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THE DYNAMIC BEHAVIORS OF A NEW IMPULSIVE PREDATOR PREY MODEL WITH IMPULSIVE CONTROL AT DIFFERENT FIXED MOMENTS

Linjun Wang, Youxiang Xie, Qicheng Deng

In this paper, we propose a new impulsive predator prey model with impulsive control at different fixed moments and analyze its interesting dynamic behaviors. Sufficient conditions for the globally asymptotical stability of the semi-trivial periodic solution and the permanence of the present model are obtained by Floquet theory of impulsive differential equation and small amplitude perturbation skills. Existences of the "infection-free" periodic solution and the "predator-free" solution are analyzed by bifurcation theory of impulsive differential equation. Finally, the analytical results presented in the work are validated by numerical simulation figures for this proposed model.

Keywords: impulsive differential equation, bifurcation theory, stability, impulsive control, persistence and extinction

Classification: 34D23, 92D30

1. INTRODUCTION

In recent years, many biologists have been concerning about the management of renewable natural resources, and also aware that it is possible to change the genetic pattern of resources by suitable stocking and harvesting which can play an important part in the permanence of ecosystems [16, 28, 32]. Some researchers have studied the effects of toxicants that emitted into the environment from industrial and household sources on biological species [2, 22, 35]. In fact, most of these ecosystems are assumed that the exogenous input of toxicants has its continuous characteristic. Moreover, there are many activities that are related to natural factors, such as planting, drought, harvesting and flooding, as well as human exploitation. These activities have deeply perturbed ecological systems. Especially, sudden changes can be mathematically described in the form of impulses. So we often remove the continuous input of toxicants from the ecological systems and replace it using a pulse perturbation. Therefore, in order to accurately describe these systems, we often exploit an impulsive differential equation. Recently, since the theory of impulsive differential equations develops very quickly, it has led to the proposal of many different kinds of population dynamical models of impulsive differential equations which have been researched by many experts [3, 7, 18, 26]. Meng

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et al. proposed a new SEIRS epidemic disease model with two profitless delays and nonlinear incidence and analyzed the dynamic behaviors of the model under pulse vaccination [29]. Jin et al. proposed a pulse vaccination SIR model with periodic infection rate, and studied the stability of the infection-free periodic solution and the existence of the positive periodic solution [38]. Gao et al. formulated an SEIRS epidemic model with time delays and pulse vaccination, and obtained the exact infection-free periodic solution of the impulsive epidemic system and proved its global attractability and persistence [24]. Zuo and Jiang investigated a stochastic non-autonomous Holling-Tanner predator-prey system with impulsive effect, and proved the existence and global attraction of the boundary periodic solution by using the comparison theorem [27]. Sun et al. presented a new integrated pest management predator-prey model, and verified the existence and stability of the order-1 periodic orbit for the proposed model [14]. Sun et al. studied a pest control scaled prey-predator model, where the yield releases of the predator and chemical control strength are dependent on the pest control level [15]. Zhang and Tan considered a stochastic predator-prey system in a polluted environment with pulse toxicant input and impulsive perturbations, and obtained a set of sufficient conditions for extinction, with weak persistence in the mean and global attraction to any positive solution of the proposed system, and estimated the conditions for the upper boundedness of the expectations of this proposed system solution [25]. For more related research works, one may refer to the references [4, 11, 19, 23, 33, 34, 36, 37, 39].

Some experts have controlled the pests by exploiting viruses and simultaneously releasing the pest population [8, 20]. First, a small amount of pathogens are introduced into a pest population with the expectation that it will generate an epidemic and that it will subsequently be endemic. The success of this method depends on the survival of the microbes which in turn depends on environmental factors. At the same time, we consider to release the pests infected in the laboratories to the pest population with periodic impulsive effect. The infected pests have little effect on the crops. The susceptible pests become infected through direct contact with the infective ones or through encountering the free-living infective stage in the environment. Thus it can infect the pest population and result in the death of them continuously. The main purpose of this paper therefore is to formulate and investigate an epidemiological model for the bio-control of a pest. In fact, the theoretical investigation and its application analysis can be found in almost every field [1, 6, 9, 10, 12, 13, 17, 21, 30]. This pest population is assumed to grow according to a logistic curve in the absence of disease [5, 31]. Moreover, many experts have studied biological model with impulsive effect, but the interesting research work obtained by comprehensive prevention and impulse control strategy at different fixed moments is very less. Therefore, in this paper, we investigate the dynamic behaviors of a new impulsive predator prey model with impulsive control at different fixed moments.

The paper is organized as follows: We establish a new impulsive predator prey model with impulsive control at different fixed moments in Section 2. In Section 3, some corresponding preliminaries are given and ready for the investigation of our proposed model. Some sufficient conditions that assure the stability and persistence of semi-trivial periodic solutions are strictly provided in Section 4. Existences of the "infection-free" periodic solution and the "predator-free" solution are sufficiently analyzed by bifurcation theory of impulsive differential equation in Section 5. In Section 6, numerical simulations validate the correctness of theoretical analysis on this new biological model.

2. THE FORMULATION OF PEST CONTROL MODEL

In Ref. [36], Xie at al. provided the insect-pathogen model. In this paper, based on this model, we suppose that the pathogens are introduced at the pulse time and the natural enemies of pest are also introduced, and then we can obtain a new biological control model which is given as follows:

$$\begin{cases} \left.\begin{array}{lll} \dot{S}(t) & = [r - a\,(S(t)+I(t))]\,S(t) - \lambda S(t)I(t) - bS(t)Y(t) & = f_1(t) \\ \dot{I}(t) & = \lambda S(t)I(t) + (r-\beta - a\,(S(t)+I(t)))\,I(t) & = f_2(t) \\ \dot{Y}(t) & = \mu bS(t)Y(t) - d_1Y(t) & = f_3(t) \end{array}\right\} \begin{array}{l} t\neq nT, \\ t \neq (n+l-1)T, \end{array} \\ \Delta I(t) \;\; = \alpha,\ t=(n+l-1)T \\ \Delta Y(t) \;\; = R,\ t = nT, \end{cases} \tag{2.1}$$

where

$$\Delta I(t) = I(t^+) - I(t),\ \Delta Y(t) = Y(t^+) - Y(t),$$

$\alpha > 0$ represents the release amount of the infected pests at $t = (n+l-1)T$; R represents the release amount of predators at $t = nT$; T is the period of the impulsive effect and $n \in N^+$, $N^+ = \{1, 2 \cdots\}$, $0 < l < 1$. Throughout this paper, it is supposed that the mortality rate is smaller than the birth rate, which means $\beta < \gamma$, considering the illness.

3. PRELIMINARIES

In order to conveniently get the expected results in the next section, it is very necessary to use the following notations:

$$R_+ = [0, +\infty)\,,$$

$$R_+^n = \{x = (x_1, x_2, \cdots, x_n) \in R^n\, |x_i \geq 0, i = 1, 2, \cdots, n\}\,,$$

$$f = (f_1, f_2, f_3)^T\,.$$

Let $V : R_+ \times R_+^3 \to R_+$, then V is said to belong to class ν_0 if:

(1) V is piecewise continuous in $((n-1)T, (n+l-1)T] \times R_+^3$, and $((n+l-1)T, nT] \times R_+^3 (n \in N_+)$. For each $x \in R_+^3$, and there exists

$$\lim_{(t,Y)\to(n\tau^+,x)} V(t,Y) = V(n\tau^+, x);$$

(2) V satisfies the local Lipschitz condition in x.

Definition 3.1.
Let $V \in v_0$, then for $(t, x) \in ((n-1)T, (n+l-1)T] \times R_+^3$ or $((n+l-1)T, nT] \times R_+^3$, the upper right derivative of $V(t, x)$ with respect to the system (2.1) is defined as

$$D^+V(t,x) = \lim_{h\to 0} \sup \frac{1}{h}\left[V(t+h, x+hf(t,x)) - V(t,x)\right].$$

The solution of system (2.1) is piecewise continuous for $x : R_+ \to R_+^3$. $x(t)$ is continuous and differentiable in $((n-1)T, (n+l-1)T]$, and $((n+l-1)T, nT]$, $n \in N_+$. $\lim_{t\to nT^+} x(t) = x(nT^+)$ and $\lim_{t\to(n+l-1)T^+} x(t) = x((n+l-1)T^+)$ exist. The smoothness characteristic of f ensures the global existence and uniqueness of solutions of system (2.1) [26].

Definition 3.2. Refer to [36].

Noticing that variable can not be negative, we hope that solutions of system (2.1) can also keep non-negative during the time that the initial condition is non-negative. Therefore, we need to introduce the following conclusions:

Lemma 3.1. Suppose $x(t) = (S(t), I(t), Y(t))$ be a solution of system (2.1). If $x_0^+ \geq 0$, then $x(t) \geq 0$ for all $t \geq 0$. In addition, if $x_0^+ > 0$, then we have $x(t) > 0$ for all $t \geq 0$.

Proof. We suppose that there exists $t^* \in (0, T]$, then $S(t) \geq 0$, $I(t) \geq 0$, $Y(t) \geq 0$, $S(t^*) = 0$, $\dot{S}(t^*) < 0$, $I(t^*) \geq 0$, and $Y(t^*) \geq 0$ for all $t \in (0, t^*)$. According to the first equation of system (2.1), we have $\dot{S}(t^*) = 0$, and this is a contradiction. Using its first equation, we can obtain

$$S(t) = \begin{cases} S(0^+)e^{\int_0^t [\gamma - a(S(\xi)+I(\xi)-\lambda I(\xi)-by(\xi))]]\mathrm{d}\xi}, & t \in (0, lT], \\ S(lT^+)e^{\int_{lT}^t [\gamma - a(S(\xi)+I(\xi)-\lambda I(\xi)-by(\xi))]\mathrm{d}\xi}, & t \in (lT, T]. \end{cases}$$

□

Obviously, $S(t) \geq 0$ for $S(0^+) \geq 0$ and when $S(0^+) > 0$, $S(t) > 0$ for t $\in (0, T]$. Moreover, we can validate it for $I(t)$ and $Y(t)$. Next we will obtain an important result about the following impulsive differential equations.

Lemma 3.2. (Panetta [10]) Assume that $V \in v_0$ satisfies the following conditions:

$$\begin{cases} D^+V(t, x) \leq g(t, V(t, x)), & t \neq nT \\ V(t, x(t^+)) \leq \psi_n(V(t, x)), & t = nT, \end{cases} \tag{3.1}$$

where $g : R_+ \times R_+ \to R$ is continuous in $(nT, (n+1)T] \times R_+$. For $u \in R_+$, $n \in N$, $\lim\limits_{(t,v)\to(nT^+,u)} g(t, v) = g(nT^+, u)$ exists and $\psi_n : R_+ \to R_+$ is non-decreasing. We assume that $r(t)$ is the maximal solution of the scalar impulsive differential equation

$$\begin{cases} \dot{u}(t) = g(t, u(t)), & t \neq nT, \\ u(t^+) = \psi_n(u(t)), & t = nT, \\ u(0^+) = u_0 \end{cases}$$

which exists on $[0, \infty)$.

Therefore, we have $V(t, x(t)) \leq r(t)$ for $t \geq t_0$ according to $V(0^+, x_0) \leq u_0$, in which $x(t)$ is a solution of system (2.1).

Next we will obtain some basic properties about the following impulsive differential equations.

Lemma 3.3. Suppose $r-\beta>0, I_0\geq 0$, then system

$$\begin{cases} \dot{I}(t)=(r-\beta-aI(t))I(t), & t\neq (n+l-1)T \\ I(t^+)=I(t)+\alpha, & t=(n+l-1)T, \\ I(0^+)=I_0, \beta>0, \end{cases} \tag{3.2}$$

has a unique positive periodic solution

$$I^*(t)=\begin{cases} \frac{I^*(0^+)e^{(r-\beta)(t-(n-1)T)}(r-\beta)}{r-\beta-aI^*(0^+)+aI^*(0^+)e^{(r-\beta)(t-(n-1)T)}}, & t\in((n-1)T,(n+l-1)T], \\ \frac{[I^*(0^+)e^{(r-\beta)lT}+\alpha]e^{(r-\beta)(t-(n+l-1)T)}(r-\beta)}{r-\beta-a[I^*(0^+)e^{(r-\beta)lT}+\alpha]+a[I^*(0^+)e^{(r-\beta)lT}+\alpha]e^{(r-\beta)(t-(n+l-1)T)}}, & t\in((n+l-1)T,nT], \end{cases}$$

in which

$$I^*(0^+)=\frac{\omega+\sqrt{\omega^2+4a\alpha(r-\beta)(e^{(r-\beta)T}-1)}}{2a(e^{(r-\beta)T}-1)}$$

and

$$\omega=(r-\beta+a\alpha)(e^{(r-\beta)T}-1).$$

In addition, any solution $I(t)$ of system (3.2) can be given as

$$\begin{aligned} I(t) \quad = \quad & \frac{I(0^+)e^{(r-\beta)t}(r-\beta)}{r-\beta-aI(0^+)+aI(0^+)e^{(r-\beta)t}}-\frac{I^*(0^+)e^{(r-\beta)t}(r-\beta)}{r-\beta-aI^*(0^+)+aI^*(0^+)e^{(r-\beta)t}} \\ & +I^*(t), t\in((n-1)T,nT], \end{aligned}$$

and satisfies $|I(t)-I^*(t)|\to 0$ as $t\to\infty$.

We consider the following equation

$$\begin{cases} \dot{Y}(t)=-d_1Y(t), & t\neq nT, \\ Y(t^+)=Y(t)+R, & t=nT, \\ Y(0^+)=Y_0, \end{cases} \tag{3.3}$$

and it is easy to check that there exists the only positive periodic solution

$$Y^*(t)=\frac{\mathrm{Re}^{-d_1(t-nT)}}{1-e^{-d_1T}}.$$

When susceptible pest $S(t)$ is absent, system (2.1) will be transformed as

$$\begin{cases} \left.\begin{array}{l} \dot{I}(t)=(r-\beta-aI(t))\,I(t) \\ \dot{Y}(t)=-d_1Y(t) \end{array}\right\} t\neq nT,\ t\neq (n+l-1)T, \\ I(t^+)=I(t)+\alpha,\ t=(n+l-1)T \\ Y(t^+)=Y(t)+R,\ t=nT. \end{cases} \tag{3.4}$$

By Lemma 3.3, positive periodic solution $(I^*(t),Y^*(t))$ of system (3.4) is given as

$$I^*(t)=\begin{cases} \frac{I^*(0^+)e^{(r-\beta)(t-(n-1)T)}(r-\beta)}{r-\beta-aI^*(0^+)+aI^*(0^+)e^{(r-\beta)(t-(n-1)T)}}, & t\in((n-1)T,(n+l-1)T], \\ \frac{[I^*(0^+)e^{(r-\beta)lT}+\alpha]e^{(r-\beta)(t-(n+l-1)T)}(r-\beta)}{r-\beta-a[I^*(0^+)e^{(r-\beta)lT}+\alpha]+a[I^*(0^+)e^{(r-\beta)lT}+\alpha]e^{(r-\beta)(t-(n+l-1)T)}}, & t\in((n+l-1)T,nT], \end{cases}$$

$$Y^*(t) = Y^*(0^+)e^{-d_1(t-(n-1)T)}, \quad (n-1)T < t \le nT,$$

in which

$$Y^*(0^+) = \frac{R}{1 - e^{-d_1 T}}.$$

Moreover, any solution $(I(t), Y(t))$ of system (3.4) with initial value $(I(0^+), Y(0^+))$ can be given as

$$I(t) = \frac{I(0^+)e^{(r-\beta)t}(r-\beta)}{r-\beta-aI(0^+)+aI(0^+)e^{(r-\beta)t}} - \frac{I^*(0^+)e^{(r-\beta)t}(r-\beta)}{r-\beta-aI^*(0^+)+aI^*(0^+)e^{(r-\beta)t}} + I^*(t)$$

for $t \in ((n-1)T, nT]$ and

$$Y(t) = (Y(0^+) - Y^*(0^+))e^{-d_1 t} + Y^*(t), \qquad (n-1)T < t \le nT.$$

It is not difficult to obtain following results.

Lemma 3.4. Assume $Z^*(t) = (I^*(t), Y^*(t))$ is a positive periodic solution of system (3.4). For each solution $Z(t) = (I(t), Y(t))$ of system (3.4) with initial value $Z(0^+) = (I(0^+), Y(0^+))$, we have $\|Z(t) - Z^*(t)\| \to 0$ as $t \to \infty$.

Considering the ultimate boundedness of solutions of system (2.1), we have the following results.

Lemma 3.5. There exists a constant $M > 0$ such that, for any solution

$$x(t) = (S(t), I(t), Y(t))$$

of system (2.1) with initial value $x(0^+) = (S(0^+), I(0^+), Y(0^+)) \in R^3_+$, we have $S(t) \le M, I(t) \le M$ and $Y(t) \le M$ with t large enough.

Proof. Set $V(t) = S(t) + I(t) + \frac{1}{\mu}Y(t)$.

By calculating the derivative of V(t) with respect to system (2.1), when $t \ne (n+l-1)T$ and $t \ne nT$, we have

$$\begin{aligned} D^+V(t) &= (r - a(S(t)+I(t)))(S(t)+I(t)) - \beta I(t) - \tfrac{d_1}{\mu}Y(t) \\ &\le -\gamma V(t) + M_0, \end{aligned}$$

where $\gamma = \min\{\beta, \frac{d_1}{\mu}\}$ and

$$M_0 = \sup_{S(t)+I(t)>0} \{[r - a(S(t)+I(t))](S(t)+I(t))\} < \infty.$$

If $t = (n+l-1)T$, we have

$$V((n+l-1)T^+) \le V((n+l-1)T) + \alpha.$$

If $t = nT$, we can obtain

$$V(nT^+) \le V(nT) + R.$$

By Lemma 3.2, we can easily have

$$\lim_{t\to\infty}\sup V(t) \le \frac{M_0}{\gamma} + \frac{(\alpha+R)e^{\gamma T}}{e^{\gamma T}-1}.$$

According to this, there exists a constant $M > 0$ such that $S(t) \le M, I(t) \le M$ and $Y(t) \le M$ for t large enough. This completes the proof. □

In order to discuss the stability of semi-trivial periodic solution of system (2.1), we need to present the Floquet theory for the linear T-periodic impulsive equation:

$$\begin{cases} \dot{h}(t) = A(t)h(t), t \ne T_k, t \in R, \\ h(t^+) = h(t) + B_k h(t), t = T_k. \end{cases} \tag{3.5}$$

Firstly, we need to use the following assumptions:

(H1) $A(\cdot) \in PC(R, C^{n\times n})$ and $A(t+T) = A(t)(t \in R)$, in which $PC(R, C^{n\times n})$ is the set of all piecewise continuous matrix functions that are right continuous at $t = T_k$, and $C^{n\times n}$ is the set of all $n \times n$ matrices.

(H2) $B_k \in C^{n\times n}$, $\det(E + B_k) \ne 0$, $T_k < T_{k+1}(k = 1, 2, 3\ldots)$.

(H3) There exists a $q \in N$, such that $B_{k+q} = B_k$ and $T_{k+q} = T_k + T(k = 1, 2, 3\ldots)$. Suppose $\Phi(t)$ is a fundamental matrix of equations (3.4), then there exists a unique non-singular matrix $M \in C^{n\times n}$ that satisfies $\Phi(t + T) = \Phi(t)M, t \in R$.

The matrix M is called the monodromy matrix of (3.4) and it corresponds to the fundamental matrix of $\Phi(t)$. All the monodromy matrices of (3.4) are similar and have the same eigenvalues. The eigenvalues $\mu_1,\mu_2,\ldots,\mu_n$ of the monodromy matrices are called the Floquet multipliers of (3.4).

Lemma 3.6. (Lakshmikantham, Bainov and Simeonov [26], Floquet Theorem). Suppose that Hypotheses (H1) – (H3) hold, then the linear T-periodic impulsive (3.4) is

(1) stable if and only if all the multipliers $\mu_j(j = 1, 2, \ldots, n)$ of (3.4) satisfy the inequality $|\mu_j| \le 1$. Moreover, for those μ_j which satisfy $|\mu_j| = 1$, there correspond simple elementary divisors;

(2) asymptotically stable if and only if all the multipliers μ_j of (3.4) satisfy $|\mu_j| < 1$;

(3) unstable if $|\mu_j| > 1$ for some $j = 1, 2, \ldots, n$.

Next we will provide some necessary definitions and lemmas about the persistence of dynamical systems, and it will be used in the following discussion of permanence of system (2.1). For more details, see [34, 36].

Let X be a metric space with metric d, and let $f : X \to X$ be a continuous map. For any $x \in X$, we represent $f^n(x) = f(f^{n-1}(x))$ for any integer $n > 1$ and $f^1(x) = f(x)$. f is said to be compact in X, if for any bounded set $H \subset X$, set $f(H) = \{f(x) : x \in H\}$ is precompact in X. f is said to be point dissipative if there is a bounded set $B_0 \subset X$ such that, for any $x \in X$, $\lim_{n\to\infty} d(f^n(x), B_0) = 0$.

For any $x_0 \in X$, the positive semi-orbit through x_0 is defined by

$$\gamma^+(x_0) = \{f^n(x_0) = x_n, n = 1, 2, \cdots\}.$$

The negative semi-orbit through x_0 is defined as a sequence $\gamma^-(x_0) = \{x_k\}$ which satisfy $f(x_{k-1}) = x_k$ for integers $k \leq 0$. The omega limit set of $\gamma^+(x_0)$ is defined by

$$\varpi(x_0) = \{y \in X : \text{there is a sequence } n_k \to \infty \text{ such that } \lim_{k\to\infty} x_{n_k} = y\},$$

and the alpha limit set of $\gamma^-(x_0)$ is defined by

$$\alpha(x_0) = \{y \in X : \text{ there is a sequence } n_k \to -\infty \text{ such that } \lim_{k\to\infty} x_{n_k} = y\}.$$

A nonempty set B $\subset X$ is said to be invariant if f(B) $\subseteq B$. A nonempty set M of B is called isolated in X if it is the maximal invariant set in a neighborhood of itself. For a nonempty set M of X, set

$$W^s(M) := \{x \in X : \lim_{n\to\infty} d(f^n(x), M)\}$$

is called the stable set of M.

Let M_1 and M_2 be two isolated invariant sets and set M_1 is said to be chained to set M_2, usually expressed as $M_1 \to M_2$, if there exists a full orbit though some $x \notin M_1 \cup M_2$ such that $\varpi(x) \subset M_2$ and $\alpha(x) \subset M_1$.

A finite sequence $\mu = \{M_1, M_2, \cdots, M_n\}$ of isolated invariant sets is called a chain if $M_1 \to M_2 \to \cdots \to M_n$, and the chain is called a cycle if $M_n = M_1$.

Let X_0 be a nonempty open set of X. We denote

$$\partial X_0 := X \backslash X_0,$$

$$M_\partial := \{x \in \partial X_0 : f^n(x) \in \partial X_0, \forall n \geq 0\}$$

Lemma 3.7. (Li et al. [34], Xie et al. [36]) Suppose $f : X \to X$ is a continuous map. It is assumed that the following conditions hold:

(C_1) Map f is compact and point dissipative and $f(X_0) \subseteq X_0$;

(C_2) There exists a finite sequence $\mu = \{M_1, M_2, \cdots, M_n\}$ of compact and isolated invariant sets in ∂X_0 such that

(1) $M_i \cap M_j = \emptyset$ for any $i, j = 1, 2, \cdots, n$ and $i \neq j$;

(2) $\Omega(M_\partial) := \bigcup_{x\in M_\partial} \varpi(x) \subset \bigcup_{i=1}^{n} M_i$;

(3) no subset of μ forms a cycle in ∂X_0;

(4) $W^s(M_i) \cap X_0 = \emptyset$ for each $1 \leq i \leq n$.

The map f is uniformly persistent with respect to $(X_0, \partial X_0)$; that is, there exists a constant $\eta > 0$ such that

$$\lim_{n\to\infty} \inf d(f^n(x), \partial X_0) \geq \eta$$

for all $x \in X_0$.

In the following section, we will investigate the stability of the susceptible pest-eradication periodic solution.

It is not difficult to have that system (2.1) has a semi-trivial periodic solution $X(t) = (0, I^*(t), Y^*(t))$, where $I^*(t)$ and $Y^*(t)$ are the positive periodic solution of the system (3.4). Exploiting the global asymptotic stability of this periodic solution, we can easily obtain the following theorem.

4. THE STABILITY AND PERSISTENCE OF SEMI-TRIVIAL PERIODIC SOLUTIONS.

Theorem 4.1. If $r > \beta, \lambda \geq$ a and

$$\int_0^T (r - (a+\lambda)I^*(t) - bY^*(t))\,\mathrm{d}t < 0,$$

then the semi-trivial periodic solution $(0, I^*(t), Y^*(t))$ of system (2.1) is globally asymptotical stable.

Proof. First, we investigate the local stability of susceptible pest eradication periodic solution $(0, I^*(t), Y^*(t))$.

Suppose $u_1(t) = S(t)$, $u_2(t) = I(t) - I^*(t)$, and $u_3(t) = Y(t) - Y^*(t)$, then we can obtain

$$\begin{array}{l} \Delta u_2(t) = 0,\ t = (n+l-1)T,\ 0 < l < 1. \\ \Delta u_3(t) = 0,\ t = nT. \end{array}$$

The corresponding linearized system is given by

$$\left\{ \begin{array}{l} \dot{u_1}(t) = (r - (a+\lambda)I^*(t) - bY^*(t))\, u_1(t) \\ \dot{u_2}(t) = (\lambda - a)I^*(t)u_1(t) + (r - \beta - 2aI^*(t))u_2(t) \\ \dot{u_3}(t) = \mu bY^*(t)u_1(t) - d_1u_3(t). \end{array} \right. \tag{4.1}$$

We assume that $\Phi(t)$ is the fundamental matrix of system (4.1), and then we have

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} r - (a+\lambda)I^*(t) - bY^*(t) & 0 & 0 \\ (\lambda - a)I^*(t) & r - \beta - 2aI^*(t) & 0 \\ \mu bY^*(t) & 0 & -d_1 \end{pmatrix} \Phi(t)$$

with $\Phi(0) = I$ a 3×3 identity matrix.

According to calculation, we have

$$\Phi(t) = \begin{pmatrix} e^{\int_0^t (r-(a+\lambda)I^*(s)-bY^*(s))\,\mathrm{d}s} & 0 & 0 \\ * & e^{\int_0^t (r-\beta-2aI^*(s))\,\mathrm{d}s} & 0 \\ ** & *** & e^{-d_1 t} \end{pmatrix}.$$

It is not required in the following discussion, so it is not necessary to give the specific forms of $*, **, ***$, which can be discarded. Obviously, the eigenvalues of matrix $M = \Phi(T)$ are given as follows [17]:

$$\lambda_2 = e^{\int_0^T (r-\beta-2aI^*(s))\,\mathrm{d}s} = \ldots < 1;$$

$$\lambda_3 = e^{-d_1 t} < 1;$$

$$\lambda_1 = e^{\int_0^T (r-(a+\lambda)I^*(s)-bY^*(s))\,\mathrm{d}s}.$$

Hence while $|\lambda_1| < 1$, i.e., $e^{\int_0^T (r-(a+\lambda)I^*(t)-bY^*(t))\,\mathrm{d}t} < 1$. According to the Floquet theory, we have that the solution $(0, I^*(t), Y^*(t))$ is locally asymptotically stable.

Next we will prove the global attractivity of $(0, I^*(t), Y^*(t))$. We first choose $\varepsilon_1 > 0$ such that

$$\delta = e^{\int_0^T [r-(a+\lambda)(I^*(t)-\varepsilon_1)-b(Y^*(t)-\varepsilon_1)]\mathrm{d}t} < 1.$$

By Lemmas 3.2 and 3.3, we have

$$I(t) \geq I^*(t) - \varepsilon_1, Y(t) \geq Y^*(t) - \varepsilon_1 \tag{4.2}$$

for sufficiently large t.

So we may suppose that (4.2) holds for all $t \geq 0$, which together with system (2.1) and equation (4.2), and then we can obtain

$$\begin{aligned} S'(t) &= (r - a(S(t) + I(t)))S(t) - \lambda S(t)I(t) - bS(t)Y(t) \\ &\leq S(t)(r - (a+\lambda)(I^*(t) + \varepsilon_1) - b(Y^*(t) + \varepsilon_1)). \end{aligned}$$

For any $t > 0$, we can choose an integer $n \geq 0$ such that $t = nT + \widetilde{t}$, where $\widetilde{t} \in [0, T)$. Integrating the above inequality from 0 to t, we have

$$\begin{aligned} S(t) &\leq S(0^+)e^{\int_0^t [r-(a+\lambda)(I^*(s)+\varepsilon_1)-b(Y^*(s)+\varepsilon_1)]\,\mathrm{d}s} \\ &= S(0^+)e^{\int_0^{nT} [r-(a+\lambda)(I^*(s)+\varepsilon_1)-b(Y^*(s)+\varepsilon_1)]ds} e^{\int_{nT}^{nT+\tilde{t}} [r-(a+\lambda)(I^*(s)+\varepsilon_1)-b(Y^*(s)+\varepsilon_1)]\,\mathrm{d}s} \\ &\leq S(0^+)e^{M_0 T}\delta^n, \end{aligned}$$

where

$$M_0 = \sup_{t\geq 0}[r - (a+\lambda)(I^*(t) + \varepsilon_1) - b(Y^*(t) + \varepsilon_1)] > 0,$$

which implies $S(t) \to 0$ as $t \to \infty$. So we can give a conclusion that the solution $(0, I^*(t), Y^*(t))$ is globally asymptotical stable. We will investigate the permanence of system (2.1) in the next part. □

Theorem 4.2. If $r - \beta > 0$, $\lambda \geq a$ and

$$\int_0^T [r - (a+\lambda)I^*(t) - bY^*(t)]\,\mathrm{d}t > 0, \tag{4.3}$$

then system (2.1) is permanent.

P r o o f . Noticing that the impulsive effects in system are periodic, system (2.1) can be regarded as periodic system with period T. So we can exploit the persistence theory of dynamical systems to analyze the permanence of system (2.1).

Herein we define

$$X = \{(S(t), I(t), Y(t)) : S(t) \geq 0,\ I(t) \geq 0,\ Y(t) \geq 0\}$$

$$X_0 = \{(S(t), I(t), Y(t)) : S(t) > 0,\ I(t) \geq 0,\ Y(t) \geq 0\},$$

and thus we have

$$\partial X_0 = X \backslash X_0 = \{(S(t), I(t), Y(t)) \in X : S(t) = 0\}.$$

According to Lemma 3.1, we can conclude that X and X_0 are positively invariant with respect to system (2.1), and ∂X_0 is relatively closed set in X.

Let $P : X \to X$ be a Poincaré map which is associated with system (2.1); that is

$$P(S_0, I_0, Y_0) = u(T, S_0, I_0, Y_0), (S_0, I_0, Y_0) \in X,$$

where $u(T, S_0, I_0, Y_0)$ is the unique solution of system (2.1) with initial value

$$u(0^+, S_0, I_0, Y_0) = (S_0, I_0, Y_0).$$

From Lemma 3.4, Poincaré map P is compact and point dissipative on X. Therefore, condition (C_1) of lemma 3.7 holds.

Suppose

$$M_\partial = \{(\mathrm{S}_0, \mathrm{I}_0, \mathrm{Y}_0) \in \partial X_0 :\ P^n(\mathrm{S}_0, \mathrm{I}_0, \mathrm{Y}_0) \in \partial X_0, n = 1, 2, \ldots\},$$

in which

$$P^n = P(P^{n-1}),\ n > 1,$$

and $P^1 = P$.

Firstly, we will prove $M_\partial = \partial X_0$.

Clearly,

$$M_\partial \subseteq \partial X_0.$$

For any $(0, I_0, Y_0) \in \partial X_0$, by $S_0 = 0$, the solution $(S(t), I(t), Y(t))$ of system (2.1) with initial value

$$(S(0^+), I(0^+), Y(0^+)) = (0, I_0, Y_0)$$

satisfies $S(t) = 0, I(t) \geq 0$, and $Y(t) \geq 0$ for all $t \geq 0$.

So for any integer $n > 0$, we have $P^n(0, I_0, Y_0) \in \partial X_0$. This implies $(0, I_0, Y_0) \in M_\partial$. Then $M_\partial = \partial X_0$ holds.

System (2.1) can be simplified as (3.4) in ∂X_0. According to Lemma 3.4, System (2.1) has globally attractive periodic solution $(0, I^*(t), Y^*(t))$ in ∂X_0. This shows that Map P has a global attractor $M_1 = \{(0, I^*(0), Y^*(0))\}$ in ∂X_0.

It is easy to check that, ∂X_0, $\{M_1\}$ is isolated, invariant, and does not form a cycle. Therefore, conditions (1) – (3) of (C_2) in Lemma 3.7 hold.

Secondly, assume $x_0 = (S_0, I_0, Y_0) \in X_0$. By the continuity of solutions with respect to the initial value, for any $\varepsilon > 0$, there is a $\delta_1 > 0$, when $\|x_0 - M_1\| < \delta_1$, we can obtain

$$\|u(t, x_0) - u(t, M_1)\| < \varepsilon, \ \forall t \in [0, T]. \tag{4.4}$$

Then we have

$$\lim_{n\to\infty} \sup d(P^n(x_0), M_1) \geq \delta_1. \tag{4.5}$$

Assume that the conclusion is not true, then we have

$$\lim_{n\to\infty} \sup d(P^n(x_0), M_1) < \delta_1$$

for some $x_0 \in X_0$.

For the sake of simplicity, we may suppose that

$$d(P^n(x_0), M_1) < \delta_1, \ \forall n > 0.$$

In addition, exploiting (4.4), we have that

$$\|u(t, P^n(x_0)) - u(t, M_1)\| < \varepsilon, \ \forall n > 0, \ t \in [0, T].$$

Then, for any $t \geq 0$, let $t = nT + \widetilde{t}$, where $\widetilde{t} \in [0, T)$ and $n = [\frac{t}{T}]$ is the greatest integer less than or equal to $\frac{t}{T}$, we can obtain

$$\|u(t, x_0) - u(t, M_1)\| = \left\|u(\widetilde{t}, P^n(x_0)) - u(\widetilde{t}, M_1)\right\| < \varepsilon. \tag{4.6}$$

Since $u(t, x_0) = (S(t), I(t), Y(t))$ and

$$u(t, M_1) = (0, I^*(t), Y^*(t)),$$

(4.6) signifies that

$$\begin{cases} 0 < S(t) < \varepsilon \\ I(t) \leq I^*(t) + \varepsilon \\ Y(t) \leq Y^*(t) + \varepsilon \end{cases} \tag{4.7}$$

for all $t \geq 0$.

According to (4.3), we can choose $\varepsilon > 0$ such that

$$\rho = e^{\int_0^T [r - a(\varepsilon + I^*(t) + \varepsilon) - \lambda(I^*(t)+\varepsilon) - b(Y^*(t)+\varepsilon)]\,\mathrm{d}t} = e^{\int_0^T \rho_0(t,\varepsilon)\,\mathrm{d}t} > 1.$$

Moreover, by inequalities (4.7), we can obtain

$$\begin{aligned} S'(t) &= S(t)[r - a(I(t) + S(t)) - \lambda I(t) - bY(t)] \\ &\geq S(t)[r - a(\varepsilon + I^*(t) + \varepsilon) - \lambda(I^*(t) + \varepsilon) - b(Y^*(t) + \varepsilon)]. \end{aligned} \tag{4.8}$$

For any $t \geq 0$, choose an integer $k \geq 0$ such that $t = kT + \widehat{t}$, where $\widehat{t} \in [0, T)$. Integrating (4.8) from 0 to t, then we have

$$\begin{aligned} S(t) &\geq S(0^+) e^{\int_0^t \rho_0(s,\varepsilon)\,\mathrm{d}s} \\ &= S(0^+) e^{k\int_0^T \rho_0(s,\varepsilon)ds + \int_{kT}^{kT+\widehat{t}} \rho_0(s,\varepsilon)\,\mathrm{d}s} \geq \frac{S(0^+)}{e^{TN_0}} \rho^k, \end{aligned}$$

in which

$$N_0 = \sup_{t\geq 0} \rho_0(t,\varepsilon) > 0.$$

Therefore

$$\lim_{t\to\infty} S(t) = \infty,$$

which is a contradiction with $0 \leq S(t) \leq \varepsilon$ for all $t \geq 0$. Hence claim (4.5) holds. This shows $W^s(M_1) \cap X_0 = \emptyset$. So condition (4) of (C_2) holds. Therefore, exploiting Lemma 3.7, P is uniformly persistent with respect to $(X_0, \partial X_0)$.

Lastly, Noticing that system (2.1) is periodic, we have that system (2.1) is uniformly persistent. By Lemma 3.5, system (2.1) is permanent. So we complete the proof of Theorem 4.2. □

5. EXISTENCE OF THE "INFECTION-FREE" PERIODIC SOLUTION AND THE "PREDATOR-FREE" SOLUTION.

5.1. Preliminaries

In this section, we will analyze the existence of the "infection-free" and the "predator-free" periodic solutions exploiting the technique used by A. Lakmeche in [1]. Preliminaries in Section 4.1 of Ref. [36] are still introduced to obtain the later results.

Lemma 5.1. (Lakmeche [1], Xie et al. [36]) If $|1-a_0^{'}| < 1$ and $d_0^{'} = 0$, then we can obtain:

(a) If $BC \neq 0$, then we have a bifurcation. Moreover, we have a bifurcation of nontrivial periodic solution of system (4.1), if $BC < 0$, and a subcritical case if $BC > 0$.

(b) If $BC = 0$, then we have an undetermined case.

5.2. Existence of the "infection-free" periodic solution

Herein we consider the following set of differential equations:

$$\begin{cases} \left.\begin{aligned} \dot{S}(t) &= \mu b S(t)Y(t) - d_1 S(t) && \triangleq F_1(S,Y) \\ \dot{Y}(t) &= (r-aY(t))Y(t) - bS(t)Y(t) && \triangleq F_2(S,Y) \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} S(t^+) &= S(t) + R && \triangleq \theta_1(S,Y) \\ Y(t^+) &= Y(t) && \triangleq \theta_2(S,Y) \end{aligned}\right\} t = nT,\ n = 0,1,2,\ldots \end{cases}$$

In order to successfully use the conclusion of Lemma 5.1, we need to compute:

$$d_0^{'} = 1 - e^{\int_0^{T_0} \frac{\partial F_2(\pi(\zeta))}{\partial Y}\,\mathrm{d}\zeta} = 1 - e^{\int_0^{T_0}(r-bS^*(\zeta))\,\mathrm{d}\zeta},$$

if $d_0^{'} = 0$, we can obtain the formula of T_0. Moreover,

$$a_0^{'} = 1 - e^{\int_0^{T_0} -d_1\,\mathrm{d}t} = 1 - e^{-d_1 T_0} > 0,$$

$$b_0' = -\int_0^{T_0} e^{\int_v^{T_0} -d_1 \, \mathrm{d}\xi} \mu b s^*(v) e^{\int_0^v -bs^*(\xi)\, \mathrm{d}\xi} \mathrm{d}v < 0.$$

Noticing $\frac{\partial \theta_1}{\partial Y} = \frac{\partial \theta_2}{\partial S} = 0$, $\frac{\partial \theta_1}{\partial S} = \frac{\partial \theta_2}{\partial Y} = 1$, $\frac{\partial^2 \theta_2}{\partial Y^2} = \frac{\partial^2 \theta_2}{\partial S \partial Y} = 0$, it is easy to verify that $C > 0$, and if

$$rT_0 - \int_0^{T_0} b\widetilde{S}(\xi)\, \mathrm{d}\xi > 0,$$

then

$$B = -\left(\frac{1}{a_0'} \tilde{S}'(T_0) \frac{\partial^2 \Phi_2(T_0, V_0)}{\partial S \partial Y} + [r - b\tilde{S}(T_0)] e^{\int_0^{T_0} [r - b\tilde{S}(\zeta)]\, \mathrm{d}\zeta} \right) < 0.$$

By Lemma 5.1, we can have the following results.

Theorem 5.1. (Lakmeche [1]) System (2.1) has a positive periodic "infection" solution if $T > T_0$ and is close to T_0, in which T_0 is the root of $d_0{}' = 0$.

This theorem illustrates that the "semi-trivial" solution becomes unstable if the periodic T is more than T_0 and close to T_0, and becomes the "infection-free" solution.
5.3. Existence of the "predator-free" periodic solution.

We consider the following set of differential equations

$$\begin{cases} \left.\begin{aligned} \dot{S}(t) &= \lambda S(t)I(t) + [r - \beta - a(S(t) + I(t))]\, S(t) && \triangleq F_1(S, I) \\ \dot{I}(t) &= [r - a(S(t) + I(t))]\, I(t) - \lambda S(t) I(t) && \triangleq F_2(S, I) \end{aligned}\right\} t \neq (n + l - 1)T, \\ \left.\begin{aligned} S(t^+) &= S(t) + \alpha && \triangleq \theta_1(S, I) \\ I(t^+) &= I(t) && \triangleq \theta_2(S, I) \end{aligned}\right\} t = (n + l - 1)T. \end{cases}$$

We first compute the following:

$$d_0' = 1 - e^{\int_0^{T_0'} \frac{\partial F_2(\pi(\zeta))}{\partial I} \mathrm{d}\zeta} = 1 - e^{rT_0' - \int_0^{T_0'} (\lambda \tilde{S}(\zeta) + a\tilde{S}(\zeta))\, \mathrm{d}\zeta}.$$

If $d_0' = 0$, we can obtain T_0'. Moreover,

$$a_0' = 1 - e^{\int_0^{T_0'} (r - \beta - 2a\tilde{S}(t))\, \mathrm{d}t} > 0,$$

$$b_0' = -\int_0^{T_0'} e^{\int_v^{T_0'} (r - \beta - 2a\tilde{S}(\zeta))\, \mathrm{d}\zeta} (\lambda - a)\tilde{S}(v) e^{\int_0^v \frac{\partial F_2}{\partial I} \mathrm{d}\zeta} \mathrm{d}v > 0.$$

Noticing $\frac{\partial \theta_1}{\partial I} = \frac{\partial \theta_2}{\partial S} = 0$, $\frac{\partial \theta_1}{\partial S} = \frac{\partial \theta_2}{\partial I} = 1$, $\frac{\partial^2 \theta_2}{\partial I^2} = \frac{\partial^2 \theta_2}{\partial S \partial I} = 0$, we can easily check that $C > 0$, and if

$$\int_0^{T_0'} [r - (\lambda + a)\tilde{S}(\zeta)]\, \mathrm{d}\zeta < 0,$$

then

$$B = -\left(\frac{1}{a_0'} \widetilde{S}'(T_0') \frac{\partial^2 \Phi_2(T_0', V_0)}{\partial S \partial I} + [r - a\widetilde{S}(T_0') - \lambda \widetilde{S}(T_0')] e^{\int_0^{T_0'} [r - a\widetilde{S}(\xi) - \lambda \widetilde{S}(\xi)]\, \mathrm{d}\xi}\right) < 0.$$

From Lemma 5.1, we can obtain the following results.

Theorem 5.2. (Lakmeche [1]) System (2.1) has a positive periodic solution "predator-free" solution if $T > T_0^{'}$, and is close to $T_0^{'}$, where $T_0^{'}$ is the root of $d_0^{'} = 0$.

This theorem illustrates that the "semi-trivial" solution becomes unstable if the period T is more than $T_0^{'}$ and close to $T_0^{'}$, and the variable $I(t)$ begins to oscillate with an amplitude, and becomes the "predator-free" solution.

6. NUMERICAL SIMULATIONS

In this section, we will use numerical simulation figures to confirm the results of our theoretical analysis and further illustrate the complex and rich characteristics of the proposed model. In recent years, many researchers have developed a great deal of management technology which is used in ecosystem and agriculture. We can conveniently take impulsive control strategy in controlling or even eliminating pests by using some advanced technology and thus can prevent crops from being harm to a great extent.

Firstly, when we choose $r = 2.1,\ \lambda = 0.1,\ a = 0.5,\ \mu = 0.8,\ \beta = 1,\ \alpha = 0.005,\ T = 1.23,\ b = 0.5,\ R = 1,\ d = 0.5,\ l = 0.3$, by verification, we can easily find that these parameters satisfy the sufficient conditions of Theorem 4.1, and thus the mature predator-extinction periodic solution is globally attractive. Its corresponding dynamic behavior can be shown in Figure 1. The phase portraits of S(t), I(t) and Y(t) is shown in Figure 1 (a). The phase portrait of S(t) and t is shown in Figure 1(b), from which we can find that S(t) goes to extinction. The phase portrait of I(t) and t is shown in Figure 1(c), from which it can be found that I(t) goes oscillatory. Figure 1(d) shows the phase portrait of Y(t) and t and it also shows that Y(t) goes oscillatory. Moreover, it can be also found that the periodic solution of system (2.1) is globally attractive.

Secondly, we analyze and discuss the permanence of system (2.1). When we choose parameters $r = 2.1, \lambda = 0.1, a = 0.5, \mu = 0.8, \beta = 1, \alpha = 0.005, T = 12, b = 0.5, R = 1, d = 0.5, l = 0.3$, we can easily check that they satisfy the sufficient conditions of Theorem 4.2 in previous Section. Therefore, system (2.1) is permanent. Figure 2 (b) exhibits the positive time series of predator species in the interval [260, 400]. The positive time series of prey species in the interval [0, 100] are clearly shown in Figure 2 (c) and (d). It can be shown from Figure 2 that time series of three species are stable if t is large enough. Therefore, these numerical simulation figures sufficiently demonstrate the permanence of system (2.1).

7. CONCLUSIONS

In this paper, we have studied a new impulsive predator prey model with impulsive control at different fixed moments. We have confirmed that If $r > \beta, \lambda \geq a$ and $\int_0^T (r-(a+\lambda)I^*(t)-bY^*(t))\,\mathrm{d}t<0$, then the semi-trivial periodic solution $(0, I^*(t), Y^*(t))$ of system (2.1) is globally asymptotical stable by using Floquet theories and small amplitude perturbation technique. Moreover, we proved that If $r - \beta < 0,\ \lambda \geq a$ and $\int_0^T [r - (a + \lambda)I^*(t) - bY^*(t)]\,\mathrm{d}t > 0$, then system (2.1) is permanent. We also obtain sufficient conditions for the existence of the "infection-free" periodic solution and the "predator-free" solution by bifurcation theory of impulsive differential equation. In addition, we can drive the susceptible pest to extinction, exploiting the impulsive control

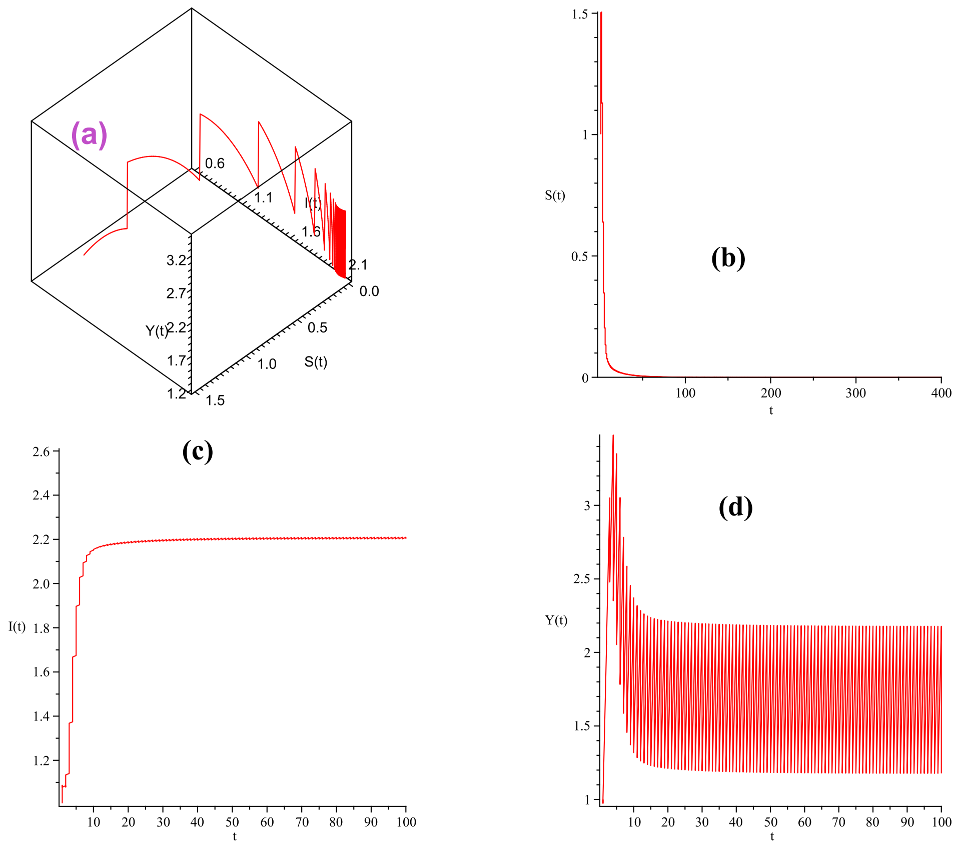

Fig. 1. Numerical solution of system (2.1) with $r = 2.1, \lambda = 0.1, a = 0.5, \mu = 0.8, \beta = 1, \alpha = 0.005, T = 1.23, b = 0.5, R = 1, d = 0.5, l = 0.3$, and all the parameters satisfy $\int_0^T (r - (a + \lambda)I^*(t) - bY^*(t))\,\mathrm{d}t < 0$. (a) Phase (S(t), I(t), Y(t)) (b) Time-series of the susceptible pest species S(t). (c) Time-series of the corresponding infected pest species I(t). (d) Time-series of natural enemies of the pest Y(t).

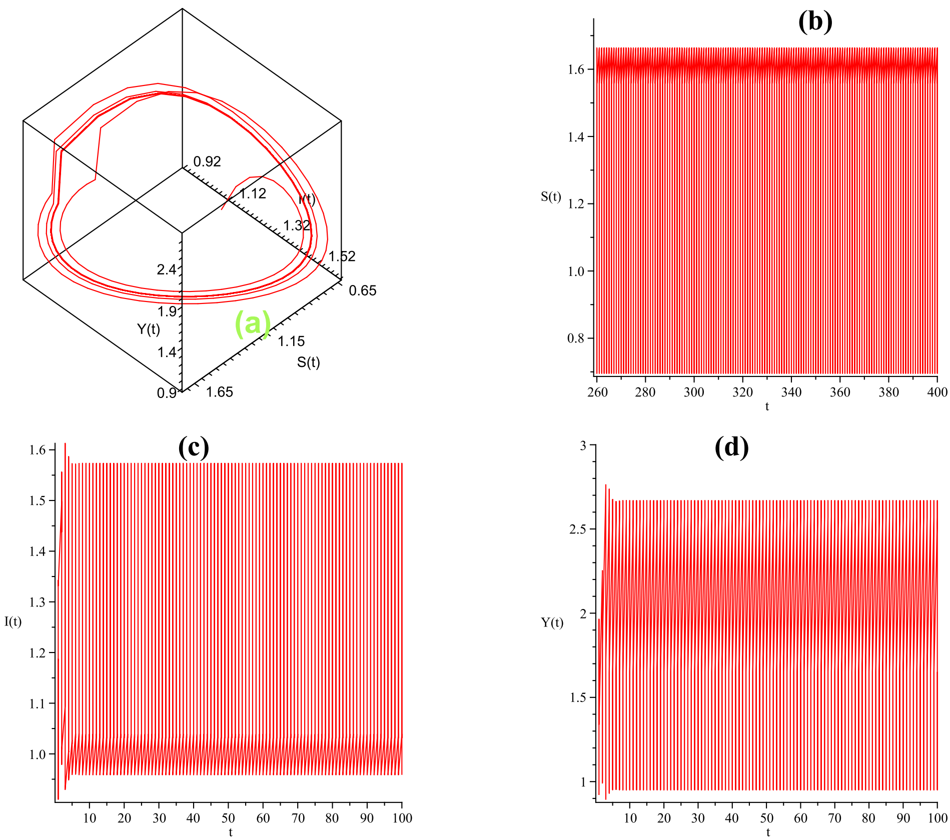

Fig. 2. Numerical solution of system (2.1) with $r = 2.1, \lambda = 0.1, a = 0.5, \mu = 0.8, \beta = 1, \alpha = 0.005, T = 12, b = 0.5, R = 1, d = 0.5, l = 0.3$, and all the parameters satisfy $\int_0^T [r - (a+\lambda)I^*(t) - bY^*(t)]\,\mathrm{d}t > 0$. (a) Phase (S(t), I(t), Y(t)). (b) Time-series of the susceptible pest species S(t). (c) Time-series of the corresponding infected pest species I(t). (d) Time-series of natural enemies of the pest Y(t).

strategy, as well as the effect of the viruses on the ecosystem environment and cost of the releasing pest which is infected in a laboratory such that $T < T_0 \approx 1.24$. Therefore, in practice, we can be sure that the reasonable impulse constant control should be taken to manage the agricultural resources. Moreover, it can effectively make the ecosystem display more unpredictable and interesting dynamic characteristics.

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Lin Jun Wang, Hubei key Laboratory of Hydroelectric Machinery Design and Maintenance, College of Mechanical and Power Engineering, China Three Gorges University, Yichang, Hubei 443002. P. R. China.
e-mail: ljwang2006@126.com

You Xiang Xie, College of Science Technology, China Three Gorges University, Yichang, Hubei 443002. P. R. China.
e-mail: xieyouxiangxie@126.com

Qi Cheng Deng, Hubei key Laboratory of Hydroelectric Machinery Design and Maintenance, College of Mechanical and Power Engineering, China Three Gorges University, Yichang, Hubei 443002. P. R. China.
e-mail: 578420767@qq.com