *57* (2012), 1420–1430. DOI:10.1109/tac.2011.2176391

Limin Zhang, School of Automation, Beijing Institute of Technology, Beijing 100081. P. R. China.
e-mail: zlm9559@bit.edu.cn

Jian Sun, School of Automation, Beijing Institute of Technology, Beijing 100081. P. R. China.
e-mail: sunjian@bit.edu.cn

Qingkai Yang, School of Automation, Beijing Institute of Technology, Beijing 100081. P. R. China.
e-mail: qingkai.yang@bit.edu.cn