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INVERSE OPTIMAL CONTROL FOR LINEARIZABLE NONLINEAR SYSTEMS WITH INPUT DELAYS

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We consider inverse optimal control for linearizable nonlinear systems with input delays based on predictor control. Under a continuously reversible change of variable, a nonlinear system is transferred to a linear system. A predictor control law is designed such that the closed-loop system is asymptotically stable. We show that the basic predictor control is inverse optimal with respect to a differential game. A mechanical system is provided to illustrate the effectiveness of the proposed method.

Keywords: nonlinear systems, inverse optimality, predictor control, input delays

Classification: 93Cxx, 93Dxx

1. INTRODUCTION

Predictor-based techniques have been developed for stabilization linear/nonlinear systems with input delays [1, 2, 3, 7, 8, 10, 14, 18], tracking control [26, 27], optimal performance analysis of networked control systems [23, 24, 25], as well as observer design for a class of nonlinear system in cascade with counter-convecting transport dynamics [9].

The inverse optimality concept is of practical importance since it allows the design of optimal control laws, which may minimize/maximize a physical quantity of interest and which may possess certain robustness margins, without the need to solve a Hamilton–Jacobi–Isaacs partial differential equation (PDE) [19].

Inverse optimality, as an design objective for delay systems was pursued by Jankovic [12, 13]. Inverse-optimal re-design of the predictor-based feedback law was presented by using a low-pass filter in [17]. Input-to-state stability (ISS) and inverse optimality of linear time-varying-delay predictor feedbacks have been investigated in [5]. The method in [5] is extended to multi-input linear systems [6]. Inverse optimal control for strict-feedforward nonlinear systems with input delays also exists [11].

If we can find coordinate changes to transform a class of nonlinear systems into linear systems, it will be very meaningful. It is revealed that the family of feedforward systems contains a substantial class that is linearizable by a diffeomorphic coordinate change in [15, 16]. An algorithm along with explicit transformations that linearizes a class of

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feedforward nonlinear systems is in [21]. Sufficient and necessary conditions are given for a control-affine mechanical system that can be transformed into a linear system by a diffeomorphism coordinate change in [20].

In this paper, we extend the result of [17] to inverse optimal control design for a class of linearizable nonlinear systems with input delays. First, using a continuously reversible change of variable, a nonlinear system is transferred to a linear system. A predictor control law is designed such that the closed-loop system is asymptotically stable. It is shown that the control law is inverse optimal with respect to a meaningful differential game. Inverse optimal control for a class of linearizable nonlinear systems with input delays has never been published from the knowledge of authors.

Notation. We use the common definitions of class $\mathcal{K}$, $\mathcal{K}_\infty$, $\mathcal{KL}$ functions from [18]. $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues, respectively of the corresponding matrices. For a vector $X \in R^n$, $|X|$ denotes its usual Euclidean norm. For a scalar function $u(\cdot,t) \in L_2(0,1)$, $\|u(t)\|$ denotes $\left(\int_0^1 u^2(x,t)\,\mathrm{d}x\right)^{1/2}$. For a scalar function $U \in L_2(0,D)$, $\|U(t)\|$ denotes $\left(\int_0^D U^2(\theta)\,\mathrm{d}\theta\right)^{1/2}$.

2. SYSTEM DESCRIPTION AND BACKSTEPPING TRANSFORMATION

Consider the system with input delay

$$\dot{Z}(t) = \varphi(Z(t), U(t-D)) \tag{1}$$

where $Z \in R^n$ is the state, $U \in R$ is the input signal delayed by D units of time, and $\varphi : R^n \times R \to R^n$ is continuously differentiable. Suppose that there exists a continuously reversible coordinate change

$$X(t) = f(Z(t)) \tag{2}$$

such that system (1) is converted into a linear system as

$$\dot{X}(t) = AX(t) + B_1 U(t-D) \tag{3}$$

with $A \in R^{n\times n}, B_1 \in R^n$.

Remark 1. Sufficient and necessary conditions are given for a control-affine mechanical system that can be transformed into a linear system by a coordinate change in Theorem 1 of [20]. More information can be found from [4] for linearization of mechanical control systems and [22] for linearization of Hamiltonian and gradient systems.

Remark 2. In [21], for a class feedforward systems

$$\dot{x} = f(x) + g(x)u \tag{4}$$

where $f = [f_1, f_2, \cdots, f_n]^T$, $g = [g_1, g_2, \cdots, g_n]^T$, and $f_j(x) = x_j \overline{f}_j(x_{j+1}, \cdots, x_n) + \widehat{f}_j(x_{j+1}, \cdots, x_n)$, $g_j(x) = x_j \overline{g}_j(x_{j+1}, \cdots, x_n) + \widehat{g}_j(x_{j+1}, \cdots, x_n)$, $\widehat{f}_j(0) = 0, \widehat{g}_j(0) = 0, f_n = 0, \overline{f}_j(0), g_n \in R/\{0\}$ can be transformed into a linear controllable system via a coordinate change.

Using the transport PDE to express the delay, system (3) can be rewritten as

$$\begin{aligned} \dot{X}(t) &= AX(t) + B_1 u(0,t) && (5)\\ u_t(x,t) &= u_x(x,t),\ x \in [0,D] && (6)\\ u(D,t) &= U(t) && (7) \end{aligned}$$

where $u(x,t) = U(x+t-D)$.

The infinite-dimensional backstepping transformation is defined as

$$w(x,t) = u(x,t) - ke^{Ax}X(t) - k\int_0^x e^{A(x-y)}B_1 u(y,t)\,\mathrm{d}y, \qquad (8)$$

for all $x \in [0,D]$. The gain vector k is selected so that $A + B_1 k$ is hurwitz.

Under the backstepping transformation, system (5)–(7) is transferred as

$$\begin{aligned} \dot{X}(t) &= (A+B_1k)X(t) + B_1 w(0,t) && (9)\\ w_t(x,t) &= w_x(x,t),\ x \in [0,D] && (10)\\ w(D,t) &= U(t) - ke^{AD}X(t) - k\int_0^D e^{A(D-y)}B_1 u(y,t)\,\mathrm{d}y. && (11) \end{aligned}$$

The inverse backstepping transformation of w is defined as follows:

$$u(x,t) = w(x,t) + ke^{(A+B_1k)x}X(t) + k\int_0^x e^{(A+B_1k)(x-y)}B_1 w(y,t)\,\mathrm{d}y, \qquad (12)$$

for all $x \in [0,D]$. With (12), system (9)–(11) is transferred to system (5)–(7).

3. ASYMPTOTICAL STABILIZATION FOR LINEARIZABLE NONLINEAR SYSTEMS

The control law for system (1) is designed as follows:

$$U(t) = \frac{c}{c+1}U_1(t) = U^*(t) \qquad (13)$$

where

$$U_1(t) = ke^{AD}f(Z(t)) + k\int_{t-D}^t e^{A(t-\theta)}B_1 U(\theta)\,\mathrm{d}\theta \qquad (14)$$

and $c > 0$ is sufficiently large, and the vector k is selected so that $A + B_1 k$ is hurwitz.

Theorem 3.1. Consider the closed-loop system (1), (13) and (14), there exist $c^* > 0$, and a class of $\mathcal{KL}$ function $\beta(s,t)$ such that for all $c > c^*$,

$$\Gamma(t) \le \beta(\Gamma(0),\, t), \quad \text{for all } t \ge 0, \qquad (15)$$

with

$$\Gamma(t) = |Z(t)| + \|U(t)\|. \qquad (16)$$

The proof of Theorem 3.1 is based on a series of technical lemmas which are given next.

First, we will prove that system (5)–(7) under the control law (13) and

$$U_1(t) = ke^{AD}X(t) + k\int_0^D e^{A(D-y)}B_1u(y,t)\,\mathrm{d}y \tag{17}$$

where k is given by (14), is exponentially stable.

Lemma 3.2. Consider system (5)–(7), together with the control law (13) and (17), for any $0 < \mu < 1$, there exist

$$\overline{\lambda} = \frac{\mu\min\{\frac{\lambda_{\min}(Q)}{2}, \frac{a_1 b}{2}\}}{\max\{\lambda_{\max}(P), \frac{a_1 e^{bD}}{2}\}}, \tag{18}$$

$$R = \frac{\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}{\min\{\lambda_{\min}(P), \frac{a_1}{2}\}}\max\{\lambda_{\max}(P), \frac{a_1 e^b}{2}\}\max\{\overline{\alpha}_1, 1+\overline{\alpha}_2\}, \tag{19}$$

and

$$c^* = \frac{\sqrt{e^{bD}\overline{a}\max\left\{\frac{a_1}{\lambda_{\min}(Q)}, \frac{1}{b}\right\}}}{\sqrt{1-\mu}}, \tag{20}$$

such that for all $c > c^*$, it holds

$$\Omega(t) \le R\Omega(0)e^{-\overline{\lambda}t}, \text{ for } t \ge 0, \tag{21}$$

with

$$\Omega(t) = |X(t)|^2 + \|u(t)\|^2. \tag{22}$$

Proof. The proof is similar to that in [5], it is omitted. □

Noting that $u(y,t) = U(y+t-D)$, it is not difficult to find that (17) can be rewritten as

$$U_1(t) = ke^{AD}X(t) + k\int_{t-D}^t e^{A(t-\theta)}B_1U(\theta)\,\mathrm{d}\theta. \tag{23}$$

Lemma 3.3. Consider the closed-loop system (3), (13), (23), for any $0 < \mu < 1$, there exist $\overline{\lambda} > 0$, $R > 0$ and $c^* > 0$, which are given by (18), (19), (20), respectively, such that for all $c > c^*$, it holds

$$\widetilde{\Omega}(t) \le R\widetilde{\Omega}(0)e^{-\overline{\lambda}t}, \quad \text{for all } t \ge 0, \tag{24}$$

with

$$\widetilde{\Omega}(t) = |X(t)|^2 + \|U(t)\|^2. \tag{25}$$

Proof. The proof is similar to that in [5], it is omitted. □

Lemma 3.4. Under the condition (2), there exist class $\mathcal{K}_\infty$ functions α_1, α_2 such that

$$|X(t)| + \|U(t)\| \le \alpha_1(|Z(t)| + \|U(t)\|), \tag{26}$$

$$|Z(t)| + \|U(t)\| \le \alpha_2(|X(t)| + \|U(t)\|). \tag{27}$$

Proof. With the help of (2), $f(\cdot)$ is continuously reversible, there exist class $\mathcal{K}_\infty$ functions $\overline{\alpha}_1$, $\overline{\alpha}_2$ such that

$$|f(Z(t))| \le \overline{\alpha}_1(|Z(t)|), \tag{28}$$

and

$$|f^{-1}(X(t))| \le \overline{\alpha}_2(|X(t)|). \tag{29}$$

So it can be deduced that

$$\begin{aligned} &|X(t)| + \|U(t)\| \\ &= |f(Z(t))| + \|U(t)\| \le \overline{\alpha}_1(|Z(t)|) + \|U(t)\| \le \alpha_1(|Z(t)| + \|U(t)\|) \end{aligned} \tag{30}$$

and

$$\begin{aligned} &|Z(t)| + \|U(t)\| \\ &= |f^{-1}(X(t))| + \|U(t)\| \le \overline{\alpha}_2(|X(t)|) + \|U(t)\| \le \alpha_2(|X(t)| + \|U(t)\|) \end{aligned} \tag{31}$$

where $\alpha_1(s) = \overline{\alpha}_1(s) + s$, $\alpha_2(s) = \overline{\alpha}_2(s) + s$. The proof is completed. □

Proof of the Theorem 3.1. Under the condition (2), it is easy to see that (23) can be described as (14). With (24), (26), (27), we get

$$\begin{aligned} |Z(t)| + \|U(t)\| &\le \alpha_2(|X(t)| + \|U(t)\|) \\ &\le \alpha_2\left(\sqrt{2(|X(t)|^2 + \|U(t)\|^2)}\right) \\ &\le \alpha_2\left(\sqrt{2R(|X(0)|^2 + \|U(0)\|^2)e^{-\overline{\lambda}t}}\right) \\ &\le \alpha_2\left(\sqrt{2R(|X(0)| + \|U(0)\|)^2 e^{-\overline{\lambda}t}}\right) \\ &\le \alpha_2\left(\sqrt{2R}(|X(0)| + \|U(0)\|)e^{\frac{-\overline{\lambda}t}{2}}\right) \\ &\le \alpha_2\left(\sqrt{2R}\alpha_1(|Z(0)| + \|U(0)\|)e^{\frac{-\overline{\lambda}t}{2}}\right) \\ &= \beta(\Gamma(0), t), \quad \text{for all } t \ge 0, \end{aligned} \tag{32}$$

where $\beta(s,t) = \alpha_2(\sqrt{2R}\alpha_1(s)e^{\frac{-\overline{\lambda}t}{2}})$. The proof is completed. □

4. INVERSE OPTIMAL DESIGN FOR LINEARIZABLE NONLINEAR SYSTEM

Theorem 4.1. Consider the closed-loop system (1), (13), (14), there exists a $c^{**} \geq c^*$ such that for all $c > c^{**}$, the control law (13), (14) minimizes the cost functional

$$J = \lim_{t\to\infty}\left(2\beta V(t) + \int_0^t \left(L(\tau) + \frac{\beta a_1 e^{bD}}{c} U^2(\tau)\right)\mathrm{d}\tau\right), \tag{33}$$

where L is a functional of $(Z(t), U(\theta))$, for all $t - D \leq \theta \leq t$, such that

$$L(t) \geq \beta\alpha_3(\Gamma(t)), \tag{34}$$

for arbitrary $\beta > 0, b > 0$, $a_1 = \frac{1}{4}\frac{\lambda_{\max}(PB_1B_1^TP)}{\lambda_{\min}(Q)}$, α_3 is a class $\mathcal{K}_\infty$ function, Γ is given by (16) and V is

$$V(t) = f(Z(t))^T P f(Z(t)) + \frac{a_1}{2}\int_0^D e^{bx} w^2(x,t)\,\mathrm{d}x. \tag{35}$$

Proof. Choose

$$\begin{aligned} L(t) = &-\tfrac{\beta a_1 e^{bD}}{c+1}U_1(t)^2 + 2\beta X^T(t)QX(t) - 4\beta X^T(t)PB_1 w(0,t) \\ &+a_1\beta w^2(0,t) + a_1\beta\textstyle\int_0^D be^{bx}w^2(x,t)\,\mathrm{d}x \end{aligned} \tag{36}$$

where $a_1 = \frac{1}{4}\frac{\lambda_{\max}(PB_1B_1^TP)}{\lambda_{\min}(Q)}$, U_1, w are given by (23), (8), respectively, and b, β are arbitrary positive scalars. Using (8) and (12), after some calculations, we get

$$\frac{|X(t)|^2 + \|u(t)\|^2}{\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}} \leq |X(t)|^2 + \|w(t)\|^2 \leq \max\{\overline{\alpha}_1, 1+\overline{\alpha}_2\}(|X(t)|^2 + \|u(t)\|^2) \tag{37}$$

where

$$\begin{aligned} \overline{\alpha}_1 &= 3\left(1 + |k|^2|B_1|^2\tfrac{e^{2|A|D}-1}{2|A|}\right), \\ \overline{\alpha}_2 &= 3|k|^2\tfrac{e^{2|A|D}-1}{2|A|}, \\ \overline{\beta}_1 &= 3\left(1 + |k|^2|B_1|^2\tfrac{e^{2|A+B_1k|D}-1}{2|A+B_1k|}\right), \\ \overline{\beta}_2 &= 3|k|^2\tfrac{e^{2|A+B_1k|D}-1}{2|A+B_1k|}. \end{aligned} \tag{38}$$

With (17), (37), we have

$$U_1(t)^2 \leq \overline{a}(|X(t)|^2 + \|w(t)\|^2) \tag{39}$$

where $\overline{a} = 2|k|^2e^{2|A|D}(1+|B_1|^2)\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}$. With (36), (39), we get

$$\begin{aligned} L(t) \geq &-\beta\tfrac{\overline{a}a_1e^{bD}}{c+1}(|X(t)|^2 + \|w(t)\|^2) + 2\beta\lambda_{\min}(Q)|X(t)|^2 \\ &-\tfrac{4\beta}{a_1}|X^T(t)PB_1|^2 + a_1b\beta\textstyle\int_0^D e^{bx}w^2(x,t)\,\mathrm{d}x \\ \geq &-\beta\tfrac{\overline{a}a_1e^{bD}}{c+1}(|X(t)|^2 + \|w(t)\|^2) + \beta\lambda_{\min}(Q)|X(t)|^2 + a_1b\beta\textstyle\int_0^D e^{bx}w^2(x,t)\,\mathrm{d}x \\ \geq &\;\beta\left(-\tfrac{\overline{a}a_1e^{bD}}{c+1} + \min\{\lambda_{\min}(\mathrm{Q}), \mathrm{a_1b}\}\right)(|X(t)|^2 + \|w(t)\|^2). \end{aligned} \tag{40}$$

Choose $c > c^{**}$ where c^{**} is such that

$$c^{**} \geq \max\left\{ \frac{2\overline{a}a_1 e^{bD}}{\min\{\lambda_{\min}(Q), a_1 b\}}, \frac{\sqrt{e^{bD}\overline{a}\max\left\{\frac{a_1}{\lambda_{\min}(Q)}, \frac{1}{b}\right\}}}{\sqrt{1-\mu}} \right\}, \tag{41}$$

for some $0 < \mu < 1$. By (37) and (41), we get from (40) that

$$\begin{aligned} L(t) &\geq \tfrac{\beta}{2}\min\{\lambda_{\min}(Q), a_1 b\}(|X(t)|^2 + \|w(t)\|^2) \\ &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1 b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}(|X(t)|^2 + \|u(t)\|^2) \\ &= \tfrac{\beta\min\{\lambda_{\min}(Q), a_1 b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}(|X(t)|^2 + \|U(t)\|^2). \end{aligned} \tag{42}$$

With (26), one has

$$|X(t)| + \|U(t)\| \geq \alpha_1^{-1}(|Z(t)| + \|U(t)\|). \tag{43}$$

By (42), (43), we get

$$\begin{aligned} L(t) &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1 b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}(|X(t)|^2 + \|U(t)\|^2) \\ &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1 b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}} \tfrac{(|X(t)| + \|U(t)\|)^2}{2} \\ &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1 b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}} \tfrac{(\alpha_1^{-1}(|Z(t)| + \|U(t)\|)^2}{2}. \end{aligned} \tag{44}$$

So (34) is obtained with $\alpha_3(s) = \frac{\min\{\lambda_{\min}(Q), a_1 b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}} \frac{(\alpha_1^{-1}(s))^2}{2}$.

With the help of (35), (36), (13), and the fact that $U^*(t) = \frac{c}{c+1}U_1(t)$, we have

$$\begin{aligned} L(t) &= -\tfrac{\beta c a_1 e^{bD}}{(c+1)^2}U_1(t)^2 + \beta a_1 e^{bD}\left(w(D,t)^2 - \tfrac{U_1(t)^2}{(c+1)^2}\right) - 2\beta\dot{V}(t) \\ &= -\tfrac{\beta c a_1 e^{bD}}{(c+1)^2}U_1(t)^2 + \beta a_1 e^{bD}\left((U(t) - U_1(t))^2 - \tfrac{U_1(t)^2}{(c+1)^2}\right) - 2\beta\dot{V}(t) \\ &= \tfrac{\beta a_1 e^{bD}}{c}(U^*(t))^2 + \beta a_1 e^{bD}\left((U(t) - U^*(t))^2 - \tfrac{2U(t)U^*(t)}{c}\right) - 2\beta\dot{V}(t). \end{aligned} \tag{45}$$

It can be deduced that

$$\begin{aligned} &\textstyle\int_0^t (L(\tau) + \frac{\beta a_1 e^{bD}}{c}U^2(\tau))\,\mathrm{d}\tau \\ &= -2\beta V(t) + 2\beta V(0) + \textstyle\int_0^t \beta a_1 e^{bD}\left(1 + \frac{1}{c}\right)(U(\tau) - U^*(\tau))^2\,\mathrm{d}\tau. \end{aligned} \tag{46}$$

We get

$$J = 2\beta V(0) + \int_0^\infty \beta a_1 e^{bD}\left(1 + \tfrac{1}{c}\right)(U(\tau) - U^*(\tau))^2\,\mathrm{d}\tau. \tag{47}$$

So the minimum of (47) is reached with

$$U(t) = U^*(t) \tag{48}$$

such that

$$J = 2\beta V(0). \tag{49}$$

The proof is completed. □

5. EXAMPLE

Consider the following mechanical system given by [20] as follows:

$$\dot{Z}_1(t) = Z_2(t) \tag{50}$$

$$\dot{Z}_2(t) = U(t-D) \tag{51}$$

$$\dot{Z}_3(t) = Z_4(t) \tag{52}$$

$$\dot{Z}_4(t) = Z_1(t)(1+Z_1(t)) + \frac{Z_1(t)Z_4(t)}{1+Z_1(t)} \tag{53}$$

where $Z_1, Z_2, Z_3, Z_4 \in R$ with $Z_1 > -1$ are the states and $U \in R$ is the input signal delayed by D units of time. Denote $X(t) = [X_1(t), X_2(t), X_3(t), X_4(t)]^T$, $Z(t) = [Z_1(t), Z_2(t), Z_3(t), Z_4(t)]^T$, with a change of variable

$$X(t) = f(Z(t)) = \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) - \frac{1}{2}(\frac{Z_4(t)}{1+Z_1(t)})^2 \\ \frac{Z_4(t)}{1+Z_1(t)} \end{bmatrix} \tag{54}$$

the plant (50)–(53) can be converted into

$$\dot{X}_1(t) = X_2(t) \tag{55}$$

$$\dot{X}_2(t) = U(t-D) \tag{56}$$

$$\dot{X}_3(t) = X_4(t) \tag{57}$$

$$\dot{X}_4(t) = X_1(t). \tag{58}$$

Denote

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \tag{59}$$

we have

$$e^{At} = \begin{bmatrix} 1 & t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{t^2}{2} & \frac{t^3}{6} & 1 & t \\ t & \frac{t^2}{2} & 0 & 1 \end{bmatrix}. \tag{60}$$

By (54), we have

$$Z(t) = f^{-1}(X(t)) = \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) + \frac{1}{2}X_3(t)^2 \\ X_4(t)(1+X_1(t)) \end{bmatrix}. \tag{61}$$

Choosing

$$k = [-9,\ -5,\ -2,\ -7] \tag{62}$$

then $A + B_1 k$ is a hurwitz matrix. Using Theorem 3.1, the control law

$$U(t) = \frac{c}{c+1} U_1(t) = U^*(t) \tag{63}$$

where $c > 0$ is sufficiently large, and

$$U_1(t) = k e^{AD} f(Z(t)) + k \int_{t-D}^{t} e^{A(t-\theta)} B_1 U(\theta)\, \mathrm{d}\theta \tag{64}$$

asymptotically stabilizes system (50)–(53). Solving the matrix equation

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -9 & -5 & -2 & -7 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}^T P + P \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9 & -5 & -2 & -7 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{65}$$

we get

$$P = \begin{bmatrix} 3.5833 & 0.4722 & 1.6389 & 3.7500 \\ 0.4722 & 0.1944 & 0.2500 & 0.5278 \\ 1.6389 & 0.2500 & 2.8056 & 3.1944 \\ 3.7500 & 0.5278 & 3.1944 & 6.4167] \end{bmatrix}. \tag{66}$$

By calculating, one has $a_1 = 0.1505$, and

$$\begin{aligned} V(t) = {} & \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) - \frac{1}{2}(\frac{Z_4(t)}{1+Z_1(t)})^2 \\ \frac{Z_4(t)}{1+Z_1(t)} \end{bmatrix}^T \\ & \times \begin{bmatrix} 3.5833 & 0.4722 & 1.6389 & 3.7500 \\ 0.4722 & 0.1944 & 0.2500 & 0.5278 \\ 1.6389 & 0.2500 & 2.8056 & 3.1944 \\ 3.7500 & 0.5278 & 3.1944 & 6.4167] \end{bmatrix} \\ & \times \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) - \frac{1}{2}(\frac{Z_4(t)}{1+Z_1(t)})^2 \\ \frac{Z_4(t)}{1+Z_1(t)} \end{bmatrix} + \frac{0.1505}{2} \int_0^D e^x w^2(x,t)\, \mathrm{d}x, \end{aligned} \tag{67}$$

$$\begin{aligned} L(t) = {} & -\frac{0.1505\beta e^D}{c+1} U_1(t)^2 + 2\beta Z_1(t)^2 + 2\beta Z_2(t)^2 + 2\beta \left(Z_3(t) - \frac{1}{2} \left(\frac{Z_4(t)}{1+Z_1(t)} \right)^2 \right)^2 \\ & + 2\beta \left(\frac{Z_4(t)}{1+Z_1(t)} \right)^2 - 4\beta \left(0.4722 Z_1(t) + 0.1944 Z_2(t) + 0.25 \left(Z_3(t) - \frac{1}{2} (\frac{Z_4(t)}{1+Z_1(t)})^2 \right) \right. \\ & \left. + 0.5278 \frac{Z_4(t)}{1+Z_1(t)} \right) w(0,t) + 0.1505\beta w^2(0,t) + 0.1505\beta \int_0^D e^x w^2(x,t)\, \mathrm{d}x. \end{aligned} \tag{68}$$

By Theorem 4.1, the control law (63), (64) minimizes the cost functional

$$J = \lim_{t\to\infty}\left(2\beta V(t) + \int_0^t \left(L(\tau) + \frac{0.1505\beta e^D}{c}U^2(\tau)\right)\,\mathrm{d}\tau\right) \tag{69}$$

for arbitrary $\beta > 0$ and $c > 0$ is sufficiently large and $V(t)$, $L(t)$ are given by (67), (68), respectively.

Responses of the states and the control law of the closed-loop system (50)–(53), (63), (64) are shown for $D = 3, c = 100$ in Fig.1. One can observe that the closed-loop system is asymptotically stable and the control law (63), (64) is inverse optimal with respect to the cost functional (69).

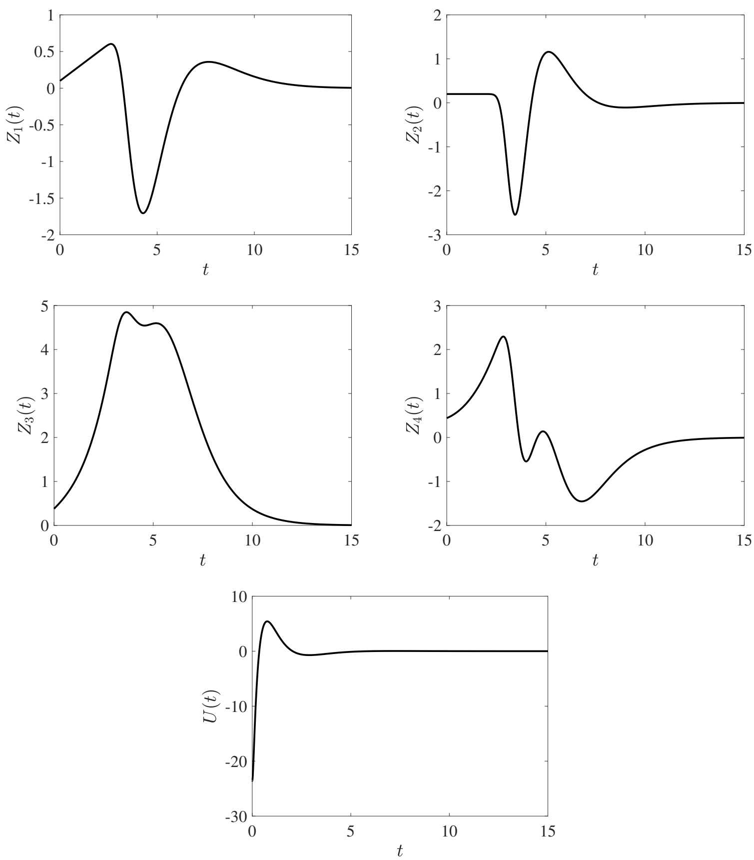

Fig. 1. Responses of the states $Z_1(t), Z_2(t), Z_3(t), Z_4(t)$ and the control law $U(t)$ of the closed-loop system (50)–(53),(63), (64) for initial conditions $Z_1(0) = 0.1, Z_2(0) = 0.2, Z_3(0) = 1.1, Z_4(0) = 0.44$ and $U(\theta) = 0$, for $\theta \in [0, 3]$.

6. CONCLUSIONS

We consider inverse optimal control design for linearizable nonlinear systems with input delays based on predictor control. A nonlinear system is transferred to a linear system with a continuously reversible coordinate change. We show that the basic predictor control is inverse optimal with respect to a meaningful differential game. A mechanical system is given to illustrate the validity of the proposed method.

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