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STABILITY ANALYSIS OF UNCERTAIN COMPLEX-VARIABLE DELAYED NONLINEAR SYSTEMS VIA INTERMITTENT CONTROL WITH MULTIPLE SWITCHED PERIODS

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In this paper, an intermittent control approach with multiple switched periods is proposed for the robust exponential stabilization of uncertain complex-variable delayed nonlinear systems with parameters perturbation, in which the considered complex systems have bounded parametric uncertainties. Based on the Lyapunov stability theory and comparison theorem of differential equations, some stability criteria are established for a class of uncertain complex delayed nonlinear systems with parameters perturbation. Finally, some numerical simulations are given to show the effectiveness and the benefits of the theoretical results.

Keywords: complex delayed system, uncertain, stabilization, intermittent control, switched

Classification: 34D06, 34D35, 34C15

1. INTRODUCTION

Since Pecora and Carroll ([22]) introduced a method to synchronize two identical chaotic systems with different initial conditions, chaos synchronization has also obtained much attention due to its potential application ([1, 7, 21, 34]) to physics, secure communication, informatics, etc. Many effective control schemes have also been proposed for the control and synchronization of chaotic systems, such as pinning control ([25]),linear separation method ([26]), output feedback ([29]), time delay feedback control ([19]), event-triggered technique ([30]), occasional bang-bang control ([24]), occasional proportional feedback ([3]), impulsive control ([31]), and intermittent control ([36]). In comparison with continuous control of chaos, the discontinuous control scheme, which includes occasional bang-bang control, occasional proportional feedback, impulsive control, and intermittent control, has attracted more interest recently due to its easy implementation in engineering. Among them, the main idea of impulsive control and intermittent control can be applied to most dynamical systems, particularly on sampling-data systems.

Recently, intermittent control has been extensively used in realizing stabilization and synchronization of chaotic systems and complex dynamical networks, for example, see ([2, 8, 9, 10, 11, 12, 13, 23, 27]) and references therein. In this type of control strategy,

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each period usually contains two types of time, one is work time where the controller is activated, and the other one is rest time where the controller is off. When rest time is zero, the intermittent control reduces to continuous control, while work time is zero means that the intermittent control becomes the impulsive control. Obviously, compared with the continuous control methods, intermittent control is more economical and efficient because the system output is measured intermittently rather than continuously. In the traditional periodically intermittent control ([2, 8, 9, 10, 27, 34]), the control period, the control time as well as the control rate was assumed to be constants. This requirement of periodicity in intermittent control is unreasonable, which can unavoidably lead to some conservatism in practical applications. Inspired the non-periodical intermittent control strategy, Liu et al. ([12, 13]) discussed the synchronization problem of complex dynamical networks via aperiodically intermittent pinning control. In ([11]), the authors discussed the pinning synchronization problem in complex dynamical networks with both non-delay and delay couplings by using intermittent control with two switched periods. Compared to intermittent control with one period, the proposed control method is less conservative and more practical. Furthermore, a new control technique for switched complex dynamical networks is proposed in ([23]), where intermittent control with multiple control periods was used to handle the synchronization problem in complex dynamical networks. Obviously, this new method is more flexible, and its application scope is wider.

However, the aforementioned literature concerned with the stabilization and synchronization problem of chaotic systems and complex dynamical networks with real variables. The objective of study in this paper is complex-variable nonlinear system which has been proposed and studied. Since Fowler et al. ([6]) introduced the complex Lorenz equations, the complex Lorenz system was also introduced to describe and simulate rotating fluids and detuned laser ([20]). Mahmoud et al. ([18]) introduced the complex Chen and complex Lü systems and showed their chaotic attractors and the stability properties of their fixed points. A new hyperchaotic complex Lorenz system was generated from the complex Lorenz system and its dynamical properties was also analyzed and studied ([16]). Luo et al. ([15]) constructed a new fractional-order complex Lorenz system and investigated its synchronization. Mahmoud et al. ([17]) constructed a modified time delay complex Lü system and investigated its dynamics, synchronization problem of the modified time delay complex system was also achieved by using the active control. In Refs. ([4, 5, 32, 33, 35]), the stabilization and synchronization problems of complex-variable impulsive system with and without delay was investigated respectively. Furthermore, taking the control cost and practical implementation into account, it is of great importance to study the stability analysis of time delay complex system by using intermittent control. However, to the best of our knowledge, most of existing works are dealing with the problem of synchronization and stabilization in complex-variable delayed systems with continuous control ([15, 16, 17, 18]). There is little analytical results about the stabilization in the form of intermittent control scheme with multiple switched periods.

From these discussions above, this paper devotes to investigate the stabilization of uncertain complex-variable delayed systems by using intermittent control methods with multiple switched periods. The main contributions of our work is given as follows: (1) We overcome the difficulty of uncertain factor with parameters perturbation; (2)

The multiple switched periods control technique is given to achieve the stabilization of complex-variable delayed system; (3) We directly discuss the stability problem of delayed complex-variable systems by constructing a positive definite function $V(t,x) = x^T H\overline{x}$ in the complex fields. Some numerical examples with simulation show the effectiveness of the proposed control scheme.

The rest of this paper is organized as follows. Section 2 describes the uncertain complex-variable delayed nonlinear systems considered in this paper and gives some necessary preliminaries. In section 3, general stability criteria for stabilization are derived analytically. In Section 4, numerical examples are given to illustrate the theoretical results. Finally some conclusions are given in Section 5.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Notations: Throughout this paper, for any complex number (or complex vector) x, the notations x^r and x^i denote its real and imaginary parts, respective, and $\overline{x}$ denotes the complex conjugate of x. The norm of any complex vector x is $||x|| = \sqrt{x^T\overline{x}}$, sup denotes the upper bound. Denote $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ as the maximal and minimal eigenvalues of matrix A respectively. A^T and $\overline{A}$ denote its transpose and its conjugate, respectively. $||A|| = \sqrt{A^T\overline{A}}$. $H \in \mathcal{H}^{n\times n}$ denotes the set of $n \times n$ Hermite matrices. Let $C([-\overline{\tau}, 0], \mathcal{C}^n)$ be a Banach space of continuous with the norm $||\varphi|| = sup_{-\overline{\tau}\leq\sigma\leq 0}||\varphi(\sigma)||$. Denote $[\varphi(t)]_{\overline{\tau}} = ([\varphi_1(t)]_{\overline{\tau}}, [\varphi_2(t)]_{\overline{\tau}}, \dots, [\varphi_n(t)]_{\overline{\tau}})$, $[\varphi_k(t)]_{\overline{\tau}} = sup_{-\overline{\tau}\leq\sigma\leq 0}||\varphi_k(t+\sigma)||$. I is a unit matrix with appropriate dimension.

Consider the following form of the controlled complex-variable delayed dynamical system

$$\begin{cases} \dot{x} = (A + \Delta A(t))x + (B + \Delta B(t))x(t - \tau(t)) \\ \qquad + f(t,x) + g(t, x(t - \tau(t))) + u, \\ x(t_0 + s) = \phi(s) \in C([-\overline{\tau}, 0], C^n), \end{cases} \tag{1}$$

where $x(t) = (x_1, x_2, \dots, x_n)^T \in \mathcal{C}^n$ is a n-dimensional state complex vector with $x_l = x_l^r + jx_l^i, l = 1, 2, \dots, n$ and $j = \sqrt{-1}$, superscripts r and i stand for the real and imaginary parts of the state complex vector x, respectively. $A, B \in \mathcal{C}^{n\times n}$, $\Delta A(t) \in \mathcal{C}^{n\times n}$ and $\Delta B(t) \in \mathcal{C}^{n\times n}$ parameters perturbation matrices bounded by $\Delta A^T(t)\overline{A(t)} \leq \gamma_1^2 I$ and $\Delta B^T(t)\overline{B(t)} \leq \gamma_2^2 I$, respectively. $f(\cdot,\cdot), g(\cdot,\cdot)$:$[0, +\infty) \times \mathcal{C}^n \to \mathcal{C}^n$ are continuous nonlinear function vectors, $\tau(t)$ is continuous functions with $0 \leq \tau(t) \leq \overline{\tau}$ ($\overline{\tau}$ is a constant). $u(t)$ is a linear state feedback controller with intermittent control. The intermittent control scheme using k-switches in a control period is shown in Figure 1 (k is finite integer).

We note that $E_1^{(m)} = [mT, mT+\theta T_1)$ (the control width in switch T_1), $E_2^{(m)} = [mT+\theta T_1, mT+T_1)$ (the non-control width in switch T_1), $E_3^{(m)} = [mT+T_1, mT+T_1+\theta T_2)$, $E_4^{(m)} = [mT+T_1+\theta T_2, mT+mT_1+mT_2)$, $\dots$, $E_{2r-1}^{(m)} = [mT+\widetilde{T_{r-1}}, mT+\widetilde{T_{r-1}}+\theta T_r)$ (the control width in switch T_r), and $E_{2r-1}^{(m)} = [mT+\widetilde{T_{r-1}}+\theta T_r, mT+\widetilde{T_r})$ (the non-control width in switch $\widetilde{T_r}$), where $m = 0, 1, 2, \dots,$ $r = 1, 2, \dots, k,$ $\widetilde{T_r} = T_1 + T_2 + \dots + T_r,$ $T = T_1 + T_2 + \dots + T_k,$ $\widetilde{T_0} = 0,$ $\widetilde{T_1} = T_1,$ $\widetilde{T_k} = T$. As show in Figure 1, $T_1, T_2, \dots, T_k$ are k periods appearing alternatively. $d_1, d_2, \dots, d_k$ are called the control widths in control periods $T_1, T_2, \dots, T_k$, respectively. θ ($0 < \theta < 1$) is called the rate of control

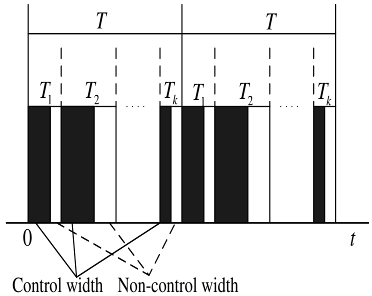

Fig. 1. An intermittent control scheme with k-switched periods.

duration in control periods, and it satisfies $\theta = \frac{d_r}{T_r}$ for $r = 1, 2, \ldots, k$, $(1-\theta)T_r$ is called the non-control widths in control period T_r. For convenience, note that $E = E_1^{(m)} \cup E_3^{(m)} \cup \ldots \cup E_{2r-1}^{(m)}, \widehat{E} = E_2^{(m)} \cup E_4^{(m)} \cup \ldots \cup E_{2r}^{(m)}$. The intermittent control with multiple switched periods are defined as follows

$$u(t) = \begin{cases} Kx, & t \in E, \\ 0, & t \in \widehat{E}. \end{cases} \tag{2}$$

Here, $K \in \mathcal{C}^{n\times n}$ is a control gain matrix.

Remark 2.1. If there is only one or two switched periods, it becomes the simple intermittent control ([2, 8, 9, 10, 27, 34]) or intermittent control with two switched periods ([11]). Obviously, when $\theta = 1$ (rest time is zero), the intermittent control with multiple switched periods (2) is degenerated to a continuous case which has been extensively proposed in the previous work ([15, 16, 17, 18]). While the intermittent control with multiple switched periods (2) turns into the impulsive control scheme ([4, 5, 14, 28, 32, 33, 35]) when $\theta \to 0$ (work time is zero).

Before stating our main results, we give some necessary assumption, definition and lemmas, which are useful in deriving stabilization criteria. Throughout this paper, we always assume that the complex vector-variable functions $f(t, x)$ and $g(t, x(t-\tau(t)))$ satisfy the following Assumption 2.2, that is,

Assumption 2.2. Suppose that there exist two positive constants L_f and L_g such that the complex-variable vector functions $f(t, x)$ and $g(t, x(t-\tau(t)))$ satisfy

$$||f(t,x) - f(t,y)|| \le L_f ||x - y||,$$

$$||g(t, x(t-\tau(t))) - g(t, y(t-\tau(t)))|| \le L_g ||x(t-\tau(t)) - y(t-\tau(t))||.$$

Lemma 2.3. (Zheng [33]) For all $X \in \mathcal{C}^n$ and $H \in \mathcal{H}^{n\times n}$, the following inequality holds:

$$\lambda_{\min}(H) X^T \overline{X} \le X^T H \overline{X} \le \lambda_{\max}(H) X^T \overline{X}.$$

Lemma 2.4. (Zheng [33]) For any $X, Y \in \mathcal{C}^n$ and constant $\zeta > 0$ if $H \in \mathcal{H}^{n\times n}$ is a positive definite matrix, then

$$X^T H\overline{Y} + Y^T H\overline{X} \leq \zeta X^T H\overline{X} + \zeta^{-1} Y^T H\overline{Y}.$$

Remark 2.5. From Lemma 2.4 , we can easily obtain that for any two complex numbers α, β and any real constant η the inequality $\alpha\overline{\beta} + \overline{\alpha}\beta \leq \eta\alpha\overline{\alpha} + \eta^{-1}\beta\overline{\beta}$ holds.

Lemma 2.6. (Li et al. [9]) Let $x : [t_0 - \overline{\tau}, +\infty) \to [0, +\infty)$ be a continuous function such that $\dot{x}(t) \leq -ax(t) + b \max x_t$ is satisfied for $\forall t \geq t_0$. If $a > b > 0$ then $x(t) \leq \max x_{t_0} \exp(-\gamma(t-t_0)), t \geq 0$, where $\max x_t = \sup_{t-\overline{\tau}\leq\xi\leq t} x(\xi)$ and $\gamma > 0$ is the smallest real root of equation $a - \gamma - b\exp(\gamma\overline{\tau}) = 0$.

Lemma 2.7. (Li et al. [9]) Let $x : [t_0 - \overline{\tau}, +\infty) \to [0, +\infty)$ be a continuous function such that $\dot{x}(t) \leq ax(t) + b \max x_t$ is satisfied for $\forall t \geq t_0$. If $a > 0, b > 0$ then $x(t) \leq \max x_t \leq \max x_{t_0} \exp((a + b)(t - t_0)), t \geq 0$, where $\max x_t = \sup_{t-\overline{\tau}\leq\xi\leq t} x(\xi)$.

Lemma 2.8. (Yang and Xu [28]) Let $0 \leq \tau(t) \leq \overline{\tau}, F(t, \mu, \overline{\mu}) : \mathcal{R}^+ \times \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be non-decreasing in $\overline{\mu}$ for each fixed $(t, \mu, \overline{\mu})$ and $I_k(\mu) : \mathcal{R} \to \mathcal{R}$ be non-decreasing in μ. Suppose that $\mu(t)$ and $\upsilon \in C([-\overline{\tau}, 0], \mathcal{R}^n)$ satisfy

$$\begin{cases} D^+\mu(t) \leq F(t, \mu, \overline{\mu}), & t \geq 0, \\ \mu(t_k) \leq I_k(\mu(t_k)), & k \in N, \end{cases}$$

$$\begin{cases} D^+\upsilon(t) \leq F(t, \upsilon, \overline{\upsilon}), & t \geq 0, \\ \upsilon(t_k) \leq I_k(\upsilon(t_k)), & k \in N, \end{cases}$$

where the right and upper Dini's derivative $D^+\mu(t)$ is defined as $D^+\mu(t) = \overline{\lim}_{h\to 0^+} \frac{\mu(t+h)-\mu(t)}{h}$, where $h \to 0^+$ means that h approaches zero from the right-hand side. Then $\mu(t) \leq \upsilon(t)$ for $-\tau \leq t \leq 0$ implies that $\mu(t) \leq \upsilon(t)$ for $t \geq 0$.

Definition 2.9. If there exist $\varepsilon > 0$ and $M > 0$ such that for any $t \geq 0$, $||x(t)|| \leq M \sup_{-\overline{\tau}\leq\theta\leq 0} ||x(\theta)|| \exp(-\varepsilon t)$, then the system (1) is said to be exponentially stable via intermittent control with multiple switched periods (2).

Definition 2.10. (Lu et al. [14]) (Average Impulsive Interval) The average impulsive interval of the impulsive sequence $\xi = \{t_1, t_2, \ldots\}$ is equal to (N_0, T_a) if there exist positive integer N_0 and positive number T_a, such that $N_\xi \geq \frac{T-t}{T_a} - N_0, \forall T > t \geq 0$, where N_ξ denotes the number of impulsive times of the impulsive sequence ξ on the interval (t, T).

3. MAIN RESULTS

The main purpose of this paper is to establish sufficient conditions under which the global exponential stability is ensured for the system (1). In this section, based on Lyapunov stability theory and intermittent control technique, an intermittent control approach with multiple switched periods is proposed. Then, we have the following results.

Theorem 3.1. Suppose that Assumption 2.2 holds and the following conditions are satisfied

(i) $A^TH + H\overline{A} + 2L_fH + L_g\zeta_1H + \zeta_2H + \zeta_3\gamma_1^2\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}H + \zeta_3^{-1}H - \zeta_4^{-1}H$
$-K\overline{H} - K^TH + g_1H \leq 0,$

(ii) $A^TH + H\overline{A} + 2L_fH + L_g\zeta_1H + \zeta_2H + \zeta_3\gamma_1^2\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}H + \zeta_3^{-1}H - \zeta_4^{-1}H - g_3H \leq 0,$

(iii) $L_g\zeta_1^{-1}H + \zeta_2^{-1}B^TH\overline{B} + \zeta_4\gamma_2^2\frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}H - g_2H \leq 0,$

(iv) $\theta\gamma > (1-\theta)(g_3 + g_2),$

where $g_1 > g_2 > 0$, $g_3 > 0$. $\gamma > 0$ is the smallest real root of equation $g_1 - \gamma - g_2\exp(\gamma\overline{\tau}) = 0$. Then the intermittent controlled complex-variable delayed chaotic system is globally exponentially stable at origin.

Proof. Select the following Lyapunov type function defined as

$$V(t,x) = X^TX. \tag{3}$$

When $t \in E$ then, the derivative of (3) along the trajectories of (1) and Assumption 2.2, we can obtain

$$\begin{aligned}
D^+V(t) &= \dot{x}^TH\overline{x} + x^TH\dot{\overline{x}} \\
&= [(A+\Delta A(t))x + (B+\Delta B(t))x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau(t))) + u]^TH\overline{x} \\
&\quad + x^TH\overline{[(A+\Delta A(t))x + (B+\Delta B(t))x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau(t))) + u]} \\
&= x^T(A^TH + H\overline{A})\overline{x} + x^T(t-\tau(t))B^TH\overline{x} + x^TH\overline{Bx(t-\tau(t))} \\
&\quad + (\Delta A(t)x)^TH\overline{x} + x^TH\overline{(\Delta A(t)x)} \\
&\quad + (\Delta B(t)x(t-\tau(t)))^TH\overline{x} + x^TH\overline{(\Delta B(t)x(t-\tau(t)))} \\
&\quad + g^T(t,x(t-\tau(t)))H\overline{x} + x^TH\overline{g(t,x(t-\tau(t)))} \\
&\quad + f^T(t,x)H\overline{x} + x^TH\overline{f(t,x)} + u^TH\overline{x} + x^TH\overline{u}.
\end{aligned}$$

From Assumption 2.2 and Lemma 2.4, we have

$$f(t,x)^TH\overline{x} + x^TH\overline{f(t,x)} \leq 2L_fx^TH\overline{x}, \tag{4}$$

$$
\begin{aligned}
&g^T(t, x(t-\tau(t)))H\overline{x} + x^T H\overline{g(t, x(t-\tau(t)))}\\
&\qquad \le L_g x(t-\tau(t))^T H\overline{x} + L_g x^T H\overline{x(t-\tau(t))}\\
&\qquad \le L_g\zeta_1 x^T H\overline{x} + L_g\zeta_1^{-1} x(t-\tau(t))^T H\overline{x(t-\tau(t))},
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&x^T(t-\tau)(B^T\overline{x} + x^T H\overline{Bx(t-\tau(t))}\\
&\qquad \le \zeta_2 x^T H\overline{x} + \zeta_2^{-1} x(t-\tau(t))^T B^T H\overline{Bx(t-\tau(t))},
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&(\Delta A(t)x)^T H\overline{x} + x^T H\overline{(\Delta A(t)x)}\\
&\qquad \le \zeta_3(\Delta A(t)x)^T H\overline{(\Delta A(t)x)} + \zeta_3^{-1} x^T H\overline{x}\\
&\qquad \le \zeta_3\gamma_1^2 \tfrac{\lambda_{\max}(H)}{\lambda_{\min}(H)} x^T H\overline{x} + \zeta_3^{-1} x^T H\overline{x},
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&(\Delta B(t)x(t-\tau(t)))^T H\overline{x} + x^T H\overline{(\Delta B(t)x(t-\tau(t)))}\\
&\qquad \le \zeta_4\gamma_2^2 \tfrac{\lambda_{\max}(H)}{\lambda_{\min}(H)} x^T(t-\tau(t))H\overline{x(t-\tau(t))} + \zeta_4^{-1} x^T H\overline{x}.
\end{aligned} \tag{8}
$$

Thus, we obtain

$$
\begin{aligned}
D^+V(t) \le{}& x^T\Big[A^T H + H\overline{A} + 2L_f H + L_g\zeta_1 H + \zeta_2 H + \zeta_3\gamma_1^2 \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)} H\\
&+ \zeta_3^{-1} H + \zeta_4^{-1} H - K\overline{H} - K^T H + g_1 H\Big]\overline{x} - g_1 x^T H\overline{x}\\
&+ x^T(t-\tau(t))\Big[L_g\zeta_1^{-1} H + \zeta_2^{-1} B^T H\overline{B} + \zeta_4\gamma_2^2 \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)} H - g_2 H\Big]\\
&\overline{x(t-\tau(t))} + g_2 x^T(t-\tau(t))H\overline{x(t-\tau(t))}\\
\le{}& -g_1 V(t) + g_2 V(t-\tau(t)).
\end{aligned} \tag{9}
$$

On the other hand, when $t \in \widehat{E}$, similarly, we can derive

$$
\dot{V}(t) \le g_3 V(t) + g_2 V(t-\tau(t)). \tag{10}
$$

Thus, we have

$$
\begin{cases}
\dot{V}(t) \le -g_1 V(t) + g_2 V(t-\tau(t)), & t \in E,\\
\dot{V}(t) \le g_3 V(t) + g_2 V(t-\tau(t)), & t \in \widehat{E}.
\end{cases} \tag{11}
$$

When $mT \le t < mT + \theta T_1$, from (11), we have

$$
\begin{aligned}
\dot{V}(t) &\le -g_1 V(t) + g_2 V(t-\tau(t))\\
&\le -g_1 V(t) + g_2\Big[\max_{t-\overline{\tau}\le\xi\le t} V(\xi)\Big].
\end{aligned}
$$

From lemma 2.6, $V(t)$ satisfies

$$
V(t) \le \Big[\max_{mT-\overline{\tau}\le\xi\le mT} V(\xi)\Big]\exp(-\gamma(t-mT)), \tag{12}
$$

here $\gamma > 0$ is the smallest real root of equation $g_1 - \gamma - g_2\exp(\gamma\tau) = 0$. When $mT + \theta T_1 \le t < mT + T_1$, we have

$$
\begin{aligned}
\dot{V}(t) &\le g_3 V(t) + g_2 V(t-\tau(t))\\
&\le g_3 V(t) + g_2\Big[\max_{t-\overline{\tau}\le\xi\le t} V(\xi)\Big].
\end{aligned}
$$

From lemma 2.7, $V(t)$ satisfies

$$V(t) \le \Big[\max_{mT+\theta T_1-\overline{\tau}\le\xi\le mT+\theta T_1} V(\xi)\Big] \exp\Big[(g_3+g_2)(t-mT-\theta T_1)\Big]. \tag{13}$$

When $mT+T_1 \le t < mT+T_1+\theta T_2$, we have

$$\dot{V}(t) \le -g_1 V(t) + g_2\Big[\max_{t-\overline{\tau}\le\xi\le t} V(\xi)\Big].$$

From lemma 2.6, $V(t)$ satisfies

$$V(t) \le \Big[\max_{mT+T_1-\overline{\tau}\le\xi\le mT+T_1} V(\xi)\Big] \exp[-\gamma(t-mT-T_1)], \tag{14}$$

where $\gamma > 0$ is the smallest real root of equation $g_1-\gamma-g_2\exp(\gamma\overline{\tau}) = 0$. When $mT+T_1+\theta T_2 \le t < mT+T_1+T_2$, we have

$$\dot{V}(t) \le g_3 V(t) + g_2\Big[\max_{t-\overline{\tau}\le\xi\le t} V(\xi)\Big].$$

From lemma 2.7, $V(t)$ satisfies

$$V(t) \le \Big[\max_{mT+T_1+\theta T_2-\overline{\tau}\le\xi\le mT+T_1+\theta T_2} V(\xi)\Big] \exp[(g_3+g_2)(t-mT-T_1-\theta T_2)]. \tag{15}$$

Similarly, we have: when $mT+\widetilde{T_{r-1}} \le t < mT+\widetilde{T_{r-1}}+\theta T_r$,

$$\dot{V}(t) \le -g_1 V(t) + g_2\Big[\max_{t-\overline{\tau}\le\xi\le t} V(\xi)\Big].$$

From lemma 2.6, $V(t)$ satisfies

$$V(t) \le \Big[\max_{mT+\widetilde{T_{r-1}}-\overline{\tau}\le\xi\le mT+\widetilde{T_{r-1}}} V(\xi)\Big] \exp[-\gamma(t-mT-\widetilde{T_{r-1}})], \tag{16}$$

where $\gamma > 0$ is the smallest real root of equation $g_1-\gamma-g_2\exp(\gamma\overline{\tau}) = 0$. When $mT+\widetilde{T_{r-1}}+\theta T_r \le t < mT+\widetilde{T_r}$, we have

$$\dot{V}(t) \le g_3 V(t) + g_2\Big[\max_{t-\overline{\tau}\le\xi\le t} V(\xi)\Big].$$

From lemma 2.7, $V(t)$ satisfies

$$V(t) \le \Big[\max_{mT+\widetilde{T_{r-1}}+\theta T_r-\overline{\tau}\le\xi\le mT+\widetilde{T_{r-1}}+\theta T_r} V(\xi)\Big] \exp\Big[(g_3+g_2)(t-mT-\widetilde{T_{r-1}}-\theta T_r)\Big]. \tag{17}$$

Now, based on above Eqs. (12)−(17), we estimate $V(t)$. When $m = 0$, the following inequalities can be derived. For $t \in [0, \theta T_1)$,

$$V(t) \le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp(-\gamma t). \tag{18}$$

For $t \in [\theta T_1, T_1)$,

$$\begin{aligned} V(t) &\le \Big[\max_{\theta T_1-\overline{\tau}\le\xi\le\theta T_1} V(\xi)\Big] \exp[(g_3+g_2)(t-\theta T_1)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(t-\theta T_1)-\gamma\theta T_1]. \end{aligned} \tag{19}$$

For $t \in [T_1, T_1+\theta T_2)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T_1-\overline{\tau}\le\xi\le T_1} V(\xi)\Big] \exp[-\gamma(t-T_1)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(1-\theta)T_1-\gamma(t-(1-\theta)T_1)]. \end{aligned} \tag{20}$$

For $t \in [T_1+\theta T_2, T_1+T_2)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T_1+\theta T_2-\overline{\tau}\le\xi\le T_1+\theta T_2} V(\xi)\Big] \exp[(g_3+g_2)(t-T_1-\theta T_2)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(t-\theta(T_1+T_2))-\gamma\theta(T_1+T_2)]. \end{aligned} \tag{21}$$

Similarly, we have: for $t \in [\widetilde{T_{r-1}}, \widetilde{T_{r-1}}+\theta T_r)$,

$$\begin{aligned} V(t) &\le \Big[\max_{\widetilde{T_{r-1}}-\overline{\tau}\le\xi\le\widetilde{T_{r-1}}} V(\xi)\Big] \exp[-\gamma(t-\widetilde{T_{r-1}})] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(1-\theta)\widetilde{T_{r-1}}-\gamma(t-(1-\theta)\widetilde{T_{r-1}})]. \end{aligned} \tag{22}$$

For $t \in [\widetilde{T_{r-1}}+\theta T_r, \widetilde{T_r})$,

$$\begin{aligned} V(t) &\le \Big[\max_{\widetilde{T_{r-1}}+\theta T_r-\overline{\tau}\le\xi\le\widetilde{T_{r-1}}+\theta T_r} V(\xi)\Big] \exp[(g_3+g_2)(t-\widetilde{T_{r-1}}-\theta T_r)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(t-\theta\widetilde{T_r})-\gamma\theta\widetilde{T_r}]. \end{aligned} \tag{23}$$

When $m = 1$, the following inequalities can be derived. For $t \in [T, T+\theta T_1)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T-\overline{\tau}\le\xi\le T} V(\xi)\Big] \exp[-\gamma(t-T)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(1-\theta)T-\gamma(t-(1-\theta))T]. \end{aligned} \tag{24}$$

For $t \in [T+\theta T_1, T+T_1)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T+\theta T_1-\overline{\tau}\le\xi\le T+\theta T_1} V(\xi)\Big] \exp[(g_3+g_2)(t-T-\theta T_1)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big] \exp[(g_3+g_2)(t-\theta(T+T_1))-\gamma\theta(T+T_1)]. \end{aligned} \tag{25}$$

For $t \in [T+T_1, T+T_1+\theta T_2)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T+T_1-\overline{\tau}\le\xi\le T+T_1} V(\xi)\Big]\exp[-\gamma(t-T-T_1)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(1-\theta)(T+T_1)-\gamma(t-(1-\theta)(T+T_1))]. \end{aligned} \tag{26}$$

For $t \in [T+T_1+\theta T_2, T+T_1+T_2)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T+T_1+\theta T_2-\overline{\tau}\le\xi\le T+T_1+\theta T_2} V(\xi)\Big]\exp[(g_3+g_2)(t-T-T_1-\theta T_2)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(t-\theta(T+T_1+T_2))-\gamma\theta(T+T_1+T_2)]. \end{aligned} \tag{27}$$

Similarly, we have: for $t \in [T+\widetilde{T_{r-1}}, T+\widetilde{T_{r-1}}+\theta T_r)$,

$$\begin{aligned} V(t) &\le \Big[\max_{T+\widetilde{T_{r-1}}-\overline{\tau}\le\xi\le T+\widetilde{T_{r-1}}} V(\xi)\Big]\exp[-\gamma(t-T-\widetilde{T_{r-1}})] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(1-\theta)(T+\widetilde{T_{r-1}}))-\gamma(t-(1-\theta)(T+\widetilde{T_{r-1}}))]. \end{aligned} \tag{28}$$

For $t \in [T+\widetilde{T_{r-1}}+\theta T_r, T+\widetilde{T_r})$,

$$\begin{aligned} V(t) &\le \Big[\max_{T+\widetilde{T_{r-1}}+\theta T_r-\overline{\tau}\le\xi\le T+\widetilde{T_{r-1}}+\theta T_r} V(\xi)\Big]\exp[(g_3+g_2)(t-T-\widetilde{T_{r-1}}-\theta T_r)] \\ &\le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(t-\theta(T+\widetilde{T_r}))-\gamma\theta(T+\widetilde{T_r})]. \end{aligned} \tag{29}$$

According to mathematical induction, we have the following estimate of $V(t)$ for any integer m. For $t \in [mT, mT+\theta T_1)$,

$$V(t) \le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(1-\theta)mT-\gamma(t-(1-\theta))mT]. \tag{30}$$

Since $mT \le t$, we have

$$V(t) \le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[-(\theta\gamma-(1-\theta)(g_3+g_2))t]. \tag{31}$$

For $t \in [mT+\theta T_1, mT+T_1)$,

$$V(t) \le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(t-\theta(mT+T_1))-\gamma\theta(mT+T_1)]. \tag{32}$$

Since $t \le mT+T_1$, we have

$$V(t) \le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[-(\theta\gamma-(1-\theta)(g_3+g_2))t]. \tag{33}$$

For $t \in [mT+T_1, mT+T_1+\theta T_2)$,

$$V(t) \le \Big[\max_{-\overline{\tau}\le\xi\le 0} V(\xi)\Big]\exp[(g_3+g_2)(1-\theta)(mT+T_1)-\gamma(t-(1-\theta)(mT+T_1))]. \tag{34}$$

Since $mT + T_1 \leq t$,

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[-(\theta\gamma - (1-\theta)(g_3+g_2))t]. \tag{35}$$

For $t \in [mT + T_1 + \theta T_2, mT + T_1 + T_2)$,

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[(g_3+g_2)(t - \theta(mT+T_1+T_2)) - \gamma\theta(mT+T_1+T_2)]. \tag{36}$$

Since $t \leq mT + T_1 + T_2$, we have

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[-(\theta\gamma - (1-\theta)(g_3+g_2))t]. \tag{37}$$

Similarly, we have: for $t \in [mT + \widetilde{T_{r-1}}, mT + \widetilde{T_{r-1}} + \theta T_r)$,

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[(g_3+g_2)(1-\theta)(mT+\widetilde{T_{r-1}}) - \gamma(t-(1-\theta)(mT+\widetilde{T_{r-1}}))]. \tag{38}$$

Since $mT + \widetilde{T_{r-1}} \leq t$, we have

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[-(\theta\gamma - (1-\theta)(g_3+g_2))t]. \tag{39}$$

For $t \in [mT + \widetilde{T_{r-1}} + \theta T_r, mT + \widetilde{T_r})$,

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[(g_3+g_2)(t - \theta(mT+\widetilde{T_r})) - \gamma\theta(mT+\widetilde{T_r})]. \tag{40}$$

Since $t \leq mT + \widetilde{T_r}$, we have

$$V(t) \leq \Big[\max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)\Big] \exp[-(\theta\gamma - (1-\theta)(g_3+g_2))t]. \tag{41}$$

In a conclusion, for any time $t \in [0, +\infty)$, it follows from (30)$-$(41) and Lemma 2.3 that

$$\lambda_{\min}(H)||x||^2 \leq V(t) \leq M\exp(-\varphi t).$$

Thus, we have

$$||x|| \leq \sqrt{\frac{M}{\lambda_{\min}(H)}}\exp(-\frac{\varphi}{2}t) \tag{42}$$

where $M = \max_{-\overline{\tau}\leq\xi\leq 0} V(\xi)$ and $\varphi = \theta\gamma - (1-\theta)(g_3+g_2)$. From condition (iv) and $0 < \theta < 1$, it is easy to obtain that the M and φ are positive constants. Therefore, the stabilization of the complex-variable chaotic delayed system (1) is realized. The proof is completed. □

In simulations, for convenience, we let $H = I, K = kI$ and $\zeta_i = 1 (i = 1, 2, 3, 4)$, so $g_1 = 2k - \lambda_{\max}(A^T + \overline{A}) - 2L_f - L_g - \gamma_1^2 - 3$, $g_2 = L_g + \lambda_{\max}(B^T\overline{B}) + \gamma_2^2$, $g_3 = 2k - g_1$.

Then, from the Theorem 3.1, we have the following corollary.

Corollary 3.2. Given θ and $\overline{\tau}$, if there exist control gain k and positive γ such that $g_1 = \gamma + g_2 \exp(\gamma\overline{\tau})$ and $\theta\gamma \geq (1-\theta)(g_3+g_2)$, then the intermittent controlled complex-variable chaotic delayed system is globally exponentially stable at origin.

Remark 3.3. In ([34]), we discussed the synchronization of the complex-variable delayed chaotic systems with discontinuous coupling by using the simple intermittent control. However, this paper investigates the stabilization of complex-variable delayed systems by employing the intermittent control with multiple switched periods. Moreover, the conditions given by Theorem 3.1 and Corollary 3.2 do not assume that the parameters $\overline{\tau}, \theta$ and T satisfy $\overline{\tau} < \theta T$ and $\overline{\tau} + \theta T < T$, which can be applied to more nonlinear systems.

When the rate of control duration $\theta \to 0$, the intermittent control works only at instants $t_i (i = 1, 2, \ldots)$. Thus, the intermittent control (2) becomes the following impulsive control

$$u(t) = \sum_{i=1}^{\infty} Kx(t)\delta(t - t_i), \tag{43}$$

in which the impulsive instant sequence $\{t_i\}_{i=1}^{\infty}$ satisfies $0 = t_0 < t_1 < \ldots < t_i < \ldots$ and $\lim_{i\to\infty} t_i = +\infty$. $\delta(\cdot)$ is the Dirac function. And the intermittent controlled system (1) is converted into the following impulsively controlled system

$$\begin{cases} \dot{x} = (A + \Delta A(t))x + (B + \Delta B(t))x(t - \tau(t)) \\ \qquad + f(t, x) + g(t, x(t - \tau(t))), & t \neq t_i, \\ \Delta x = x(t_i^+) - x(t_i^-) = Kx, & i = 1, 2, \ldots \end{cases} \tag{44}$$

where $x(t_i^+) = \lim_{t\to t_i^+} x(t)$, $x(t_i^-) = \lim_{t\to t_i^-} x(t)$. Without loss of generality, we assume that $\lim_{t\to t_i^+} x(t) = x(t_i)$, which means that the solution of (44) is right continuous at time t_i.

By employing the concept of average impulsive interval ([14]) as well as the comparison principle, several sufficient conditions given are easy to be obtained.

Theorem 3.4. Suppose that Assumption 2.2 holds. If the following inequalities hold

$$||(I + K)^T H\overline{(I + K)}||^2 \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)} \leq \eta < 1 \tag{45}$$

and

$$\frac{\ln \eta}{T_a} + g_3 + \eta^{-N_0} g_2 < 0, \tag{46}$$

where g_2 and g_3 satisfies the conditions (ii-iv) in Theorem 3.1, then the impulsive controlled uncertain complex-variable chaotic delayed system is globally exponentially stable at origin.

Proof. When $t \in [t_{i-1}, t_i)$, $i \in N$, differentiating $V(t)$ defined in (3) along the solution of (44), we obtain

$$\dot{V}(t) \leq g_3 V(t) + g_2 V(t - \tau(t)). \tag{47}$$

On the other hand, when $t = t_i$, we have

$$\begin{aligned} V(t_i) &= ((I+K)x(t_i^-))^T H\overline{((I+K)x(t_i^-))} \\ &\le ||(I+K)^T H\overline{(I+K)}||^2 \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)} x(t_i^-)^T H\overline{x(t_i^-)} \\ &\le \eta x(t_i^-)^T H\overline{x(t_i^-)}. \end{aligned} \tag{48}$$

For any $\varepsilon > 0$, let $\upsilon(t)$ be an unique solution of the following impulsive delayed system

$$\begin{cases} \dot{\upsilon}(t) = g_3\upsilon(t) + g_2\upsilon(t-\tau(t)) + \varepsilon, & t \neq t_i, \\ \upsilon(t_i) = \eta\upsilon(t_i^-), & i \in N, \\ \upsilon(s) = ||\phi(s)||^2, & -\tau \le s \le 0, \end{cases} \tag{49}$$

where $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T$.

Since $V(s) \le ||\phi(s)||^2$ for $-\overline{\tau} \le s \le 0$, it follows from Lemma 2.8 that $0 \le V(t) \le \upsilon(t)$, for $t \ge 0$.

By the formula for the variation of parameters, one obtains $\upsilon(t)$ from (49) that

$$\upsilon(t) = \omega(t,0)\upsilon(0) + \int_0^t \omega(t,s)(g_2\upsilon(s-\tau(s)) + \varepsilon)\,\mathrm{d}s \tag{50}$$

where $\omega(t,s), 0 \le s \le t$, is Cauchy matrix of the linear system

$$\begin{cases} \varsigma(t) = g_3\varsigma(t), & t \neq t_i, \\ \upsilon(t_i) = \eta\upsilon(t_i^-), & i \in N. \end{cases} \tag{51}$$

According to the representation of the Cauchy matrix, we get the following estimation of $\omega(t,s)$. Since the average impulsive interval of the impulsive sequence $\xi = \{t_1, t_2, \ldots\}$ is equal to (N_0, T_a), we have $N_\xi \ge \frac{t-s}{T_a} - N_0, \forall t > s \ge 0$. Denote $a = -g_3 - \frac{\ln\eta}{T_a}$, note that $0 < \eta < 1$, we obtain

$$\begin{aligned} \omega(t,s) &= e^{g_3(t-s)}\Pi_{s<t_k\le t}\eta \\ &= e^{g_3(t-s)}\eta^{N_\xi(t,s)} \\ &\le e^{(-a-\frac{\ln\eta}{T_a})(t-s)}\eta^{-N_0}\eta^{\frac{t-s}{T_a}} \\ &= \eta^{-N_0}e^{-a(t-s)}, 0 \le s < t. \end{aligned} \tag{52}$$

For simplicity, let $\sigma = \eta^{-N_0}\sup_{-\overline{\tau}\le s\le 0}\{||\phi(s)||^2\}$, from (50) and (52), one has

$$\begin{aligned} \upsilon(t) &\le \eta^{-N_0}e^{-at}\upsilon(0) + \int_0^t e^{-a(t-s)}[b\upsilon(s-\tau(s)) + \eta^{-N_0}\varepsilon]\,\mathrm{d}s \\ &\le \sigma e^{-at} + \int_0^t e^{-a(t-s)}[b\upsilon(s-\tau(s)) + \eta^{-N_0}\varepsilon]\,\mathrm{d}s. \end{aligned} \tag{53}$$

Denote $h(\lambda)=\lambda-a+be^{\lambda\overline{\tau}}, b=\eta^{-N_0}g_2$, from (46), one has $a>0, b>0, a-b>0, h(0)<0, h(+\infty)>0, h'(\lambda)=1+b\overline{\tau}e^{\lambda\overline{\tau}}>0$, then $h(\lambda)=0$ has an unique solution $\lambda>0$. Since $\eta^{-N_0}\varepsilon>0, \lambda>0, a-b>0$ and $\eta^{-1}>1$, we derive that

$$v(t)\le\eta^{-N_0}\sup_{-\overline{\tau}\le s\le 0}v(s)<\sigma e^{-\lambda t}+\frac{\varepsilon}{\eta^{N_0}(a-b)}, -\overline{\tau}\le t\le 0.$$

In the following, we will prove the following inequality holds

$$v(t)<\sigma e^{-\lambda t}+\frac{\varepsilon}{\eta^{N_0}(a-b)}, t\ge 0. \tag{54}$$

If it is not true, there exists a constant $t^*>0$ such that

$$v(t)\ge\sigma e^{-\lambda t}+\frac{\varepsilon}{\eta^{N_0}(a-b)}, t\ge t^*, \tag{55}$$

and

$$v(t)<\sigma e^{-\lambda t}+\frac{\varepsilon}{\eta^{N_0}(a-b)}, t<t^*. \tag{56}$$

From inequalities (53) and (56), we obtain

$$\begin{aligned} v(t^*) &\le \sigma e^{-at^*}+\int_0^{t^*}e^{-a(t^*-s)}\Big[bv(s-\tau(s))+\eta^{-N_0}\varepsilon\Big]\mathrm{d}s \\ &\le \sigma e^{-at^*}+e^{-at^*}\int_0^{t^*}e^{as}\Big[b\sigma e^{-\lambda(s-\tau(s))}+\frac{b\varepsilon}{\eta^{N_0}(a-b)}+\frac{\varepsilon}{\eta^{N_0}}\Big]\mathrm{d}s \\ &\le e^{-at^*}\Big(\sigma+b\sigma e^{\lambda\tau}\int_0^{t^*}e^{(a-\lambda)s}\,\mathrm{d}s+\frac{a\varepsilon}{\eta^{N_0}(a-b)}\int_0^{t^*}e^{as}\,\mathrm{d}s\Big) \\ &= e^{-at^*}\Big[\sigma+\sigma(e^{(a-\lambda)t^*}-1)+\frac{\varepsilon}{\eta^{N_0}(a-b)}(e^{at^*}-1)\Big] \\ &< \sigma e^{-\lambda t^*}+\frac{\varepsilon}{\eta^{N_0}(a-b)} \end{aligned} \tag{57}$$

which contradicts with inequality (55). Consequently, inequality (54) holds. Let $\varepsilon\to 0$, then we have $V(t)\le v(t)\le\sigma e^{-\lambda t}$ for $t\ge 0$. Therefore, the controlled system (54) is exponentially asymptotically stable. The proof is completed. □

Let the impulses gain matrix $K=kI\in\mathcal{R}^{n\times n}$ in Theorem 3.4, the following corollary holds.

Corollary 3.5. Suppose that Assumption 2.2 holds. If there exist a constant k such that the following conditions hold $-2<k<0$ and $\frac{2\ln|1+k|}{T_a}+g_3+(1+k)^{-2N_0}g_2<0$, where g_2 and g_3 satisfies the conditions (ii-iv) in Theorem 3.1, then the impulsive controlled complex-variable chaotic delayed system is globally exponentially stable at origin.

Remark 3.6. From conditions of the Theorem 3.4 and Corollary 3.5, we note that the more uncertain the system parameters, the shorter the average impulsive interval should be designed. If the bounds of parametric uncertainties are unavailable, Theorem 3.4 and Corollary 3.5 still can ensure the stabilization if the length of average impulsive interval is chosen to be small enough. This implies that the proposed scheme has certain degree of robustness.

Remark 3.7. In this paper, an intermittent control approach with multiple switched periods is considered to investigate the stabilization of complex nonlinear systems. The obtained results can be actually generalized to the case of real-variable systems. This point will be further verified by numerical example 4.3.

4. NUMERICAL SIMULATIONS

In this subsection, the illustrative examples are provided to show the effectiveness of the stabilization of delayed complex systems with parameters perturbation. We consider the following delayed complex Chua's circuit system

$$\begin{cases} \dot{x}_1 = 0.5p(x_2 + \overline{x_2}) - p(1+b)x_1 - g(x) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -qx_2(t - \overline{\tau}) \end{cases} \tag{58}$$

where $g(x) = 0.5p(a-b)(|x_1+1| - |x_1-1|)$. When the parameters $p = 2.5, q = 0.4, a = -61/44, b = 3/4, \overline{\tau} = 0.01$, the delayed complex Chua's system exhibits periodic behavior. According to Lemma 2.4, one has $||g(x) - g(y)|| \leq 0.5|p(a-b)|||x-y||$, then $L_f = 5.3409, L_g = 0$. So that Assumption 2.2 holds. Take the parameters perturbation matrices as the following

$$\Delta A(t) = \begin{pmatrix} -0.4\cos t & 0.4\cos t & 0 \\ 0 & 0.5\cos t & 0 \\ 0 & 0 & -0.6\cos t \end{pmatrix}, \ \Delta B(t) = \begin{pmatrix} 0.4\sin t & 0 & 0 \\ 0 & 0.5\sin t & 0 \\ 0 & 0 & 0.6\sin t \end{pmatrix},$$

it is obviously that $\Delta A(t)^T \Delta A(t) \leq 0.41I, \Delta B(t)^T \Delta B(t) \leq 0.36I$.

Example 4.1. Consider the intermittent control scheme. For simplicity, in the numerical simulations, the initial states of the complex-variable delayed system is $(2, 1-5j, 3-4j)$. The intermittent control is

$$u(t) = \begin{cases} kx, & t \in E_1^m \cup E_3^m, \\ 0, & t \in E_2^m \cup E_4^m. \end{cases}$$

Let $\gamma_1^2 = 0.41, \gamma_2^2 = 0.36, k = 10, \theta = 0.6, T = 0.8, T_1 = 0.1, T_2 = 0.3, T_3 = 0.4$. By solving conditions (i)−(iii), we can obtain $g_1 = 5.3987, g_3 = 14.6013, g_2 = 0.52$. According to Corollary 3.2, all conditions are satisfied, so this system is robustly exponentially stable. This is verified by the simulation results shown in Figure 2. There results show that the stabilization has been achieved according to the intermittent control with multiple switched periods.

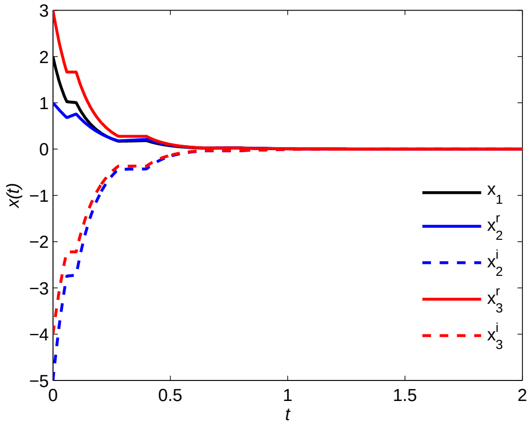

Fig. 2. Trajectories of real and imaginary parts of $x_i(t)$ with $k = 10, \theta = 0.6, T = 0.8, T_1 = 0.1, T_2 = 0.3, T_3 = 0.4$.

Example 4.2. Consider the impulsive scheme. Taking the impulsive signal $\xi = \{t_1, t_2, \ldots\}$ satisfy (52) with average impulsive interval $T_a = 0.02$ and $N_0 = 3$, the impulsive strength

$$K = \begin{pmatrix} -0.5 & 0.5 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{pmatrix}.$$

Then, we have $\eta = 0.6545$, $\frac{\ln \eta}{T_a} + g_3 + \eta^{-N_0} g_2 = -4.7382 < 0$, so all conditions in Theorem 3.4 are satisfied. Figure 3 displays the stabilization impulsively controlled complex system. The results show that the stabilization has been achieved.

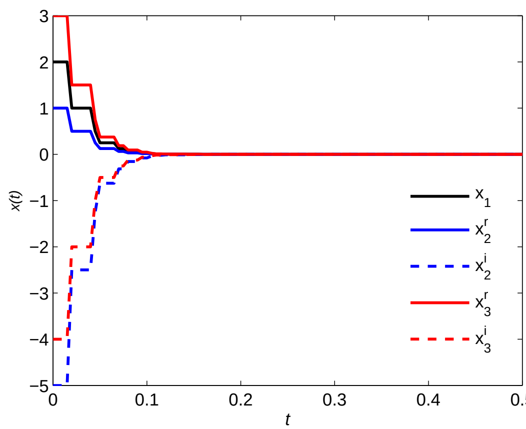

Fig. 3. Trajectories of real and imaginary parts of $x_i(t)$ with impulse.

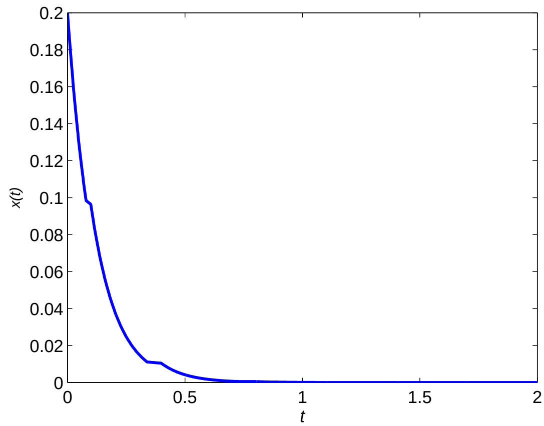

Fig. 4. Trajectories of $x_i(t)$ with
$k = 8, \theta = 0.8, T = 0.8, T_1 = 0.1, T_2 = 0.3, T_3 = 0.4.$

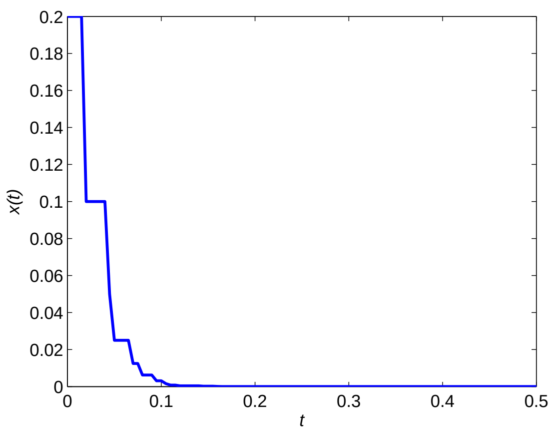

Fig. 5. Trajectories of $x_i(t)$ with impulse.

Example 4.3. Consider the controlled Ikeda oscillator described by the following equation $\dot{x} = -ax + b\sin(x(t - \overline{\tau})) + u(t)$, which exhibits chaotic behavior when the parameters $a = 1, b = 4$ and $\overline{\tau} = 2$. In this example, we can calculate $L_f = 0$ and $L_g = 1$. $u(t)$ is the controller 2 and 43, respectively. For numerical simulation, we select $k = 8, \theta = 0.8, T = 0.8, T_1 = 0.1, T_2 = 0.3, T_3 = 0.4$. According to Theorem 3.1, time delay real-variable chaotic systems can reach stabilization. The state trajectories are shown in Figure 4. According to Theorem 3.4, the other conditions are chosen to be the same as example 4.2, Figure 5 shows the trajectories of Ikeda oscillator under impulsive control. From the numerical simulations, it is easy to find that the obtained results in this paper can be used to research the stabilization of real variable systems.

Remark 4.4. From the numerical results, it is noted that, the controlled complex variable delayed system with parameters perturbation has quickly and perfectly achieved exponential stability. Compared with the existing results ([4, 5, 32]), the time involution of stabilization is much shorter. On the other hand, in the most of articles ([4, 5, 15, 16, 17, 18, 32]), a common approach studying the stability and synchronization of complex-variable systems is to separate them into real parts and imaginary parts, and rewrite them as two equivalent real variable systems, then discuss the synchronization problem by use of the stability criteria of real systems, but it is very lengthy and complicated. For instance, the time delay chaotic complex Lü system ([14]) is to separate it into real parts and imaginary parts, and rewrite it as a five dimensions time delay real system. Then discuss the stability by use of the stability criteria of real system such as Ref. ([9]), but it is very complicated, lengthy and tedious. In this paper, we directly discuss the stability problem of delayed complex-variable systems by constructing a positive definite function $V(t) = x^T H\overline{x}$ in the complex fields. Therefore, Theorems 3.1 and 3.4 generalize some know results in the literatures ([15, 16, 17, 18]).

5. CONCLUSION

In this paper, the stabilization problem of a class of uncertain time delay complex-variable nonlinear systems has been investigated by Lyapunov stability theory and inequality techniques. Two types of dynamic control schemes are proposed to guarantee stabilization, including periodically intermittent control method and impulsive control method. We consider the stabilization of complex-variable delayed nonlinear systems via intermittent control multiple switched periods. Furthermore, when the control width index $\theta \to 0$, the impulsive control scheme as special case is also given. Sufficient conditions for stabilization are obtained based on the stability theory and comparison theorem of differential equations. The theoretical results show that complex-variable delayed systems can achieve stabilization even if complex systems are switched off sometimes. Finally, the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the proposed schemes. Note that, in this paper, we do not think about the information of time-delay, so the Lyapunov function constructed is conservative. In the future, we will take the time-delay influence into account and extend the proposed method to further study the stabilization of T-S fuzzy complex-variable delayed nonlinear systems.

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