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DYNAMIC COVERAGE CONTROL DESIGN OF MULTI-AGENT SYSTEMS UNDER ELLIPSE SENSING REGIONS

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This paper studies the dynamic coverage control problem for cooperative region reconnaissance where a group of agents are required to reconnoitre a given region. The main challenge of this problem is that the sensing region of each agent is an ellipse. This modeling results in asymmetric(directed) interactions among agents. First, the region reconnaissance is formulated as a coverage problem, where each point in the given region should be surveyed until a preset level is achieved. Then, a coverage control law is designed that minimizes coverage performance index by finite switches between nominal control laws and perturbation control law. Finally, numerical simulations are provided to indicate the efficiency of the proposed control law.

Keywords: coverage, multi-agent systems, region reconnaissance, ellipse sensing region

Classification: 49J20, 93C05

1. INTRODUCTION

In the past decade, there has been growing interest in reconnaissance problem with mobile sensor platforms in two-dimensional space. In [2], a class of region reconnaissance problem was proposed, which the search area need to be cover repeatedly to maximize the probability of detection of the region. In [17, 20], the target reconnaissance problem was considered. In [9], a problem that each small area in the polygon area needed to be re-covered every time has been considered and a cleaning problem was raised to solve a region reconnaissance problem in [12]. In [6], authors detailed studied of region reconnaissance issues which has been defined as the fact that multiple agents to cover the area, while ensuring that the search interval for all small areas is minimized and a feasible solution was given. However, region reconnaissance problem, in the sense of active exploration of an unknown domain, is an open problem in the field of *Unmanned Aerial Vehicle* (UAV) networks in three-dimensional space.

As an important research direction in cooperative control, cooperative coverage problems of multiple agents have drawn much attention to the researchers in recent years, in which a group of agents cooperatively monitor and achieve some certain tasks in a static or dynamical environment. The coverage control of multiple agents has many practi-

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cal applications in such as search and rescuer, environmental exploration, surveillance, environmental monitor, fire spread control and so on, especially in sensor [23] or robot network areas [4, 14]. Meanwhile, a few of coverage problems have been considered in applications involving UAV and *Autonomous Underwater Vehicle* (AUV). Extensive research has been conducted in the field of area search by UAVs equipped with cameras [1, 3]. Also, the coverage control problem for underwater applications using a fleet of cooperative submarines with vision-based cameras has been studied in [19]. Based on the assumption that the fleet was operating in a plane perpendicular to the direction of gravity (i. e., no vertical motion), a gradient descent control law was designed so that the coverage goal was achieved. Coverage control can be broadly classified as region coverage, boundary coverage and target coverage [10]. Up to now, there are different optimization formulations are proposed for region coverage problem. For example, Voronoi partition was adopted in [5, 8], where the authors introduced the locational cost function as an index to optimize sensor locations. [22] considered the sweep coverage of discrete time multi-robot networks with general topologies and provided a decentralized coverage algorithm, which incorporates two operations: workload partition and sweeping. [18] designed the ant robot, which uses smell traces to mark the covered areas and help navigation. By this method, complete coverage of the region can be achieved even if the environment changes. In addition, a coverage error function was proposed to formulate the dynamic coverage problem in [11], and centralized control laws were designed to minimize the coverage error function. In [16], the authors provided more insight on some technical issues regarding the derivative of the coverage error function introduced in [11], and control laws were proposed to solve the dynamic coverage problem for agents with affine nonlinear dynamics.

However, one common thread in the earlier works on coverage control is the consideration of circular sensing region for each agent. Although this model is suitable for agent carrying sensors such as laser rang finder, it is not realistic for agents equipped with *Charge Coupled Device* (CCD) camera. Generally, the sensing region of the CCD camera is an ellipse [21], which is the instantaneous detection area of the CCD camera. This modeling results in asymmetric(directed) interactions among agents. Meanwhile, the overlapping between the ellipse sensing regions of different agents is not considered.

This article studies the dynamic coverage problem for a group of UAVs in three-dimensional space, which is motivated by region reconnaissance application using CCD camera to reconnoitre a given region until each point in the given region is surveyed to a certain preset level. The main challenge of this problem is that the sensing region of UAV is an ellipse so that the orientation with respect to the yaw axis of UAV is needed to be considered. This work is an extension of our previous work [13] from circular sensing region to ellipse sensing region. The UAVs are assumed to have downwards facing CCD camera with a conical field of view, which creates an ellipse sensing region on the ground. The ellipse sensing region is dependent on the altitude of UAV. UAVs at higher altitude can cover more region but the reconnaissance ability is lower compared to UAVs at lower altitude. In order to avoid overlapping between the ellipse sensing regions of different agents, a novel coverage performance index is addressed and a reconnaissance ability-based sensed region partitioning is given. Then, a coverage control law is designed based on sensed region partitioning to drive the agent network to minimize the coverage performance index and guarantee all the agents remaining within a predefined altitude range. To verify the effectiveness of proposed control law, some simulations are carried out.

The rest of this paper is organized as follows. In Section 2, the region reconnaissance is formulated as a coverage problem. In Section 3, a coverage control law is designed and its convergence is proved . Simulation results are provided to illustrate the performance of the proposed control law in Section 4. Section 5 concludes the paper and describes directions for future work.

2. PROBLEM FORMULATION

In this section, the region reconnaissance is formulated as a dynamic coverage problem. First, a simplified dynamics model of agent is given. Then, the reconnaissance ability of an agent is described by a function with respect to its altitude. Finally, a coverage performance index is constructed.

2.1. Dynamics model of each agent

In this paper, we consider a scenario in which a group of agents cooperatively reconnoitre a given region using their vision sensors. Before our problem formulation, a global coordinate frame $OXYZ$ is given shown in Figure 1.

For simplicity, each agent is considered to be a point mass. We consider a group of n agents, and the $i-$th agent is denoted by A_i. The dynamic model of A_i is described by

$$\begin{cases} \dot{x}_i = u_{i,x}, \\ \dot{y}_i = u_{i,y}, \\ \dot{z}_i = u_{i,z}, \\ \dot{\theta}_i = u_{i,\theta}, \end{cases} \tag{1}$$

where $i \in I_n = \{1, \cdots, n\}$, $X_i = [x_i, y_i, z_i]^{\mathrm{T}}$ is the position vector of agent A_i, θ_i is an orientation with respect to the yaw axis, $u_{i,x}, u_{i,y}$ and $u_{i,z}$ are the control inputs of A_i along the axis of OX, OY and OZ, which are assumed to be bounded, i. e., $\|u_{i,x}\| \leq u_{i,x}^{\max}$,$\|u_{i,y}\| \leq u_{i,y}^{\max}$ and $\|u_{i,z}\| \leq u_{i,z}^{\max}$, $u_{i,\theta}$ is the control input of the orientation θ_i, z_i denotes the altitude of A_i where the altitude is within a constraint. The minimum and maximum altitude of A_i are denoted by $z_{\min}^i$ and $z_{\max}^i$, respectively, i. e., $z_i \in [z_{\min}^i, z_{\max}^i]$, $i \in I_n$. It is assumed that $z_{\min}^i$ and $z_{\max}^i$ are the same for all agents, i. e., $z_{\min}^i = z_{\min}$ and $z_{\max}^i = z_{\max}$. The minimum altitude $z_{\min}$ is assumed $z_{\min} > 0$ to ensure all the agents will fly above ground obstacles, whereas the maximum altitude $z_{\max}$ is determined by the reconnaissance ability of A_i.

2.2. Reconnaissance ability function

Let $q_i = [x_i, y_i]^{\mathrm{T}}$ be the projection position of the center of A_i in the OXY plane. An illustration of cooperative reconnaissance of n agents can be shown in Figure 1, where X_i is the position vector of A_i, Ω denotes the given region under reconnaissance in the plane OXY. $\mathcal{S}_i$ is ellipse sensing region of A_i in the OXY plane, C_i is the centre of the ellipse $\mathcal{S}_i$, θ_i is the orientation with respect to the yaw axis.

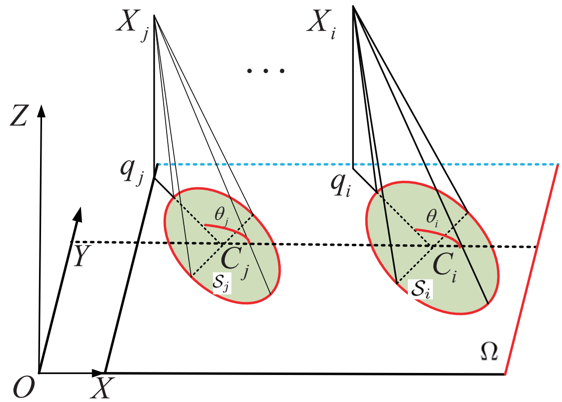


Fig. 1. Reconnaissance coverage concept.

First, a base sensing region of A_i is considered. Without loss of generality, we define a base sensing region as the region surveyed by an agent positioned at $X_{i0} = [x_{i,0}, y_{i0}, z_{i0}]^{\mathrm{T}}$ with an orientation $\theta_i = 0$ and $z_{i0} = z_i$ shown in Figure 2. In Figure 2, $\mathcal{S}_{i0}$ is the base sensing region of A_i, X_{i0} is the position of base agent A_{i0} and q_{i0} is the projection of X_{i0} on the ground, $C_{i0} = C_i$ is the centre of the region $\mathcal{S}_{i0}$, the line $X_{i0}C_{i0}$ is the detection range of the sensing cone of A_{i0}, α is the horizontal line-of-sight angle which is shown as $\angle AX_{i0}B$, γ is the vertical angle of sight which is shown as $\angle CX_{i0}D$, σ the angle of pitch of each CCD camera which is fixed. Let $(x_{C_{i0}}, y_{C_{i0}})$ is the coordinate of centre of the ellipse $\mathcal{S}_{i0}$. The region $\mathcal{S}_{i0}$ can be expressed as (2)

$$\mathcal{S}_{i0}(X_{i0}) = \left\{ x', y' \middle| \frac{(x' - x_{C_{i0}})^2}{a_i^2} + \frac{(y - y_{C_{i0}})^2}{b_i^2} \le 1 \right\} \tag{2}$$

where

$$a_i = \frac{z_i \tan(0.5\alpha)}{\sin(\sigma + 0.5\gamma)}$$

is minor semi-axis,

$$b_i = \frac{1}{2} z_i [\tan(0.5\pi - \sigma) - \tan(0.5\pi - \sigma - \gamma)]$$

is major semi axis.

Then, the sensing region of an agent located at $X_i = [x_i, y_i, z_i]^{\mathrm{T}}$ with an orientation θ_i can be derived by rotating around the base sensing region $\mathcal{S}_{i0}(X_{i0})$, which is given as follows

$$\mathcal{S}_i(X_i, \theta_i) = \{x, y | (x, y)^{\mathrm{T}} = \mathbf{R}(\theta_i)(x', y')^{\mathrm{T}}, (x', y') \in \mathcal{S}_{i0}\}, \tag{3}$$

where $\mathbf{R}(\theta_i)$ is the 2×2 rotation matrix.

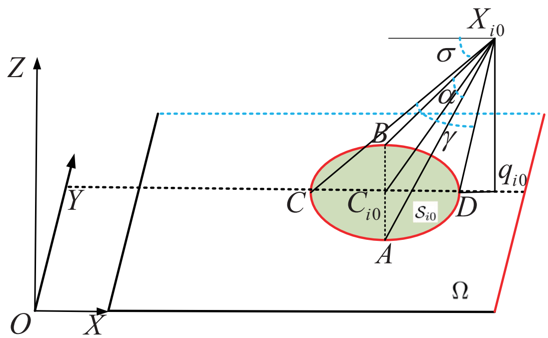


Fig. 2. Base sensing region of A_i.

Last, a function $g(z_i) : [z_{\min}, z_{\max}] \to [0,1]$ is defined to describe reconnaissance ability of A_i, which is dependent on the altitude constraints $z_{\min}$,$z_{\max}$ of A_i. The function $g(z_i)$ is required to have the following properties:

(i) The reconnaissance ability with respect to A_i should be zero outside $\mathcal{S}_i(X_i, \theta_i)$, i. e., $g(z_i) = 0$ if $q \notin \mathcal{S}_i$;

(ii) Due to the decrease of reconnaissance ability while the increase of its altitude, $g(z_i)$ should be a decreasing function of z_i;

(iii) Since the altitude z_i is within a constraint $z_{\min} \le z_i \le z_{\max}$, thus, $g(z_i) = 1$ when $z_i = z_{\min}$ and $g(z_i) = 0$ when $z_i = z_{\max}$;

(iv) $g(z_i)$ is first order differentiable with respect to z_i.

(v) $\frac{\partial g(z_i)}{\partial z_i}$ exists within $\mathcal{S}_i(X_i, \theta_i)$. The integrals over some part of $\partial\mathcal{S}_i(X_i, \alpha)$ are not zero.

The definition of the reconnaissance ability function is not unique. In this paper, the simplest reconnaissance ability function is selected, which is the uniform one, i. e. the reconnaissance ability function is the same for all points in $\mathcal{S}_i(X_i, \theta_i)$. In this paper, we define the function $g(z_i)$ as follows:

$$g(z_i) = \begin{cases} \frac{E_z^2 - (z_i - z_{\min})^2}{E_z^2}, & if\ q \in \mathcal{S}_i; \\ 0, & if\ q \notin \mathcal{S}_i. \end{cases} \tag{4}$$

where $E_z = z_{\max} - z_{\min}$.

The derivative $\frac{\partial g(z_i)}{\partial z_i} : [z_{\min}, z_{\max}] \to [g_d^{\min}, 0]$ is as follows

$$g_d(z_i) \triangleq \frac{\partial g(z_i)}{\partial z_i} = \begin{cases} \frac{-2(z_i - z_{\min})}{E_z^2}, & if\ q \in \mathcal{S}_i; \\ 0, & if\ q \notin \mathcal{S}_i. \end{cases} \tag{5}$$

where $g_d^{\min} = -\frac{2}{E_z}$.

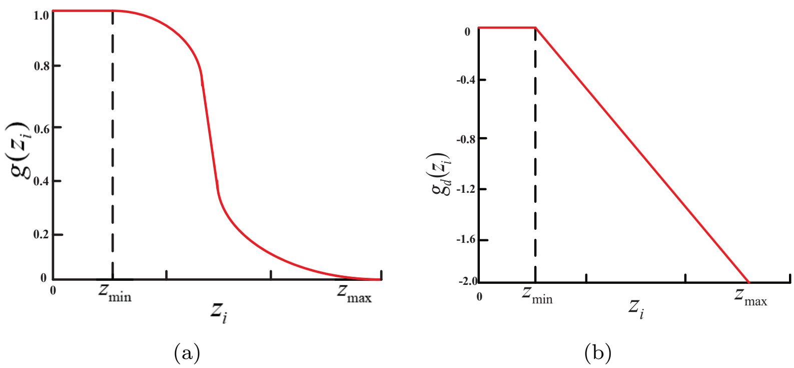


Fig. 3. (a)Reconnaissance ability function, (b)the derivative of reconnaissance ability function.

The function $g(z_i)$ is shown in Figure 3(a) and the derivative of this function can be seen in Figure 3(b). As shown in Figure 3(a), this function has the property that $g(z_{\text{min}}) = 1$ and $g(z_{\text{max}}) = 0$. In addition, $g(z_i)$ is first order differentiable with respect to z_i, or $\frac{\partial g(z_i)}{\partial z_i}$ exists within $\mathcal{S}_i$, which is required in the control law design in Section 3. It should be noted that $g(z_i)$ and $f_d(z_i)$ are continuous functions of z_i.

2.3. Coverage performance index

The aim of the region reconnaissance problem under consideration is to cover a given region using the sensing region of multi-agent such that all the points in the region are surveyed until a preset level is achieved.

Each agent reconnoitres points within the given region Ω and collect useful information using its visual sensor. For a fixed point, the information collected by the visual sensor of the agent increases as the time increases. Thus, the effective coverage achieved by A_i surveying one point in the region Ω from an initial time $t_0 = 0$ to time t is defined as follows

$$\Upsilon_i(q, t) = \int_0^t g(z_i(\tau))\mathrm{d}\tau. \tag{6}$$

From (6), it can be seen that the accumulation of information is proportional to time. When a fixed point within the given region Ω is reconnoitred by multiple agents, in order to avoid overlapping between the ellipse sensing regions of different agents, the agent with the highest reconnaissance capability is selected as the effective reconnaissance agent to reconnoitre the fixed point, which can be described as (7).

$$\Upsilon_{I_n}(q, t) = \int_0^t \max_{i \in I_n} g(z_i(\tau))\, \mathrm{d}\tau. \tag{7}$$

From (7), it can be seen that the accumulation of information also increases with the increase of time.

Let $C^*(q)$ be the desired effective coverage level of each point in region Ω. The definition of effective coverage of all points in the region Ω by n agents is given as follows:

Definition 2.1. We say that the effective coverage of all the points in region Ω can be achieved by the multi-agent networks, if $\Upsilon_{I_n}(q,t) = C^*(q)$ at some time t for $\forall q \in \Omega$.

Let q denote one point in Ω. A space density function $\Phi : \Omega \to \mathbb{R}+$ is assigned to describe the probability that some event takes place over Ω. Then, the coverage performance index for region reconnaissance by multi-agent networks can be described as

$$H(t) = \int_{\Omega} h(C^*(q) - \Upsilon_{I_n}(q,t))\Phi(q)\,\mathrm{d}q, \tag{8}$$

where $h(x)$ is a penalty function that is positive definite, twice differentiable, strictly convex on $(0, C^*(q)]$ and satisfies $h(x) = h'(x) = 0$, for all $x \le 0$. Positive definite and strict convexity here refer to that $h(x) = h'(x) = h''(x) > 0$, for all $x \in (0, C^*(q)]$. This definition for $h(x)$ is chosen to prevent a negative contribution to the coverage performance index at the points $q|\Upsilon_{I_n}(q,t) > C^*(q)$. $H(t)$ can be regard as a measure of how better the coverage provided by the agents network in $\mathbb{R}^3$ is. As $H(t) \to 0$ means that each point in region Ω has been covered effectively. Then $H(t) = 0$, means that the mission is accomplished. Thus we are interested in minimizing $H(t)$ by designing control law of each agent in the next section.

3. CONTROL LAW DESIGN

In this section, a coverage control law is proposed which consists of nominal control laws and a perturbation control law. Under the nominal control laws, A_i can be driven to a condition where all points in its sensing region are fully covered. However, the coverage performance index function $H(t)$ is not driven to zero. Meanwhile, the sensing region of each agent is an ellipse so that the orientation with respect to the yaw axis of an agent is needed to be considered. Hence, the perturbation control law is designed which can drive A_i to a desired angle to guarantee driving the system away from the condition where all points in its sensing region are fully covered.

Before designing the control law, a region partitioning method is developed. Based on the region partitioning method, a coverage control law is designed.

3.1. Sensed region partitioning

In order to avoid overlapping between sensing regions of different agents, a partitioning scheme is given based on the reconnaissance ability. The partitioning scheme is achieved in a manner similar to [15], where only the subset of Ω sensed by all the agents is partitioned. It means that $\cup_i^n \mathcal{S}_i$ is partitioned by (9) and A_i is assigned a cell

$$W_i \triangleq \{q \in \Omega : g(z_i) \ge g(z_j), i \ne j\} \tag{9}$$

with the equality holding true only $z_i = z_j$, so that the cells W_i comprise a complete tessellation of the sensed region $\cup_i^n \mathcal{S}_i$.

First, the general case of $\partial W_j \cap \partial W_i$ is considered. $\partial W_j \cap \partial W_i$ is either an arc of $\partial \mathcal{S}_i$ if $z_i < z_j$ or of $\partial \mathcal{S}_j$ if $z_i > z_j$. An example of partitioning with this case illustrated can be seen in Figure 4, where the black dashed lines denote the major axis of each ellipse sensing region , the boundaries of the sensing regions $\partial \mathcal{S}_i$ are composed of blue dashed lines and solid line with different colours. A_1, A_2 and A_3 illustrate the general case. Then, the case where $z_i = z_j$ is considered. Two situations are included: (i) if ∂W_j and ∂W_i have two intersection points, $\partial W_j \cap \partial W_i$ is chosen arbitrarily as the line segment defined by the two intersection points of $\partial \mathcal{S}_i$ and $\partial \mathcal{S}_j$; (ii) if ∂W_j and ∂W_i have four intersection points, $\partial W_j \cap \partial W_i$ is chosen arbitrarily as the line segment defined by the four intersection points of $\partial \mathcal{S}_i$ and $\partial \mathcal{S}_j$. Hence, the resulting cells consist of circular arcs and line segments; The illustration of this case can be seen in Figure 4, where A_4 and A_5 are at the same altitude and ∂W_4 and ∂W_5 have two intersection points, A_6 and A_7 are at the same altitude and ∂W_6 and ∂W_7 have four intersection points. Last, if the sensing region of A_i is contained within the sensing region of $A_j, j \neq i$, i. e. $\mathcal{S}_i \cap \mathcal{S}_j = \mathcal{S}_i$, then $W_i = \mathcal{S}_i$ and $W_j = \mathcal{S}_j \setminus \mathcal{S}_i$. The illustration of this case can be seen in Figure 4, where the sensing region of A_9 contains the sensing region of A_8.

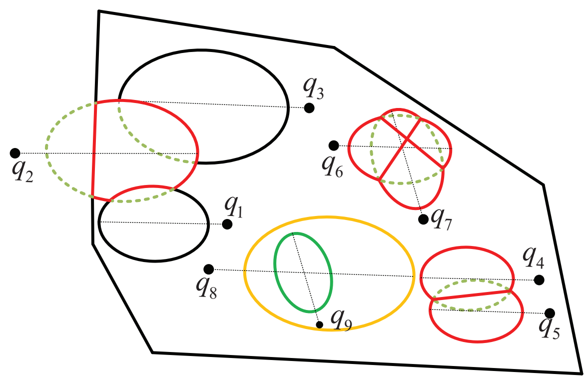


Fig. 4. Region partitioning examples.

Remark 3.1. The aforementioned partitioning is a complete tessellation of the sensed region $\cup_i^n \mathcal{S}_i$. However it is not a complete tessellation of Ω. The resulting cells W_i are compact but they are not always convex. It is also possible that a cell W_i consists of multiple disjoint regions, such as the cell of A_5 shown in red in Figure 4. The given region not assigned by the partitioning scheme is denoted as $\mathcal{O} = \Omega \setminus \cup_i^n \mathcal{S}_i$.

3.2. Nominal control law

Let $u_{i,q} = [u_{i,x}, u_{i,y}]^{\mathrm{T}} \in \mathbb{R}^2$. Then, $u_{i,q}$, $u_{i,z}$ and $u_{i,\theta}$ are the corresponding control inputs for A_i. The coverage control law consists of nominal control laws and a perturbation control law. In this subsection, the nominal control laws of A_i are designed. Let $\bar{u}_{i,q}$, $\bar{u}_{i,z}$ and $\bar{u}_{i,\theta}$ denote the nominal control laws of A_i. Based on the agent's dynamics (1), the reconnaissance ability (4) and the coverage performance index (8), the nominal control laws are presented as follows.

$$\begin{aligned}\bar{u}_{i,q} =& - k_{i,q}\{\int_{\partial W_i \cap \mathcal{O}} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)\Phi(q)\,\mathrm{d}q \\ &+ \sum_{j\neq i}\int_{\partial W_i \cap \partial W_j} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t))(g(z_i) - g(z_j))\Phi(q)\,\mathrm{d}q\end{aligned} \tag{10}$$

$$\begin{aligned}\bar{u}_{i,z} =& - k_{i,z}\int_{\partial W_i \cap \mathcal{O}} \digamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)\Phi(q)\,\mathrm{d}q \\ &- k_{i,z}\sum_{j\neq i}\int_{\partial W_i \cap \partial W_j} \digamma_i^i n_i h(C^*(q) - \Upsilon_{I_n}(q,t))(g(z_i) - g(z_j))\Phi(q)\,\mathrm{d}q \\ &- k_{i,z}\int_{W_i} h'(C^*(q) - \Upsilon_{I_n}(q,t)) g_d\Phi(q)\,\mathrm{d}q\end{aligned} \tag{11}$$

$$\begin{aligned}\bar{u}_{i,\theta} =& - k_{i,\theta}\int_{\partial W_i \cap \mathcal{O}} \urcorner_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)\Phi(q)\,\mathrm{d}q \\ &- k_{i,\theta}\sum_{j\neq i}\int_{\partial W_i \cap \partial W_j} \urcorner_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t))(g(z_i) - g(z_j))\Phi(q)\,\mathrm{d}q\end{aligned} \tag{12}$$

where Γ_i^i, $\digamma_i^i$ and $\urcorner_i^i$ are the Jacobian matrix of the points $q \in \partial W_i$ with respect to q_i, z_i and θ_i defined in Appendix, n_i is the outward pointing normal vector of W_i defined in Appendix, k_{q_i}, k_{z_i} and $k_{i,\theta}$ are positive constants.

Before proceeding, the following condition is introduced.

Condition 3.2. $C^*(q) = \Upsilon_{I_n}(q,t), \forall q \in W_i, i \in I_n$.

This condition describes a coverage condition where all points in the sensing region of A_i are satisfactorily covered. The nominal control laws (10), (11) and (12) result in Condition 3.2 satisfaction. Then the lemma is given as follows.

Lemma 3.3. Consider the agent's dynamics (1) and the reconnaissance ability (4), the nominal control laws (10), (11) and (12) drive each agent converge to Condition 3.2.

P r o o f. Consider the function

$$V(t) = -\dot{H}(t) \tag{13}$$

where

$$\dot{H}(t) = -\int_{\Omega} h'(C^*(q) - \Upsilon_{I_n}(q,t)) \max_{i\in I_n} g(z_i)\Phi(q)\,\mathrm{d}q. \tag{14}$$

In terms of the properties of $g(z_i)$, we have $g(z_i) \neq 0$ if $q \in \mathcal{S}_i$ and $g(z_i) = 0$ if $q \notin \mathcal{S}_i$. Thus, $\dot{H}(t)$ can be rewritten as

$$\dot{H}(t) = -\int_{\cup_i^n \mathcal{S}_i} h'(C^*(q) - \Upsilon_{I_n}(q,t)) \max_{i\in I_n} g(z_i)\Phi(q)\,\mathrm{d}q. \tag{15}$$

By utilizing partitioning scheme (9), the function $V(t)$ can be written as

$$.V(t) = \sum_{i \in I_n} \int_{W_i} h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i) \Phi(q) \, \mathrm{d}q. \tag{16}$$

Then, the time derivative of $V(t)$ can be obtained as follows

$$\begin{aligned} \dot{V}(t) = & - \sum_{i \in I_n} \int_{W_i} h''(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)^2 \Phi(q) \, \mathrm{d}q \\ & + \sum_{i \in I_n} \left[\frac{\partial V(t)}{\partial q_i} \dot{q}_i + \frac{\partial V(t)}{\partial z_i} \dot{z}_i + \frac{\partial V(t)}{\partial \theta_i} \dot{\theta}_i \right]. \end{aligned} \tag{17}$$

By using the Leibniz integral rule [7], we obtain

$$\begin{aligned} \frac{\partial V(t)}{\partial q_i} = & \sum_{i \in I_n} [\int_{\partial W_i} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i) \Phi(q) \, \mathrm{d}q \\ & + \int_{W_i} h'(C^*(q) - \Upsilon_{I_n}(q,t)) \frac{\partial g(z_i)}{\partial q_i} \Phi(q) \, \mathrm{d}q] \end{aligned} \tag{18}$$

where Γ_i^i denotes the Jacobian matrix with respect to q_i of the points $q \in \partial W_i$,

$$\Gamma_i^i = \frac{\partial q}{\partial q_i}, q \in \partial W_i, i \in I_n.$$

According to its constituent components, $\frac{\partial V(t)}{\partial q_i}$ can be rewritten as

$$\begin{aligned} \frac{\partial V(t)}{\partial q_i} = & \int_{\partial W_i} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i) \Phi(q) \, \mathrm{d}q \\ & + \int_{W_i} h'(C^*(q) - \Upsilon_{I_n}(q,t)) \frac{\partial g(z_i)}{\partial q_i} \Phi(q) \, \mathrm{d}q \\ & + \sum_{j \neq i} [\int_{\partial W_j} \Gamma_j^i n_j h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_j) \Phi(q) \, \mathrm{d}q \\ & + \int_{W_j} h'(C^*(q) - \Upsilon_{I_n}(q,t)) \frac{\partial g(z_j)}{\partial q_i} \Phi(q) \, \mathrm{d}q]. \end{aligned} \tag{19}$$

Since $\frac{\partial g(z_j)}{\partial q_i} = \frac{\partial g(z_i)}{\partial q_i} = 0$, we obtain

$$\begin{aligned} \frac{\partial V(t)}{\partial q_i} = & \int_{\partial W_i} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i) \Phi(q) \, \mathrm{d}q \\ & \sum_{j \neq i} \int_{\partial W_j} \Gamma_j^i n_j h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_j) \Phi(q) \, \mathrm{d}q \end{aligned} \tag{20}$$

whose two terms indicate how a movement of A_i affects the boundary of its cell and the boundaries of the cells of other agents. It is clear that only the cells W_j which have a common boundary with W_i will be affected and only at that common boundary.

$$\partial W_i = \{W_i \cap \partial\Omega\} \cup \{\partial W_i \cap \partial\mathcal{O}\} \cup \{\cup_{j\neq i}(\partial W_i \cap \partial W_j)\}. \tag{21}$$

These sets represent the parts of ∂W_i that lie on the boundary of Ω, the boundary of

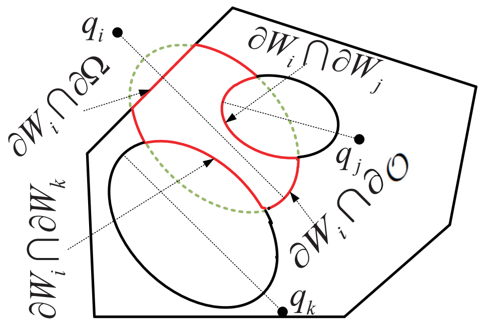


Fig. 5. Boundary decomposition into disjoint sets.

the agent's sensing region and the parts that are between the boundary of the cell of A_i and ∂W_j. For instance, the decomposition of ∂W_i can be seen in Figure 5 with the sets $\partial W_i \cap \partial\Omega$, $\partial W_i \cap \partial\mathcal{O}$ and $\cup_{j\neq i}(\partial W_i \cap \partial W_j)$ appearing in solid red.

For $q \in \partial\Omega$, it holds that $\Gamma_i^i = 0_{2\times 2}$ since the region Ω is static. Additionally, since the common boundary $\partial W_i \cap W_j$, $W_i \cap \partial W_j$ and $\partial W_i \cap \mathcal{O}$ are affected by the movement of A_i, $\frac{\partial V(t)}{\partial q_i}$ can be simplified as

$$\begin{aligned}\frac{\partial V(t)}{\partial q_i} &= \int_{\partial W_i \cap \mathcal{O}} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)\Phi(q)\,\mathrm{d}q \\ &\quad + \sum_{j\neq i}\int_{\partial W_j \cap \partial W_i} \Gamma_j^i n_j h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_j)\Phi(q)\,\mathrm{d}q \\ &\quad + \sum_{j\neq i}\int_{\partial W_i \cap \partial W_j} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)\Phi(q)\,\mathrm{d}q.\end{aligned} \tag{22}$$

Because the boundary $\partial W_i \cap \partial W_j$ is common among A_i and A_j, it holds true that $\Gamma_i^i = \Gamma_j^i$ when evaluated over it. Also, it is true that $n_j = -n_i$ when $q \in \partial W_i \cap \partial W_j$. Finally, the sums and the integrals within them can be combined, producing the final form of the planar control law

$$\begin{aligned}\frac{\partial V(t)}{\partial q_i} &= \int_{\partial W_i \cap \mathcal{O}} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) g(z_i)\Phi(q)\,\mathrm{d}q \\ &\quad + \sum_{j\neq i}\int_{\partial W_j \cap \partial W_i} \Gamma_i^i n_i h'(C^*(q) - \Upsilon_{I_n}(q,t)) (g(z_i) - g(z_j))\Phi(q)\,\mathrm{d}q.\end{aligned} \tag{23}$$

Similarly, $\frac{\partial V(t)}{\partial z_i}$ is given as follows,

$$\begin{aligned}\frac{\partial V(t)}{\partial z_i} = &\sum_{i\in I_n}\Big[\int_{\partial W_i}\digamma_i^i n_i h'(C^*(q)-\Upsilon_i(q,t))\Phi(q)g(z_i)\,\mathrm{d}q \\ &+\int_{W_i}h'(C^*(q)-\Upsilon_i(q,t))\frac{\partial g(z_i)}{\partial z_i}\Phi(q)\,\mathrm{d}q\end{aligned} \tag{24}$$

and (24) can rewritten as

$$\begin{aligned}\frac{\partial V(t)}{\partial z_i} = &\int_{\partial W_i}\digamma_i^i n_i h'(C^*(q)-\Upsilon_{I_n}(q,t))\Phi(q)g(z_i)\,\mathrm{d}q \\ &+\int_{W_i}h'(C^*(q)-\Upsilon_{I_n}(q,t))\frac{\partial g(z_i)}{\partial z_i}\Phi(q)\,\mathrm{d}q \\ &+\sum_{j\neq i}[\int_{\partial W_j}\digamma_j^i n_j h'(C^*-\Upsilon_{I_n}(q,t))\Phi(q)g(z_j)\,\mathrm{d}q \\ &+\int_{W_j}h'(C^*(q)-\Upsilon_{I_n}(q,t))\frac{\partial g(z_j)}{\partial z_i}\Phi(q)\,\mathrm{d}q]\end{aligned} \tag{25}$$

where $\digamma_j^i$ denotes the Jacobian matrix with respect to z_i of the points $q \in \partial W_j$.

Since $\frac{\partial g(z_j)}{\partial z_i} = 0$, we have

$$\begin{aligned}\frac{\partial V(t)}{\partial z_i} = &\int_{\partial W_i}\digamma_i^i n_i h'(C^*(q)-\Upsilon_{I_n}(q,t))\Phi(q)g(z_i)\,\mathrm{d}q \\ &+\int_{W_i}h'(C^*(q)-\Upsilon_{I_n}(q,t))\frac{\partial g(z_i)}{\partial z_i}\Phi(q)\,\mathrm{d}q \\ &+\sum_{j\neq i}\int_{\partial W_j}\digamma_j^i n_j h'(C^*(q)-\Upsilon_{I_n}(q,t))g(z_j)\Phi(q)\,\mathrm{d}q.\end{aligned} \tag{26}$$

It is easy to find that $\digamma_j^i = \digamma_i^i$ if $q \in \partial W_i \cap \partial W_j$. Also, it is true that $n_j = -n_i$ when $q \in \partial W_i \cap \partial W_j$. Then, the altitude control law (26) can be rewritten as

$$\begin{aligned}\frac{\partial V(t)}{\partial z_i} = &\int_{\partial W_i\cap\mathcal{O}}\digamma_i^i n_i h'(C^*(q)-\Upsilon_{I_n}(q,t))g(z_i)\Phi(q)\,\mathrm{d}q \\ &+\sum_{j\neq i}\int_{\partial W_i\cap\partial W_j}\digamma_i^i n_i h(C^*(q)-\Upsilon_{I_n}(q,t))(g(z_i)-g(z_j))\Phi(q)\,\mathrm{d}q \\ &+\int_{W_i}h'(C^*(q)-\Upsilon_{I_n}(q,t))\frac{\partial g(z_i)}{\partial z_i}\Phi(q)\,\mathrm{d}q.\end{aligned} \tag{27}$$

Similarly, using the boundary decomposition (21) and the fact that $\frac{\partial g(z_i)}{\partial \theta_i} = 0$, we obtain

$$\begin{aligned}\frac{\partial V(t)}{\partial \theta_i} = &\int_{\partial W_i\cap\mathcal{O}}\daleth_i^i n_i h'(C^*(q)-\Upsilon_{I_n}(q,t))g(z_i)\Phi(q)\,\mathrm{d}q \\ &+\sum_{j\neq i}\int_{\partial W_i\cap\partial W_j}\daleth_i^i n_i h'(C^*(q)-\Upsilon_{I_n}(q,t))(g(z_i)-g(z_j))\Phi(q)\,\mathrm{d}q.\end{aligned} \tag{28}$$

Compare the forms of the nominal control laws of A_i with (23), (27) and (28) we have

$$\bar{u}_{i,q} = -k_{q_i}\frac{\partial V(t)}{\partial q_i}, \quad \bar{u}_{i,z} = -k_{z_i}\frac{\partial V(t)}{\partial z_i}, \quad \bar{u}_{i,\theta} = -k_{\theta_i}\frac{\partial V(t)}{\partial \theta_i}. \tag{29}$$

Substituting (29) into (17), we can obtain

$$\begin{aligned}\dot{V}(t) = &-\sum_{i\in I_n}\int_{W_i} h''(C^*(q) - \Upsilon_{I_n}(q,t))g(z_i)^2\Phi(q)\,\mathrm{d}q \\ &- k_{q_i}(\frac{\partial V(t)}{\partial q_i})^2 - k_{z_i}(\frac{\partial V(t)}{\partial z_i})^2 - k_{\theta_i}(\frac{\partial V(t)}{\partial \theta_i})^2.\end{aligned} \tag{30}$$

It is clear that $\dot{V}(t) \le 0$. The equality holding true only $C^*(q) = \Upsilon_{I_n}(q,t), \forall q \in W_i, i \in I_n$ and $V(t) \ge 0$ and $V(t) = 0$ if and only if $C^*(q) = \Upsilon_{I_n}(q,t), \forall q \in W_i, i \in I_n$. Thus, Condition 3.2 is achieved under the control laws (10), (11) and (12).

□

Remark 3.4. For the initial locations of A_i, one of the following statements holds:

i) the interior of sensing region of different agents intersects with the boundary of $\partial\Omega$, i. e., $\partial\mathcal{S}_i \cap \partial\Omega \neq \emptyset$, $\forall i \in I_n$; or

ii) the sensing region is inside the given region Ω.

Note that if i) and ii) are violated, the control law (10), (11) and (12) will be zero all the time.

3.3. Perturbation control law

Note that the satisfaction of Condition 3.2 does not necessarily imply the convergence of the coverage performance index $H(t)$ to the neighborhood of zero. When Condition 3.2 holds, the control effort of the nominal control law (10), (11) and (12) remain zero. Thus, a perturbation control law is needed to be designed to guarantee driving the system away from Condition 3.2.

Let t_s be the time at which Condition 3.2 holds and $H(t_s) > 0$. That is, t_s is the time of entering into Condition 3.2 with $H(t_s) \neq 0$. Define the following set:

$$\Omega_q(t_s) = \{q \in \Omega : 0 < \Upsilon_{I_n}(q,t) < C^*\}. \tag{31}$$

Let $\bar{\Omega}_q(t_s)$ be the closure of $\Omega_q(t_s)$. For A_i, let $\Omega^i_\theta(t_s)$ denote the set of angle between $\vec{C_i q}$ and $\vec{OX}$ for all the points in $\bar{\Omega}_q(t_s)$, that is,

$$\Omega^i_\theta(t_s) = \{\hat{\theta}_i : \hat{\theta}_i =< \vec{C_i q}, \vec{OX} >, q \in \bar{\Omega}_q(t_s)\}. \tag{32}$$

Let $\theta_i(t_s)$ denote the angle when Condition 3.2 holds, $\hat{\Omega}^i_\theta(t_s)$ denote the set of angles in $\Omega^i_\theta(t_s)$ that minimize the included angle between $\theta_i(t_s)$ and $\Omega^i_\theta(t_s)$, that is,

$$\hat{\Omega}^i_\theta(t_s) = \{\theta^*_i(t_s) \in \Omega^i_\theta(t_s) : \theta^*_i(t_s) = \mathrm{argmin}_{\hat{\theta}_i \in \hat{\Omega}^i_\theta(t)}\|\theta_i(t_s) - \hat{\theta}_i(t_s)\|\}. \tag{33}$$

Note that the set $\hat{\Omega}^i_\theta(t_s)$ may include two angles. In this case, a rule can be assigned to pick up an angle $\theta_i^*(t_s)$. This rule is that we choose a clockwise angle between the two angles. Consider a simple perturbation control law shown as

$$\hat{u}_{i,\theta}(t) = -\hat{k}_{\theta_i}\big(\theta_i(t) - \theta_i^*(t_s)\big), t > t_s. \tag{34}$$

According to linear systems theory, the feedback control law (34) will result in having $\theta_i^*(t_s)$, for some $i \in I_n$. When Condition 3.2 holds and $H(t) > 0$, this control law will drive A_i towards its associated angle $\theta_i^*(t_s)$.

Remark 3.5. Note that the projection of the centre of an agent is outside of its ellipse sensing region. Thus, if perturbation control law is designed similar to [19] to drive the projection of the centre of an agent to a point in Ω with unsatisfactory coverage level, it may lead to some points in the given region are not effective coverage. Meanwhile, the given region Ω is a bounded region on the OXY plane, it is difficult to obtain the desired point for z_i outside of the OXY plane. To sum up, perturbation control laws for $u_{i,z}$ and $u_{i,q}$ are not designed.

3.4. Overall control strategy

Under the control laws (10), (11) and (12), all agents in the system are in continuous motion as long as the state described in Condition 3.2 is avoided. Whenever Condition 3.2 holds with $H(t) \neq 0$, the system has to be perturbed by switching to (34) that ensures violating Condition 3.2. Once away from Condition 3.2, the controller is switched back to the nominal control in (10), (11) and (12). Only when both Condition 3.2 and $H(t) = 0$ are satisfied, the switch is not needed.

To sum up, we can present the main result.

Theorem 3.6. Consider the agent's dynamics (1) and their reconnaissance ability (4), the control laws

$$u_{i,\theta} = \begin{cases} \bar{u}_{i,\theta}, & \text{if Condition 3.2 does not hold;} \\ \hat{u}_{i,\theta}, & \text{if Condition 3.2 holds;} \end{cases} \tag{35}$$

$$u_{i,z} = \bar{u}_{i,z} \tag{36}$$

$$u_{i,q} = \bar{u}_{i,q} \tag{37}$$

drive the $H(t) \to 0$ as $t \to \infty$.

Proof. Under the nominal control laws, the A_i is driven to the state described in Condition 3.2. Whenever Condition 3.2 holds and the coverage performance index function $H(t)$ is not driven to a neighborhood of zero, A_i is perturbed from Condition 3.2 by switching to the perturbation control law (34). Once away from Condition 3.2, the controller is switched back to the nominal control laws (10), (11) and (12). This procedure is repeated until the set $\Omega_q(t)$ is empty. When the set $\Omega_q(t)$ is empty, via the definition of $\Omega_q(t)$, one has:

$$\lim_{t\to\infty} \Upsilon_{I_n}(q,t) \geq C^*, \forall q \in \Omega. \tag{38}$$

It implies that $\forall q \in \Omega, h(C^*(q) - \Upsilon_{I_n}(q,t)) = 0$. Then, we have $H(t) \to 0$ as $t \to \infty$.

Next, we prove that switching between the nominal control laws and the perturbation control law is finite. Consider the following function

$$H(t) = \int_{\Omega} h(C^*(q) - \Upsilon_{I_n}(q,t))\Phi(q)\,\mathrm{d}q \geq 0 \tag{39}$$

$$\dot{H}(t) = -\int_{\Omega} h'(C^*(q) - \Upsilon_{I_n}(q,t)) \max_{i\in I_n} g(z_i)\Phi(q)\,\mathrm{d}q \leq 0. \tag{40}$$

When the nominal control laws $\bar{u}_{i,q}$ and $\bar{u}_{i,z}$ are employed, $\dot{H}(t) < 0$ always holds since Condition 3.2 is not satisfied. Thus, the function $H(t)$ decreases by an amount of non-zero value during the applications of $\bar{u}_{i,q}$, $\bar{u}_{i,z}$ and $\bar{u}_{i,\theta}$. It implies that a region of measure larger than zero is covered after every single switch (from $\hat{u}_{i,\theta}$ to $\bar{u}_{i,\theta}$) and since the region Ω is compact, there will be no infinite switch. Thus, finite switching will be performed to guarantee that the set $\Omega_q(t)$ is empty. Clearly, if Ω is open or unbounded, there is no guarantee that after each switch the nonzero measure region covered by the formation will eventually cover the entire region. This is the main reason for requiring that Ω be compact. □

Remark 3.7. Note that if Condition 3.2 holds, the control law $\hat{u}_{i,\theta}$ is in effect. Once it converges to θ_i^*, and $H(t) = 0$, then the goal is met and the control converges to $\hat{u}_{i,\theta} = 0$. If $H(t) \neq 0$, the controller switches back to $\bar{u}_{i,q}$, $\bar{u}_{i,z}$ and $\bar{u}_{i,\theta}$. Switching will recur until $H(t) = 0$.

4. NUMERICAL SIMULATIONS

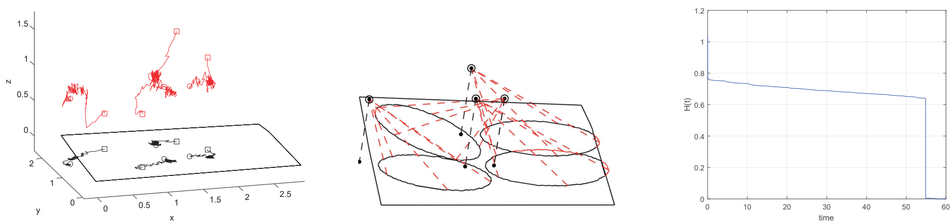
In this section, some simulations are carried out to demonstrate the effectiveness of the proposed control law. In order to show the advantage of the proposed control law, we consider the following two different cooperative region reconnaissance scenarios.

The given region Ω is a square region of side length $d = 1.7$ units length, the desired effective coverage level $C^* = 0.3$, the control gains in the control laws (35),(36) and (37) are set to be $k_{q_i} = 0.8$, $k_{z_i} = 1.2$, $k_{\theta_i} = 1.0$ and $\hat{k}_{q_i} = 0.22$. It is assume that $z_{\min} = 0.4$, $z_{\max} = 2.7$, $\alpha = \frac{1}{9}\pi$, $\gamma = \frac{2}{9}\pi$ and $\sigma = \frac{1}{6}\pi$ for all agents.

First, we need to execute region sensed partitioning method to obtain construction of cell W_i, and then agent begins to execute its control laws to obtain $u_{i,q}$, $u_{i,z}$ and $u_{i,\theta}$. In order to implement the partitioning method and control laws in an algorithmic manner, the region Ω and cells W_i need to be approximated by polygons. In order to calculate the value of the control law, several line integrals have to be calculated numerically as well as one double integral. The line integrals are calculated as sums, each term of which is evaluated on arc ∂W_i , $\partial W_i \bigcap \partial W_j$ or $\partial W_i \bigcap \mathcal{O}$. The double integral is just the area of the corresponding cell W_i and can be calculated simply as the area of the polygonal approximation of that cell. Meanwhile, a simple first order Euler scheme is used to integrate with respect to time.

Case 4.1. Four agents are considered to reconnoitre the given region Ω.

In this example, four agents are considered to reconnoitre the given region Ω. In the example, the space density function $\Phi(q)$ is set equal to unity. The trajectories of four agents and their projections on Ω is shown in Figure 6(a). The initial positions of the agents are marked by squares and their final positions by circles. It can be seen that guarantees all the agents remain within a predefined altitude range [0.2,2.0]. As it can be seen in Figure 6(b), each agent is approximated as a point mass and final network configuration is given. Figure 6(c) shows the $H(t)$ with switching control converge to zero. However, the addition of more agents may result in better convergence rate of coverage performance index. Thus, in the Case 2, seven agents is considered to reconnoitre the given region Ω.



(a) (b) (c)

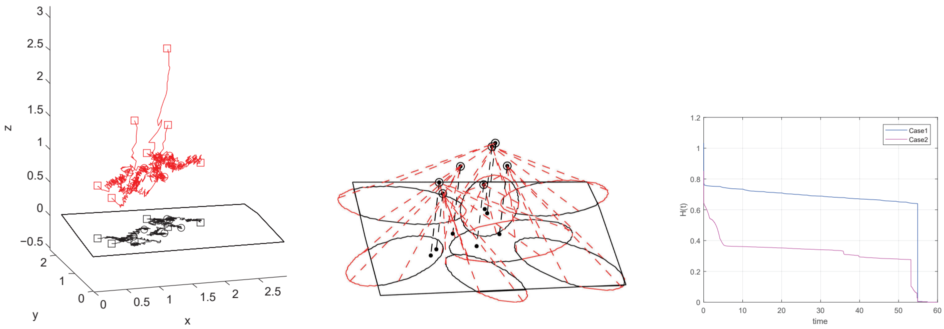
Fig. 6. (a) The trajectories of four agents and their projections on Ω; (b) Final network configuration of four agents and (c) The change of coverage performance index $H(t)$ with respect to time.

Case 4.2. Seven agents are considered to reconnoitre the given region Ω.

In this example, seven agents are considered to reconnoitre the given region Ω. The space density function $\Phi(q)$ is set equal to unity which is identical to the Case 1. The trajectories of seven agents in three-dimensional space and their projections on OXY are shown in Figure 7(a). The initial positions of the agents are marked by squares and their final positions by circles. It can be seen that guarantees all the agents remain within a predefined altitude range [0.2,2.8]. As it can be seen in Figure 7(b), each agent is approximated as a point mass and final network configuration is shown. Figure 7(c) shows the $H(t)$ with switching control converge to zero. Compare the convergence rate of coverage performance index of Case 2 with that of Case 1, the convergence rate of coverage performance index of Case 2 is faster than that of Case 1 shown in Figure 7(c). It is easy to find that the addition of more agents will result in significantly better convergence rate of coverage performance index.

5. CONCLUSIONS

A region reconnaissance problem of multi-agent systems is considered in this manuscript in which a group of agents reconnoitre a given region until each point in the given region is surveyed to a certain preset level. The region reconnaissance of multi-agent systems is



(a) (b) (c)

Fig. 7. (a) The trajectories of seven agents and their projections on Ω; (b) Final network configuration of seven agents and (c) The changes of coverage performance index $H(t)$ with respect to time.

formulated as a coverage problem. The sensing region is described as an ellipse and the reconnaissance ability function of each agent is expressed by a function of the altitude of agent. A novel coverage performance index is addressed to avoid overlapping between sensing regions of different agents. Based on reconnaissance ability-based sensed region partitioning scheme, a coverage control law is proposed which minimizes the coverage performance index and guarantees all the agents remaining within a predefined altitude range. Simulation studies are carried out to validate the efficiency of the proposed control law.

For future investigation, there are still some interesting questions:

i) The region reconnaissance in geometrically complex environment need to be considered, such as the urban areas and mountainous.

ii) Theoretical analysis on the relationship between number of the agents and convergence rate of the coverage performance index is needed.

APPENDIX

The parametric equation of the boundary of $\mathcal{S}_i$ defined in (3) is

$$\begin{aligned}\rho_i(k):\begin{bmatrix} x \\ y \end{bmatrix} &= \mathbf{R}(\theta_i)\begin{bmatrix} x_{i0} - z_i\tan(\frac{\pi}{2}-\sigma-\frac{\gamma}{2}) + a_i\cos k_i \\ y_{i0} + b_i \sin k_i \end{bmatrix} \\ &= \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \mathbf{R}(\theta_i)\begin{bmatrix} -z_i\tan(\frac{\pi}{2}-\sigma-\frac{\gamma}{2}) + a_i\cos k_i \\ b_i\sin k_i \end{bmatrix}\end{aligned} \tag{41}$$

where $\mathbf{R}(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}$ is the rotation matrix, $k_i \in [0, 2\pi], \theta_i \in [0, 2\pi]$.

The outward pointing normal vector n_i is given by

$$n_i = \frac{1}{\sqrt{\frac{x^2}{a_i^4} + \frac{y^2}{b_i^4}}} \mathbf{R}(\theta_i) \begin{bmatrix} \frac{x}{a_i^2} \\ \frac{y}{b_i^2} \end{bmatrix}. \tag{42}$$

It can be shown that

$$\Gamma_i^i = \begin{bmatrix} \frac{\partial x}{\partial x_i} & \frac{\partial x}{\partial y_i} \\ \frac{\partial y}{\partial x_i} & \frac{\partial y}{\partial y_i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbb{I}_{2\times 2} \tag{43}$$

and similarly that

$$F_i^i = \frac{\partial q}{\partial z_i} = \begin{bmatrix} \frac{\partial x}{\partial z_i} \\ \frac{\partial y}{\partial z_i} \end{bmatrix} = \mathbf{R}(\theta_i) \begin{bmatrix} -\tan(\frac{\pi}{2} - \sigma - \frac{\gamma}{2}) + \cos k_i \frac{\tan(0.5\alpha)}{\sin(\sigma + 0.5\gamma)} \\ \sin k_i \frac{1}{2}[\tan(0.5\pi - \sigma) - \tan(0.5\pi - \sigma - \gamma)] \end{bmatrix}, \tag{44}$$

where $q \in \partial W_i$. And

$$\urcorner_i^i = \frac{\partial q}{\partial \theta_i} = \begin{bmatrix} \frac{\partial x}{\partial \theta_i} \\ \frac{\partial y}{\partial \theta_i} \end{bmatrix} = \begin{bmatrix} -\tan(\frac{\pi}{2} - \sigma - \frac{\gamma}{2}) \sin\theta_i + a_i \cos k_i \sin\theta_i + b_i \sin k_i \cos\theta_i \\ \tan(\frac{\pi}{2} - \sigma - \frac{\gamma}{2}) \cos\theta_i - a_i \cos k_i \cos\theta_i + b_i \sin k_i \sin\theta_i \end{bmatrix}, \tag{45}$$

where $q \in \partial W_i$.

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