ROBUST OPTIMAL PID CONTROLLER DESIGN FOR ATTITUDE STABILIZATION OF FLEXIBLE SPACECRAFT

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This paper presents a novel robust optimal control approach for attitude stabilization of a flexible spacecraft in the presence of external disturbances. An optimal control law is formulated by using concepts of inverse optimal control, proportional-integral-derivative control and a control Lyapunov function. A modified extended state observer is used to compensate for the total disturbances. High-gain and second order sliding mode algorithms are merged to obtain the proposed modified extended state observer. The second method of Lyapunov is used to demonstrate its properties including the convergence rate and ultimate boundedness of the estimation error. The proposed controller can stabilize the attitude control system and minimize a cost functional. Moreover, this controller achieves robustness against bounded external disturbances and the disturbances caused by the elastic vibration of flexible appendages. Numerical simulations are provided to demonstrate the performance of the developed controller.

Keywords: robust optimal control, inverse optimal control, control Lyapunov function, extended state observer, flexible spacecraft

Classification: 93C10, 93C95, 93D15

1. INTRODUCTION

Attitude control methodologies for spacecraft are of prime importance in communication, navigation, remote sensing, and other space-related missions. The problem of optimal attitude control has increased dramatically among researchers (see e. g., [1, 9, 18]). Optimal attitude control problem involves the design of a stabilizing feedback control law which is optimal with respect to a given cost functional. Various nonlinear optimal control techniques have been increasing considered for wide range of control system applications. Optimal and adaptive quaternion feedback [24] has also used for large angle maneuvers of spacecraft. The state dependent Riccati equation (SDRE) technique is a suboptimal control method which is easy to implement [2]. Stansbery and Clourtier [28] developed a position and attitude controller using the SDRE method. The SDRE and a neural network were integrated by Xin and Balakrishnan [29] to develop a robust optimal attitude controller for spacecraft in the presence an uncertain moment of inertia. However, the main drawback of SDRE method is the task of solving the

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Riccati equation repetitively at every integration step. It is a formidable task to use the SDRE method if the system order is higher. Xin and his colleagues [30, 31] have applied the $\theta - D$ technique to produce a suboptimal controller for integrated position and attitude maneuvers.

An inverse optimal control technique is an alternative approach that provides an optimal controller without actually solving the Hamilton-Jacobi-Bellman (HJB) equation [5, 14, 23]. Inverse optimal control approaches were first addressed by Krstic and Tsiotras [15] to design optimal attitude control laws. Park [19] presented an inverse optimal and robust attitude control law for a rigid spacecraft. Luo et al. [17] proposed adaptive H_∞ inverse optimal control method to solve the attitude tracking control problem of spacecraft.

In practical situation, the model parameters of the spacecraft may be uncertain and the spacecraft is always affected by external disturbances. A potential method often used to compensate for uncertainties and disturbances is a control method based on an extended state observer (ESO) [6, 8]. ESO has been widely applied to handle various kinds of engineering control problems such as flight control and chemical process control. In [16] the ESO based disturbance rejection control approach has been used for attitude tracking of a rigid spacecraft. However, this controller did not consider the vibration effect of flexible appendages in the attitude control design.

An optimal sliding mode control (OSMC) is one of effective control methods to design a robust optimal controller. SMC is well known for nonlinear robust control method to deal with a system with modelling uncertainty and an external disturbance problem [32]. SMC has been employed in [3, 4, 11] to design controllers for attitude motion of flexible spacecraft. Pukdeboon and Zinober [21] have used the OSMC technique to solve the optimal control problem for spacecraft attitude tracking system. Later, OSMC has been used in [22] to develop robust optimal attitude controllers for a flexible spacecraft.

However, the optimal controllers mentioned above have some disadvantages. The SDRE controller usually provides only local asymptotic stability whereas the main drawback of the optimal Lyapunov approach is that it is difficult to find a Lyapunov function to satisfy the partial differential equation derived from the Krasovskii theorem.

In this paper, a novel robust optimal control scheme for flexible spacecraft is designed in the presence of external disturbances to achieve robust optimal attitude stabilization. A new inverse optimal controller for the attitude stabilizing systems is constructed based on Sontag-type formula [26, 27], proportional-integral-derivative (PID) control and a control Lyapunov function (CLF) [13, 20]. Then, the total disturbance including the vibration of flexible appendages and external disturbance is estimated by a modified version of the traditional ESO in [7].

In this paper the ultimate boundedness of estimation error of an adapted ESO is guaranteed by using Lyapunov's theorem. The proposed new attitude controller for flexible spacecraft enforces attitude motion, robustness, and optimality with respect to a family of cost functionals and achieves disturbance rejection.

The main contributions of this paper include:

(I) An inverse optimal PID control method for flexible spacecraft attitude regulation maneuvers is proposed for the first time in this paper. A new CLF for the spacecraft motion system is constructed by applying the backstepping method.

(II) A new disturbance observer is designed based on the high gain second-order sliding mode control algorithm. The stability of the proposed compensator is analyzed by the Lyapunov framework.

This paper is organized as follows. Section 2 introduces the basic of the inverse optimality approach which is required for the following discussion. In Section 3 we formulate the dynamics and kinematics of a flexible spacecraft [3, 33]. The problem statement and control objective are also provided. Section 4 proposes an inverse optimal PID control design to achieve the asymptotic convergence of error system states to zero. In Section 5, a modified ESO method is developed to estimate the total disturbance including the vibration of flexible appendages and external disturbances. The ultimate boundedbess of the estimation error is guaranteed using Layapunov's theorem. Numerical results are given in Section 6 to illustrate the performance of the developed control law. In Section 7, we present conclusions.

2. INVERSE OPTIMAL CONTROLLER DESIGN

We consider nonlinear dynamic systems affine in the control variable

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathcal{R}^n$ is a state vector, $u \in \mathcal{R}^m$ denotes the input vector, $f : \mathcal{R}^n \to \mathcal{R}^n$ is a continuous function with $f(0) = 0$, and $g : \mathcal{R}^n \to \mathcal{R}^{n\times m}$ represents a matrix-valued function.

The aim is to demonstrate that a state feedback $u = k(x)$ is optimal with respect to a cost functional

$$I = \int_0^\infty L(x,u)\,\mathrm{d}t = \int_0^\infty (l(x) + u^T R(x)u)\,\mathrm{d}t, \tag{2}$$

where $l(x)$ is a positive semi-definite loss function and $R : \mathcal{R}^n \to \mathcal{R}^{n\times n}$ is a symmetric positive definite weight. The term $L(x,u)$ is required to be determined from the feedback $k(x)$ and a Lyapunov function $V(x)$ associated with the closed-loop system 1.

Let us denote $L_f V(x)$ and $L_g V(x)$ as the Lie derivatives of V along the solution of system (1). It should be noted that the Lie derivative of V with respect to $h : \mathcal{R}^n \to \mathcal{R}^n$ is defined as $L_h V(x) = \frac{\partial V}{\partial x} h$.

A smooth positive definite function $V(x)$ is called as a control Lyapunov function (CLF) of the system (1) if it satisfies the following property

$$L_g V = 0 \Longrightarrow L_f V < 0, \quad \forall x \neq 0, \tag{3}$$

Based on the Sontag's formula [26] and a CLF $V(x)$, a finite horizon optimal controller with integrated cost given in (2) is provided by

$$\kappa^*(x) = -\beta(x)(L_g V)^T \tag{4}$$

with

$$\beta(x) = \begin{cases} c_0 + \dfrac{a(x) + \sqrt{a(x)^2 + (b(x)^T b(x))^2}}{b(x)^T b(x)} & \text{if} \quad \|b(x)\| \neq 0 \\ c_0 & \text{if} \quad \|b(x)\| = 0 \end{cases} \tag{5}$$

where c_0 is a positive constant, $a(x) = L_f V$ and $b(x) = (L_g V)^T$.

Remark 2.1. Based on CLFs, and inverse optimal control method, a stabilizing feedback controller is designed first and then shown to be optimal with respect to a cost functional that imposed penalties on the state and control input.

3. NONLINEAR MODEL OF SPACECRAFT AND PROBLEM FORMULATION

3.1. Kinematic equation

We now briefly explain the use of quaternions for description of the attitude. We define the quaternion $Q = [q_0 \quad q^T]^T \in \mathcal{R} \times \mathcal{R}^3$ with $q = [q_1 \quad q_2 \quad q_3]^T \in \mathcal{R}^3$ The kinematic equation for the attitude can then be expressed as (see, [33])

$$\dot{Q} = \frac{1}{2} \begin{bmatrix} -q^T \\ T(Q) \end{bmatrix} \omega, \tag{6}$$

where $T(Q) = q^\times + q_0 I_3$ with I_3 the 3×3 identity matrix. Note that the scalar q_0 and the three-dimensional vector q must satisfy the condition

$$q^T q + q_0^2 = 1. \tag{7}$$

3.2. Flexible spacecraft dynamics

The equation governing a flexible spacecraft is expressed as [3]

$$\begin{aligned} J\dot{\omega} + \delta^T \ddot{\eta} &= -\omega^\times \left(J\omega + \delta^T \dot{\eta} \right) + u + d & (8) \\ \ddot{\eta} + C\dot{\eta} + K\eta &= -\delta\dot{\omega}, & (9) \end{aligned}$$

where $J =\in \mathcal{R}^{3\times 3}$ denotes the symmetric inertia matrix of the whole spacecraft, $\delta \in \mathcal{R}^{N\times 3}$ is the coupling matrix between the central rigid body and the flexible attachments with N being the mode number, $\eta \in \mathcal{R}^N$ is the modal displacement. $u \in \mathcal{R}^3$ denotes the control torques and $d \in \mathcal{R}^3$ represents the external disturbances. The stiffness K and the damping C are defined as

$$K = \text{diag}\left(\omega_{ni}^2, i = 1, 2, \ldots, N\right) \tag{10}$$

$$C = \text{diag}\left(2\zeta_i \omega_{ni}, i = 1, 2, \ldots, N\right) \tag{11}$$

with damping ζ_i and natural frequency ω_{ni}.

Considering (8), let us define an auxiliary variable

$$\vartheta = \delta\omega + \dot{\eta}. \tag{12}$$

The first time derivative of ϑ is

$$\begin{aligned} \dot{\vartheta} &= \delta\dot{\omega} + \ddot{\eta} \\ &= -C\vartheta + C\delta\omega - K\eta. \end{aligned} \tag{13}$$

Substituting (13) into (8), one can obtain

$$(J - \delta^T \delta)\dot{\omega} = -\omega^\times J\omega + u(t) + \xi(t) + d(t), \tag{14}$$

where $\xi(t)$ represents the total coupling effect term defined by

$$\xi(t) = \delta^T \begin{bmatrix} K & C \end{bmatrix} \begin{bmatrix} \eta \\ \vartheta \end{bmatrix} - \delta^T C\delta\omega - \omega^\times \delta^T (\vartheta - \delta\omega). \tag{15}$$

Now, we have obtain the dynamic equation as

$$(J - \delta^T \delta)\dot{\omega} = -\omega^\times J\omega + u + \bar{d} \tag{16}$$

where $\bar{d} = \xi(t) + d(t)$ denotes the total disturbance including coupling effect term and bounded external disturbance. Letting $J_0 = J - \delta^T \delta$, (16) becomes

$$\dot{\omega} = -J_0^{-1}\omega^\times J\omega + J_0^{-1}u + \tilde{d}(t), \tag{17}$$

where $\tilde{d}(t) = J_0^{-1}\bar{d}(t)$.

To develop our robust optimal control method, the following assumption is required.

Assumption 3.1. The ith component of the total disturbance $\tilde{d}(t)$ in (50) and its first time derivative $\dot{\tilde{d}}(t)$ are unknown but bounded, i. e.

$$\max(|\tilde{d}_i(t)|) \leq \bar{D}_1 \text{ and } \max(|\dot{\tilde{d}}_i(t)|) \leq \bar{D}_2, \; i = 1, 2, 3,$$

where $\bar{D}_1$ and $\bar{D}_2$ are positive constants.

3.3. Problem formulation

The attitude stabilization of a flexible spacecraft is considered. The control objective is to design a robust optimal controller that stabilizes the attitude motion. Thus, it is required to design a state feedback controller that minimizes the cost functional (2) and forces the states of the closed-loop system consisting of equations (6) and (13) to the equilibrium when $t \to \infty$. This implies that

$$\lim_{t\to\infty} q(t) = 0, \quad \lim_{t\to\infty} q_0(t) = 1, \quad \text{and} \quad \lim_{t\to\infty} \omega(t) = 0. \tag{18}$$

4. OPTIMAL PID CONTROLLER DESIGN

This section presents an inverse optimal PID nonlinear controller design for solving the attitude stabilization problem of a flexible spacecraft. The basic concepts of inverse optimal control approach and CLF are employed. In this section, the backstepping technique is used to find a CLF associated with the attitude stabilization system and PID methods are used to solve the optimal control problem.

4.1. Backstepping method

We suppose that ω is the virtual input to the subsystem (6) and defined the stabilizing function such that

$$\alpha = -K_P q - K_I \varepsilon, \tag{19}$$

where K_P and K_I are the symmetric and positive definite matrices. In (19), ε is the integral variable defined as

$$\varepsilon = \int_0^t q(\tau)\,\mathrm{d}\tau. \tag{20}$$

We now define the error between the state ω and the desired control α such that

$$z = \omega - \alpha. \tag{21}$$

Using (21), (6) becomes

$$\dot{Q} = \frac{1}{2}\begin{bmatrix} -q^T \\ T(Q) \end{bmatrix}(z+\alpha). \tag{22}$$

Also, the first time derivative of z is obtained as

$$\begin{aligned} \dot{z} \;&=\; J_0^{-1}\big(-(z+\alpha)^{\times}J(z+\alpha)\big) + \frac{1}{2}K_P T(Q)(z+\alpha) \\ &\quad + K_I q + J_0^{-1}u + J_0^{-1}\bar{d}. \end{aligned} \tag{23}$$

Consider the following candidate Lyapunov function

$$\mathcal{V}_1 = \gamma q^T q + \gamma(q_0-1)^2 + \frac{\gamma}{2}\varepsilon^T K_I \varepsilon. \tag{24}$$

The time derivative of (24) along the trajectories of the closed-loop system becomes

$$\begin{aligned} \dot{\mathcal{V}}_1 \;&=\; \gamma q^T\Big(T(Q)(z+\alpha)\Big) - \gamma q^T(q_0-1)(z+\alpha) + \gamma\varepsilon^T K_I q \\ &=\; -\gamma q^T K_P q + \gamma q^T z \end{aligned} \tag{25}$$

From (25), obviously $q \to 0$ and $q_0 \to 1$ as $t \to \infty$ when $z = 0$.

Remark 4.1. In this paper, using backstepping method, the variable z consists of ω, q and ε. From (6), one can have $\omega = 2T(Q)^{-1}\dot{q}$. This implies that the term ω already presents the derivative term of q, so the term $K_D\dot{q}$ have been omitted in this case. In this case, $\dot{q}$ is implicitly included into the relation.

Next, we first propose an inverse optimal controller for the flexible spacecraft motion equations in the absence of uncertainties and disturbances.

4.2. Inverse optimal PID control

Letting $x = [\varepsilon^T \;\; q^T \;\; z^T]^T$, the flexible spacecraft motion equations (22) and (23) with the integral variable ε can be rewritten as

$$\dot{x} = F(x) + B(x)u + D, \tag{26}$$

where

$$F(x) = \begin{bmatrix} q \\ \frac{1}{2}T(Q)(z+\alpha) \\ \Xi(q_0, q, \alpha, z) \end{bmatrix} \tag{27}$$

$$B(x) = \begin{bmatrix} 0_{3\times 3} \\ 0_{3\times 3} \\ J_0^{-1} \end{bmatrix} \quad \text{and} \quad D = \tilde{d}, \tag{28}$$

where

$$\begin{aligned} \Xi(q_0, q, \alpha, z) \quad &= \quad -J_0^{-1}(z+\alpha)^\times J(z+\alpha) + \frac{1}{2}K_P T(Q)(z+\alpha) \\ &\quad +K_I q. \end{aligned}$$

In the absence of the disturbance D, we now propose the following inverse optimal controller

$$u = \kappa_\gamma(x) = -\gamma\lambda(x)z, \tag{29}$$

with $\lambda(x)$ being a function defined as

$$\lambda(x) = \begin{cases} 1 + \left[\dfrac{\psi + \sqrt{\psi^2 + (z^T z)^2}}{\|z\|^2}\right] z^T, & \|z\| \neq 0 \\ 1, & \|z\| = 0, \end{cases} \tag{30}$$

where

$$\psi(x) = z^T \Big((\gamma I_3 + J_0 K_I)q + \mu z - (z+\alpha)^\times (J(z+\alpha) + \frac{1}{2} J_0 K_P T(Q)(z+\alpha) \Big)$$

with γ and μ begin positive constants. In the following theorem, we show that our chosen Lyapunov function is a CLF for the system (26).

Theorem 4.2. If the following function $\mathcal{V}_2$ is expressed as,

$$\mathcal{V}_2 = \mathcal{V}_1 + \frac{1}{2} z^T J_0 z, \tag{31}$$

then $\mathcal{V}_2$ is a CLF for the system (26).

P r o o f. Since J_0 is symmetric positive definite, we can write $\mathcal{V}_2$ as

$$\mathcal{V}_2 = x^T \Omega x + \gamma(q_0 - 1)^2 \tag{32}$$

$$\Omega = \frac{1}{2}\begin{bmatrix} \gamma K_I & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \gamma I_3 & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & J_0 \end{bmatrix} \tag{33}$$

with $0_{n\times n}$ being the $n \times n$ zero matrix. If $\gamma > 0$ is chosen, then this ensures that $\mathcal{V}_2$ is positive definite.

Computing the time-derivative of $\mathcal{V}_2$ along the trajectories of the closed-loop system in equations (22) and (23), we obtain

$$\begin{aligned} \dot{\mathcal{V}}_2 &= \dot{\mathcal{V}}_1 + z^T J_0 \dot{z} \\ &= -\gamma q^T K_P q + \gamma q^T z + z^T\Big(-(\alpha+z)^{\times} J(\alpha+z) \\ &\quad +\frac{1}{2} J_0 K_P (q_0 I_3 + q^{\times})(z+\alpha) + J_0 K_I q + u\Big) \\ &= -\gamma q^T K_P q - \mu z^T z + z^T\Big((\gamma I_3 + J_0 K_I) q + \mu z + u \\ &\quad -(\alpha+z)^{\times} J(\alpha+z) + \frac{1}{2} J_0 K_P (q_0 I_3 + q^{\times})(z+\alpha)\Big) \\ &= -\vartheta^T \Pi \vartheta + z^T\Big((\gamma I_3 + J_0 K_I) q + \mu z \\ &\quad -(\alpha+z)^{\times} J(\alpha+z) + \frac{1}{2} J_0 K_P (q_0 I_3 + q^{\times})(z+\alpha)\Big) \\ &\quad + z^T u, \end{aligned} \tag{34}$$

where μ is a positive constant,

$$\Pi = \begin{bmatrix} \gamma K_P & 0_{3\times3} \\ 0_{3\times3} & \mu I_3 \end{bmatrix} \text{ and } \vartheta = \begin{bmatrix} q \\ z \end{bmatrix}. \tag{35}$$

We know that $\dot{\mathcal{V}}_2$ can be written as

$$\dot{\mathcal{V}}_2 = L_f \mathcal{V}_2 + L_g \mathcal{V}_2 u. \tag{36}$$

Comparing (36) to (34), one obtains

$$L_g \mathcal{V}_2 = z^T \tag{37}$$

where $0_{1\times3}$ is the 1×3 zero matrix.

If $L_g\mathcal{V}_2 = 0_{1\times3}$ then $z^T = 0_{1\times3}$ Therefore, when $L_g\mathcal{V}_2 = 0_{1\times3}$, we have

$$\begin{aligned} L_f \mathcal{V}_2 &= \dot{\mathcal{V}}_2 \\ &= -\vartheta^T \Pi \vartheta \end{aligned} \tag{38}$$

Evidently $L_f\mathcal{V}_2 < 0$, $\forall x \neq 0$. Therefore, the function $\mathcal{V}_2$ is a CLF for the system (26). □

Consider the system (26) in the absence of disturbance vector D. Then the dynamic feedback controller u in (29) stabilizes the system (26).

Next, a new optimal control law is developed. We show that the proposed feedback law can stabilize the spacecraft system (26) and minimize the cost functional (39).

Theorem 4.3. The feedback control law (29) with $\gamma \geq 1$ solves the inverse optimal assignment problem for the attitude control system (26) by stabilizing the spacecraft system (26) and minimizing the cost functional

$$L_a = \lim_{T\to\infty}\left[\mathcal{V}_2(x(T)) + \int_0^T (l(x,\frac{\gamma}{2}) + \frac{1}{\gamma}u^T Ru)\,\mathrm{d}t\right], \tag{39}$$

where $l(x)$ is defined by

$$\begin{aligned} l(x) &= -z^T\Big[(\gamma I_3 + J_0K_I)q + \mu z - (\alpha+z)^\times J(\alpha+z) \\ &\quad +\frac{1}{2}J_0K_P(q_0I_3+q^\times)(\alpha+z)\Big] + \vartheta^T\Pi\vartheta \\ &\quad +z^TR^{-1}z. \end{aligned} \tag{40}$$

Proof. The proof consists of two parts. For the first part,it is required to ensure that for $\gamma \geq \frac{1}{2}$, the controller $u = \kappa_\gamma$ globally asymptotically stabilizes the origin of the system (26). Consider the smooth positive-definite radially unbounded function $\mathcal{V}_2(x)$ defined in (31) as the Lyapunov function. The derivative of $\mathcal{V}_2(x)$ along the system trajectories of the system (26) is

$$\begin{aligned} \dot{\mathcal{V}}_2 &= L_f\mathcal{V}_2 + L_B\mathcal{V}_2\kappa_\gamma \\ &= \psi(x) - \gamma\lambda(x)z^Tz. \end{aligned} \tag{41}$$

Since $\mathcal{V}_2(x)$ is a CLF for the system, when $z = 0$, one obtains $\dot{\mathcal{V}}_2(x) < 0$ for all $x \neq 0$. When $\psi(x) < 0$, it is obvious that $\dot{\mathcal{V}}_2 < 0$ is obtained. Suppose $z \neq 0$ and $\psi(x) \geq 0$. We can rewrite $\dot{\mathcal{V}}_2$ as

$$\begin{aligned} \dot{\mathcal{V}}_2(x) &= (1-\gamma)\psi(x) - \gamma\sqrt{\psi^2 + (z^Tz)^2} - \gamma z^Tz \\ &= \frac{(1-2\gamma)\psi(x)^2 - \gamma^2(z^Tz)^2}{(1-\gamma)\psi(x) + \gamma\sqrt{\psi^2+(z^Tz)^2}} - \gamma z^Tz. \end{aligned} \tag{42}$$

Obviously, $\dot{\mathcal{V}}_2 < 0$ is obtained for all $\gamma \geq \frac{1}{2}$. the controller $u = \kappa_\gamma(x)$ globally asymptotically stabilizes the origin of the system (26).

For the second part, it is required that $u = \kappa_\gamma(x)$ is an inverse optimal control. Setting $l(x,\gamma) = -\dot{\mathcal{V}}_2$, one obtains $l(x,\gamma) > 0$ for all $x \neq 0$. We consider an intermediate cost functional given by

$$I_0(x) = \int_0^T (u+\gamma\lambda(x)z)^TR(x)(u+\gamma\lambda(x)z)\,\mathrm{d}t, \quad \gamma \geq 1. \tag{43}$$

Evidently, an optimal control $u^* = -\gamma\lambda(x)z$ minimizes I_0. Let $2\lambda(x)R(x) = I_n$. Using the fact that $u^T L_B \mathcal{V}_2 = \dot{\mathcal{V}}_2(x) - L_f \mathcal{V}_2$, one obtains

$$\begin{aligned} I_0(x) &= \int_0^T (u^T R(x) u + \gamma^2 \lambda(x)^2 z^T R(x) z + 2\gamma\lambda(x) u^T R(x) z)\,\mathrm{d}t \\ &= \int_0^T (u^T R(x) u + \frac{\gamma^2}{4} z^T R(x)^{-1} z + \gamma(\dot{\mathcal{V}}_2(x) - L_f \mathcal{V}_2))\,\mathrm{d}t \\ &= \int_0^T (u^T R(x) u - \gamma\big[\psi(x) + (\frac{\gamma}{2}\lambda(x) z)^T z\big] + \gamma \dot{\mathcal{V}}_2(x))\,\mathrm{d}t \\ &= \int_0^T (\gamma l(x, \frac{\gamma}{2}) + u^T R(x) u + \gamma \dot{\mathcal{V}}_2(x))\,\mathrm{d}t. \end{aligned} \tag{44}$$

Since $\int_0^T \dot{\mathcal{V}}_2(x)\,\mathrm{d}t = \mathcal{V}_2(x(T)) - \mathcal{V}_2(x(0))$ and the optimal value function I_0^* is equal to zero, one has

$$\mathcal{V}_2(x(0)) = \frac{1}{\gamma} \int_0^T L(x, u)\,\mathrm{d}t + \mathcal{V}_2(x(T)), \tag{45}$$

where $L(x, u) = \gamma l(x, \frac{\gamma}{2}) + u^T R(x) u$. Thus, $u^* = \kappa_\gamma(x)$ is an optimal control for following functional

$$I = \int_0^T (l(x, \frac{\gamma}{2}) + \frac{1}{\gamma} u^T R(x) u)\,\mathrm{d}t + \mathcal{V}_2(x(T)), \quad \gamma \geq 1. \tag{46}$$

Because $\mathcal{V}_2(x(T)) \to 0$ as $T \to \infty$, $\mathcal{V}_2(x(0)) = \int_0^\infty (\gamma l(x, \frac{\gamma}{2}) + \frac{1}{\gamma}(u^*)^T R(x) u^*)\,\mathrm{d}t$. Thus, $I_0^* = \mathcal{V}_2(x(0))$. This completes the proof. □

5. MODIFIED EXTENDED STATE OBSERVER

Because of the great advances in nonlinear control theory, the observer-based controller has been considered as one of the most common approaches in industrial applications. The extended state observer (ESO) presented in [6, 8] is a very useful method that has high efficiency in accomplishing nonlinear dynamic estimation. In this section the modified version of the conventional ESO is designed and the finite-time stability of the proposed ESO system is investigated using the strict Lyapunov function.

We can rewrite (23) as

$$\dot{z} = f + Gu + \tilde{d}, \tag{47}$$

where

$$f = \Xi(q_0, q, \alpha, z), \quad G = J_0^{-1} \quad \text{and} \quad \tilde{d} = G\bar{d}. \tag{48}$$

From (47) the proposed robust optimal PID control is designed as

$$u = u^* - G^{-1}\hat{d}, \tag{49}$$

where $\hat{d}$ is the estimate of the disturbance vector $\tilde{d}$. Clearly, from (49) if $\hat{d} \to \tilde{d}$, then the control law u is the same as u^* presented in Section 5.

Using the idea of ESO, a nonlinear ESO can be designed for estimating the disturbances D. We add an extended state χ to the state equations to represent the total disturbances $\bar{d}$. The system (47) then becomes

$$\begin{aligned} \dot{z} &= f + Gu + \tilde{d} \\ \dot{\chi} &= g(t), \end{aligned} \tag{50}$$

where the function $g(t)$ is the estimated derivative of the disturbances $\tilde{d}$.

It is not necessary the known the value of $\bar{g}$ since we can make the ultimate boundedness of the estimation error sufficiently small by proper selecting the control parameters.

Then the modified ESO for the system (47) is proposed to be as follows

$$\begin{aligned} E_1 &= Z_1 - z \\ \dot{Z}_1 &= Z_2 + f + Gu - \epsilon^{-1}\lambda_1 E_1 \\ \dot{Z}_2 &= -g(t) - \epsilon^{-1}\lambda_2 \mathrm{sign}^{\alpha}(E_1) - \epsilon^{-1}\lambda_3 E_1 \\ &\quad -\epsilon^{-1}\lambda_4 \mathrm{sign}(E_1), \end{aligned} \tag{51}$$

where E_1 is the estimation error of the ESO, Z_1 and Z_2 are the observer output, and $\lambda_1 = \mathrm{diag}(\lambda_{11}, \lambda_{12}, \lambda_{13})$, $\lambda_2 = \mathrm{diag}(\lambda_{21}, \lambda_{22}, \lambda_{23})$, $\lambda_3 = \mathrm{diag}(\lambda_{31}, \lambda_{32}, \lambda_{33})$ and $\lambda_4 = \mathrm{diag}(\lambda_{41}, \lambda_{42}, \lambda_{43})$ with $\lambda_{1i} > 0$, $\lambda_{2i} > 0$, $\lambda_{3i} > 0$ and $\lambda_{4i} > 0$ are the observer gains. Here, the function $\mathrm{sign}(E_1)$ and $\mathrm{sign}^{\alpha}(E_1)$ are defined as

$$\mathrm{sign}(E_1) = \begin{bmatrix} \mathrm{sign}(E_{11}) \\ \mathrm{sign}(E_{12}) \\ \mathrm{sign}(E_{13}) \end{bmatrix}$$

and

$$\mathrm{sign}^{\alpha}(E_1) = \begin{bmatrix} |E_{11}|^{\alpha}\mathrm{sign}(E_{11}) \\ |E_{12}|^{\alpha}\mathrm{sign}(E_{12}) \\ |E_{13}|^{\alpha}\mathrm{sign}(E_{13}) \end{bmatrix}$$

with $\alpha \in (0, 1)$.

Note that the proposed modified ESO presented by (51) has been presented to estimate the total disturbance. The total disturbance is composed of the vibration effect and external disturbance.

Theorem 5.1. Let Assumption 3.1 hold. Consider the system (50) with the adaptive ESO (51). Then there exist positive observer gains λ_{1i}, λ_{2i}, λ_{3i} and λ_{4i} $(i = 1, 2, 3)$ and $\alpha \in (0, 1)$ such that the ultimate boundedness of the estimation error is ensured.

Proof. Letting $e_1 = E_1$ and $e_2 = Z_2 - \tilde{d}$ the observer error dynamics can be transformed to the scalar form $(i = 1, 2, 3)$ as

$$\begin{aligned} \dot{e}_{1i} &= \epsilon^{-1}\Big(e_{2i} - \lambda_{1i} e_{1i}\Big) \\ \dot{e}_{2i} &= -g_i(t) - \epsilon^{-1}\lambda_{2i}|e_{1i}|^{\alpha}\mathrm{sign}(e_{1i}) - \epsilon^{-1}\lambda_{3i} e_{1i} \\ &\quad -\epsilon^{-1}\lambda_{4i}\mathrm{sign}(e_{1i}). \end{aligned} \tag{52}$$

We define $\nu = [|e_{1i}|^{\frac{\alpha+1}{2}}\text{sign}(e_{1i}) \quad |e_{1i}|^{\frac{1}{2}} \quad e_{1i} \quad e_{2i}]^T$. To prove the stability, we select the Lyapunov function

$$\dot{\mathcal{V}}_3 = \frac{1}{2}\nu^T \Pi \nu, \tag{53}$$

where

$$\Pi_1 = \frac{1}{2}\begin{bmatrix} \frac{4\lambda_{2i}}{\alpha+1} & 0 & 0 & 0 \\ 0 & 4\lambda_{4i} & 0 & 0 \\ 0 & 0 & 2\lambda_{3i}+\lambda_{1i}^2 & \lambda_{1i} \\ 0 & 0 & \lambda_{1i} & 2 \end{bmatrix}.$$

Taking the time derivative to both sides of (53), we obtain

$$\begin{aligned} \dot{\mathcal{V}}_3 &= 2\lambda_{2i}|e_{1i}|^\alpha \text{sign}(e_1)\dot{e}_{1i} + (\lambda_{3i} + \frac{1}{2}\lambda_{1i}^2)e_{1i}\dot{e}_{1i} \\ &\quad +2e_{2i}\dot{e}_{2i} - \lambda_{1i}e_{2i}\dot{e}_{1i} - \lambda_{1i}e_{1i}\dot{e}_{2i} + 2\lambda_{4i}\text{sign}(e_1)\dot{e}_{1i} \\ &= 2\lambda_{2i}|e_{1i}|^\alpha \text{sign}(e_1)\epsilon^{-1}(e_{2i} - \lambda_{1i}e_{1i}) + (\lambda_{3i} + \frac{1}{2}\lambda_{1i}^2)e_{1i} \\ &\quad \times\epsilon^{-1}(e_{2i} - \lambda_{1i}e_{1i}) + 2e_{2i}\epsilon^{-1}\Big(- \lambda_{2i}|e_{1i}|^\alpha \text{sign}(e_1) \\ &\quad -\lambda_{3i}e_{1i} - \lambda_{4i}\text{sign}(e_1) - \epsilon g(t)\Big) \\ &\quad -\epsilon^{-1}\lambda_{1i}e_{2i}(e_{2i} - \epsilon^{-1}\lambda_{1i}e_{1i}) + 2\lambda_{4i}\text{sign}(e_{1i}) \\ &\quad \times\epsilon^{-1}(e_{2i} - \lambda_{1i}e_{1i}) - \lambda_{1i}e_{1i}\epsilon^{-1}\Big(- \lambda_{2i}|e_{1i}|^\alpha \text{sign}(e_{1i}) \\ &\quad -\lambda_{3i}e_{1i} - \lambda_{4i}\text{sign}(e_1) - \epsilon g(t)\Big), \end{aligned} \tag{54}$$

which can be further written as

$$\begin{aligned} \dot{\mathcal{V}}_3 &= \epsilon^{-1}\Big(- \lambda_{1i}\lambda_{2i}|e_{1i}|^{\alpha+1} - (\lambda_{1i}^3 + \lambda_{1i}\lambda_{3i})e_{1i}^2 - \lambda_{1i}^2 e_{1i}e_{2i} \\ &\quad -\lambda_{1i}e_{2i}^2 + \lambda_{1i}^2 - \lambda_{1i}\lambda_{4i}|e_{1i}|\Big) + \lambda_{1i}e_{1i}g_i(t) \\ &\quad -2e_{2i}g_i(t) \end{aligned} \tag{55}$$

After some manipulation, the derivative of $\mathcal{V}_3$ can be written as follows:

$$\dot{\mathcal{V}}_3 = -\epsilon^{-1}\nu^T\Pi_2\nu + \varrho\nu \tag{56}$$

where

$$\Pi_2 = \lambda_{1i}\begin{bmatrix} \lambda_{2i} & 0 & 0 & 0 \\ 0 & \lambda_{4i} & 0 & 0 \\ 0 & 0 & \lambda_{1i}^2+\lambda_{3i} & \lambda_{1i} \\ 0 & 0 & \lambda_{1i} & 1 \end{bmatrix} \tag{57}$$

and

$$\varrho = \begin{bmatrix} 0 & 0 & \lambda_{1i}g_i(t) & -2g_i(t) \end{bmatrix}. \tag{58}$$

Letting $L = [0 \quad \lambda_{1i}\bar{D}_2 \quad \bar{D}_2]^T$, one obtains

$$\dot{\mathcal{V}}_3 \leq -\epsilon^{-1}\sigma_{\min}(\Pi_2)\|\nu\|^2 + \|L\|\|\nu\|. \tag{59}$$

From (59) it follows that

$$\sigma_{\min}(\Pi_1)\|\nu\|^2 \leq \mathcal{V}_3^2 \leq \sigma_{\max}(\Pi_1)\|\nu\|^2 \tag{60}$$

and (59) becomes

$$\dot{\mathcal{V}}_3 \leq -\epsilon^{-1}\frac{\sigma_{\min}(\Pi_2)}{\sigma_{\max}(\Pi_1)}\mathcal{V}_3 + \frac{\|L\|}{\sqrt{\sigma_{\max}(\Pi_1)}}\mathcal{V}_3^{\frac{1}{2}}. \tag{61}$$

Considering $W^2 = \epsilon^2\mathcal{V}_3$, it is obtained that

$$\dot{W} \leq -\epsilon^{-1}\beta_1 W + \epsilon\beta_2, \tag{62}$$

where $\beta_1 = \frac{\sigma_{\min}(\Pi_2)}{2\sigma_{\max}(\Pi_1)}$ and $\beta_2\frac{\|L\|}{2\sqrt{\sigma_{\max}(\Pi_1)}}$. We obtain that $\dot{W}$ in (62) ensures ultimate boundedness. If $\epsilon \to 0$ the ultimate bound of the error also tends to zero. This completes the proof. □

Remark 5.2. It should be noticed that conditions for the stability of the modified ESO (43) have been obtained in terms of positive gains λ_{1i}, λ_{2i}, λ_{3i}, λ_{4i} and $\alpha \in (0,1)$ in the equation (45) for the estimation errors. When suitable gains are chosen, Z_2 will be a precise estimate of $\tilde{d}$ and the bound of the estimation error E_2 is very small and tend to zero if $\epsilon \to 0$.

Using the results from the ESO system, Z_2 is the good estimated disturbance. Thus, for the control law (49), we use $\hat{d}$ that is determined by $\hat{d} = Z_2$. Thus, the proposed inverse optimal PID control can be obtained as.

$$u = u^* - G^{-1}Z_2. \tag{63}$$

Note that the proposed controller (63) is designed by combing the inverse optimal controller (29) and the result of estimated disturbance. With suitable control gains defined by the inverse optimal control approach, the optimal PID controller (63) contains both optimality and robustness performance to attenuate external disturbances.

6. SIMULATION RESULTS

An example of attitude control of flexible spacecraft [10] is presented with numerical simulations to compare the inverse optimal PID controller with the optimal Lyapunov sliding mode controller (OLSMC) in [22]. The spacecraft is assumed to have the nominal inertia matrix

$$J = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} \quad \text{kg} \cdot \text{m}^2$$

and coupling matrices

$$\delta = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11678 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{bmatrix} \quad \mathrm{kg}^{1/2} \cdot \mathrm{m/s}^2$$

respectively. The first four elastic modes have been considered in the model used for simulating spacecraft at $\omega_{n1} = 0.7681$, $\omega_{n2} = 1.1038$, $\omega_{n3} = 1.8733$, $\omega_{n4} = 2.5496$ rad/sec with damping $\xi_1 = 0.0056$, $\xi_2 = 0.0086$, $\xi_3 = 0.013$, $\xi_1 = 0.025$. The initial states of the rotation motion are given by

$$Q(0) = \begin{bmatrix} 0.8832 \\ 0.3 \\ -0.3 \\ 0.2 \end{bmatrix} \quad \omega(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \mathrm{rad/sec},$$

$$\eta(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \dot{\eta}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The attitude control problem is considered in the presence of external disturbance $d(t)$.

$$d(t) = 0.5 \begin{bmatrix} 0.3\cos(0.1t) + 0.1 \\ 0.15\sin(0.1t) + 0.3\cos(0.1t) \\ 0.3\sin(0.1t) + 0.1 \end{bmatrix} \quad \mathrm{N-m}. \tag{64}$$

The proposed inverse optimal PID controller is obtained by using (63) with the modified ESO (51). For the proposed inverse optimal PID controller, the control parameters are set as $K_P = 2I_3$, $K_I = 0.4I_3$, $\gamma = 60$ and $\mu = 0.5$, $\lambda_{1i} = 5.0$, $\lambda_{2i} = 1.5$, $\lambda_{3i} = 1.0$ and $\lambda_{4i} = 0.5$. Also, the same control parameters in the OLSMC method in [22] are selected. Note that the values of the four elastic modes can be obtained by using (12) and (13). The disturbance can be obtained by using (64). However, in this paper, the proposed disturbance observer is used to compensate for the total disturbance consisting of the vibration effect and external disturbance. Thus, the total disturbance should be plotted.

Simulation results are conducted to compare the performance of both control methods. In Figures 1 and 2, responses of quaternion and angular velocity components converge to zero after 100 seconds. The components of angular velocity vector are not smooth. From Figure 3 one can see that OLSMC in [22] stabilizes the closed-loop system of flexible spacecraft. Figures 4 and 5 show the modal displacements $(\eta_1 - \eta_4)$ in which oscillations are reduced slowly.

On the other hand, Figure 6 shows that the inverse optimal PID controller achieves good responses of the quaternions which converge to zero in about 80 seconds. Similarly, from Figure 7 one can see that the angular velocities go to zero after 80 seconds. The responses of angular velocities are smoother and the proposed controller gives quicker

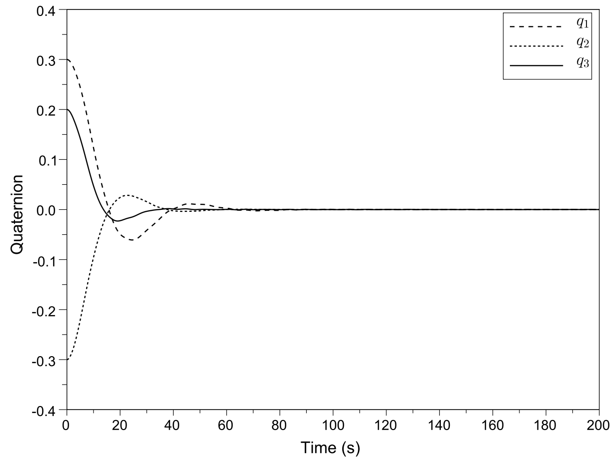

Fig. 1. Quaternion vector under OLSMC in [22].

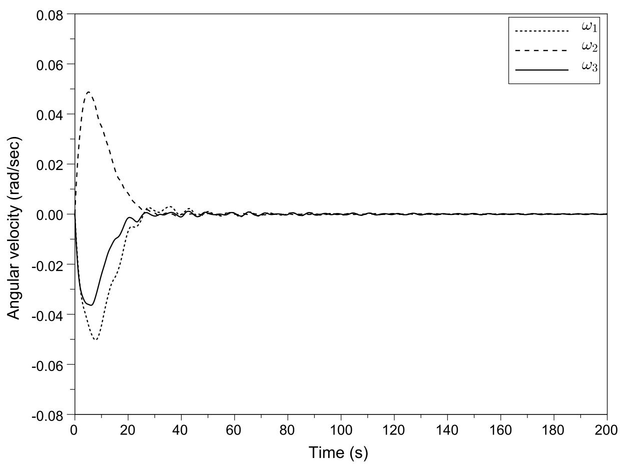

Fig. 2. Angular velocity vector under OLSMC in [22].

convergence rates when compared with those obtained from OLSMC. As shown in Figure 8 the control torques obtained by the proposed inverse optimal PID controller are smoother than ones obtained by OLSMC in [22]. Our proposed control method has the proposed modified ESO presented by (51) using to compensate for the total disturbance consisting of the vibration effect and external disturbance. Thus, smoother control responses are obtained. As shown in Figures 9 – 11, the components of the to-

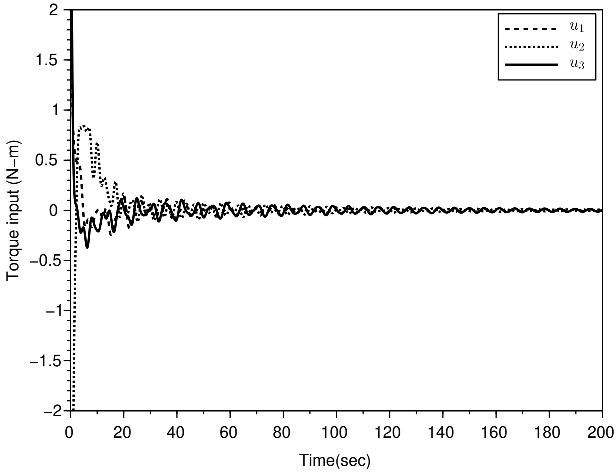

Fig. 3. Control torques under OLSMC in [22].

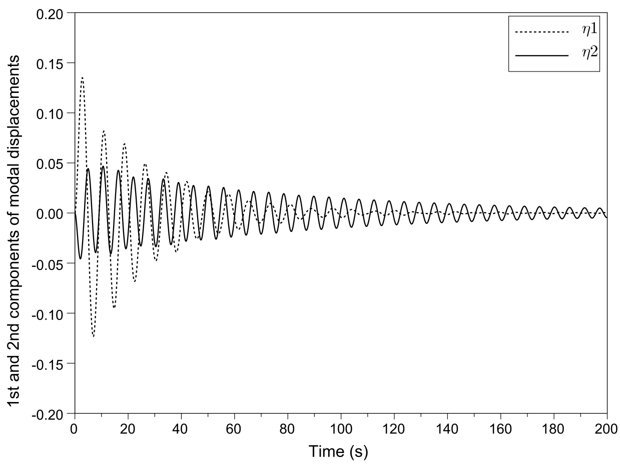

Fig. 4. Modal displacements under OLSMC in [22].

tal disturbance $\tilde{d}$ and the estimated total disturbance $\hat{d}$ are presented. The estimated disturbance $\hat{d}$ converges to the total disturbance $\tilde{d}$. The proposed ESO can properly estimate the values of the total disturbance.

Simulation results obtained by OLSMC and proposed inverse optimal PID controller are compared. One can see that the proposed inverse optimal PID controller offers smoother attitude velocity responses and better responses of modal displacements. In

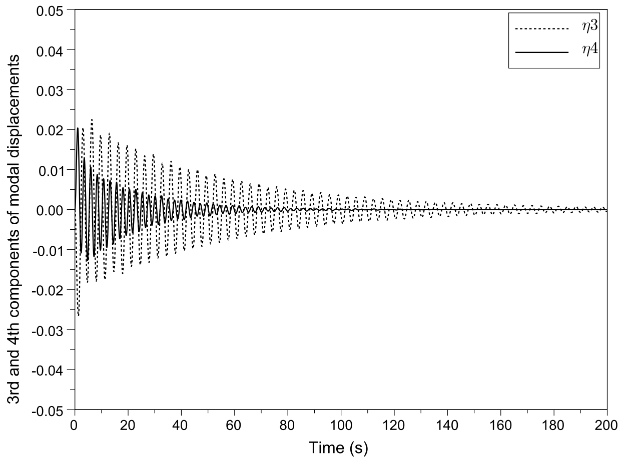

Fig. 5. Modal displacements under OLSMC in [22].

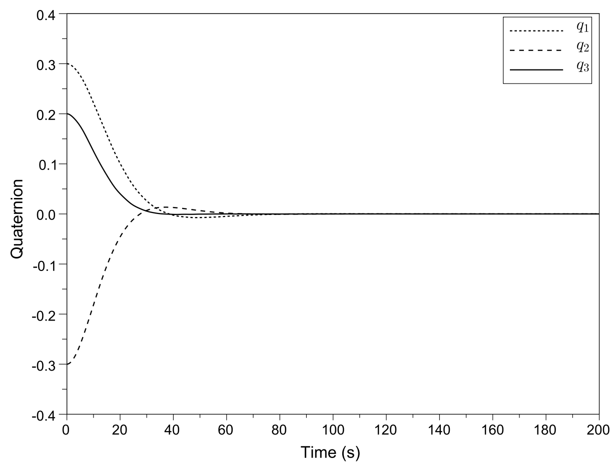

Fig. 6. Quaternion vector under controller (63).

view of these simulation results, the proposed inverse optimal PID controller is considered as a more useful approach for general cases of attitude regulation problems of a flexible spacecraft.

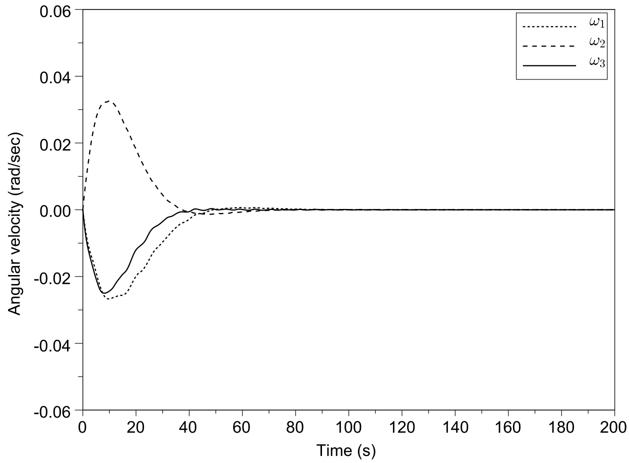

Fig. 7. Angular velocity vector under controller (63).

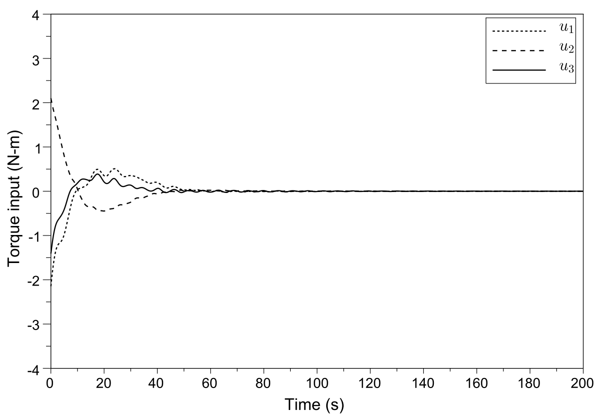

Fig. 8. Control torques under controller (63).

7. CONCLUSION

A robust optimal PID control algorithm for a flexible spacecraft in the presence of unknown bounded external disturbances has been successfully designed. This controller is designed based on CLF, PID and inverse optimal control schemes. The modified ESO is developed to compensate for external disturbances and vibration of flexible appendages. The second method of Lyapunov is used to demonstrate its properties including the convergence rate and ultimate boundedness of the estimation error. The proposed controller

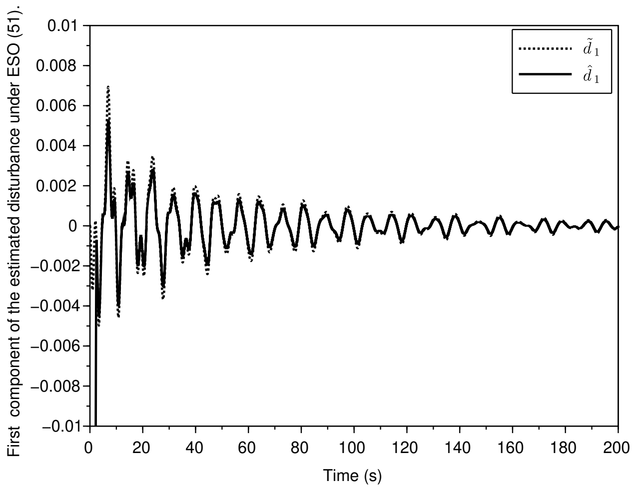

Fig. 9. First component of the estimated total disturbance.

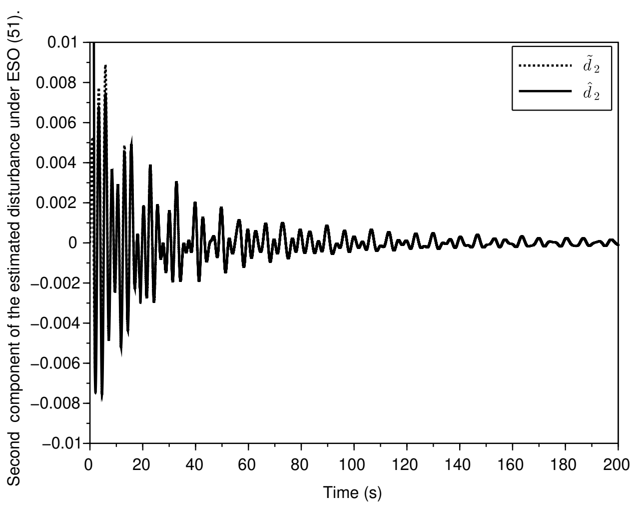

Fig. 10. Second component of the estimated total disturbance.

can be expressed as the sum of an optimal controller and estimated disturbance. It is shown that the developed controller can stabilize the attitude control system and minimize a cost functional. Numerical simulations are performed to verify the performance of the developed control method. The robust optimal control for attitude stabilization of flexible spacecraft has rarely been studied and most developed control methods usually consider only the optimal control problem or robust control problem. In contrast to

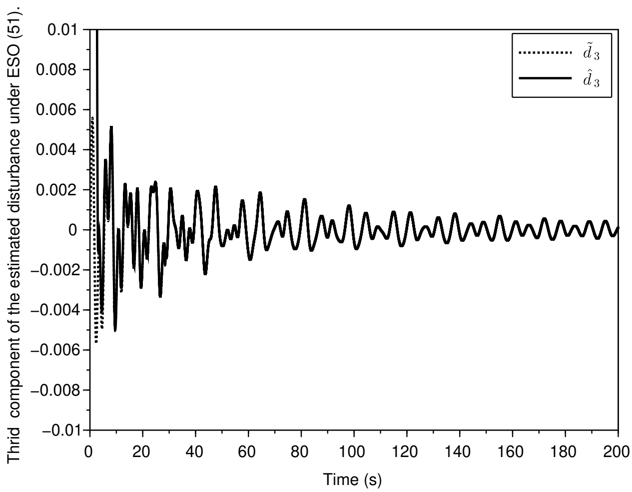

Fig. 11. Third component of the estimated total disturbance.

the previous control algorithms, the proposed control method in this paper is optimal with respect to a cost functional. Moreover, with the developed ESO, this controller can effectively suppress external disturbances. Thus, the proposed PID optimal control method offers both optimality and robustness. However, the main limitation of the proposed control method is that the total disturbance including external disturbance and the disturbance caused by the elastic vibration of flexible appendages and its first time derivative are required to be bounded.

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DECLARATION OF CONFLICTING INTEREST

The author confirms that there are no known conflicts of interest associated with this manuscript and there has been no significant financial support for this work that could have influenced its outcome.

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REFERENCES

[1] S. Bharadwaj, M. Osipchuk, K. D. Mease, and F. C. Park: Geometry and inverse optimality in global attitude stabilization. J. Guidance Control Dynamic *21* (1998), 930–939. DOI:10.2514/2.4327

[2] J. R Cloutier: State-Dependent Riccati equation techniques: An overview. In: Proc. American Control Conference, Albuquerque 1997, pp. 932–936. DOI:10.1109/acc.1997.609663

[3] S. DiGennaro: Output stabilization of flexible spacecraft with active vibration supression. IEEE Aerop. Electron. Syst. Mag. *39* (2003), 747–759. DOI:10.1109/taes.2003.1238733

[4] J. Erdong and S. Zhaowei: Passivity-based control for a flexible spacecraft in the presence of disturbance. Int. J. Non-linear Mechanics *45* (2010), 348–356. DOI:10.1016/j.ijnonlinmec.2009.12.008

[5] R. A. Freeman and P. V. Kokotović: Inverse optimality in robust stablilzation. SIAM J. Control Optim. *34* (1996), 1365–1391. DOI:10.1137/s0363012993258732

[6] B. Z. Guo and Z. Zhao: On the convergence of an extended state observer for nonlinear systems with uncertainty. Syst. Control Lett. *60* (2011), 420–430. DOI:10.1016/j.sysconle.2011.03.008

[7] J. Han and Y. Huang: Analysis and design for the second order nonlinear continuous extended states observer. Chinese Sci. Bull. *45* (2000), 1938–1944. DOI:10.1007/bf02909682

[8] J. Han: From PID to active disturbance rejection control. IEEE Trans. Ind. Electron. *56* (2009), 900–906. DOI:10.1109/tie.2008.2011621

[9] N. M Horri, P. Palmer, and M. Roberts: Gain scheduled inverse optimal satellite attitude control. IEEE Trans. Aerospace Electron. Systems *48* (2012), 2437–2457. DOI:10.1109/taes.2012.6237602

[10] Q. Hu: Sliding mode attitude control with L_2-gain performance and vibration reduction of flexible spacecraft with actuator dynamics. Acta Astronautica *67* (2010), 572–583. DOI:10.1016/j.actaastro.2010.04.018

[11] Q. Hu and G. Ma: Varaible structure control and active vibration suppression of flexible spacecraft during attitude maneuver. Aerospace Science Technol. *9* (2005), 307–317. DOI:10.1016/j.ast.2005.02.001

[12] H. K. Khalil: Nonlinear Systems. Prentice-Hall Press, 1996.

[13] M. Krstić and P. V. Kokotović: Control Lyapunov functions for adaptive nonlinear stabilization. Systems Control Lett. *26* (1995), 17–23. DOI:10.1016/0167-6911(94)00107-7

[14] M. Krstić and Z. H. Li: Inverse optimal design of input-to-state stabilizing nonlinear controllers. IEEE Trans. Automat. Control *43* (1998), 336–350. DOI:10.1109/9.661589

[15] M. Krstić and M. P. Tsiotras: Inverse optimal stabilization of a rigid spacecraft. IEEE Trans. Automat. Control *44* (1999), 1042–1045. DOI:10.1109/9.763225

[16] K. Lu, Y. Xia, and M. FU: Controller design for rigid spacecraft attitude tracking with actuator saturation. Inform. Sci. *220* (2013), 343–366. DOI:10.1016/j.ins.2012.07.039

[17] W. Luo, Y. C. Chung, and K. V. Ling: Inverse optimal adaptive control for attitude tracking spacecraft. IEEE Trans. Automat. Control *50* (2005), 1639–1654. DOI:10.1109/tac.2005.858694

[18] Y. Park: Robust and optimal attitude stabilization of spacecraft with external disturbances. Aerospace Sci. Technol. *9* (2005), 253–259. DOI:10.1016/j.ast.2005.01.002

[19] Y. Park: Inverse optimal and robust nonlinear attitude control of rigid spacecraft. Aerospace Sci. Technol. *28* (2013), 257–265. DOI:10.1016/j.ast.2012.11.006

[20] J. A. Primb, V. Nevistić, and J. C. Doyle: Nonlinear optimal control : a control Lyapunov function and receding horizon perspective. Asian J. Control *1* (1999), 14–24. DOI:10.1111/j.1934-6093.1999.tb00002.x

[21] C. Pukdeboon and A. S. I. Zinober: Control Lyapunov function optimal sliding mode controllers for attitude tracking of spacecraft. J. Franklin Inst. *349* (2012), 456–475. DOI:10.1016/j.jfranklin.2011.07.006

[22] C. Pukdeboon: Optimal sliding mode controllers for attitude stabilization of flexible spacecraft. Math. Problems Engrg. *2011* (2011) Article ID 863092, 1–20. DOI:10.1155/2011/863092

[23] R. Sepulchre, R. A. Freeman, and P. V. Kokotović: Constructive Nonlinear Control. Springer–Verlag, New York 1997. DOI:10.1007/978-1-4471-0967-9

[24] R. Sharma and A. Tewari: Optimal nonlinear tracking of spacecraft attitude maneuvers. IEEE Trans. Control Systems Technol. *12* (2004), 677–682. DOI:10.1109/tcst.2004.825060

[25] M. D. Shuster: A survey of attitude representations. J. Astronaut. Sci. *41* (1993), 439–517.

[26] E. D. Sontag: A universal construction of Artstein's theorem on bonlinear stabilization. Systems Control Lett. *13* (1989), 117–123. DOI:10.1016/0167-6911(89)90028-5

[27] E. D. Sontag: Mathematical Control Theory, Deterministic Finite Dimensional Systems. Second edition. Springer-Verlag, New York 1998. DOI:10.1007/978-1-4612-0577-7

[28] D. T. Stansbery and J. R Cloutier: Position and attitude control of a spacecraft using the state-dependent Riccati equation techniques. In: Proc. American Control Conference, Chicago 2000. DOI:10.1109/acc.2000.879525

[29] M. Xin and S. N. Balakrishnan: State dependent Riccati equation based spacecraft attitude control. In: Proc. 40th AIAA Aerospace Sciences Meeting and Exhibit, Reno 2002. DOI:10.2514/6.2002-1071

[30] M. Xin, S. N. Balakrishnan, and D. T. Stansbery: Spacecraft position and attitude control with $\theta - D$ technique. In: Proc. 42th AAIA Aerospace Sciences Meeting and Exhibit, Reno 2004. DOI:10.2514/6.2004-540

[31] M. Xin and H. Pan: Nonlinear optimal control of spacecraft approaching a tumbling target. Aerospace Sci. Technol. *15* (2011), 79–89. DOI:10.1016/j.ast.2010.05.009

[32] V. I. Utkin: Sliding Modes in Control and Optimization. Springer–Verlag, Berlin 1992.

[33] J. R. Wertz: Spacecraft Attitude Determination and Control. Kluwer Academic, Dordrecht, London 1978.

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