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NON-FRAGILE ESTIMATION FOR DISCRETE-TIME T–S FUZZY SYSTEMS WITH EVENT-TRIGGERED PROTOCOL

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This paper investigates the non-fragile state estimation problem for a class of discrete-time T–S fuzzy systems with time-delays and multiple missing measurements under event-triggered mechanism. First of all, the plant is subject to the time-varying delays and the stochastic disturbances. Next, a random white sequence, the element of which obeys a general probabilistic distribution defined on $[0, 1]$, is utilized to formulate the occurrence of the missing measurements. Also, an event generator function is employed to regulate the transmission of data to save the precious energy. Then, a non-fragile state estimator is constructed to reflect the randomly occurring gain variations in the implementing process. By means of the Lyapunov–Krasovskii functional, the desired sufficient conditions are obtained such that the Takagi–Sugeno (T–S) fuzzy estimation error system is exponentially ultimately bounded in the mean square. And then the upper bound is minimized via the robust optimization technique and the estimator gain matrices can be calculated. Finally, a simulation example is utilized to demonstrate the effectiveness of the state estimation scheme proposed in this paper.

Keywords: Takagi–Sugeno fuzzy system, exponentially ultimately boundness, non-fragile estimation, robust optimization

Classification: 93B35, 93C42, 93E10, 93E11

1. INTRODUCTION

Over the past half century, the nonlinear systems have received great concern due to the fact that nonlinearity is ubiquitous in practical world. One interesting idea is to linearize the nonlinear system globally through linearization feedback, but the method is not so adaptable that it can only be performed in a finite class of nonlinear systems; another is to linearize the nonlinear system in a small range around a certain working point, and the resulting linear system is to be analyzed. Unfortunately, when the system is far from the operating point, it is difficult to ensure the desired performance. To address these challenges, the fuzzy logic models [35] have been proposed to be an effective approach to dealing with the complex nonlinear systems, which could approximate smooth nonlinear systems to specified accuracy with compact set. Based on the local linearity, many complex nonlinear systems can be represented by T–S fuzzy models. As a result, there witnessed a rapidly growing interest in T–S fuzzy systems and many important results

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have been available, such as the stability and stabilization problems [18, 24], the control problem [12, 33, 45, 48] and the filtering problem [27, 25, 40, 41, 42, 43].

It is usually assumed that the estimator/filter to be designed can be accurately implemented to achieve the desirable performance index. Unfortunately, in complex industrial production processes, there are many unavoidable interference factors, such as the analog-to-digital conversion calculations, the finite word-length, and component aging or damaging, etc. These factors would cause unavoidable impact on the filter parameters during implementation [17, 39]. Sometimes even small changes of the filter parameters can result in the degradation of the estimation performance or even the collapse of the estimator. Therefore, many scholars have begun to study how to design the estimator to maintain the stability of the corresponding closed-loop system when the parameters change within a certain range, but still meeting the corresponding performance requirements. This is the non-fragile estimator, which has been studied in [16, 17, 30, 36, 39, 43]. Among them, a design method of non-fragile filter has been investigated in [16] for uncertain systems with stochastic uncertainties and incomplete measurements. A non-fragile filter has been designed in [36] for discrete time-delayed neural networks with parameter uncertainties. A non-fragile H_∞ filter with randomly occurring gain variations has been studied in [17] for a class of time-delay fuzzy systems. Different from [17, 36], the upper bound of performance index involve the parameter uncertainties due to the introduction of the non-fragile filter. As such, the robust optimization technique needs to be utilized to optimize the uncertain upper bound.

As is well known, various networked-induced phenomena could degrade the performance of the system or deteriorate the execution accuracy of designed estimator without enough careful consideration [47]. In the past years, a number of investigation results have been published, such as the communication delays [15, 28, 29, 44], missing measurements [22, 23, 26, 37] and data packet dropouts [42]. For example, H_∞ filtering problem has been investigated in [37] for a class of discrete-time networked systems with randomly occurring distributed state delays. The distributed H_∞ filtering problem has been studied on T–S fuzzy system in [42], where the uncertain packet dropout rate is considered. It is worth mentioning that there exists only a small number of reported results associated with the T–S fuzzy system with networked-induced phenomena, compared with the rich literature on results for networked control systems. According to our investigations, it is due primarily to the complexity and difficulty from the T–S fuzzy system itself.

Nowadays, the utilization of the event-triggered protocol has become more and more popular in the networked control areas [2, 5, 6, 8, 7, 13, 20, 21, 38, 27, 34, 31]. Only when a certain condition of the event generator function is satisfied, data transmission can be allowed to the filters/estimators. Compared with the traditional data transmission, the event-triggered communication protocol may reduce the network resource occupancy due to discarding a part of measurements in the transmission and sacrificing a certain of performance. The distributed H_∞ state estimation problem has been studied for a class of state-saturated systems with randomly occurring mixed delays in [20]. As special event-triggered protocols, the Round-Robin protocol has been introduced to study the output-feedback control problem in [5] and distributed filtering problem in [1]. Moreover, the weighted Try-Once-Discard protocol has been employed to investigate the fusion estimation problem in [32]. Clearly, the introduction of event-triggering protocol would deteriorate the system performance. Unfortunately, it is not directly reflected in the mentioned references. Moreover, the non-fragile state estimation problem of T–S fuzzy systems under the event-triggered mechanism has not been well investigated. These problems inspired us to explore the answers in this paper.

As stated above, a larger number of results has been reviewed on the state estimation problems for the T–S fuzzy systems. It is worth noting that, little attention has been posed on the estimation problem with both event-triggered protocols and missing measurements. For this purpose, in this paper, we focus on the non-fragile state estimation problem for a class of T–S time-delay fuzzy systems with event-triggered protocols and successive missing measurements. The main contributions of this paper are summarized as follows:

1) the non-fragile state estimation problem is investigated for a class of time-delayed fuzzy systems with Gaussian white noises and event-triggered protocols;

2) sufficient conditions are constructed by means of the intensive stochastic analysis techniques to guarantee the desired exponentially ultimately bounded in mean square;

3) the desirable upper bound in the performance index is minimized by utilizing the robust optimization method.

Notation: The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m*n}$ denotes the set of all real matrices with dimension $m*n$. The superscript "T" stands for matrix transposition. I and 0 represent the Identity matrix and zero matrix, if no confusion is caused. $P>0$ means that matrix P is real symmetric and positive definite; $\|A\|$ refers to the norm of a matrix A defined by $\|A\|^2 = \mathrm{trace}(\mathrm{A}^{\mathrm{T}}\mathrm{A})$. In symmetric block matrices or complex matrix expressions, an aster risk $(*)$ is employed to represent a term that is induced by symmetry, and diag$\{\ldots\}$ stands for a block-diagonal matrix. In addition, $\mathbb{E}\{x\}$ denote expectation of the stochastic variable x. Prob$\{\cdot\}$ means the occurrence of the event "$\cdot$". If matrices A and B have the same dimensions, $A \circ B$ denotes the Hadamard product, where $[A\circ B]_{ij} = [A_{ij}\cdot B_{ij}]$. $\mathrm{Vec}_m^t[x_t]$ refers to such a column vector $[x_1 \ \ldots \ x_m]^T$. $\mathrm{diag}_m\{P\} \triangleq \mathrm{diag}\{\underbrace{P,\ldots,P}_{m}\}$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of discrete-time T–S fuzzy time varying time-delay system with stochastic noises:

Plant Rule i. If $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and ... and $\theta_r(k)$ is M_{ir}, then

$$\begin{cases} x(k+1) = A_i x(k) + B_i x(k-\tau(k)) + D_{1i}\omega(k), \\ z(k) = L_i x(k), \\ x(k) = \phi(k), \forall k \in Z^-, \ i = 1,\ldots,r, \end{cases} \tag{1}$$

where M_{ij} is the fuzzy model, r is the number of If-then rules, $\theta(k) = [\theta_1(k), \theta_2(k), \ldots, \theta_r(k)]$ is the premise variable vector, $x(k) \in \mathbb{R}^n$ represents the state vector, $\omega(k) \in \mathbb{R}^p$ is a zero-mean Gaussian white noise sequence on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, $\mathbb{E}\{\omega(k)\omega^T(k)\} = Q = \mathrm{diag}\{q_1^2, \ldots, q_p^2\}$ $(q_l > 0, l = 1, \ldots, p)$, $z(k) \in \mathbb{R}^l$ is the interested output vector, and $\tau(k)$ denotes the time-varying time-delay. A_i, B_i, D_{1i} and L_i are known constant matrices with compatible dimensions. $\phi(k)(k \in \mathbb{Z}^-)$ is the random initial state satisfying $\sup_{k\in\mathbb{Z}^-} \mathbb{E}\{\|\phi(k)\|^2\} < \infty$.

The time-varying delay $\tau(k)$ satisfies

$$\tau_{\min} \leqslant \tau(k) \leqslant \tau_{\max} \tag{2}$$

where $\tau_{\min}$ and $\tau_{\max}$ are constant positive integers representing the lower and upper bounds on the time-delay, respectively.

In this paper, the measurement output with multiple missing measurements is described as

$$\begin{aligned} y(k) &= \Xi C_i x(k) + D_{2i}\omega(k) \\ &= \sum_{l=1}^{m} \beta_l C_{il} x(k) + D_{2i}\omega(k) \end{aligned} \tag{3}$$

where $y(k) \in \mathbb{R}^m$ is the actual measurement signal of (1), $\Xi := \mathrm{diag}\{\beta_1, \ldots, \beta_m\}$ with $\beta_l (l = 1, 2, \ldots, m)$ being m uncorrelated random variables, C_i, D_{2i} and C_{il} are known matrices with compatible dimensions, and $C_{il} := \mathrm{diag}\{\underbrace{0, \ldots, 0}_{l-1}, 1, \underbrace{0, \ldots, 0}_{m-l}\}C_i$. It is assumed that β_l has the probabilistic density function $p_l(s)(l = 1, 2, \ldots, m)$ on the interval $[0, 1]$ with mathematical expectation μ_l and variance δ_l^2 $(\delta_l > 0, \forall l)$. Note that β_l could satisfy any discrete probabilistic distribution on the interval $[0, 1]$, which can cover the widely used Bernoulli distribution. In the sequel, denote $\bar{\Xi} = \mathbb{E}\{\Xi\}$ and $\tilde{\Xi} = \Xi - \bar{\Xi}$, one then has $\mathbb{E}\{\tilde{\Xi}\} = 0$ and $\mathrm{Var}\{\tilde{\Xi}\} = \hat{\Xi}$, and

$$\bar{\Xi} = \mathrm{diag}\{\mu_1, \ldots, \mu_m\}, \qquad \hat{\Xi} = \mathrm{diag}\{\delta_1^2, \ldots, \delta_m^2\}.$$

According to the above formulation, the T–S fuzzy system to be considered is summarized as follows:

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} h_i(\theta(k))[A_i x(k) + B_i x(k - \tau(k)) + D_{1i}\omega(k)], \\ y(k) = \sum_{i=1}^{r} h_i(\theta(k))[\Xi C_i x(k) + D_{2i}\omega(k)], \\ z(k) = \sum_{i=1}^{r} h_i(\theta(k)) L_i x(k) \end{cases} \tag{4}$$

where the fuzzy basis function are given by

$$h_i(\theta(k)) = \frac{\upsilon_i(\theta(k))}{\sum_{i=1}^{r} \upsilon_i(\theta(k))}, \qquad \upsilon_i(\theta(k)) = \prod_{j=1}^{r} M_{ij}(\theta_j(k))$$

with $M_{ij}(\theta_j(k))$ representing the grade of membership of $\theta_j(k)$ in M_{ij}. Here, $\upsilon_i(\theta(k))$ has the following basic properties:

$$\upsilon_i(\theta(k)) \geqslant 0, i = 1, 2, \ldots, r, \qquad \sum_{i=1}^{r} h_i(\theta(k)) > 0,$$

and therefore

$$h_i(\theta(k)) \geqslant 0, i = 1, 2, \ldots, r, \qquad \sum_{i=1}^{r} h_i(\theta(k)) = 1.$$

In what follows, define $h_i := h_i(\theta(k))$ for brevity.

In order to decrease the energy consumption, the well-known event-triggered protocol is introduced in this paper. Next, define an event generator function with the following type:

$$\psi(y(k), y(k_t), \lambda) = (y(k) - y(k_t))^T \Omega (y(k) - y(k_t)) - \lambda \tag{5}$$

where k_t is the latest triggered time up to the current sampling instant k, Ω is a given symmetric positive definite weighting matrix, and λ is a positive adjustable threshold which determines the triggered frequency. Based on (5), the measurement output will be transmitted to the corresponding estimator if and only if $\psi(y(k), y(k_t), \lambda) > 0$.

Denote the triggering instant series as $S_0 < S_1 < S_2 < \ldots < S_m < S_{m+1} < \ldots$, and $k_t = S_m$ when $k \in [S_m, S_{m+1})$. As such, the triggered time instant can be determined iteratively as

$$S_{m+1} = \min\left\{k | k > S_m, \psi(y(k), y(k_t), \lambda) > 0\right\}. \tag{6}$$

In this paper, the following fuzzy non-fragile estimator is constructed for the fuzzy system (4):

Estimator Rule i: If $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and $\ldots$ and $\theta_r(k)$ is M_{ir}, then

$$\begin{cases} \hat{x}(k+1) = (A_{fi} + \Delta A_i(k))\hat{x}(k) + (K_i + \Delta K_i(k))(y(k_t) - \Xi C_i \hat{x}(k)), \\ \quad \hat{z}(k) = L_i \hat{x}(k) \end{cases} \tag{7}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the estimation, A_{fi} and K_i are the estimator gain matrices to be determined, and the time-varying matrices $\Delta A_i(k)$ and $\Delta K_i(k)$ represent norm-bounded parameter uncertainties that satisfy constraints:

$$\Delta A_i(k) = H_i F_i(k) N_a, \quad \Delta K_i(k) = H_i F_i(k) N_k \tag{8}$$

where H_i, N_a and N_k are known constant matrices with appropriate dimensions, and $F_i(k)$ are unknown matrices satisfying

$$F_i^T(k) F_i(k) \leqslant I. \tag{9}$$

Remark 2.1. Sometimes, the estimator to be designed can not achieve the desirable design objective due to many unavoidable interference factors in complex industrial production processes, which would cause unavoidable adverse impact on the estimator parameters during implementation. To resist these adverse impacts, it is very necessary to allow a certain of variation range of the estimator gains. For this purpose, the well-known norm-bounded parameter uncertainties are employed to formulate the allowable range. This type of estimators is called as the non-fragile estimator [16, 17, 36, 43]. In addition, random variables with known expectations and variances can also be utilized to characterize the allowable range. This type of estimators is called as the resilient estimator [14, 30, 39].

Define $\rho(k) = y(k_t) - y(k)$, which is the difference of the measurements between the latest triggered instant and the current sampling instant. Then, the event triggered function (5) can be rewritten as

$$\psi(\rho(k), y(k), \lambda) := \rho^T(k)\Omega\rho(k) - \lambda. \tag{10}$$

Moreover, the estimator (7) can be reformulated as

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{r} h_i[(A_{fi} + \Delta A_i(k))\hat{x}(k) + (K_i + \Delta K_i(k))(y(k) + \rho(k) - \bar{\Xi}C_i\hat{x}(k))], \\ \hat{z}(k) = \sum_{i=1}^{r} h_i L_i \hat{x}(k). \end{cases} \tag{11}$$

Letting $e(k) = x(k) - \hat{x}(k)$ and $z_e(k) = z(k) - \hat{z}(k)$, from (6), (7) and (8), the following estimation error dynamics can be obtained:

$$\begin{cases} e(k+1) = \sum_{i,j=1}^{r} h_i h_j[(A_{fj} - K_j\bar{\Xi}C_j)e(k) + (\Delta A_j(k) - \Delta K_j(k)\bar{\Xi}C_j)e(k) \\ \qquad + (A_i - A_{fj} - K_j\bar{\Xi}(C_i - C_j))x(k) \\ \qquad - (\Delta A_j(k) + \Delta K_j(k)\bar{\Xi}(C_i - C_j))x(k) \\ \qquad - (K_j + \Delta K_j(k))\rho(k) - (K_j + \Delta K_j(k))\tilde{\Xi}C_i x(k) \\ \qquad + (D_{1i} - (K_j + \Delta K_j(k))D_{2i})\omega(k) + B_i x(k - \tau(k))], \\ z_e(k) = \sum_{i,j=1}^{r} h_i h_j[L_j e(k) + (L_i - L_j)x(k)]. \end{cases} \tag{12}$$

Then, let $\eta(k) = [x^T(k) \;\; e^T(k)]^T$. From (1) and (12), one can get an augmented system:

$$\begin{cases} \eta(k+1) = \sum_{i,j=1}^{r} h_i h_j[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k - \tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\tilde{\Xi}\bar{C}_i\eta(k) \\ \qquad + (D_{ij} + \Delta\bar{K}_{ij}(k))\omega(k) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k)], \\ z_e(k) = \sum_{i,j=1}^{r} h_i h_j \bar{L}_{ij}\eta(k) \end{cases} \tag{13}$$

where

$$A_{ij} = \begin{bmatrix} A_i & 0 \\ A_i - A_{fj} - K_j\bar{\Xi}(C_i - C_j) & A_{fj} - K_j\bar{\Xi}C_j \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i & 0 \\ B_i & 0 \end{bmatrix},$$

$$\Delta A_{ij}(k) = \begin{bmatrix} 0 & 0 \\ -\Delta A_j(k) - \Delta K_j(k)\bar{\Xi}(C_i - C_j) & \Delta A_j(k) - \Delta K_j(k)\bar{\Xi}C_j \end{bmatrix},$$

$$D_{ij} = \begin{bmatrix} D_{1i} \\ D_{1i} - K_jD_{2i} \end{bmatrix}, \quad \bar{K}_j = \begin{bmatrix} 0 \\ -K_j \end{bmatrix}, \quad \Delta\bar{K}_j(k) = \begin{bmatrix} 0 \\ -\Delta K_j(k) \end{bmatrix},$$

$$\bar{L}_{ij} = \begin{bmatrix} L_i & 0 \\ L_i - L_j & L_j \end{bmatrix}, \quad \Delta\bar{K}_{ij}(k) = \begin{bmatrix} 0 \\ -\Delta K_j(k)D_{2i} \end{bmatrix}, \quad \bar{C}_i = \begin{bmatrix} C_i^T \\ 0 \end{bmatrix}^T.$$

In the following, the definition of exponentially ultimately bounded is presented.

Definition 2.2. The estimation error dynamics (13) are said to be exponentially ultimately bounded in mean square if there exist scalars $\delta > 0, \sigma > 0$ and $\alpha \in (0,1)$ such that

$$\mathbb{E}\{\|z_e(k)\|_R^2\} \leq \delta\alpha^k + \sigma \tag{14}$$

where α is called the decay rate and σ is the asymptotic upper bound of $\mathbb{E}\{\|z_e(k)\|_R^2\}$.

To this end, our aim in this paper is to deal with the non-fragile estimation problem of the fuzzy system (4) with admissible time-delays and missing measurements under the event-triggered schedule. In the other words, the target is to design the estimator of the form (7) such that the augmented estimating error dynamics is exponentially ultimately bounded in mean square.

3. MAIN RESULTS

Before giving our main results, some useful lemmas are firstly presented.

Lemma 3.1. (Guan and Chen [12]) For any real matrices X_{ij} for $i, j, s, t = 1, 2, \dots, r$ and $\Lambda > 0$ with appropriate dimensions, we have

$$\sum_{i,j,s,t=1}^{r} h_ih_jh_sh_tX_{ij}^T\Lambda X_{st} \leq \sum_{i,j=1}^{r} h_ih_jX_{ij}^T\Lambda X_{ij}. \tag{15}$$

Next, a set of sufficient conditions is provided in the following theorem, which ensures the existence of an exponentially ultimately bounded state estimator in mean square for the T–S fuzzy system (4).

Theorem 3.2. Consider the T–S fuzzy system (4) with given estimator parameters. Let a constant $\gamma \in (0,1)$, the estimator gains $\bar{K}_j$, positive definite matrix $R > \bar{L}_{ij}^T\bar{L}_{ij}$ be given. The estimation error dynamics (13) is exponentially ultimately bounded in mean square if there exist positive definite matrices $P > R$ and $Q > 0$, and two positive scalars ν_{ij} and λ_{ij} such that the following inequalities hold for any i and j:

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & \Pi_{ij}^{12} & \Pi_{ij}^{13} \\ * & \Pi_{ij}^{22} & \Pi_{ij}^{23} \\ * & * & \Pi_{ij}^{33} \end{bmatrix} < 0 \tag{16}$$

where

$$
\begin{aligned}
\Pi_{ij}^{11} =&(A_{ij}+\Delta A_{ij}(k))^T P(A_{ij}+\Delta A_{ij}(k)) - (1-\gamma)P + (\tau_{\max}-\tau_{\min}+1)Q \\
&+ \bar{C}_i^T \mathbf{1}_{m\times m}^T[(\bar{K}_j+\Delta\bar{K}_j(k))^T P(\bar{K}_j+\Delta\bar{K}_j(k)) \circ \widehat{\Xi}]\mathbf{1}_{m\times m}\bar{C}_i, \\
\Pi_{ij}^{12} =&(A_{ij}+\Delta A_{ij}(k))^T P\bar{B}_i, \quad \Pi_{ij}^{13} = (A_{ij}+\Delta A_{ij}(k))^T P(\bar{K}_j+\Delta\bar{K}_j(k)), \\
\Pi_{ij}^{22} =&\bar{B}_i^T P\bar{B}_i - (1-\gamma)^{\tau_{\max}}Q, \quad \Pi_{ij}^{23} = \bar{B}_i^T P(\bar{K}_j+\Delta\bar{K}_j(k)), \\
\Pi_{ij}^{33} =&(\bar{K}_j+\Delta\bar{K}_j(k))^T P(\bar{K}_j+\Delta\bar{K}_j(k)) - \nu_{ij}\Omega, \\
\delta =&\mathbb{E}\{\eta^T(0)P\eta(0) + \sum_{l=-\tau(0)}^{-1}(1-\gamma)^{-l-1}\eta^T(l)Q\eta(l) \\
&+ \sum_{m=-\tau_{\max}+1}^{-\tau_{\min}}\sum_{l=m}^{-1}(1-\gamma)^{-l-1}\eta^T(l)Q\eta(l)\}, \\
\sigma_{ij} =&\frac{1}{\gamma}(\mathbf{1}_{p\times 1}^T[\Phi_{ij}\circ Q]\mathbf{1}_{p\times 1} + \nu_{ij}\lambda), \\
\Phi_{ij} =&D_{ij}^T(P^{-1} - \lambda_{ij}^{-1}\mathcal{N}_i\mathcal{N}_i^T)^{-1}D_{ij} + \lambda_{ij}\mathcal{H}_j^T\mathcal{H}_j, \\
\mathcal{N}_i =&\begin{bmatrix} 0 \\ N_k D_{2i} \end{bmatrix}.
\end{aligned}
$$

P r o o f. Construct the following Lyapunov–Krasovskii functional:

$$
V(k) = V_1(k) + V_2(k) + V_3(k) \tag{17}
$$

where

$$
\begin{aligned}
V_1(k) &= \eta^T(k)P\eta(k), \\
V_2(k) &= \sum_{l=k-\tau(k)}^{k-1}(1-\gamma)^{k-l-1}\eta^T(l)Q\eta(l), \\
V_3(k) &= \sum_{m=k-\tau_{\max}+1}^{k-\tau_{\min}}\sum_{l=m}^{k-1}(1-\gamma)^{k-l-1}\eta^T(l)Q\eta(l).
\end{aligned}
$$

Computing the expectation of the difference of $V_i(k)$ $(1\le i\le 3)$ along the trajectory of (13), one obtains via Lemma 3.1

$$
\begin{aligned}
&\mathbb{E}\{V_1(k+1) - V_1(k)\} \\
=&\mathbb{E}\Big\{\sum_{i,j,s,t=1}^{r} h_i h_j h_s h_t\Big[(A_{ij}+\Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j+\Delta\bar{K}_j(k))\rho(k) \\
&+ (\bar{K}_j+\Delta\bar{K}_j(k))\widetilde{\Xi}\bar{C}_i\eta(k) + (D_{ij}+\Delta\bar{K}_{ij}(k))\omega(k)\Big]^T P\Big[(A_{st}+\Delta A_{st}(k))\eta(k) \\
&+ \bar{B}_s\eta(k-\tau(k)) + (\bar{K}_t+\Delta\tilde{K}_t(k))\rho(k) + (\bar{K}_t+\Delta\bar{K}_t(k))\widetilde{\Xi}\bar{C}_s\eta(k)
\end{aligned}
$$

$$
\begin{aligned}
&+ (D_{st} + \Delta\bar{K}_{st}(k))\omega(k)\Big] - \eta^T(k)P\eta(k) \\
\leq &\sum_{i,j=1}^{r} h_i h_j \Big[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k)\Big]^T P \\
&\times \Big[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k)\Big] - \eta^T(k)P\eta(k) \\
&+ \sum_{i,j=1}^{r} h_i h_j \mathbb{E}\{\eta^T(k)\bar{C}_i^T\widetilde{\Xi}^T(\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))\widetilde{\Xi}\bar{C}_i\eta(k)\} \\
&+ \sum_{i,j=1}^{r} h_i h_j \mathbb{E}\{\omega^T(k)(D_{ij} + \Delta\bar{K}_{ij}(k))^T P(D_{ij} + \Delta\bar{K}_{ij}(k))\omega(k)\}. \qquad (18)
\end{aligned}
$$

According to the proposition 1 in [13], one has

$$
\begin{aligned}
&\mathbb{E}\{\eta^T(k)\bar{C}_i^T\widetilde{\Xi}^T(\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))\widetilde{\Xi}\bar{C}_i\eta(k)\} \\
=&\eta^T(k)\bar{C}_i^T\mathbf{1}_{m\times m}^T[((\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))) \circ \mathbb{E}\{\widetilde{\Xi}\widetilde{\Xi}^T\}]\mathbf{1}_{m\times m}\bar{C}_i\eta(k) \\
=&\eta^T(k)\bar{C}_i^T\mathbf{1}_{m\times m}^T[((\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))) \circ \widehat{\Xi}]\mathbf{1}_{m\times m}\bar{C}_i\eta(k), \qquad (19)
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathbb{E}\{\omega^T(k)(D_{ij} + \Delta\bar{K}_j(k))^T P(D_{ij} + \Delta\bar{K}_j(k))\omega(k)\} \\
=&\mathbf{1}_{p\times 1}^T[(D_{ij} + \Delta\bar{K}_{ij}(k))^T P(D_{ij} + \Delta\bar{K}_{ij}(k)) \circ Q]\mathbf{1}_{p\times 1} \\
=:&\ \mathbf{1}_{p\times 1}^T[\bar{\Phi}_{ij} \circ Q]\mathbf{1}_{p\times 1} \qquad (20)
\end{aligned}
$$

where $\bar{\Phi}_{ij} = (D_{ij} + \Delta\bar{K}_{ij}(k))^T P(D_{ij} + \Delta\bar{K}_{ij}(k))$.

Subsisting (19) and (20) into (18), one has

$$
\begin{aligned}
&\mathbb{E}\{V_1(k+1) - V_1(k)\} \\
\leq &\sum_{i,j=1}^{r} h_i h_j \Big[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k)\Big]^T \\
&\times P\Big[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k)\Big] \\
&+ \sum_{i,j=1}^{r} h_i h_j \eta^T(k)\bar{C}_i^T\mathbf{1}_{m\times m}^T[((\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))) \circ \widehat{\Xi}]\mathbf{1}_{m\times m}\bar{C}_i\eta(k) \\
&- \eta^T(k)P\eta(k) + \sum_{i,j=1}^{r} h_i h_j \mathbf{1}_{p\times 1}^T[\bar{\Phi}_{ij} \circ Q]\mathbf{1}_{p\times 1}.
\end{aligned}
$$

Also, one can calculate

$$
\mathbb{E}\{V_2(k+1) - V_2(k)\}
$$

$$
\begin{aligned}
&= \mathbb{E}\Big\{ \sum_{l=k+1-\tau(k+1)}^{k} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) - \sum_{l=k-\tau(k)}^{k-1} (1-\gamma)^{k-l-1}\eta^T(l)Q\eta(l)\Big\} \\
&= \eta^T(k)Q\eta(k) + \sum_{l=k+1-\tau(k+1)}^{k-\tau(k)} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) + \sum_{l=k+1-\tau(k)}^{k-1} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) \\
&\quad - \sum_{l=k-\tau(k)}^{k-1} (1-\gamma)^{k-l-1}\eta^T(l)Q\eta(l) - (1-\gamma)^{\tau(\mathrm{k})}\eta^T(k-\tau(k))Q\eta(k-\tau(k)) \\
&\le -\gamma V_2(k) + \eta^T(k)Q\eta(k) - (1-\gamma)^{\tau_{\max}}\eta^T(k-\tau(k))Q\eta(k-\tau(k)) \\
&\quad + \sum_{l=k-\tau_{\max}+1}^{k-\tau_{\min}} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l). \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&\mathbb{E}\{V_3(k+1) - V_3(k)\} \\
&= \mathbb{E}\Big\{ \sum_{m=k+2-\tau_{\max}}^{k+1-\tau_{\min}} \sum_{l=m}^{k} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) - \sum_{m=k+1-\tau_{\max}}^{k-\tau_{\min}} \sum_{l=m}^{k-1} (1-\gamma)^{k-l-1}\eta^T(l)Q\eta(l)\Big\} \\
&= \sum_{m=k+1-\tau_{\max}}^{k-\tau_{\min}} \sum_{l=m}^{k-1} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) + (\tau_{\max} - \tau_{\min})\eta^{\mathrm{T}}(\mathrm{k})\mathrm{Q}\eta(\mathrm{k}) \\
&\quad - \sum_{m=k+1-\tau_{\max}}^{k-\tau_{\min}} \sum_{l=m}^{k-1} (1-\gamma)^{k-l-1}\eta^T(l)Q\eta(l) - \sum_{l=k+1-\tau_{\max}}^{k-\tau_{\min}} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) \\
&= -\gamma V_3(k) - \sum_{l=k+1-\tau_{\max}}^{k-\tau_{\min}} (1-\gamma)^{k-l}\eta^T(l)Q\eta(l) + (\tau_{\max} - \tau_{\min})\eta^T(k)Q\eta(k). \qquad (22)
\end{aligned}
$$

According to the event generator function (10), the positive scalar ν_{ij} can be introduced such that the following inequality holds:

$$
-\nu_{ij}\rho^T(k)\Omega\rho(k) + \nu_{ij}\lambda \ge 0. \qquad (23)
$$

Thus, noticing (18)–(23), one further obtains that

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(k)\} \\
&= \mathbb{E}\{V(k+1) - V(k)\} \\
&\le \sum_{i,j=1}^{r} h_i h_j \Big\{ \Big[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k) \Big]^T \\
&\quad \times P\Big[(A_{ij} + \Delta A_{ij}(k))\eta(k) + \bar{B}_i\eta(k-\tau(k)) + (\bar{K}_j + \Delta\bar{K}_j(k))\rho(k) \Big] \\
&\quad - (1-\gamma)\eta^T(k)P\eta(k) - (1-\gamma)^{\tau_{\max}}\eta^T(k-\tau(k))Q\eta(k-\tau(k)) \\
&\quad + \eta^T(k)\bar{C}_i^T\mathbf{1}_{m\times m}^T[((\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))) \circ \widehat{\Xi}]\mathbf{1}_{m\times m}\bar{C}_i\eta(k)
\end{aligned}
$$

$$
\begin{aligned}
&+ (\tau_{\max} - \tau_{\min} + 1)\eta^T(k)Q\eta(k) - \gamma V(k) + \mathbf{1}_{p\times 1}^T[\bar{\Phi}_{ij} \circ Q]\mathbf{1}_{p\times 1} \\
&- \nu_{ij}\rho^T(k)\Omega\rho(k) + \nu_{ij}\lambda\Big\} \\
\leq& \sum_{i,j=1}^{r} h_i h_j \Big\{ - \gamma V(k) + \bar{\eta}^T(k)\Pi_{ij}\bar{\eta}(k) + \nu_{ij}\lambda + \mathbf{1}_{p\times 1}^T[\bar{\Phi}_{ij} \circ Q]\mathbf{1}_{p\times 1}\Big\}.
\end{aligned} \tag{24}
$$

where $\bar{\eta}(k) = [\eta^T(k) \quad \eta^T(k-\tau(k)) \quad \rho^T(k)]^T$.

Then, it follows from (16) that (24) can further become

$$
\mathbb{E}\{V(k+1)\} \leq (1-\gamma)\mathbb{E}\{V(k)\} + \sum_{i,j=1}^{r} h_i h_j \Big[\nu_{ij}\lambda + \mathbf{1}_{p\times 1}^T[\bar{\Phi}_{ij} \circ Q]\mathbf{1}_{p\times 1}\Big]. \tag{25}
$$

Next, one can obtain the following decomposition of $\Delta\bar{K}_{ij}(k)$:

$$
\Delta\bar{K}_{ij}(k) = \begin{bmatrix} 0 \\ -H_j F_j(k) N_k D_{2i} \end{bmatrix} = \bar{H}_j \mathcal{F}_j(k)\mathcal{N}_i. \tag{26}
$$

By means of the lemma 1 in [46], one has

$$
\begin{aligned}
&\bar{\Phi}_{ij} \\
=&(D_{ij} + \Delta\bar{K}_{ij}(k))^T P(D_{ij} + \Delta\bar{K}_{ij}(k)) \\
=&(D_{ij} + \mathcal{H}_j\mathcal{F}_j(k)\mathcal{N}_i)^T P(D_{ij} + \mathcal{H}_j\mathcal{F}_j(k)\mathcal{N}_i) \\
\leq& D_{ij}^T(P^{-1} - \lambda_{ij}^{-1}\mathcal{N}_i\mathcal{N}_i^T)^{-1}D_{ij} + \lambda_{ij}\bar{H}_j^T\bar{H}_j \\
=&\Phi_{ij}.
\end{aligned} \tag{27}
$$

According to (25) and (27), one can derive the following formula:

$$
\begin{aligned}
&\mathbb{E}\{V(k)\} \\
\leq&(1-\gamma)^k\mathbb{E}\{V(0)\} + \frac{1}{\gamma}\sum_{i,j=1}^{r} h_i h_j(\mathbf{1}_{p\times 1}^T[\Phi_{ij} \circ Q]\mathbf{1}_{p\times 1} + \nu_{ij}\lambda)(1-(1-\gamma)^k) \\
\leq&(1-\gamma)^k\mathbb{E}\{V(0)\} + \frac{1}{\gamma}\sum_{i,j=1}^{r} h_i h_j(\mathbf{1}_{p\times 1}^T[\Phi_{ij} \circ Q]\mathbf{1}_{p\times 1} + \nu_{ij}\lambda).
\end{aligned} \tag{28}
$$

Meanwhile, it follows from $\bar{L}_{ij}^T\bar{L}_{ij} \leq R$ and $P \geq R$ that

$$
\mathbb{E}\{z_e^T(k)z_e(k)\} = \mathbb{E}\{\eta^T(k)\bar{L}_{ij}^T\bar{L}_{ij}\eta(k)\} \leq \mathbb{E}\{\eta^T(k)R\eta(k)\}, \tag{29}
$$

and

$$
\mathbb{E}\{V(k)\} \geq \mathbb{E}\{\eta^T(k)P\eta(k)\} \geq \mathbb{E}\{\eta^T(k)R\eta(k)\}. \tag{30}
$$

From (28), (29) and (30), one then can derive

$$
\mathbb{E}\{\|z_e(k)\|_R^2\}
$$

$$\begin{aligned}&\leq (1-\gamma)^k \mathbb{E}\{V(0)\} + \frac{1}{\gamma}\sum_{i,j=1}^{r} h_i h_j (\mathbf{1}_{p\times 1}^T[\Phi_{ij}\circ Q]\mathbf{1}_{p\times 1} + \nu_{ij}\lambda)\\ &= \delta(1-\gamma)^k + \sum_{i,j=1}^{r} h_i h_j \sigma_{ij}. \end{aligned} \tag{31}$$

Then, letting $\sigma = \max_{i,j}\sigma_{ij}$, one has

$$\mathbb{E}\{\|z_e(k)\|_R^2\} \leq \delta(1-\gamma)^k + \sigma.$$

Consequently, the estimator (7) is an exponentially ultimately bounded state estimator in mean square for the T–S fuzzy system (4). Therefore, the proof is now complete. □

Actually, the desirable parameters in Definition 2.2 are calculated in Theorem 3.2. In the following, one needs to optimize the upper bound of the performance index and then calculates the desirable gain parameters.

4. THE OPTIMIZATION OF THE UPPER BOUND

Actually, the optimization of the upper bound is very critical for improving the performance of the design scheme. As discussed before, the upper bound is given as:

$$\delta(1-\gamma)^k + \sum_{i,j=1}^{r} h_i h_j \sigma_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (\delta(1-\gamma)^k + \sigma_{ij}). \tag{32}$$

It is noted that $\delta(1-\gamma)^k$ characterizes the convergent speed of the norm of the estimation errors toward the upper bound σ. Moreover, δ and γ are all given constants. Naturally, $\delta(1-\gamma)^k$ would tend to zero with the increase of time instant k. As such, it is unnecessary to optimize $\delta(1-\gamma)^k$. That is, only σ_{ij} is needed to be optimized. Combining Theorem 3.2, an optimization problem can be established as follows:

$$\begin{aligned}&\min\ \sigma_{ij}\\ &s.t.\ (16).\end{aligned} \tag{33}$$

Recalling the complicated expression of σ_{ij}, which includes the parameter uncertainties characterized by the norm bounded inequalities, it is impossible to directly consider the optimization problem (33). To address this problem, the robust optimization technique is adopted to change (33) into a new solvable optimization problem and the corresponding procedure is arranged as follows. First of all, let

$$\sigma_{ij} \leq \varrho \tag{34}$$

where $\varrho > 0$ is a real decision variable.

Then, the optimization problem (33) can be reformulated as

$$\begin{aligned}&\min\ \varrho\\ &s.t.\ (16)\ \text{and}\ (34)\end{aligned} \tag{35}$$

In the following, let ϱ temporally be given and change (16) and (34) into feasible linear matrix inequalities by means of the schur complement lemma and S-Procedure. Moreover, the parameter gains of the non-fragile state estimator (7) are designed for the T–S fuzzy system (4).

Theorem 4.1. Consider the T–S fuzzy system (4) and let $0 < \gamma < 1$, $R = \mathrm{diag}\{R_1, R_2\}$, $R_1 \geq 0, R_2 \geq 0$, $R \geq \bar{L}_{ij}^T \bar{L}_{ij}$ and $\varrho > 0$ be given. A desired estimator of the form (7) exists, if there exist positive definite matrices $P_1 > R_1, P_2 > R_2$ and $Q > 0$, and positive constant scalars ε_{ij}, ρ_{ij} and λ_{ij} satisfying:

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & 0 & 0 & \Pi_{ij}^{14} & \Pi_{ij}^{15} & 0 & 0 & 0 \\ * & \Pi_{ij}^{22} & 0 & 0 & \Pi_{ij}^{25} & 0 & 0 & 0 \\ * & * & \Pi_{ij}^{33} & 0 & \Pi_{ij}^{35} & 0 & 0 & 0 \\ * & * & * & \Pi_{ij}^{44} & 0 & \Pi_{ij}^{46} & 0 & 0 \\ * & * & * & * & \Pi_{ij}^{55} & 0 & \Pi_{ij}^{57} & \Pi_{ij}^{58} \\ * & * & * & * & * & \Pi_{ij}^{66} & 0 & 0 \\ * & * & * & * & * & * & \Pi_{ij}^{77} & 0 \\ * & * & * & * & * & * & * & \Pi_{ij}^{88} \end{bmatrix} < 0, \tag{36}$$

$$\Psi_{ij} = \begin{bmatrix} \Psi_{ij}^{11} & \Psi_{ij}^{12} & 0 \\ * & \Psi_{ij}^{22} & \Psi_{ij}^{23} \\ * & * & \Psi_{ij}^{33} \end{bmatrix} \geq 0, \tag{37}$$

where

$$\begin{aligned}
\Pi_{ij}^{11} =& -(1-\gamma)P + (\tau_{\max} - \tau_{\min} + 1)Q + \rho_{ij} N_{akij}^T N_{akij} + \varepsilon_{ij} \mathcal{N}_{ki}^T \mathcal{N}_{ki}, \\
\Pi_{ij}^{14} =& \mathrm{vec}_s^t [\delta_t \bar{K}_j F_{mt} \mathbf{1}_{m\times m} \bar{C}_i]^T, \quad \Pi_{ij}^{15} = \tilde{A}_{ij}^T, \\
\Pi_{ij}^{22} =& -(1-\gamma)^{\tau_{\max}} Q, \quad \Pi_{ij}^{25} = \tilde{B}_i^T, \\
\Pi_{ij}^{33} =& -\nu_{ij}\Omega + \varepsilon_{ij} N_k^T N_k, \quad \Pi_{ij}^{44} = -\mathrm{diag}_m\{P\}, \quad \Pi_{ij}^{35} = \tilde{K}_j^T, \\
\Pi_{ij}^{55} =& -P, \quad \Pi_{ij}^{46} = \tilde{\mathcal{H}}_j, \quad \Pi_{ij}^{58} = \tilde{H}_{aj}, \\
\Pi_{ij}^{57} =& \tilde{H}_j, \quad \Pi_{ij}^{66} = -\varepsilon_{ij} I, \quad \Pi_{ij}^{77} = -\varepsilon_{ij} I, \quad \Pi_{ij}^{88} = -\rho_{ij} I, \\
\Psi_{ij}^{11} =& \gamma\varrho - \nu_{ij}\lambda - \sum_{l=1}^{p} q_l^2 \mathbf{1}_{p\times 1}^T F_{pl}^T \lambda_{ij} \bar{H}_j \bar{H}_j^T F_{pl} \mathbf{1}_{p\times 1}, \\
\Psi_{ij}^{12} =& [q_1 \mathbf{1}_{p\times 1}^T F_{p1}^T D_{ij}^T P \ldots q_p \mathbf{1}_{p\times 1}^T F_{pp}^T D_{ij}^T P], \\
\Psi_{ij}^{22} =& \mathrm{diag}_p\{P\}, \quad \Psi_{ij}^{23} = \mathrm{diag}_p\{\mathcal{N}_i\}, \\
\Psi_{ij}^{33} =& \mathrm{diag}_p\{\lambda_{ij} I\}, \quad \mathcal{H}_j = \mathrm{diag}_m\{\tilde{H}_j\}, \\
\mathcal{N}_{ki} =& \bar{N}_k \bar{\mathcal{C}}_i, \quad \bar{\mathcal{C}}_i = \mathrm{Vec}_m^t[\delta_t F_{mt} \mathbf{1}_{m\times m} \bar{C}_i], \quad \bar{N}_k = \mathrm{diag}_m\{N_k\}, \\
N_{akij} =& \mathrm{diag}\{N_a + N_k \bar{\Xi}(C_i - C_j), N_a - N_k \bar{\Xi} C_j\}, \\
F_{mt} =& \mathrm{diag}\{\underbrace{0, \ldots, 0}_{t-1}, 1, \underbrace{0, \ldots, 0}_{m-t}\},
\end{aligned}$$

$$F_{pt} = \mathrm{diag}\{\underbrace{0,\dots,0}_{t-1},1,\underbrace{0,\dots,0}_{p-t}\},$$
$$\tilde{A}_{ij} = \begin{bmatrix} P_1A_i & 0 \\ P_2A_i - \bar{A}_{fj} - K_{2j}\bar{\Xi}(C_i - C_j) & \bar{A}_{fj} - K_{2j}\bar{\Xi}C_j \end{bmatrix},$$
$$\tilde{B}_i = \begin{bmatrix} P_1B_i & 0 \\ P_2B_i & 0 \end{bmatrix}, \quad \tilde{H}_j = \begin{bmatrix} 0 \\ -P_2H_j \end{bmatrix}, \quad \tilde{H}_{aj} = \begin{bmatrix} 0 & 0 \\ -P_2H_j & P_2H_j \end{bmatrix},$$
$$\tilde{D}_{ij} = \begin{bmatrix} P_1D_{1i} \\ P_2D_{1i} - K_{2j}D_{2i} \end{bmatrix}, \quad P = \mathrm{diag}\{P_1, P_2\}, \quad \tilde{K}_j = \begin{bmatrix} 0 \\ -K_{2j} \end{bmatrix}.$$

Moreover, the estimator parameters are given as follows:

$$\begin{bmatrix} A_{fj} & K_j \end{bmatrix} = P_2^{-1}[\bar{A}_{fj} \quad K_{2j}]. \tag{38}$$

P r o o f. By utilizing the denotation of F_{mt}, one can obtain that

$$\begin{aligned} &\bar{C}_i^T \mathbf{1}_{m\times m}^T[(\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k)) \circ \widehat{\Xi}]\mathbf{1}_{m\times m}\bar{C}_i \\ =&\sum_{t=1}^{m} \delta_t^2 \bar{C}_i^T \mathbf{1}_{m\times m}^T F_{mt}^T(\bar{K}_j + \Delta\bar{K}_j(k))^T P(\bar{K}_j + \Delta\bar{K}_j(k))F_{mt}\mathbf{1}_{m\times m}\bar{C}_i. \end{aligned} \tag{39}$$

According to the Schur Complement Lemma, one has

$$\bar{\Pi}_{ij} = \begin{bmatrix} \bar{\Pi}_{ij}^{11} & 0 & 0 & \bar{\Pi}_{ij}^{14} & \bar{\Pi}_{ij}^{15} \\ * & \bar{\Pi}_{ij}^{22} & 0 & 0 & \bar{\Pi}_{ij}^{25} \\ * & * & \bar{\Pi}_{ij}^{33} & 0 & \bar{\Pi}_{ij}^{35} \\ * & * & * & \bar{\Pi}_{ij}^{44} & 0 \\ * & * & * & * & \bar{\Pi}_{ij}^{55} \end{bmatrix} < 0 \tag{40}$$

where

$$\begin{aligned} \bar{\Pi}_{ij}^{11} =& -(1-\gamma)P + (\tau_{\max} - \tau_{\min} + 1)Q, \\ \bar{\Pi}_{ij}^{14} =& \mathrm{Vec}_m^t[\delta_t(\bar{K}_j + \Delta\bar{K}_j(k))F_{mt}\mathbf{1}_{m\times m}\bar{C}_i]^T, \\ \bar{\Pi}_{ij}^{15} =& (A_{ij} + \Delta A_{ij}(k))^T, \quad \bar{\Pi}_{ij}^{22} = -(1-\gamma)^{\tau_{\max}}Q, \\ \bar{\Pi}_{ij}^{25} =& \bar{B}_i^T, \quad \bar{\Pi}_{ij}^{35} = (\bar{K}_j + \Delta\bar{K}_j(k))^T, \\ \bar{\Pi}_{ij}^{33} =& -\nu_{ij}\Omega, \quad \bar{\Pi}_{ij}^{44} = -\mathrm{diag}_m\{P^{-1}\}, \quad \bar{\Pi}_{ij}^{55} = -P^{-1}. \end{aligned}$$

Noticing that $\Delta A_j(k) = H_jF_j(k)N_a$ and $\Delta K_j(k) = H_jF_j(k)N_k$, we can get the following decompositions:

$$\begin{aligned}
\Delta A_{ij}(k) =& \begin{bmatrix} 0 & 0 \\ -\Delta A_j(k) - \Delta K_j(k)\bar{\Xi}(C_i - C_j) & \Delta A_j(k) - \Delta K_j(k)\bar{\Xi}C_j \end{bmatrix} \\
=& \begin{bmatrix} 0 & 0 \\ -H_jF_j(k)N_a - H_jF_j(k)N_k\bar{\Xi}(C_i - C_j) & H_jF_j(k)N_a - H_jF_j(k)N_k\bar{\Xi}C_j \end{bmatrix} \\
=& \begin{bmatrix} 0 & 0 \\ -H_j & H_j \end{bmatrix} \begin{bmatrix} F_j(k) & 0 \\ 0 & F_j(k) \end{bmatrix} \begin{bmatrix} N_a + N_k\bar{\Xi}(C_i - C_j) & 0 \\ 0 & N_a - N_k\bar{\Xi}C_j \end{bmatrix} \\
=& H_{aj}\mathcal{F}_{1j}(k)N_{akij}, \\
\Delta\bar{K}_j(k) =& \begin{bmatrix} 0 \\ -H_jF_j(k)N_k \end{bmatrix} = \begin{bmatrix} 0 \\ -H_j \end{bmatrix} F_j(k)N_k = \bar{H}_jF_j(k)N_k,
\end{aligned}$$

$$\begin{aligned}
\mathrm{Vec}_m^t[\delta_t\Delta\bar{K}_j(k)F_{mt}\mathbf{1}_{m\times m}\bar{C}_i] =& \mathrm{diag}_m\{\bar{H}_j\}\mathrm{diag}_m\{F_j(k)\}\mathrm{Vec}_m^t[\delta_tN_kF_{st}\mathbf{1}_{m\times m}\bar{C}_i] \\
=& \bar{\mathcal{H}}_j\mathcal{F}_{2j}(k)\mathcal{N}_{ki}.
\end{aligned}$$

As such, one has

$$\bar{\Pi}_{ij} = \hat{\Pi}_{ij} + M_1F_1N_1 + N_1^TF_1^TM_1^T + M_2F_2N_2 + N_2^TF_2^TM_2^T \tag{41}$$

where

$$\hat{\Pi}_{ij} = \begin{bmatrix} \bar{\Pi}_{ij}^{11} & 0 & 0 & \hat{\Pi}_{ij}^{14} & \hat{\Pi}_{ij}^{15} \\ * & \bar{\Pi}_{ij}^{22} & 0 & 0 & \bar{\Pi}_{ij}^{25} \\ * & * & \bar{\Pi}_{ij}^{33} & 0 & \hat{\Pi}_{ij}^{35} \\ * & * & * & \bar{\Pi}_{ij}^{44} & 0 \\ * & * & * & * & \bar{\Pi}_{ij}^{55} \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \bar{\mathcal{H}}_j & 0 \\ 0 & \bar{H}_j \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ H_{aj} \end{bmatrix},$$

$$\begin{aligned}
\hat{\Pi}_{ij}^{14} =& \mathrm{vec}_n^t[\delta_t\bar{K}_jF_{mt}\mathbf{1}_{m\times m}\bar{C}_i], \quad \hat{\Pi}_{ij}^{15} = A_{ij}^T, \quad \hat{\Pi}_{ij}^{35} = \bar{K}_j^T, \\
N_2 =& \begin{bmatrix} \mathcal{N}_{ki} & 0 & 0 & 0 & 0 \\ 0 & 0 & N_k & 0 & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} \mathcal{F}_{2j}(k) & 0 \\ 0 & F_j(k) \end{bmatrix}, \\
N_1 =& \begin{bmatrix} N_{akij} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad F_1 = \mathcal{F}_{1j}(k).
\end{aligned}$$

According to the well-known S-Procedure, one has

$$\bar{\Pi}_{ij} = \hat{\Pi}_{ij} + M_1F_1N_1 + N_1^TF_1^TM_1^T + M_2F_2N_2 + N_2^TF_2^TM_2^T < 0$$

if and only if there exist $\rho_{ij} > 0$ and $\varepsilon_{ij} > 0$ such that

$$\hat{\Pi}_{ij} + \rho_{ij}^{-1}M_1M_1^T + \rho_{ij}N_1^TN_1 + \varepsilon_{ij}^{-1}M_2M_2^T + \varepsilon_{ij}N_2^TN_2 < 0.$$

By utilizing the Schur complement lemma, one has

$$\begin{bmatrix} \Pi_{ij}^{11} & 0 & 0 & \hat{\Pi}_{ij}^{14} & \hat{\Pi}_{ij}^{15} & 0 & 0 & 0 \\ * & \Pi_{ij}^{22} & 0 & 0 & \bar{\Pi}_{ij}^{25} & 0 & 0 & 0 \\ * & * & \Pi_{ij}^{33} & 0 & \hat{\Pi}_{ij}^{35} & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_{ij}^{44} & 0 & \bar{\Pi}_{ij}^{46} & 0 & 0 \\ * & * & * & * & \bar{\Pi}_{ij}^{55} & 0 & \bar{\Pi}_{ij}^{57} & \bar{\Pi}_{ij}^{58} \\ * & * & * & * & * & \Pi_{ij}^{66} & 0 & 0 \\ * & * & * & * & * & * & \Pi_{ij}^{77} & 0 \\ * & * & * & * & * & * & * & \Pi_{ij}^{88} \end{bmatrix} < 0 \tag{42}$$

where

$$\Pi_{ij}^{33} =\bar{\Pi}_{ij}^{33} + \varepsilon_{ij} N_k^T N_k, \quad \bar{\Pi}_{ij}^{46} = \bar{\mathcal{H}}_j, \quad \bar{\Pi}_{ij}^{57} = \bar{H}_j, \quad \bar{\Pi}_{ij}^{58} = H_{aj}.$$

Then, perform the congruent transformation with $\text{diag}\{I, I, I, \underbrace{P, \dots, P}_{m}, P, I, I\}$, and denote

$$\begin{aligned}
\tilde{K}_j =& P\bar{K}_j, \quad \bar{A}_{fj} = P_2 A_{fj}, \quad \tilde{A}_{ij} = PA_{ij}, \quad \tilde{D}_{ij} = PD_{ij},\\
\tilde{B}_i =& P\bar{B}_i, \quad \tilde{H}_j = P\bar{H}_j, \quad \tilde{H}_{aj} = PH_{aj}, \quad K_{2j} = P_2 K_j, \quad \tilde{\mathcal{H}}_j = \text{diag}_m\{P\}\bar{\mathcal{H}}_j,
\end{aligned}$$

one can obtain (36).

The following step is to deal with the inequality:

$$\sigma_{ij} \leq \varrho, \tag{43}$$

which implies that

$$\mathbf{1}_{p\times 1}^T [\Phi_{ij} \circ Q] \mathbf{1}_{p\times 1} + \nu_{ij}\lambda \leq \gamma\varrho, \tag{44}$$

or equivalently

$$\begin{aligned}
&\sum_{l=1}^{p} q_l^2 \mathbf{1}_{p\times 1}^T F_{pl}^T D_{ij}^T (P^{-1} - \lambda_{ij}^{-1} \mathcal{N}_i \mathcal{N}_i^T)^{-1} D_{ij} F_{pl} \mathbf{1}_{p\times 1}\\
\leq& \gamma\varrho - \nu_{ij}\lambda - \sum_{l=1}^{p} q_l^2 \mathbf{1}_{p\times 1}^T F_{pl}^T \lambda_{ij} \bar{H}_j^T \bar{H}_j F_{pl} \mathbf{1}_{p\times 1}.
\end{aligned} \tag{45}$$

According to Schur complement Lemma, one has

$$\begin{bmatrix} \Psi_{ij}^{11} & \bar{\Psi}_{ij}^{12} & 0 \\ * & \bar{\Psi}_{ij}^{22} & \Psi_{ij}^{23} \\ * & * & \Psi_{ij}^{33} \end{bmatrix} \geq 0 \tag{46}$$

where

$$\bar{\Psi}_{ij}^{12} =[q_1 \mathbf{1}_{p\times 1}^T F_{p1}^T D_{ij}^T \dots q_p \mathbf{1}_{p\times 1}^T F_{pp}^T D_{ij}^T], \quad \bar{\Psi}_{ij}^{22} = \text{diag}_p\{P^{-1}\}.$$

Performing the congruent transformation with $\text{diag}\{I, \underbrace{P, \dots, P}_{p}, \underbrace{I, \dots, I}_{p}\}$, one has (37) through some algebraic computations. The proof can therefore be complete. □

By means of the Schur complement Lemma and S-Procedure, (16) and (34) has been changed into (36) and (37) in Theorem 4.1. By utilizing these results, the optimization problem (35) can be reformulated as follows:

$$\begin{aligned} &\min \varrho \\ &s.t. \text{ (36) and (37).} \end{aligned} \tag{47}$$

To summarize the procedure discussed so far, the following algorithm is summarized to solve the addressed minimization problem.

Algorithm 1: Fuzzy Non-Fragile Estimation	
Step 1.	Set parameters γ, $R_1 \geq 0, R_2 \geq 0$, $\upsilon_i(\theta(k))$, $\mu_1, \dots, \mu_m, \delta_1^2, \dots, \delta_m^2$, λ and Ω.
Step 2.	Compute the parameter matrices K_i and A_{fi} at the sampling instant k by solving the optimization problem (47) subject to (36) and (37).
Step 3.	Compute the parameter matrices K_i and A_{fi} and $\hat{x}(k)$ according to (11) via $h_i(\theta(k))$.
Step 4.	Stop.

Remark 4.2. This paper develops the non-fragile estimation scheme for a class of T–S fuzzy systems with time-varying delays, multiple missing measurements and event-triggering protocol. The aim of this paper is to guarantee the estimation error system satisfy the desirable exponential ultimately boundedness. Meanwhile, the ultimate upper bound of the performance index is optimized to obtain the better performance behavior. Due to the impact of non-fragile estimator, the upper bound of the performance index involving the parameter uncertainties is optimized via the robust optimization technique. The obtained optimization problem subject to a set of linear matrix inequalities can be easily solved by MATLAB toolbox. It is worth noting that the main result presented in optimization problem (47) is quite comprehensive, which includes the upper bound to be optimized, time-delays, event-triggering protocol and the initial values, which reflect the impact of various factors.

5. AN ILLUSTRATIVE EXAMPLE

In this section, a numerical example is provided to verify the effectiveness of the state estimation approach proposed in this paper. For a delayed T–S fuzzy model (4) with time-varying delays and multiple missing measurements, the parameters are given as follows:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 0.9987 & 0.9024 & 0 \\ 0 & 0.8100 & 0 \\ 0 & 0 & -0.452 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9 & 0.8167 & 0 \\ 0 & 0.3501 & 0 \\ 0 & 0 & -0.352 \end{bmatrix}, \\
B_1 &= \mathrm{diag}\{-0.06, 0.06, 0.06\}, \quad B_2 = \mathrm{diag}\{-0.1, 0.1, -0.1\}, \\
C_1 &= C_2 = \mathrm{diag}\{0, 0.9, 0\}, \quad D_{11} = D12 = \mathrm{diag}\{0.1, 0.1, 0.1\}, \\
D_{21} &= D_{22} = \begin{bmatrix} 0 & 0.1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \\
H_1 &= \mathrm{diag}\{0.5, 0, 0\}, \quad H_2 = \mathrm{diag}\{0.1, 0.1, 0.1\}, \\
N_k &= N_a = \mathrm{diag}\{0.1, 0.1, 0.1\}, \quad L_1 = L_2 = \begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix}, \\
\tau(k) &= 2 + \frac{(1 + (-1)^k)}{2}, \quad \tau_{\min} = 2, \quad \tau_{\max} = 3.
\end{aligned}
$$

Let the probabilistic density functions of β_1, β_2 and β_3 in $[0,1]$ be described as:

$$p_1(s_1)=\begin{cases}0, s_1=0\\ 0.1, s_1=0.5\\ 0.9, s_1=1\end{cases},\quad p_2(s_2)=\begin{cases}0.1, s_2=0\\ 0.1, s_2=0.5\\ 0.8, s_2=1\end{cases},\quad p_3(s_3)=\begin{cases}0.2, s_3=0\\ 0, s_3=0.5\\ 0.8, s_3=1\end{cases},$$

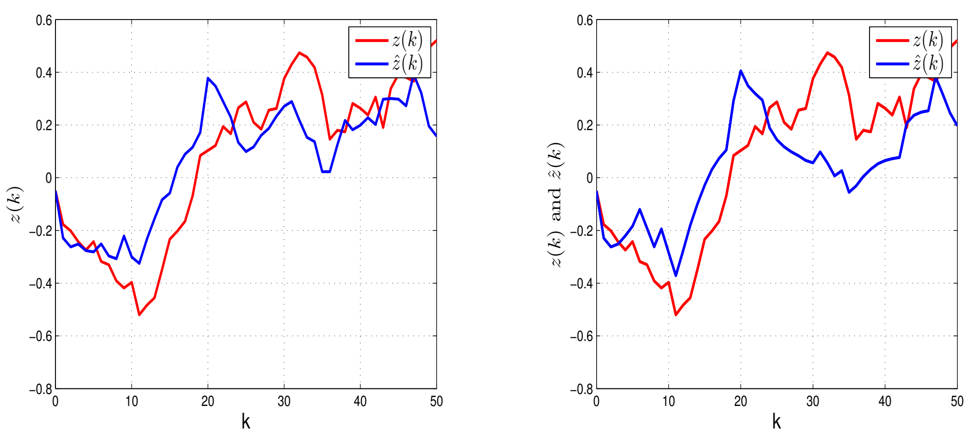
from which the expectations and variances can be easily calculated as $\mu_1=0.95, \mu_2=0.85, \mu_3=0.8$, $\sigma_1^2=0.0225$, $\sigma_2^2=0.1025$ and $\sigma_3^2=0.16$. The membership functions are assumed to be

$$h_1(x_1(k))=\begin{cases}1, |x_1(k)|<\pi/18\\ -|x_1(k)|/6+\pi/18, \pi/18\le|x_1(k)|<\pi/3\\ 1-h_2(x_1(k)), \text{otherwise}\end{cases}$$

$$h_2(x_1(k))=\begin{cases}0, |x_1(k)|<\pi/18\\ 1-h_1(x_1(k)), \pi/18\le|x_1(k)|<\pi/3\\ -3|x_1(k)|/2+\pi/2, \text{otherwise}\end{cases}$$

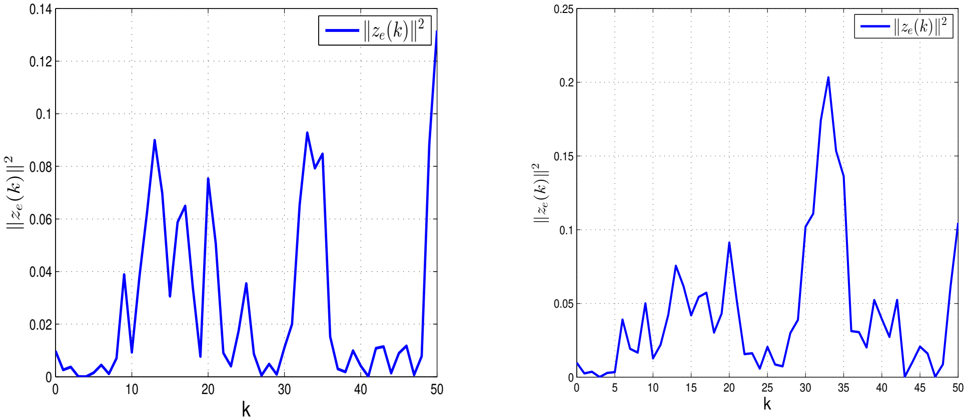
where $x_i(k)$ is the ith element of $x(k)$.

The event-triggered thresholds are set as $\lambda=0$ and $\lambda=0.1$ with the weighted matrix $\Omega=\text{diag}\{1,1,1\}$. By solving the optimization problem (47) subject to LMIs (36) and (37), one can obtain the gain matrices of the state estimator (7). Take the uncertain matrices in (7) as $F_1(k)=\text{diag}_3\{\sin(k)\}$ and $F_2(k)=\text{diag}_3\{\sin(k)\}$. Choose the initial conditions as $x(0)=\hat{x}(0)=[\ -0.1\quad -0.2\quad -0.3\ ]^T$, $x(-1)=[0.5\ 0.5\ 0.5]^T$ and $x(-2)=[0.5\ 0.5\ 0.5]^T$. Simulation results are shown in Figures 1−3 under the two cases: $\lambda=0$ and $\lambda=0.1$, where Figures 1(a)−1(b) illustrate $z(k)$ and its estimation $\hat{z}(k)$, Figures 2(a)−2(b) depict $\|z_e(k)\|^2$, and Figures 3(a)−3(b) indicate the event-triggered instants, respectively. From these figures, it can be observed that the designed estimation scheme proposed in this paper is indeed effective.



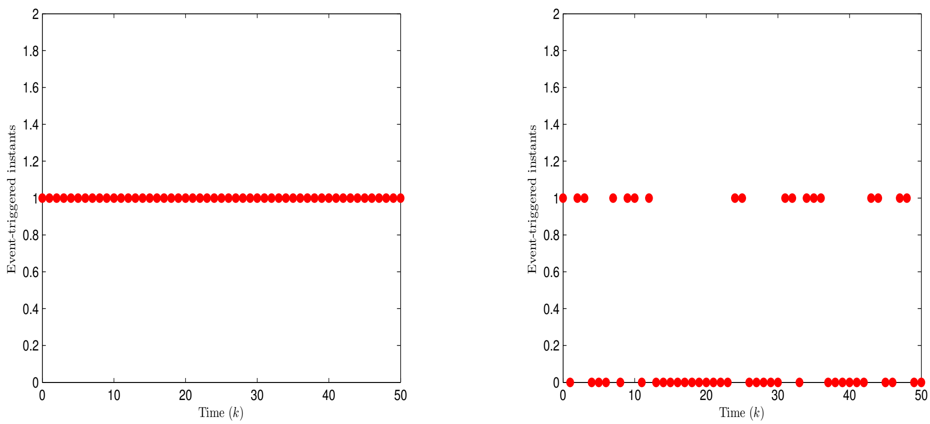
(a) $z(k)$ and $\hat{z}(k)$ when $\lambda=0$. (b) $z(k)$ and $\hat{z}(k)$ when $\lambda=0.1$.

Fig. 1. The comparison between $z(k)$ and $\hat{z}(k)$.



(a) $\|z_e(k)\|^2$ when $\lambda = 0$. (b) $\|z_e(k)\|^2$ when $\lambda = 0.1$.

Fig. 2. The comparison of $\|z_e(k)\|^2$.



(a) Event triggered instants when $\lambda = 0$. (b) Event triggered instants when $\lambda = 0.1$.

Fig. 3. The comparison of event-triggered instants.

6. CONCLUSIONS

The non-fragile state estimation algorithm has been designed for a class of uncertain discrete-time fuzzy systems with time-delays and multiple missing measurements under the event-triggered protocol. Both the time-varying delays and the stochastic disturbances have entered into the Fuzzy plant model. An individual random variable sequence satisfying a certain probabilistic distribution in the interval $[0,1]$ has been utilized to formulate the randomly occurring missing measurements. An event generator function has been introduced to determine whether the measurement can be transmitted to the estimator or not. The non-fragile estimator has been constructed by employing the well-known norm-bounded uncertainties to characterize the occurring gain variations of estimator gains. With the help of Lyapunov–Krasovskii functional, a set of sufficient conditions has been designed such that the T–S fuzzy estimation error system is exponentially ultimately bounded in the mean square. Moreover, the corresponding upper bound has been optimized and the estimator gain matrices have been computed. Finally, a simulation example has been utilized to verify the effectiveness of the state estimation technique proposed in this paper. In future, we would like to investigate the distributed state estimation problem inspired by [1, 4, 10].

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