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NEW RESULTS ON STABILITY OF PERIODIC SOLUTION FOR CNNS WITH PROPORTIONAL DELAYS AND D OPERATOR

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The problems related to periodic solutions of cellular neural networks (CNNs) involving D operator and proportional delays are considered. We shall present Topology degree theory and differential inequality technique for obtaining the existence of periodic solution to the considered neural networks. Furthermore, Laypunov functional method is used for studying global asymptotic stability of periodic solutions to the above system.

Keywords: periodic solution, D operator, existence, stability

Classification: 34D05, 34D20

1. INTRODUCTION

The cellular neural networks(CNNs) was first requested back in 1988 by Chua and Yang [6], which is investigated in various fields of science and technology. The possibility of wide practical applications of CNNs explains the still growing interests of many researchers, the recent literatures on this subject which can be found in [3, 13, 14, 23, 30–33, 36]. Oscillation and instability of CNNs may be caused by the delay, the research of the dynamic properties for delayed neural network has been attracted broad attention by many authors, see e. g. [8, 10, 24, 37, 38]. In generally, the neural networks contain different types of time delays, such as constant delays, time-varying delays, finite(or infinite) delays and distributed delays. A mass of results have been obtained for the delayed neural networks. For example, by using impulsive control Guan [11] studied the problem of whole power estimate synchronization of confused neural networks with unbounded delay and obtained the conditions for the synchronization of the considered networks with proportional lag. In [34], the authors took into consideration for a kind of robust synchronization problem of the neural networks with uncertain parameters, and obtained global robust exponential stability by using the theory of a particular delay equation. Furthermore, a global optimization control arithmetic is established for estimating the stable interval. Li, Huang and Zhu [15] gave some novel sufficient conditions for global uniform stability of cellular neural networks with mixed delays by the use of mathematical technique and Lyapunov functional method.

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Recently, CNNs involving proportional delays has been received close attention. Proportional lags occurring in neural networks have caused widespread concern due to their numerous applications in a variety of subjects of study, such as quantum electrodynamics, electric simulator and applied mathematics on algebraic structures. Liu [16] studied stability issue for a CNNs with discrete proportional delays and continuous divulgation delays. Yu [28] obtained global exponential constriction for a class of neutral delay differential equations with proportional delays. After that, Yu [29] further studied a class of HCNNs with neutral mixed delays. As proportional lag is a unbounded delay, the traditional methods for bounded delays can not be applied to CNNS with proportional lags. Hence, some new techniques should be developed for dealing with CNNS with unbounded delays.

Stimulated by the previous discussions, the main target of this paper is to derive some sufficient criteria on the global asymptotic stability for the following CNNs with proportional lags and D operator:

$$\begin{cases} (A_i x_i)'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t) g_j(x_j(q_{ij}t)) + I_i(t), \\ x_i(t) = \phi_i(t),\ t \in [-\tau, 0],\ i = 1, 2, \ldots, n, \end{cases} \tag{1.1}$$

where $\tau = \max\{\gamma(t), q_{ij}, i, j = 1, 2, \ldots, n\}$, A_i is D operator defined by

$$(A_i x_i)(t) = x_i(t) - p_i(t)x_i(t - \gamma(t)),\ i = 1, 2, \ldots, n, \tag{1.2}$$

where $x_i(t)$ and $I_i(t)$ stand for the energizing and external input of the ith neuron in the $I-$lever at time t, respectively, $c_i(t)$ stands for the rate with which the ith unit will recompound its potential to the relaxation state when disconnected from the network and outside inputs at time t, $a_{ij}(t)$ refers the power of the jth unit on the ith unit at time t, $b_{ij}(t)$ refers the power of the jth unit on the ith unit at time $q_{ij}t$, $f_j(x)$ and $g_j(x)$ describe the energizing of the ith neuron, proportionallags factors q_{ij} meets $0 < q_{ij} < 1$ and $q_{ij}t = t - (1 - q_{ij})t$ in which $(1 - q_{ij})t$ is the modification delay functions.

Throughout this paper, we always presume take for granted that $p_i(t)$, $c_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $\gamma(t)$, $I_i(t)$ are continuously periodic functions defined on $t \in [0, \infty)$ with period T, $c_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $\gamma(t)$ and $I_i(t)$ are all positive functions on $t \in [0, \infty)$. Moreover, $f_j(x_j)$ and $g_j(x_j)$ meet continuous condition on the $\mathbb{R}$.

Remark 1.1. Remember the research of neutral-type neural networks, we determine find out ascertain which the neutral character in neural networks expresses by the derivatives of system states or D operator. For example, Guo and Zhang etc. [12] discussed the under complex-valued bidirectional associative memory (BAM) neutral-type neural networks:

$$\begin{cases} z_1(t) = -D_1 z_1(t) + A_1 f(z_2(t)) + B_1 f(z_2(t - \tau(t))) + C_1 z_1'(t - \sigma(t)) + I_1, \\ z_2(t) = -D_2 z_2(t) + A_2 g(z_1(t)) + B_2 g(z_1(t - \tau(t))) + C_2 z_2'(t - \sigma(t)) + I_2, \\ z_1(s) = \zeta_(s),\ z_2(s) = \xi_(s). \end{cases} \tag{1.3}$$

Zhang, Qiu and Xiong [35] studied the following stochastic stability problem for a class of neutral-type neural networks with additive continuous delay and robust Markov jump:

$$
\begin{aligned}
&x'(t) - C_{r(t)}x'(t-\delta_3(t)) = -B_{r(t)}x(t) + A_{r(t)}f(x(t)),\\
&+ A_{\delta r(t)}f(x(t-\delta_1(t)-\delta_2(t))),\\
&x(t_0+\theta) = \phi(\theta), \quad \theta \in [-\max\{\delta_1+\delta_2, \delta_3\}, 0].
\end{aligned}
\tag{1.4}
$$

Aouiti etc. [1] studied the existence and exponential stability problems of pseudo almost periodic solutions for bidirectional associative memory neural networks (BAMNNs) with continuous lags and impulsive terms:

$$
\begin{aligned}
{x_i}'(t) &= -c_i(t)x_i(t-\alpha_i(t)) + \sum_{j=1}^{m} a_{ji}^{(1)} f_j^{(1)}(y_j(t-\tau_{ji}(t)))\\
&+ \sum_{j=1}^{m} a_{ji}^{(2)} f_j^{(2)}(y_j'(t-\sigma_{ji}(t)))\\
&+ \sum_{j=1}^{m} a_{ji}^{(3)} \int_0^{\infty} N_{ji}^{(1)}(s) f_j^{(3)}(y_j(t-s)) + \gamma_i(t), \ t \neq t_k,\\
&\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-), \ t = t_k,, \ k \in \mathbb{Z}, \ i = 1, \dots, n,\\
{y_j}'(t) &= -b_j(t)y_j(t-\beta_j(t)) + \sum_{j=1}^{m} b_{ij}^{(1)} g_i^{(1)}(y_j(t-\xi_{ij}(t)))\\
&+ \sum_{j=1}^{m} b_{ij}^{(2)} g_i^{(2)}(x_i'(t-\varsigma_{ij}(t)))\\
&+ \sum_{j=1}^{m} b_{ij}^{(3)} \int_0^{\infty} N_{ij}^{(2)}(s) g_i^{(3)}(x_i(t-s)) + \vartheta_i(t), \ t \neq t_k,\\
&\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-), \ t = t_k,, \ k \in \mathbb{Z}, \ j = 1, \dots, m.
\end{aligned}
\tag{1.5}
$$

Obviously, (1.3)-(1.5) show the neutral properties based on the terms $z_1'(t-\sigma(t))$, $z_2'(t-\sigma(t))$, $x'(t-\delta_3(t))$, $f_j^{(2)}(y_j'(t-\sigma_{ji}(t)))$ and $g_i^{(2)}(x_i'(t-\varsigma_{ij}(t)))$ which are not D operator. For more results about neutral-type neural networks, see [7, 17, 19, 20, 22]. However, the literatures for the neural networks with D operator are few. We only notice some literatures for studying the neural networks with D operator. In [27], Yao studied a model of cellular neural networks with neutral type lags and D operator and obtained the stability results of solutions for the proposed neural networks by using differential inequality and mathematical analysis techniques. Xiao [25] dealt with a class of high-order neutral neural networks system with proportional lagss and D operator and showed that all solutions of the considered system tend exponentially to zero vector. In this paper, we will further study the neutral-type neural networks when neutral term is showed by $Ax(t)$ which is defined by (1.2).

We give the main contribution of the present paper as follows:

- For obtaining the existence and stability results of periodic solution to (1.1), the properties of neutral operator A_i in (1.2) are taken into account in the neural networks, which is different other papers for studying neutral-type neural networks see e. g. [1, 7, 16, 17, 19, 20, 22, 28, 29, 35].

- Different from most of the existing methods, we develop a new unified framing to cope with the D operator, proportional delays(unbounded delays). It is worth pointing out that our main methods are also valid for the case of the non-neutral systems.
- Based on properties of neutral operator A_i, a new Lyapunov functional has been constructed which is completely different from the comparable ones of the past work.

The following sections are formed as follows: Section 2 gives some lemmas and notations. In Section 3, sufficient conditions are obtained for existence of periodic solutions to system (1.1). Asymptotic stability of system (1.1) is given in Sections 4. Section 5 presents an numerical example to substantiate the applicability of the results obtained. Finally, Section 6 offers some conclusions.

2. PRELIMINARIES

In this section, the necessary lemmas and notations are provided.

Lemma 2.1. Let X and Y are two Banach spaces, and $\mathcal{L} : D(\mathcal{L}) \subset X \to Y$, be a Fredholm operator involving index zero. Furthermore, assume that $\Omega \subset X$ is an open bounded set and $\mathcal{N} : \bar{\Omega} \to Y$ is $\mathcal{L}$-compact on $\bar{\Omega}$. if all the following conditions hold:

(1) $\mathcal{L}x \neq \lambda \mathcal{N}x,\ \forall x \in \partial\Omega \cap D(\mathcal{L}),\ \forall \lambda \in (0,1)$,

(2) $\mathcal{N}x \notin Im\mathcal{L},\ \forall x \in \partial\Omega \cap Ker\mathcal{L}$,

(3) $deg\{JQ\mathcal{N}, \Omega \cap Ker\mathcal{L}, 0\} \neq 0$,

where $J : ImQ \to Ker\mathcal{L}$ is an isomorphism. Then the operator equation $\mathcal{L}x = \mathcal{N}x$ has a solution on $\bar{\Omega} \cap D(\mathcal{L})$.

Let

$$c_\infty = \max_{t\in\mathbb{R}} |c(t)|, \quad c_0 = \min_{t\in\mathbb{R}} |c(t)|,$$

$$C_T = \{y | y \in C(\mathbb{R},\mathbb{R}),\ y(t+T) \equiv y(t),\ \forall t \in \mathbb{R}\}$$

with the norm

$$||\omega|| = \max_{t\in[0,T]} |\omega(t)|, \quad \forall \omega \in C_T.$$

It is easy to see that C_T is a Banach space. Define linear operator:

$$\mathcal{A} : C_T \to C_T, \quad [\mathcal{A}x](t) = x(t) - c(t)x(t - \tau(t)),\ \forall t \in \mathbb{R},$$

where $c(t)$ and $\tau(t)$ are $T-$periodic continuous functions.

Lemma 2.2. (Cheng and Li [5]) Assume that $|c(t)| < 1$, then the difference operator $\mathcal{A}$ exists continuous inverse operator $\mathcal{A}^{-1}$ on C_T, satisfying

$$|(\mathcal{A}^{-1}x)(t)| \leq \frac{||x||}{1-c_\infty}, \quad \forall x \in C_T.$$

Lemma 2.3. (Xin and Cheng [26]) Assume that $|c(t)| > 1$ and $\tau'(t) < 1$, then the difference operator $\mathcal{A}$ exists continuous inverse operator $\mathcal{A}^{-1}$ on C_T, satisfying

$$|(\mathcal{A}^{-1}x)(t)| \leq \frac{||x||}{c_0-1}, \quad \forall x \in C_T.$$

3. EXISTENCE OF PERIODIC SOLUTIONS

Theorem 3.1. Assume that $\int_0^T I_i(t)\,\mathrm{d}t = 0,\ i = 1,2\ldots,n,\ \gamma'(t) < 1$, and there exist constants $L_j^f \geq 0,\ L_j^g \geq 0$ and $K \geq 0$ such that

(H$_1$) $$|f_j(x_j)| \leq L_j^f, \quad |g_j(x_j)| \leq L_j^g,\ j = 1,2,\ldots,n.$$

(H$_2$) $x_i f_i(x_i) < 0$ and $x_i g_i(x_i) < 0$ for $x_i \in (-\infty,-K) \cup (K,+\infty),\ i = 1,2,\ldots,n.$

Then neural networks system (1.1) has at least one $T-$periodic solution, provided that the following conditions hold:
When $|p_i|_\infty < 1,\ i = 1,2,\ldots,n,$

$$\frac{|c_i|_\infty}{1-|p_i|_\infty} + \frac{|p_i'|_\infty}{(1-|p_i|_\infty)|1-\gamma'|_\infty} < 1 \tag{3.1}$$

and

$$\Gamma_{1i}T < 1, \tag{3.2}$$

where

$$|1-\gamma'|_\infty = \max_{t\in\mathbb{R}}\{|1-\gamma'(\tilde{\gamma}(t))|\},\ \tilde{\gamma}(t) \text{ is inverse function of } t-\gamma(t),$$

$$\Gamma_{1i} = \left(\frac{|c_i|_\infty}{1-|p_i|_\infty} + \frac{|p_i'|_\infty}{(1-|p_i|_\infty)|1-\gamma'|_\infty}\right)\Big/\left(1-\frac{|c_i|_\infty}{1-|p_i|_\infty} - \frac{|p_i'|_\infty}{(1-|p_i|_\infty)|1-\gamma'|_\infty}\right),$$

or when $|p_i|_0 > 1,\ i = 1,2,\ldots,n,$

$$\frac{|c_i|_\infty}{|p_i|_0-1} + \frac{|p_i'|_\infty}{(|p_i|_0-1)|1-\gamma'|_\infty} < 1 \tag{3.3}$$

and

$$\Gamma_{1i}'T < 1, \tag{3.4}$$

where

$$\Gamma_{1i}' = \left(\frac{|c_i|_\infty}{|p_i|_0-1} + \frac{|p_i'|_\infty}{(|p_i|_0-1)|1-\gamma'|_\infty}\right)\Big/\left(1-\frac{|c_i|_\infty}{|p_i|_0-1} - \frac{|p_i'|_\infty}{(|p_i|_0-1)|1-\gamma'|_\infty}\right).$$

P r o o f. For studying (1.1) by Lemma 2.1, denote

$$X = \{x | x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^\top \in C(\mathbb{R}, \mathbb{R}^n),\ x(t+T) \equiv x(t),\ t > 0\}$$

involving the norm $||x|| = \max\{|x_i|_\infty,\ i = 1, 2, \ldots, n\}$. Let

$$L : D(L) \subset X \to ImL \subset X, \quad (Lx)(t) = (Ax)'(t), \tag{3.5}$$

here $(Ax)(t) = ((A_1x_1)(t), \ldots, (A_nx_n)(t))^\top$, $D(L) = \{x : x \in X,\ (Ax)' \in X\}$. It is easy to see that $KerL = \mathbb{R}^n$, $ImL = \{x : x \in X,\ \int_0^T x(s)\,\mathrm{d}s = \mathbf{0}\}$ is closed in Banach space X and $dimKerL = condimImL = n$. Hence, L is a Fredholm operator involving index zero. We defined the following operators:

$$P : X \to KerL, \quad Q : X \to X/ImL,$$

$$Px = \frac{1}{T}\int_0^T x(s)\,\mathrm{d}s, \quad Qy = \frac{1}{T}\int_0^T y(s)\,\mathrm{d}s,$$

and

$$L_p = L|_{X \cap KerP} : X \cap KerP \to ImL.$$

Obviously, L_p exists its right inverse operator L_P^{-1} which has the form:

$$(L_P^{-1}y)(t) = \int_0^T G(t,s)y(s)\,\mathrm{d}s, \quad \forall y \in ImL,$$

where

$$G(t,s) = \begin{cases} \frac{s(t-T)}{T}, & 0 \le < s < t \le T, \\ \frac{t(s-T)}{T}, & 0 \le < t < s \le T. \end{cases}$$

Let $(Nx)(t) = ((N_1x_1)(t), \ldots, (N_nx_n)(t))^\top$. Define the operator N_i by

$$\begin{aligned} &N_i : X \to X, (N_ix_i)(t) = -c_i(t)x_i(t) + \sum_{j=1}^n a_{ij}(t)f_j(x_j(t)) \\ &+ \sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t)) + I_i(t), \qquad i = 1, 2, \ldots, n. \end{aligned} \tag{3.6}$$

Consider the operator equation as follows:

$$Lx = \lambda Nx, \quad \lambda \in (0,1), \tag{3.7}$$

where the operators L and N are defined by (3.5) and (3.6), respectively. $\forall x \in X$ and x is an arbitrary $T-$periodic solution of the operator equation (3.7), then $x(t)$ satisfies the following equations:

$$(A_ix_i)'(t) + \lambda c_i(t)x_i(t) - \lambda\sum_{j=1}^n a_{ij}(t)f_j(x_j(t)) - \lambda\sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t)) = 0. \tag{3.8}$$

Integrate both sides of the system (3.8) over $[0, T]$, then

$$\begin{aligned}&\int_0^T [-c_i(t)x_i(t)]\,\mathrm{d}t + \int_0^T \sum_{j=1}^n a_{ij}(t)f_j(x_j(t))\,\mathrm{d}t + \int_0^T \sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t))\,\mathrm{d}t\\ &= \int_0^T [-c_i(t)x_i(t)]\,\mathrm{d}t + \sum_{j=1}^n \int_0^T a_{ij}(t)f_j(x_j(t))\,\mathrm{d}t + \frac{1}{q_{ij}}\sum_{j=1}^n \int_0^{q_{ij}T} a_{ij}\Big(\frac{t}{q_{ij}})g_j(x_j(t)\Big)\,\mathrm{d}t\\ &= 0.\end{aligned} \tag{3.9}$$

We assert that there exists a point $t_1 \in [0, T]$ such that

$$|x_i(t_1)| \le K,\ i = 1, 2, \ldots, n. \tag{3.10}$$

In reality, if $|x_i(t)| > K,\ \forall t \in [0, T]$, then by assumption (H_2) we drive that

$$\int_0^T [-c_i(t)x_i(t)]\,\mathrm{d}t + \int_0^T \sum_{j=1}^n a_{ij}(t)f_j(x_j(t))\,\mathrm{d}t + \int_0^T \sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t))\,\mathrm{d}t \ne 0$$

which contradicts ro the system (3.9). Hence (3.10) holds. Furthermore, $\forall x \in X$, by (3.10) we drive that

$$\begin{aligned}|x_i|_\infty &= \max_{t\in[0,T]}\left|x_i(t_1) + \int_{t_1}^t x_i'(s)\,\mathrm{d}s\right|\\ &\le |x_i(t_1)| + \int_0^T |x_i'(s)|\,\mathrm{d}s\\ &\le K + \int_0^T |x_i'(s)|\,\mathrm{d}s,\ i = 1, 2, \ldots, n.\end{aligned} \tag{3.11}$$

In view of assumption (H_1) and (3.8), it follows that

$$\begin{aligned}|(A_ix_i(t))'| &\le c_i(t)|x_i(t)| + \sum_{j=1}^n[|a_{ij}(t)f_j(x_j(t))| + |b_{ij}(t)g_j(x_j(q_{ij}t))|] + |I_i(t)|\\ &\le |c_i|_\infty|x_i|_\infty + \sum_{j=1}^n[L_j^f|a_{ij}|_\infty + L_j^g|b_{ij}|_\infty] + |I_i|_\infty.\end{aligned}$$

Thus,

$$|(A_ix_i)'|_\infty \le |c_i|_\infty|x_i|_\infty + e_i,\ i = 1, 2, \ldots, n, \tag{3.12}$$

where $e_i = \sum_{j=1}^n[L_j^f|a_{ij}|_\infty + L_j^g|b_{ij}|_\infty] + |I_i|_\infty$. From $(A_ix_i)(t) = x_i(t) - p_i(t)x_i(t-\gamma(t))$, we also obtain that

$$\begin{aligned}(A_ix_i)'(t) &= \Big(x_i(t) - p_i(t)x_i(t-\gamma(t))\Big)'\\ &= x_i'(t) - p_i'(t)x_i(t-\gamma(t)) - p_i(t)x_i'(t-\gamma(t))(1-\gamma'(t))\\ &= (A_ix_i')(t) + p_i(t)x_i'(t-\gamma(t))\gamma'(t) - p_i'(t)x_i(t-\gamma(t)).\end{aligned} \tag{3.13}$$

Next, we consider the boundedness of $|x_i|_\infty$ as two cases.

Case 1. If $|p_i(t)| < 1,\ i = 1, 2, \ldots, n$, from Lemma 2.2, (3.12) and (3.13), we deduce that

$$\begin{aligned}\int_0^T |x_i'(t)|\,\mathrm{d}t &= \int_0^T |(A_i^{-1}A_i x_i')(t)|\,\mathrm{d}t \le \frac{1}{1-|p_i|_\infty}\int_0^T |(A_i x_i')(t)|\,\mathrm{d}t \\ &= \frac{1}{1-|p_i|_\infty}\int_0^T |(A_i x_i)'(t) - p_i(t)x_i'(t-\gamma(t))\gamma'(t) + p_i'(t)x_i(t-\gamma(t))|\,\mathrm{d}t \\ &\le \left(\frac{|c_i|_\infty}{1-|p_i|_\infty} + \frac{|p_i'|_\infty}{(1-|p_i|_\infty)|1-\gamma'|_\infty}\right)\int_0^T |x_i(t)|\,\mathrm{d}t \\ &+ \frac{|\gamma'|_\infty |p_i|_\infty}{(1-|p_i|_\infty)|1-\gamma'|_\infty}\int_0^T |x_i'(t)|\,\mathrm{d}t + \frac{e_i T}{1-|p_i|_\infty}.\end{aligned} \tag{3.14}$$

Based on (3.1) and (3.14), then

$$\begin{aligned}\int_0^T |x_i'(t)|\,\mathrm{d}t &\le \Gamma_{1i}\int_0^T |x_i(t)|\,\mathrm{d}t + \Gamma_{2i} \\ &\le \Gamma_{1i}T|x_i|_\infty + \Gamma_{2i},\end{aligned} \tag{3.15}$$

where Γ_{1i} is defined by (3.2) and

$$\Gamma_{2i} = \left(\frac{e_i T}{1-|p_i|_\infty}\right)\Bigg/\left(1 - \frac{|c_i|_\infty}{1-|p_i|_\infty} - \frac{|p_i'|_\infty}{(1-|p_i|_\infty)|1-\gamma'|_\infty}\right) > 0.$$

So from (3.11) and (3.15), it follows that

$$|x_i|_\infty \le K + \Gamma_{1i}T|x_i|_\infty + \Gamma_{2i}.$$

Thus, by (3.2) we have

$$|x_i|_\infty \le \frac{K+\Gamma_{2i}}{1-\Gamma_{1i}T} := M_{1i} \quad \text{(independent of } \lambda \text{ and } x). \tag{3.16}$$

Case 2. If $|p_i(t)| > 1,\ i = 1, 2, \ldots, n$, similar to the above proof, by (3.3) and (3.4) we have

$$|x_i|_\infty \le \frac{K+\Gamma_{2i}'}{1-\Gamma_{1i}'T} := M_{2i} \quad \text{(independent of } \lambda \text{ and } x), \tag{3.17}$$

where Γ_{1i}' is defined by (3.4) and

$$\Gamma_{2i}' = \left(\frac{e_i T}{|p_i|_0 - 1}\right)\Bigg/\left(1 - \frac{|c_i|_\infty}{|p_i|_0 - 1} - \frac{|p_i'|_\infty}{(|p_i|_0 - 1)|1-\gamma'|_\infty}\right).$$

Using (3.16) and (3.17), we get

$$||x|| = \max\{|x_i|_0,\ i = 1, 2, \ldots, n\} \le \max\{M_{1i}, M_{2i}\} := \widetilde{M}.$$

Take $\Omega = \{x \in X : ||x|| < \widetilde{M}+1\}$. Then, $\forall x \in \Omega$, we know that the first condition of Lemma 2.1 is true. Furthermore, let $x \in \partial\Omega \cap KerL$, obviously, x is a constant vector in $\mathbb{R}^n$. There exists i such that $|x_i| = \widetilde{M}+1$ and $|x_j| < \widetilde{M}+1$ for $j \neq i$. We assert that $QNx \neq \mathbf{0}$ for all $x \in \partial\Omega \cap KerL$. In fact, if $QNx = \mathbf{0}$, then

$$\int_0^T [-c_i(t)x_i(t)]\,\mathrm{d}t + \int_0^T \sum_{j=1}^n a_{ij}(t)f_j(x_j(t))\,\mathrm{d}t + \int_0^T \sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t))\,\mathrm{d}t = 0$$

However, use $|x_i| = M_i + 1 > K$ and assumption (H_2), then

$$\int_0^T [-c_i(t)x_i(t)]\,\mathrm{d}t + \int_0^T \sum_{j=1}^n a_{ij}(t)f_j(x_j(t))\,\mathrm{d}t + \int_0^T \sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t))\,\mathrm{d}t \neq 0$$

which is a contradiction. Hence, for any $x \in \partial\Omega \cap KerL$, $QNx \neq \mathbf{0}$ holds. Thus, the second condition of Lemma 2.1 is true. Let

$$H(x,\mu) = -\mu x + (1-\mu)QNx, \ \mu \in [0,1],$$

then we assert $H(\mu, x) \neq \mathbf{0}$ for all $x \in \partial\Omega \cap KerL$. On the contrary, if the above equality is not true, then

$$\mu x_i = \frac{1-\mu}{T}\int_0^T [-c_i(t)x_i + \sum_{j=1}^n [a_{ij}(t)f_j(x_j) + b_{ij}(t)g_j(q_{ij}x_j))]\,\mathrm{d}t.$$

If $x_i = \widetilde{M}+1$ or $x_i = -(\widetilde{M}+1)$, assumption (H_2) results in the following inequality:

$$\mu x_i \neq \frac{1-\mu}{T}\int_0^T [-c_i(t)x_i + \sum_{j=1}^n [a_{ij}(t)f_j(x_j) + b_{ij}(t)g_j(q_{ij}x_j))]\,\mathrm{d}t$$

which is a contradiction. Thus, $H(x,\mu) \neq \mathbf{0}$ for any $x \in \partial\Omega \cap KerL$. Using the property of topological degree and taking J to be the identity mapping $I : ImQ \to KerL$, we have

$$\begin{aligned} \deg\{JQN, \Omega \cap KerL, 0\} &= \deg\{H(\cdot,0), \Omega \cap KerL, 0\} \\ &= \deg\{H(\cdot,1), \Omega \cap KerL, 0\} \\ &= -1 \neq 0. \end{aligned}$$

So, we obtain that condition (3) of Lemma 2.1 holds. Therefore, based on Lemma 2.1, it follows that the operator equation $Lx = Nx$ has at least one $T-$periodic solution x in $\bar{\Omega}$. Namely, the neural networks (1.1) has at least one $T-$periodic solution. □

Remark 3.1. Since system (1.1) contains neutral terms, for obtaining the results of Theorem 3.1, we give some proper conditions (including (H_1) and (H_2)) and develop some inequality techniques in the proof of Theorem 3.1.

4. ASYMPTOTIC STABILITY OF PERIODIC SOLUTIONS

Now, we discuss global asymptotic stability of periodic solutions for system (1.1).

Definition 4.1. Assume that $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^\top$ is a periodic solution of system (1.1) and $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^\top$ is any solution of system (1.1) satisfying

$$\lim_{t\to+\infty} \sum_{i=1}^{n} |x_i(t) - x_i^*(t)| = 0.$$

Then, $x^*(t)$ is global asymptotic stable.

For convenience of studying global asymptotic stability of system (1.1), let $I_i(t) = 0,\ f_i(0) = g_i(0) = 0,\ i = 1, 2, \ldots, n$. Then (1.1) has the following form:

$$(A_i x_i)'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n}[c_{ij}(t)f_j(x_j(t)) + b_{ij}(t)g_j(x_j(q_{ij}t))],\ i = 1, 2, \ldots, n, \quad (4.1)$$

and $x = \mathbf{0}$ is the equilibrium point of (4.1). Here, we give the main Theorem:

Theorem 4.1. Assume that conditions of Theorem 3.1 hold. Suppose further that the following assumptions hold:

(H_3) there is $R_{1j}^f > 0,\ R_{2j}^g > 0$ and $\delta > 0$ such that

$$|f_j(x) - f_j(y)| \le R_{1j}^f |x-y|,\ \ |g_j(x) - g_j(y)| \le R_{2j}^g |x-y|,\ \forall x, y \in \mathbb{R},\ j = 1, 2, \ldots, n;$$

$$x_i f_i(x_i) < -\delta |x_i|^2 \ \text{ for } x_i \in (-\infty, -K) \cup (K, +\infty),\ \ i = 1, 2, \ldots, n,$$

where K is defined by (H_2).

(H_4) Let

$$\xi_i = \lim_{t\to+\infty} \inf \Big[2\check{c}_i + 2\check{a}_{ii}\delta - \alpha_i - \beta_i - |p_i|_\infty |c_i|_\infty - \vartheta_i - \kappa_i - \eta_i(t)\Big] > 0,\ i = 1, 2, \ldots, n,$$

where

$$\check{c}_i = \min_{t\in\mathbb{R}}\{c_i(t)\},\ \check{a}_{ii} = \min_{t\in\mathbb{R}}\{a_{ii}(t)\},\ i = 1, 2, \ldots, n, \quad (4.2)$$

$$\alpha_i = \sum_{j=1}^{n} |a_{ij}|_\infty R_{1j}^f,\ \beta_i = \sum_{j=1}^{n} |b_{ij}|_\infty R_{2j}^g,\ i = 1, 2, \ldots, n, \quad (4.3)$$

$$\vartheta_i = (|p_i|_\infty + 1) \sum_{j=1}^{n} \frac{|b_{ij}|_\infty R_{2j}^g}{q_{ij}},\ i = 1, 2, \ldots, n, \quad (4.4)$$

$$\kappa_i = (|p_i|_\infty + 1) \sum_{j=1}^{n} |a_{ij}|_\infty R_{1j}^f,\ i = 1, 2, \ldots, n, \quad (4.5)$$

$$\eta_i(t) = (|p_i|_\infty |c_i|_\infty + |p_i|_\infty \alpha_i + |p_i|_\infty \beta_i)\omega(t),\ i = 1, 2, \ldots, n, \quad (4.6)$$

where $\omega(t) = \frac{1}{1-\gamma'(\tilde{\gamma}(t))}$, $\tilde{\gamma}(t)$ is inverse function of $t - \gamma(t)$.

Then the neural networks system (4.1) has unique $T-$periodic solution $x^*(t) = \mathbf{0}$ which is global asymptotic stable.

Proof. Assumptions of Theorem 3.1 and 4.1 imply that (4.1) has unique $T-$periodic solution $x^*(t) = \mathbf{0}$. Suppose $x(t)$ be any solution of (4.1). Let

$$V_i(t) = (A_i x_i)^2(t), \quad i = 1, 2, \ldots, n. \tag{4.7}$$

Using derivation of (4.7) along the solution of (4.1) and assumption (H_3) give

$$\begin{aligned}
V_i'(t) &= -2c_i(t)x_i^2(t) + 2x_i(t)\sum_{j=1}^n a_{ij}(t)f_j(x_j(t)) + 2x_i(t)\sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t)) \\
&\quad + 2p_i(t)c_i(t)x_i(t)x_i(t-\gamma(t)) - 2p_i(t)x_i(t-\gamma(t))\sum_{j=1}^n a_{ij}(t)f_j(x_j(t)) \\
&\quad - 2p_i(t)x_i(t-\gamma(t))\sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t)) \\
&\le -2\check{c}_i x_i^2(t) - 2\check{a}_{ii}\delta|x_i|^2 + 2|x_i(t)|\sum_{j=1, j\neq i}^n |a_{ij}|_\infty R_{1j}^f |x_j(t)| + 2|x_i(t)| \\
&\qquad \sum_{j=1}^n |b_{ij}|_\infty R_{2j}^g |x_j(q_{ij}t))| \\
&\quad + 2|p_i|_\infty |c_i|_\infty |x_i(t)||x_i(t-\gamma(t))| + 2|p_i|_\infty |x_i(t-\gamma(t))|\sum_{j=1}^n |a_{ij}|_\infty R_{1j}^f |x_j(t)| \\
&\quad + 2|p_i|_\infty |x_i(t-\gamma(t))|\sum_{j=1}^n |b_{ij}|_\infty R_{2j}^g |x_j(q_{ij}t))| \\
&\le -2\check{c}_i x_i^2(t) - 2\check{a}_{ii}\delta|x_i|^2 + \alpha_i x_i^2(t) \\
&\qquad + \sum_{j=1, j\neq i}^n |a_{ij}|_\infty R_{1j}^f x_j^2(t) + \beta_i x_i^2(t) + \sum_{j=1}^n |b_{ij}|_\infty R_{2j}^g x_j^2(q_{ij}t) \\
&\quad + |p_i|_\infty |c_i|_\infty x_i^2(t) + |p_i|_\infty |c_i|_\infty x_i^2(t-\gamma(t)) \\
&\quad + |p_i|_\infty \alpha_i x_i^2(t-\gamma(t)) + |p_i|_\infty \sum_{j=1, j\neq i}^n |a_{ij}|_\infty R_{1j}^f x_j^2(t) \\
&\quad + |p_i|_\infty \beta_i x_i^2(t-\gamma(t)) + |p_i|_\infty \sum_{j=1}^n |b_{ij}|_\infty R_{2j}^g x_j^2(q_{ij}t)) \\
&= -(2\check{c}_i + 2\check{a}_{ii}\delta - \alpha_i - \beta_i - |p_i|_\infty |c_i|_\infty)x_i^2(t) + (|p_i|_\infty |c_i|_\infty + |p_i|_\infty \alpha_i + |p_i|_\infty \beta_i) \\
&\qquad x_i^2(t-\gamma(t)) \\
&\quad + (|p_i|_\infty + 1)\sum_{j=1, j\neq i}^n |a_{ij}|_\infty R_{1j}^f x_j^2(t) + (|p_i|_\infty + 1)\sum_{j=1}^n |b_{ij}|_\infty R_{2j}^g x_j^2(q_{ij}t),
\end{aligned} \tag{4.8}$$

where $\check{c}_i,\ \check{a}_{ii},\ \alpha_i,\ \beta_i,\ i=1,2,\ldots,n$ are defined by (4.2) and (4.3). Define further that

$$V_{\gamma_i}(t)=(|p_i|_\infty|c_i|_\infty+|p_i|_\infty\alpha_i+|p_i|_\infty\beta_i)\int_{t-\gamma(t)}^{t}\omega(s)x_i^2(s)\,\mathrm{d}s \tag{4.9}$$

and

$$V_{q_{ij}}(t)=(|p_i|_\infty+1)\sum_{j=1}^{n}\frac{|b_{ij}|_\infty R_{2j}^g}{q_{ij}}\int_{q_{ij}t}^{t}x_i^2(s)\,\mathrm{d}s. \tag{4.10}$$

By (4.9) and (4.10) we have

$$V'_{\gamma_i}(t)=(|p_i|_\infty|c_i|_\infty+|p_i|_\infty\alpha_i+|p_i|_\infty\beta_i)[\omega(t)x_i^2(t)-x_i^2(t-\gamma(t))] \tag{4.11}$$

and

$$V'_{q_{ij}}(t)=(|p_i|_\infty+1)\sum_{j=1}^{n}\frac{|b_{ij}|_\infty R_{2j}^g}{q_{ij}}\left[x_i^2(t)-\frac{1}{q_{ij}}x_i^2(q_{ij}t)\right], \tag{4.12}$$

where $\omega(t)=\frac{1}{1-\gamma'(\tilde{\gamma}(t))}$, $\tilde{\gamma}(t)$ is inverse function of $t-\gamma(t)$. Choose the Lyapunov functional for (4.1) in the following form:

$$V(t)=\sum_{i=1}^{n}[V_i(t)+V_{\gamma_i}(t)+V_{q_{ij}}(t)],\ j=1,2,\ldots,n. \tag{4.13}$$

In view of (4.8), (4.11) and (4.12), derivating (4.13) along the solution of (4.1) gives

$$\begin{aligned}
V'(t)\le\sum_{i=1}^{n}\Big[&-(2\check{c}_i+2\check{a}_{ii}\delta-\alpha_i-\beta_i-|p_i|_\infty|c_i|_\infty)x_i^2(t)\\
&+(|p_i|_\infty|c_i|_\infty+|p_i|_\infty\alpha_i+|p_i|_\infty\beta_i)\omega(t)x_i^2(t)\\
&+(|p_i|_\infty+1)\sum_{j=1,j\ne i}^{n}|a_{ij}|_\infty R_{1j}^f x_j^2(t)+(|p_i|_\infty+1)\sum_{j=1}^{n}\frac{|b_{ij}|_\infty R_{2j}^g}{q_{ij}}x_i^2(t)\Big]\\
=-\sum_{i=1}^{n}\Big[&(2\check{c}_i+2\check{a}_{ii}\delta-\alpha_i-\beta_i-|p_i|_\infty|c_i|_\infty-\vartheta_i-\eta_i(t))x_i^2(t)\\
&-(|p_i|_\infty+1)\sum_{j=1,j\ne i}^{n}|a_{ij}|_\infty R_{1j}^f x_j^2(t)\Big]\\
\le-\sum_{i=1}^{n}\Big[&2\check{c}_i+2\check{a}_{ii}\delta-\alpha_i-\beta_i-|p_i|_\infty|c_i|_\infty-\vartheta_i-\kappa_i-\eta_i(t)\Big]x_i^2(t),
\end{aligned} \tag{4.14}$$

where $\vartheta_i,\ \eta_i(t),\ \kappa_i$ are defined by (4.4)-(4.6). Assumption (H_4) yields, for any $\varepsilon>0$ and $\xi_i-\varepsilon>0$, there exists a positive constant L (enough large) such that

$$2\check{c}_i+2\check{a}_{ii}\delta-\alpha_i-\beta_i-|p_i|_\infty|c_i|_\infty-\vartheta_i-\kappa_i-\eta_i(t)\ge\xi_i-\varepsilon \text{ for } t>L,$$

which together with (4.14) gives

$$V'(t)\le-\sum_{j=1}^{n}(\xi_i-\varepsilon)x_i^2(t)<0 \text{ for } t>L. \tag{4.15}$$

Integrating both sides of (4.15) from L to $+\infty$ gives

$$V(t)+\int_{L}^{+\infty}\sum_{j=1}^{n}(\xi_i-\varepsilon)x_i^2(s)\,\mathrm{d}s\le V(0).$$

Due to Barbalat's Lemma [4], then

$$\lim_{t\to+\infty}\sum_{i=1}^{n}|x_i(t)|=0.$$

The proof of Theorem 4.1 is now completed. □

Remark 4.1. In this paper, a new Lyapunov functional has been constructed which is completely different from the comparable ones of the past work by using neutral-type operator $A_i (i = 1, 2, \ldots, n)$. In [2], Arik studied the problem for stability of neutral-type dynamical neural networks involving delay parameters and constructed the following Lyapunov functional:

$$\begin{aligned} V(x(t),x'(t),t) &= \sum_{i=1}^{n} p_i\Big(1-\alpha sgn(x_i(t)))sgn(x_i'(t))\Big)|x_i(t)| \\ &+\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}p_j\alpha\int_{t-\xi_i}^{t}|x_j'(s)|\,\mathrm{d}s+\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\int_{t-\tau_{ij}}^{t}b_{ij}|f_j(x_j(s))|\,\mathrm{d}s \\ &+k\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t}|x_j(s)|\,\mathrm{d}s. \end{aligned}$$

Samli etc. [21] obtained a generalized global stability criteria for delayed Cohen-Grossberg neural networks of neutral-type and constructed the following Lyapunov functional:

$$\begin{aligned} V(z(t)) &= [z(t)+Ez(t-\tau)]^{\top}[z(t)+Ez(t-\tau)] \\ &+\sum_{i=1}^{n}k_i\int_{t-\tau_i}^{t}z_i^2(s)\,\mathrm{d}s+\sum_{i=1}^{n}h_i\int_{t-\tau_i}^{t}g_i^2(z_i^2(s))\,\mathrm{d}s \\ &+\rho\sum_{i=1}^{n}\int_{t-\tau_i}^{t}z_i^2(s)\,\mathrm{d}s. \end{aligned}$$

Ozcan [18] considered a Cohen-Grossberg neural networks of neutral-type with multiple delays and constructed the following Lyapunov functional:

$$\begin{aligned} V(t) &= \sum_{i=1}^{n}\left[\Big(z_i(t)-\sum_{j=1}^{n}e_{ij}z_j(t-\tau_{ij})\Big)^2\right] \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}(\rho_j l_i|b_{ji}|+\rho_j\phi_i|e_{ji}|)\int_{t-\tau_{ji}}^{t}z_i^2(s)\,\mathrm{d}s \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}(\rho_j l_k|a_{jk}||e_{ji}|+\rho_j l_k b_{jk}|e_{ji}|)\int_{t-\tau_{ji}}^{t}z_i^2(s)\,\mathrm{d}s \\ &+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\rho_k l_i|b_{ki}||e_{kj}|+\eta\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t}z_i^2(s)\,\mathrm{d}s. \end{aligned}$$

The above Lyapunov functionals are different from the corresponding one in the present paper.

5. A NUMERICAL EXAMPLE

In this section, a numerical example is given to illustrate the effectiveness of the results obtained in the present paper.

Example 5.1. Consider the following neutral-type neural networks:

$$\begin{cases} (A_1x_1)'(t) = -c_1(t)x_1(t) + \sum_{j=1}^{2}[a_{1j}(t)f_j(x_j(t)) + b_{1j}(t)g_j(x_j(q_{1j}t))], \\ (A_2x_2)'(t) = -c_2(t)x_2(t) + \sum_{j=1}^{2}[a_{2j}(t)f_j(x_j(t)) + b_{2j}(t)g_j(x_j(q_{2j}t))], \end{cases} \tag{5.1}$$

where

$$\begin{bmatrix} A_1x_1(t) \\ A_2x_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - \frac{1}{2}x_1(t - \frac{1}{4}\cos 2t) \\ x_2(t) - \frac{1}{2}x_2(t - \frac{1}{4}\cos 2t) \end{bmatrix},$$

$\gamma(t) = \frac{1}{4}\cos 2t$, then $1 - \gamma'(t) = 1 + \frac{1}{2}\sin 2t > 0$,

$$p_i(t) = 0.5 < 1, \quad q_{ij} = 0.5 < 1, \ i, j = 1, 2,$$

$$f_j(x_j) = g_j(x_j) = \frac{-0.2x_j}{x_j^2 + 1}, \quad R_{1j}^f = R_{2j}^g = 0.2, \ j = 1, 2.$$

$$\begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{100}cos2t + \frac{101}{100} \\ -\frac{1}{100}\sin^2 t + +\frac{101}{100} \end{bmatrix} > 0,$$

$$\begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{10} + \frac{1}{10}\sin^2 t & \frac{1}{10} + \frac{1}{10}\cos^2 t \\ \frac{1}{10} + \frac{1}{10}\cos^2 t & \frac{1}{10} + \frac{1}{10}\sin^2 t \end{bmatrix} > 0,$$

$$\begin{bmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{4}\cos^2 t + \frac{1}{4} & \frac{1}{2} + \frac{1}{2}\cos^2 t \\ \frac{1}{2}\cos^2 t + \frac{1}{4} & \frac{1}{4} + \frac{1}{4}\cos^2 t \end{bmatrix} > 0.$$

After a simple calculation, it follows that

$$\check{c}_1 = \min_{t\in\mathbb{R}}\{c_1(t)\} = 1, \ \check{c}_2 = \min_{t\in\mathbb{R}}\{c_2(t)\} = 1, \ \check{a}_{ii} = \min_{t\in\mathbb{R}}\{a_{ii}(t)\} = 0.1, \ i = 1, 2,$$

$$\alpha_1 = \sum_{j=1}^{2}|a_{1j}|_\infty R_{1j}^f = 0.08, \ \alpha_2 = \sum_{j=1}^{2}|a_{2j}|_\infty R_{2j}^f = 0.08,$$

$$\beta_1 = \sum_{j=1}^{2}|b_{1j}|_\infty R_{2j}^g = 0.3, \ \beta_2 = \sum_{j=1}^{2}|b_{2j}|_\infty R_{2j}^g = 0.25,$$

$$\vartheta_1 = (|p_1|_\infty + 1)\sum_{j=1}^{2}\frac{|b_{1j}|_\infty R_{2j}^g}{q_{1j}} = 0.9, \ \vartheta_2 = (|p_2|_\infty + 1)\sum_{j=1}^{2}\frac{|b_{2j}|_\infty R_{2j}^g}{q_{2j}} = 0.75,$$

$$\kappa_1 = (|p_1|_\infty + 1)\sum_{j=1}^{2} |a_{1j}|_\infty R^f_{1j} = 0.12,\ \ \kappa_2 = (|p_2|_\infty + 1)\sum_{j=1}^{2} |a_{2j}|_\infty R^f_{1j} = 0.12,$$

$$\eta_1(t) = (|p_1|_\infty |c_1|_\infty + |p_1|_\infty \alpha_1 + |p_1|_\infty \beta_1)\omega(t) = \frac{7}{10}\omega(t),$$

$$\eta_2(t) = (|p_2|_\infty |c_2|_\infty + |p_2|_\infty \alpha_2 + |p_2|_\infty \beta_2)\omega(t) = \frac{27}{40}\omega(t),$$

$$\omega(t) = \frac{1}{1-\gamma'(\tilde{\gamma}(t))} = \frac{1}{1+0.5\sin(2\tilde{\gamma}(t))},$$

where $\tilde{\gamma}(t)$ is inverse function of $t-\gamma(t)$. Choose $\delta = 20$, then

$$\xi_1 = \lim_{t\to+\infty}\inf\left[2\check{c}_1 + 2\check{a}_{11}\delta - \alpha_1 - \beta_1 - |p_1|_\infty |c_1|_\infty - \vartheta_1 - \kappa_1 - \eta_1(t)\right] = 3.04 > 0,$$

$$\xi_2 = \lim_{t\to+\infty}\inf\left[2\check{c}_2 + 2\check{a}_{22}\delta - \alpha_2 - \beta_2 - |p_2|_\infty |c_2|_\infty - \vartheta_2 - \kappa_2 - \eta_2(t)\right] = 2.875 > 0.$$

By Theorem 4.1, system (5.1) has an unique periodic solution which is global asymptotic stable. The corresponding numerical simulations are presented in Figure 1.

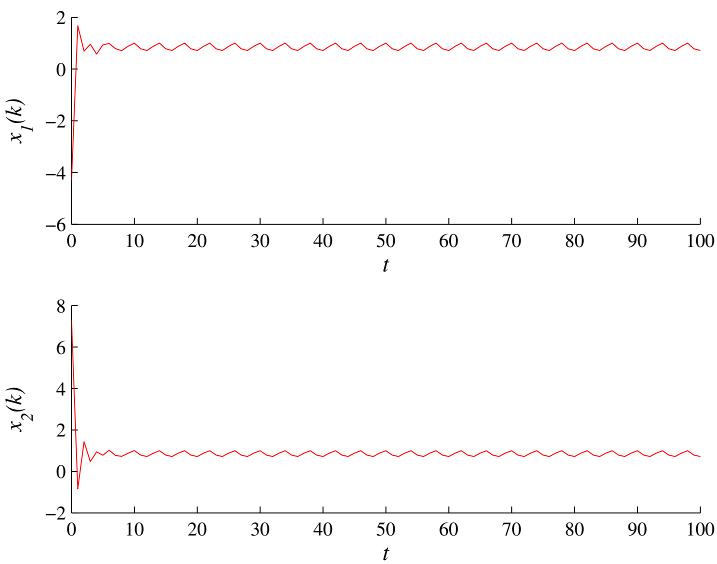

Fig. 1. Asymptotic stable periodic solution for neutral-type neural networks (5.1).

6. CONCLUSIONS

In this paper, we proposed some valuable results about existence and global asymptotic stability of periodic solution for a class of CNNs involving proportional lags and D operator. Since the proportional delay is unbounded, the key idea is how to deal with unbounded delays. We develop new technique which avoid the difficulties caused by unbounded lags. In the last past decades, CNNs with different types of delays have been studied and obtained lots of results on the dynamics of CNNs, including the stability, instability, oscillation and attractivity. The time delay as an inherent feature of signal transmission between different neurons, is one of the main sources for causing dynamic properties of neural networks, the dynamics of CNNs with time delay has been extensively investigated. However, we find that, rather than occurring in the system states, time delays can also appear in the derivatives of system states. This kind of time delays is referred to as the neutral time delays that can be found a variety of applications including transmission lines, electronic circuit system, chemical reactors and Lotka–Volterra systems. Note that, up to now, most neutral-type neural networks expressed the neutral character by the derivatives of system states. In this paper, system (1.1) expresses neutral character by D operator which is different from the other most literatures. Furthermore, we obtain some existence and stability results of periodic solution by using properties of D operator which is novel and of significance.

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