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ASYNCHRONOUS SAMPLING-BASED LEADER-FOLLOWING CONSENSUS IN SECOND-ORDER MULTI-AGENT SYSTEMS

Zhengxin Wang, Yuanzhen Feng, Cong Zheng, Yanling Lu, and Lijun Pan

This paper studies the leader-following consensus problem of second-order multi-agent systems with directed topologies. By employing the asynchronous sampled-data protocols, sufficient conditions for leader-following consensus with both constant velocity leader and variable velocity leader are derived. Leader-following quasi-consensus can be achieved in multi-agent systems when all the agents sample the information asynchronously. Numerical simulations are provided to verify the theoretical results.

Keywords: leader-following, multi-agent systems, consensus, asynchronous sampling

Classification: 93D05, 93C57

1. INTRODUCTION

Recently, there has been considerable interest in consensus of multi-agent systems [17, 20, 23, 24, 25, 32], which is the fundamental problem for the coordinated control of multi-agent systems. According to the absence and presence of a leader, consensus can be classified as leaderless consensus [2, 13, 28, 33] and leader-following consensus [8, 12, 14, 15, 18, 22, 27, 29, 37].

Leader-following consensus is a common phenomenon in the nature and also can be viewed as a tracking problem meaning that the followers of multi-agent systems can track the leader. Based on a neighbor-based local controller, a tracking consensus problem for multi-agent systems with an active leader and variable topology was studied in [15]. By proposing the distributed observers, a leader-follower consensus problem for a multi-agent system was solved in [14]. In [22], a leader-following coordination problem for a multi-agent system with a varying-velocity leader was investigated. In [37], both fixed and switching topologies were considered for a leader-following consensus problem of multi-agent systems. Based on the pinning control, sufficient criteria for guaranteeing leader-following consensus of nonlinear multi-agent systems were obtained in [27]. Finite-time consensus for leader-following multi-agent systems was studied in [8] and [18]. Quasi-consensus (or bounded consensus) for leader-following multi-agent systems was investigated in [29].

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Many control schemes have been adopted to solve the consensus problem of multi-agent systems, such as pining control, sampling control and impulsive control. Due to the digital technology of controller implementation and low communication cost, sampled-data controls are usually employed for the networked systems. The sampled-data controls of both deterministic sampling [5, 13, 19, 35] and stochastic sampling [11, 26] are effective to consensus for multi-agent systems. Most sampling controls of the aforementioned results are synchronous, that is, all the agents sample the information at the same sampling instants. Asynchronous sampling, means that all the agents differ in the sampling instants, is more practical than synchronous sampling. Therefore, a number of literatures studied the consensus problems of multi-agent systems by adopting asynchronous sampling controls. Generally, two kinds of asynchronous sampling schemes have been proposed. One is that all the agents have different sampling instants with each other but each agent and its neighbours must sample the information synchronously at each itself sampling instant [5, 6]. The other type demands that all the agents differ in their sampling instants and each agent only samples the information at its own sampling instants [3, 31, 36]. Motivated by the most recent work of [3], this paper employs the asynchronous sampled-data protocols to solve the leader-following consensus problems for second-order multi-agent systems.

Usually, consensus or complete synchronization is an objective for the networked systems [9, 13, 23, 27, 28, 32]. As the heterogeneity of self-dynamics [3, 10, 30] or the asynchronization of information update [29, 34], complete consensus of the networked systems usually can not be reached if only static controls are adopted. Instead, only quasi-consensus (-synchronization) or bounded consensus (synchronization) in the networked systems can be achieved. By applying the asynchronous sampling protocols, this paper investigates the leader-following quasi-consensus problems of second-order multi-agent systems over the directed topologies.

The main contributions are listed as follows. First, asynchronous sampled-data-based protocols are proposed. All the agents differ in sampling instants. Furthermore, each agent is only sampled at its own sampling instants and not available at sampling instants of its neighbours. Second, sufficient criteria for leader-following quasi-consensus of multi-agent systems are derived. The leaders with both constant velocity and variable velocity are considered. In addition, the upper bound of quasi-consensus errors is solved.

The rest of this paper is organized as follows. Section 2 provides some preliminaries. In Section 3, the problem is first formulated. Next, sampling-based leader-following consensus is solved for second-order multi-agent systems with a leader of constant velocity. Then, second-order multi-agent systems with a leader of variable velocity are further studied. Some sufficient conditions for leader-following quasi-consensus are derived. Some simulation results are given to illustrate the theoretical results in Section 4. Finally, conclusions are summarized in Section 5.

2. PRELIMINARIES

In this section, notations, basic concepts of graph theory and supporting lemmas are introduced.

2.1. Notations

The following notations are adopted throughout the paper. $\mathbb{R}^n$ denotes the n-dimensional Euclidian space, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. Let 0_n be the $n\times 1$ column vector with all entries equal to zero. Let O_n and I_n be the $n\times n$ zero matrix and identity matrix, respectively. $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ denotes a block-diagonal matrix with square matrix A_i being its ith diagonal block matrix. A^T means transpose for a real matrix A. The symbol $*$ in a symmetric matrix represents the symmetric elements. The matrix inequality $P > 0$ means that the symmetric matrix P is positive definite. Let $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ be the minimal and maximal eigenvalues of a symmetric square matrix P, respectively. Denote the Euclidian norm by $\|x\|$ for all $x \in \mathbb{R}^n$.

2.2. Graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a digraph with the set of vertices $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_N\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N} \in \mathbb{R}^{N\times N}$ with $a_{ij} \geq 0$. An directed edge from ν_j to ν_i of $\mathcal{G}$ is denoted by the ordered pair of vertices $e_{ji} = (\nu_j, \nu_i)$. $e_{ji} \in \mathcal{E}$ if and only if $a_{ij} > 0$. It is assumed that $a_{ii} = 0$. The set of neighbors of vertex ν_i is represented as $\mathcal{N}_i = \{\nu_j | e_{ji} \in \mathcal{E}\}$. A directed path from ν_j to ν_i is a sequence of distinct vertices $\{\nu_{\ell_1}, \nu_{\ell_2}, \ldots, \nu_{\ell_r}\}$ with $\nu_{\ell_1} = \nu_j$ and $\nu_{\ell_r} = \nu_i$ such that $(\nu_{\ell_i}, \nu_{\ell_{i+1}}) \in \mathcal{E}$, $i = 1, 2, \ldots, \ell$. A digraph $\mathcal{G}$ has a spanning tree if there exists at least one vertex which has a directed path to all the other vertices in digraph $\mathcal{G}$. The Laplacian matrix $L = [l_{ij}]_{N\times N}$ of graph $\mathcal{G}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$.

2.3. Supporting lemmas

Lemma 2.1. (Gu et al. [7]) For any positive definite matrix $M \in \mathbb{R}^{n\times n}$ and a scalar ρ, and vector function $z : [0, \rho] \to \mathbb{R}^n$ such that the integrations in the following are well defined, then one has

$$\rho \int_0^\rho z^T(t) M z(t)\,\mathrm{d}t \geq \left(\int_0^\rho z(t)\,\mathrm{d}t\right)^T M \left(\int_0^\rho z(t)\,\mathrm{d}t\right).$$

Lemma 2.2. (Park et al. [21]) For given positive integers n and m, a scalar $\lambda \in (0, 1)$, a given $n\times n$ matrix $R > 0$, two matrices W_1 and W_2 in $\mathbb{R}^{n\times m}$, and for all $\xi \in \mathbb{R}^m$, define the function $f(\lambda, R)$ as

$$f(\lambda, R) = \frac{1}{\lambda}\xi^T W_1^T R W_1 \xi + \frac{1}{1-\lambda}\xi^T W_2^T R W_2 \xi.$$

If there exists a matrix $S \in \mathbb{R}^{n\times n}$ such that $\begin{bmatrix} R & S \\ * & R \end{bmatrix} > 0$, then the following inequality holds

$$\min_{\lambda\in(0,1)} f(\lambda, R) \geq \xi^T \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ * & R \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \xi.$$

Lemma 2.3. (Boyd et al. [1]) The following linear matrix inequality (LMI)

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x),\ R(x) = R^T(x)$, is equivalent to either of the following conditions:
(1) $Q(x) > 0,\ R(x) - S^T(x)Q^{-1}(x)S(x) > 0$;
(2) $R(x) > 0,\ Q(x) - S(x)R^{-1}(x)S^T(x) > 0$.

Lemma 2.4. (Horn and Johnson [16]) Let $A \in \mathbb{R}^{n\times n}$ be a real symmetric matrix. One has

$$\lambda_{\min}(A)x^Tx \le x^TAx \le \lambda_{\max}(A)x^Tx, \quad \text{for all } x \in \mathbb{R}^n.$$

3. MAIN RESULTS

In this section, leader-following quasi-consensus in multi-agent systems with asynchronous sampling information is studied. Sufficient conditions for guaranteeing quasi-consensus are presented.

3.1. Model formulation

Consider a second-order multi-agent system composed of N agents. The dynamics of the ith agent is governed by the following second-order equations

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \qquad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(t), v_i(t)$ and $u_i(t) \in \mathbb{R}^n$ are the position state, velocity state and control protocol of the ith node, respectively.

The leader of multi-agent system (1) is described by

$$\dot{x}_0(t) = v_0(t), \tag{2}$$

where $x_0(t), v_0(t) \in \mathbb{R}^n$ are the position and velocity states of the leader, respectively.

Definition 3.1. The multi-agent system (1) is said to achieve leader-following consensus with the leader (2) if

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0 \ \text{ and } \ \lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0,$$

hold for all initial values, $i = 1, 2, \ldots, N$.

Definition 3.2. The multi-agent system (1) is said to achieve leader-following quasi-consensus with the leader (2) if there exists a constant $\varepsilon > 0$ such that

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| \le \varepsilon \ \text{ and } \ \lim_{t\to\infty} \|v_i(t) - v_0(t)\| \le \varepsilon,$$

hold for all initial values, $i = 1, 2, \ldots, N$.

For simplicity of theoretical analysis, let the dimension of all agents be $n = 1$. However, the results still hold for $n > 1$ by applying the Kronecker product.

3.2. Sampling-based leader-following consensus of second-order multi-agent systems with a leader of constant velocity

It is assumed that the velocity of the leader is a constant $v_0(t) \equiv v_0$. Consider an asynchronous sampled-data protocol which has different sampling instants for different agents. That is, suppose that the sampling instants of agent i are $\{t_k^i\}_{t=0}^{\infty}$ satisfying $0 = t_0^i < t_1^i < t_2^i < \cdots < t_k^i < \cdots$, $\lim_{k\to\infty} t_k^i = \infty$ and $0 < t_{k+1}^i - t_k^i \le h_i$ with h_i being the upper bound of sampling period, $i = 0, 1, 2, \ldots, N$. Let $h = \max\{h_0, h_1, h_2, \ldots, h_N\}$. Specifically, the following consensus protocol is employed,

$$\begin{aligned} u_i(t) \;\; = \;\; & \alpha \sum_{j\in\mathcal{N}_i} a_{ij}\left[x_j(t^j_{k_j(t)}) - x_i(t_k^i)\right] + \beta \sum_{j\in\mathcal{N}_i} a_{ij}\left[v_j(t^j_{k_j(t)}) - v_i(t_k^i)\right] \\ & + d_i\left[\alpha(x_0(t^0_{k_0(t)}) - x_i(t_k^i)) + \beta(v_0 - v_i(t_k^i))\right], \end{aligned} \tag{3}$$

for $t \in [t_k^i, t_{k+1}^i)$, $k \in \mathbb{N}$, $i = 1, 2, \ldots, N$, where α and β are coupling strengths, d_i is the pinning gain. $d_i > 0$ if the ith agent is pinned by the leader, otherwise $d_i = 0$. $k_j(t) = \max\{k | t_k^j \le t\}$ means that the latest sampling information of node j is $k_j(t)$th sampling at time t.

Remark 3.3. Based on the asynchronous sampled-data protocol (3), the sampling instants of all the agents are different with each other. Specifically, all the agents need not share the same sampling instants. In other words, if t_k^i is the kth sampling instant for the agent i, then t_k^i may be not the sampling instant for the agent j. For $t \in [t_k^i, t_{k+1}^i)$, the agent i has completed the kth sampling, but the agent j has completed the $k_j(t)$th sampling at instant t. In a word, each agent is only sampled at its own sampling instants and not available at sampling instants of its neighbours.

Let $\widehat{x}_i(t) = x_i(t) - x_0(t)$ and $\widehat{v}_i(t) = v_i(t) - v_0$. It follows from leader-following multi-agent systems (1)–(2) and the protocol (3) that

$$\begin{cases} \dot{\widehat{x}}_i(t) = \widehat{v}_i(t), \\ \dot{\widehat{v}}_i(t) = \alpha \sum\limits_{j\in\mathcal{N}_i} a_{ij}\left[\widehat{x}_j(t^j_{k_j(t)}) - \widehat{x}_i(t_k^i)\right] + \beta \sum\limits_{j\in\mathcal{N}_i} a_{ij}\left[\widehat{v}_j(t^j_{k_j(t)}) - \widehat{v}_i(t_k^i)\right] \\ \qquad - d_i\left[\alpha\widehat{x}_i(t_k^i) + \beta\widehat{v}_i(t_k^i)\right] + \delta_i(t), \quad t \in [t_k^i, t_{k+1}^i), \end{cases} \tag{4}$$

where

$$\delta_i(t) = \alpha \sum_{j\in\mathcal{N}_i} a_{ij}\left[x_0(t^j_{k_j(t)}) - x_0(t_k^i)\right] + \alpha d_i\left[x_0(t^0_{k_0(t)}) - x_0(t_k^i)\right], \; i = 1, 2, \ldots, N.$$

Let $\tau_i(t) = t - t_k^i$ for $t \in [t_k^i, t_{k+1}^i), i = 1, 2, \ldots, N$. For $t \in [t_k^i, t_{k+1}^i)$, one has

$$\begin{cases} \dot{\widehat{x}}_i(t) = \widehat{v}_i(t), \\ \dot{\widehat{v}}_i(t) = -\alpha \sum\limits_{j=1}^{N} l_{ij}\widehat{x}_j(t - \tau_j(t)) - \beta \sum\limits_{j=1}^{N} l_{ij}\widehat{v}_j(t - \tau_j(t)) \\ \qquad - \alpha d_i\widehat{x}_i(t - \tau_i(t)) - \beta d_i\widehat{v}_i(t - \tau_i(t)) + \delta_i(t), \quad i = 1, 2, \ldots, N. \end{cases} \tag{5}$$

Let $D = \operatorname{diag}\{d_1, d_2 \ldots, d_N\}$ and $H = L + D$ with N column vectors denoted by $\hbar_1, \hbar_2, \ldots, \hbar_N$. Let $\widehat{x}(t) = [\widehat{x}_1(t), \widehat{x}_2(t), \ldots, \widehat{x}_N(t)]^T$, $\widehat{v}(t) = [\widehat{v}_1(t), \widehat{v}_2(t), \ldots, \widehat{v}_N(t)]^T$, and $\delta(t) = [\delta_1(t), \delta_2(t), \ldots, \delta_N(t)]^T$. Then, (5) can be rewritten as the following compact matrix form as

$$\begin{cases} \dot{\widehat{x}}(t) = \widehat{v}(t), \\ \dot{\widehat{v}}(t) = -\alpha \sum\limits_{j=1}^{N} \hbar_j \widehat{x}_j(t - \tau_j(t)) - \beta \sum\limits_{j=1}^{N} \hbar_j \widehat{v}_j(t - \tau_j(t)) + \delta(t). \end{cases} \tag{6}$$

Denote $H_j = [\underbrace{0_N, \ldots, 0_N}_{j-1}, \hbar_j, \underbrace{0_N, \ldots, 0_N}_{N-j}], j = 1, 2, \ldots, N$. Rewrite system (6) as

$$\begin{cases} \dot{\widehat{x}}(t) = \widehat{v}(t), \\ \dot{\widehat{v}}(t) = -\alpha \sum\limits_{j=1}^{N} H_j \widehat{x}(t - \tau_j(t)) - \beta \sum\limits_{j=1}^{N} H_j \widehat{v}(t - \tau_j(t)) + \delta(t). \end{cases} \tag{7}$$

Let $\eta(t) = [\widehat{x}^T(t), \widehat{v}^T(t)]^T$. Then, one can obtain the following compact system for $t \geq 0$,

$$\dot{\eta}(t) = B\eta(t) - \sum_{n=1}^{N} C_n \eta(t - \tau_n(t)) + \Delta(t), \tag{8}$$

where

$$B = \begin{bmatrix} O_N & I_N \\ O_N & O_N \end{bmatrix}, \quad C_n = \begin{bmatrix} O_N & O_N \\ \alpha H_n & \beta H_n \end{bmatrix}, \quad \Delta(t) = \begin{bmatrix} 0_N \\ \delta(t) \end{bmatrix}.$$

Assumption 3.4. The leader has a directed path to each follower.

Theorem 3.5. $\delta(t)$ is bounded if $v_0(t) \equiv v_0$ and $t_{k+1}^i - t_k^i \leq h$ for $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots$, where h, v_0 are given constants.

P r o o f. It follows from the definition of $\delta_i(t)$ that

$$\begin{aligned} \|\delta_i(t)\| &= \left\| \alpha \sum_{j \in \mathcal{N}_i} a_{ij} \left[x_0(t_{k_j(t)}^j) - x_0(t_k^i) \right] + \alpha d_i \left[x_0(t_{k_0(t)}^0) - x_0(t_k^i) \right] \right\| \\ &\leq \alpha \sum_{j \in \mathcal{N}_i} a_{ij} \left\| x_0(t_{k_j(t)}^j) - x_0(t_k^i) \right\| + \alpha d_i \left\| x_0(t_{k_0(t)}^0) - x_0(t_k^i) \right\|. \end{aligned} \tag{9}$$

Since $|t_{k_j(t)}^j - t_k^i| < h$ and $|t_{k_0(t)}^0 - t_k^i| < h$, one has $\left\| x_0(t_{k_j(t)}^j) - x_0(t_k^i) \right\| \leq \|v_0\| h$ and $\left\| x_0(t_{k_0(t)}^0) - x_0(t_k^i) \right\| \leq \|v_0\| h$. According to (9), one derives

$$\|\delta_i(t)\| \leq \alpha h \left(\sum_{j \in \mathcal{N}_i} a_{ij} + d_i \right) \|v_0\| = \alpha h \|v_0\| (l_{ii} + d_i), \quad i = 1, 2, \ldots, N. \tag{10}$$

Therefore, there exists a positive constant δ such that $\|\Delta(t)\| = \|\delta(t)\| \leq \delta$. □

Similar to [3], denote a block entry matrix composed of $2N+2$ blocks by

$$\mathbb{I}_n = [\underbrace{O_{2N}, \dots, O_{2N}}_{n-1}, I_{2N}, \underbrace{O_{2N}, \dots, O_{2N}}_{2N+2-n}].$$

Let $\mathbb{I}_{m,n} = \mathbb{I}_m - \mathbb{I}_n$.

Theorem 3.6. Suppose that Assumption 3.4 holds. For given constants $a > 0$ and $h_n > 0$, the trajectories of system (8) exponentially converge into a compact set

$$E = \left\{ e \in \mathbb{R}^{2N} \,\middle|\, \|e\| \leq \sqrt{\frac{b\delta^2}{a\lambda_{\min}(P)}} \right\}$$

if there exist matrices $P > 0$, $Q_n > 0$, $R_n > 0$, S_n and a constant $b > 0$ such that

$$\begin{bmatrix} R_n & S_n \\ * & R_n \end{bmatrix} > 0, \quad \begin{bmatrix} \Psi & \Phi^T \mathcal{R}_1 \\ * & -\mathcal{R}_2 \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned}
&\Phi = B\mathbb{I}_1 - \sum_{n=1}^{N} C_n \mathbb{I}_{1+N+n} + \mathbb{I}_{2N+2},\\
&\Omega_n = \begin{bmatrix} \mathbb{I}_{1+N+n,1+n} \\ \mathbb{I}_{1,1+N+n} \end{bmatrix}^T \begin{bmatrix} R_n & S_n \\ * & R_n \end{bmatrix} \begin{bmatrix} \mathbb{I}_{1+N+n,1+n} \\ \mathbb{I}_{1,1+N+n} \end{bmatrix}, \quad n = 1, 2, \dots, N,\\
&\Psi = \mathbb{I}_1^T P\Phi + \Phi^T P\mathbb{I}_1 + \mathbb{I}_1^T \left(aP + \sum_{n=1}^{N} Q_n \right) \mathbb{I}_1 - \sum_{n=1}^{N} e^{-ah_n} \mathbb{I}_{1+n}^T Q_n \mathbb{I}_{1+n}\\
&\qquad - \sum_{n=1}^{N} e^{-ah_n} \Omega_n - b\mathbb{I}_{2N+2}^T \mathbb{I}_{2N+2},\\
&\mathcal{R}_1 = [h_1 R_1, h_2 R_2, \dots, h_N R_N],\\
&\mathcal{R}_2 = \operatorname{diag}\{R_1, R_2, \dots, R_N\}.
\end{aligned}$$

P r o o f. Construct Lyapunov–Krasovskii functional

$$V(t, \eta(t)) = V_1(t, \eta(t)) + V_2(t, \eta(t)) + V_3(t, \eta(t)), \tag{12}$$

where

$$\begin{aligned}
V_1(t, \eta(t)) &= \eta^T(t) P \eta(t),\\
V_2(t, \eta(t)) &= \sum_{n=1}^{N} \int_{t-h_n}^{t} e^{a(s-t)} \eta^T(s) Q_n \eta(s) \, \mathrm{d}s,\\
V_3(t, \eta(t)) &= \sum_{n=1}^{N} h_n \int_{-h_n}^{0} \int_{t+\theta}^{t} e^{a(s-t)} \dot{\eta}^T(s) R_n \dot{\eta}(s) \, \mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

with positive definite matrices $P > 0$, $Q_n > 0$ and $R_n > 0$, $n = 1, 2, \ldots, N$.

Let $W(t) = \dot{V}(t, \eta(t)) + aV(t, \eta(t)) - b\Delta^T(t)\Delta(t)$. Take the derivative of $V(t)$ along the trajectory of (8), one has

$$W(t) \le 2\eta^T(t)P\dot{\eta}(t) + \eta^T(t)\left(aP + \sum_{n=1}^{N} Q_n\right)\eta(t) - \sum_{n=1}^{N} e^{-ah_n}\eta^T(t-h_n)Q_n\eta(t-h_n)$$
$$+ \sum_{n=1}^{N} h_n^2\dot{\eta}^T(t)R_n\dot{\eta}(t) - \sum_{n=1}^{N} e^{-ah_n}h_n\int_{t-h_n}^{t}\dot{\eta}^T(s)R_n\dot{\eta}(s)\,\mathrm{d}s - b\Delta^T(t)\Delta(t). \tag{13}$$

Defining the variable

$$y(t) = [\eta^T(t), \gamma_1^T(t), \gamma_2^T(t), \Delta^T(t)]^T,$$

where $\gamma_1(t) = [\eta^T(t-h_1), \eta^T(t-h_2), \ldots, \eta^T(t-h_N)]^T$ and $\gamma_2(t) = [\eta^T(t-\tau_1(t)), \eta^T(t-\tau_2(t)), \ldots, \eta^T(t-\tau_N(t))]^T$.
Based on Lemma 2.1, one gets

$$\begin{aligned}
&-h_n\int_{t-h_n}^{t}\dot{\eta}^T(s)R_n\dot{\eta}(s)\,\mathrm{d}s \\
=&-h_n\int_{t-h_n}^{t-\tau_n(t)}\dot{\eta}^T(s)R_n\dot{\eta}(s)\,\mathrm{d}s - h_n\int_{t-\tau_n(t)}^{t}\dot{\eta}^T(s)R_n\dot{\eta}(s)\,\mathrm{d}s \\
\le&-\frac{h_n}{h_n-\tau_n(t)}[\eta(t-\tau_n(t))-\eta(t-h_n)]^T R_n[\eta(t-\tau_n(t))-\eta(t-h_n)] \qquad (14)\\
&-\frac{h_n}{\tau_n(t)}[\eta(t)-\eta(t-\tau_n(t))]^T R_n[\eta(t)-\eta(t-\tau_n(t))] \\
\le&\frac{-h_n}{h_n-\tau_n(t)}y^T(t)\mathbb{I}_{1+N+n,1+n}^T R_n\mathbb{I}_{1+N+n,1+n}y(t) - \frac{h_n}{\tau_n(t)}y^T(t)\mathbb{I}_{1,1+N+n}^T R_n\mathbb{I}_{1,1+N+n}y(t).
\end{aligned}$$

Together with Lemma 2.2, one yields

$$-h_n\int_{t-h_n}^{t}\dot{\eta}^T(s)R_n\dot{\eta}(s)\,\mathrm{d}s \le -y^T(t)\Omega_n y(t), \tag{15}$$

where

$$\Omega_n = \begin{bmatrix} \mathbb{I}_{1+N+n,1+n} \\ \mathbb{I}_{1,1+N+n} \end{bmatrix}^T \begin{bmatrix} R_n & S_n \\ * & R_n \end{bmatrix} \begin{bmatrix} \mathbb{I}_{1+N+n,1+n} \\ \mathbb{I}_{1,1+N+n} \end{bmatrix}.$$

Since $0 \le \tau_n(t) < h_n$, $y(t) - y(t-\tau_n(t)) = 0$ if $\tau_n(t) = 0$. Therefore, (15) holds similarly for

$$\Omega_n = \begin{bmatrix} \mathbb{I}_{1,1+n} \\ O_{2N(2N+2)} \end{bmatrix}^T \begin{bmatrix} R_n & S_n \\ * & R_n \end{bmatrix} \begin{bmatrix} \mathbb{I}_{1,1+n} \\ O_{2N(2N+2)} \end{bmatrix} = \mathbb{I}_{1,1+n}^T R_n\mathbb{I}_{1,1+n}.$$

Rewrite (8) as $\dot{\eta}(t) = \Phi y(t)$, where $\Phi = B\mathbb{I}_1 - \sum_{n=1}^{N} C_n\mathbb{I}_{1+N+n} + \mathbb{I}_{2N+2}$. Combining (13) – (15) yields

$$W(t) \le y^T(t)\left[\Psi + \Phi^T\left(\sum_{n=1}^{N} h_n^2 R_n\right)\Phi\right] y(t). \tag{16}$$

According to Lemma 2.3, $\Psi + \Phi^T\left(\sum_{n=1}^{N} h_n^2 R_n\right)\Phi < 0$ is equivalent to the second inequality of (11). Therefore, $W(t) < 0$ for all $y(t) \neq 0$. Applying the comparison principle or Proposition 1 of [4] derives $\lim_{t\to\infty}\eta^T(t)P\eta(t) < b\delta^2/a$. Furthermore, it follows from Lemma 2.4 that

$$\lim_{t\to\infty}\|\eta(t)\|^2 < \frac{b\delta^2}{a\lambda_{\min}(P)}.$$

Therefore, leader-following quasi-consensus in the multi-agent system (1) and the leader (2) is achieved. □

3.3. Sampling-based leader-following consensus of second-order multi-agent systems with a leader of variable velocity

It is assumed that the velocity of the leader is variable. Assume that the dynamics of the leader is given by

$$\begin{cases} \dot{x}_0(t) = v_0(t), \\ \dot{v}_0(t) = a_0(t), \end{cases} \tag{17}$$

where $a_0(t) \in \mathbb{R}$ is continuous and bounded. Apply the consensus control protocol as follows

$$\begin{aligned} u_i(t) \;=\; & \alpha\sum_{j\in\mathcal{N}_i} a_{ij}\left[x_j(t^j_{k_j(t)}) - x_i(t^i_k)\right] + \beta\sum_{j\in\mathcal{N}_i} a_{ij}\left[v_j(t^j_{k_j(t)}) - v_i(t^i_k)\right] \\ & + d_i[\alpha(x_0(t^0_{k_0(t)}) - x_i(t^i_k)) + \beta(v_0(t^0_{k_0(t)}) - v_i(t^i_k))], \end{aligned} \tag{18}$$

for $t \in [t^i_k, t^i_{k+1}),\ k \in \mathbb{N},\ i = 1, 2, \ldots, N$.

Let $\widehat{x}_i(t) = x_i(t) - x_0(t)$ and $\widehat{v}_i(t) = v_i(t) - v_0(t)$. It follows from leader-following multi-agent systems (1) – (2) and the protocol (3) that

$$\begin{cases} \dot{\widehat{x}}_i(t) = \widehat{v}_i(t) \\ \dot{\widehat{v}}_i(t) = \alpha\sum\limits_{j\in\mathcal{N}_i} a_{ij}\left[\widehat{x}_j(t^j_{k_j(t)}) - \widehat{x}_i(t^i_k)\right] + \beta\sum\limits_{j\in\mathcal{N}_i} a_{ij}\left[\widehat{v}_j(t^j_{k_j(t)}) - \widehat{v}_i(t^i_k)\right] \\ \qquad - d_i(\alpha\widehat{x}_i(t^i_k) + \beta\widehat{v}_i(t^i_k)) + \widetilde{\delta}_i(t), \quad t \in [t^i_k, t^i_{k+1}), \end{cases} \tag{19}$$

where

$$\begin{aligned} \widetilde{\delta}_i(t) = & \alpha\sum_{j\in\mathcal{N}_i} a_{ij}\left[x_0(t^j_{k_j(t)}) - x_0(t^i_k)\right] + \beta\sum_{j\in\mathcal{N}_i} a_{ij}\left[v_0(t^j_{k_j(t)}) - v_0(t^i_k)\right] \\ & + \alpha d_i\left[x_0(t^0_{k_0(t)}) - x_0(t^i_k)\right] + \beta d_i\left[v_0(t^0_{k_0(t)}) - v_0(t^i_k)\right], \quad i = 1, 2, \ldots, N. \end{aligned}$$

Then, one can obtain the following compact system for $t \geq 0$,

$$\dot{\eta}(t) = B\eta(t) - \sum_{n=1}^{N} C_n \eta(t - \tau_n(t)) + \widetilde{\Delta}(t) \tag{20}$$

where

$$\widetilde{\Delta}(t) = \begin{bmatrix} 0_N \\ \widetilde{\delta}(t) \end{bmatrix}.$$

Assumption 3.7. Suppose that $v_0(t)$ and $a_0(t)$ are bounded.

Theorem 3.8. Under Assumption 3.7, $\widetilde{\delta}(t)$ is bounded if $t_{k+1}^i - t_k^i \leq h$ for $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots$, where h is a given constant.

Proof. The proof is similar to Theorem 3.5 and thus is omitted here. □

According to Theorem 3.8, there is a positive constant $\widetilde{\delta}$ satisfying $\|\widetilde{\delta}(t)\| \leq \widetilde{\delta}$. Similar to Theorem 3.6, the following result is a straightforward consequence.

Theorem 3.9. Suppose that Assumptions 3.4 and 3.7 hold. For given constants $a > 0$ and $h_n > 0$, the trajectories of system (20) exponentially converge into a compact set

$$E = \left\{ e \in \mathbb{R}^{2N} \,\middle|\, \|e\| \leq \sqrt{\frac{b\widetilde{\delta}^2}{a\lambda_{\min}(P)}} \right\}$$

if there exist matrices $P > 0$, $Q_n > 0$, $R_n > 0$, S_n and a constant $b > 0$ such that

$$\begin{bmatrix} R_n & S_n \\ * & R_n \end{bmatrix} > 0, \quad \begin{bmatrix} \Psi & \Phi^T \mathcal{R}_1 \\ * & -\mathcal{R}_2 \end{bmatrix} < 0, \tag{21}$$

where $\Phi, \Omega_n, \Psi, \mathcal{R}_1$ and $\mathcal{R}_2$ are the same as Theorem 3.6, $n = 1, 2, \ldots, N$.

Remark 3.10. Since the protocols (3) and (18) are asynchronous, $\delta(t)$ and $\widetilde{\delta}(t)$ usually can not vanish. That is, only quasi-consensus can be achieved for second-order multi-agent systems by employing asynchronous sampled-data controls. By adopting synchronous sampled-data control [13, 19, 35], consensus can be reached for multi-agent systems.

4. NUMERICAL RESULTS

Example 4.1. Leader-following quasi-consensus with a leader of constant velocity.

Without loss of generality, consider a multi-agent system composed of three agents with one dimension described by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \quad i = 1, 2, 3, \end{cases} \tag{22}$$

where $x_i(t), v_i(t)$ and $u_i(t) \in \mathbb{R}$ are the position state, velocity state and control protocol of the ith agent, respectively.

The dynamics of the leader is given by

$$\dot{x}_0(t) = 0.25.$$

The interaction topology is a directed graph as shown in Figure 1, where 0 represents the leader and $1, 2, 3$ are the followers.

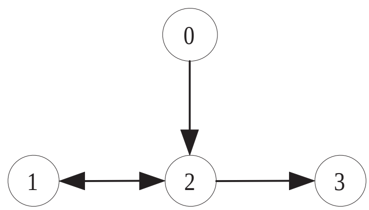

Fig. 1. Directed graph of three agents.

By choosing all the weights as 1 and observing from Figure 1, one has

$$L = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0 & & \\ & 1 & \\ & & 0 \end{bmatrix}.$$

Set $\alpha = 0.5$, $\beta = 0.8$ and $a = 0.1$. By solving LMI (11), an allowable bound of sampling intervals is derived as $h = 0.5277$. For simplicity, asynchronous period sampling is considered. Therefore, we can choose sampling periods as $h_1 = 0.5, h_2 = 0.4, h_3 = 0.3$ and $h_0 = 0.1$ for the followers and the leader, respectively. Select the initial values $x(0) = [20, -15, 10]$, $v(0) = [8, 3, -15]$ and $x_0(0) = 2$. The evolutions of positions and velocities of the followers and the leader are shown in Figures 2 and 3.

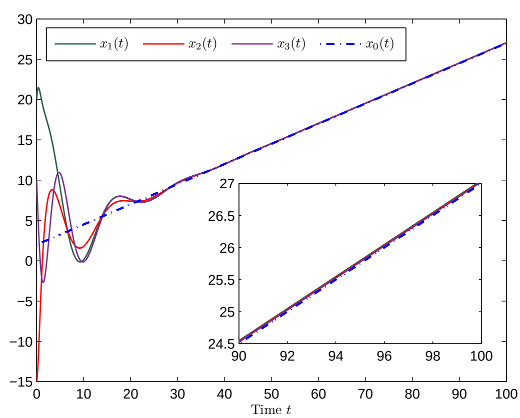

Fig. 2. Positions of four agents.

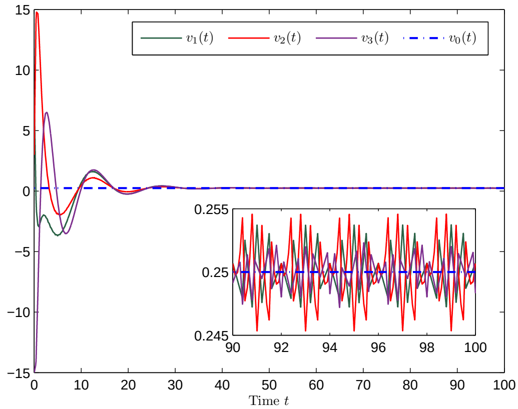

Fig. 3. Velocities of four agents.

It follows from Figures 2 and 3 that the leader moves uniformly with a constant velocity $v_0(t) \equiv 0.25$ but the followers vibrate near the leader. In particular, the position and velocity states of the followers vibrate in the bounded ranges of the position and velocity states of the leader, respectively.

The evolutions of position errors and velocity errors between the followers and the leader are shown in Figures 4 and 5.

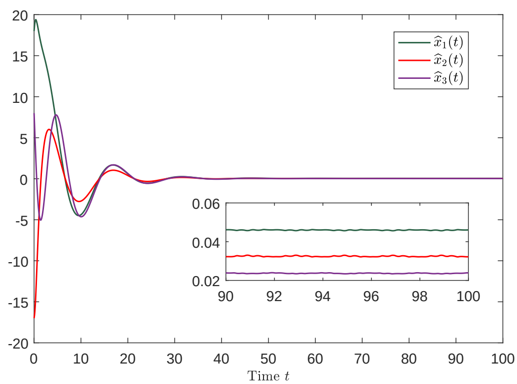

Fig. 4. Position errors of three agents.

As shown in Figures 4 and 5, the position errors and velocity errors between the leader and the followers tend towards the small neighborhoods of the origin. Therefore, leader-following quasi-consensus in multi-agent systems is achieved.

Example 4.2. Leader-following quasi-consensus with a leader of variable velocity.

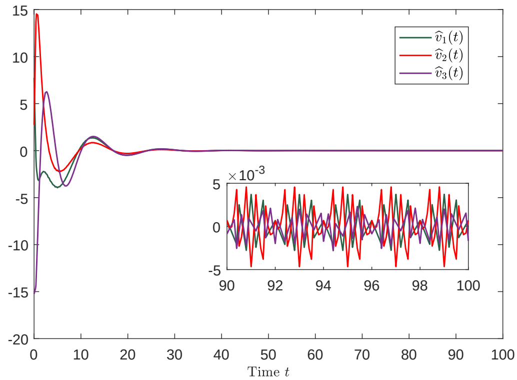

Fig. 5. Velocity errors of three agents.

Consider a leader-following multi-agent system composed of three follower agents and one leader as shown in Figure 1. The dynamics of three follower agents are described by (22). The dynamics of the leader is given by

$$\begin{cases} \dot{x}_0(t) = v_0(t), \\ \dot{v}_0(t) = a_0(t), \end{cases} \tag{23}$$

where the variable velocity is chosen as $v_0(t) = \sin(t) + \frac{t}{t+1}$. Therefore, Assumptions 3.4 and 3.7 hold. Take the same parameters as Example 4.1. The initial values for the followers and the leader are selected as $x(0) = [15, -20, 5]$, $v(0) = [-10, 5, 15]$ and $x_0(0) = 10$, $v_0(0) = 0$, respectively. The evolutions of positions and velocities of the followers and the leader are shown in Figures 6 and 7.

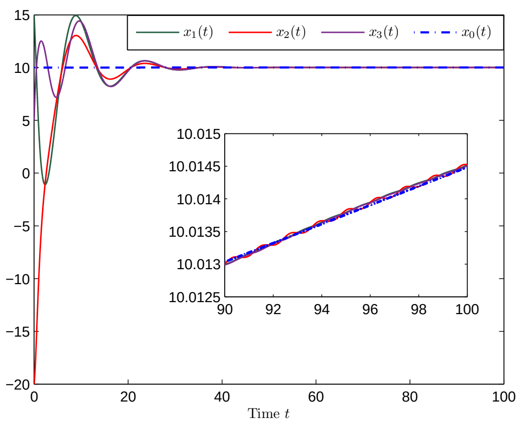

Fig. 6. Positions of four agents.

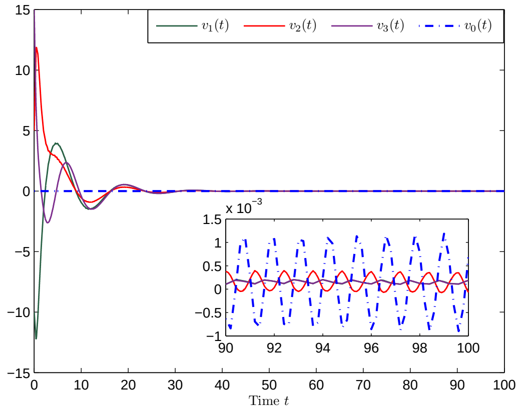

Fig. 7. Velocities of four agents.

The evolutions of position errors and velocity errors between the followers and the leader are shown in Figures 8 and 9.

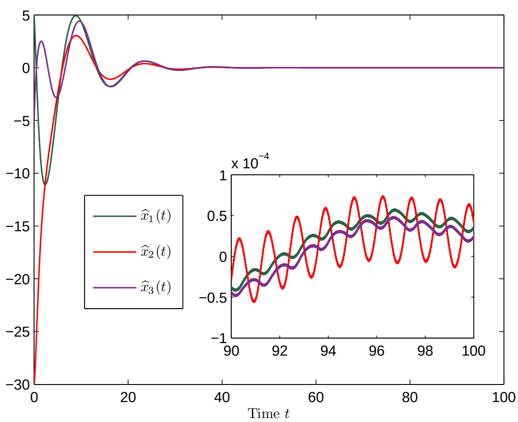

Fig. 8. Position errors of three agents.

It follows from Figures 6 and 7 that the leader moves with a variable velocity but the followers gradually move near the leader. By Figures 8 and 9, the states of the followers converge into the bounded ranges of the states of the leader ultimately. In other words, leader-following quasi-consensus is reached for multi-agent systems with a leader of variable velocity.

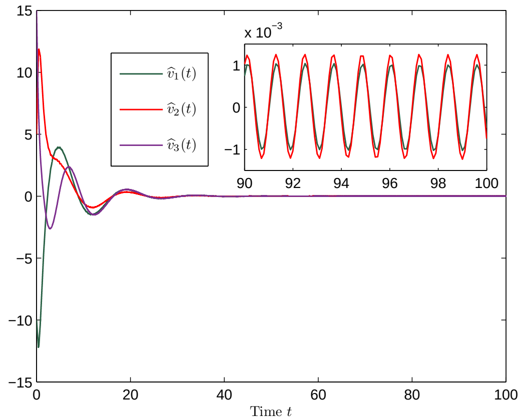

Fig. 9. Velocity errors of three agents.

5. CONCLUSION

In this paper, the leader-following consensus problem is investigated for second-order multi-agent systems with both constant velocity leader and variable velocity leader. Based on a precondition that all the agents have different sampling instants with each other, the asynchronous sampled-data protocols are proposed. Some sufficient conditions for leader-following quasi-consensus are obtained. All the follower agents can track the leader with a bounded range in leader-following multi-agent systems. Numerical simulations show the effectiveness of the theoretical results.

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Zhengxin Wang, Corresponding author. College of Science, Nanjing University of Posts and Telecommunications, Nanjing 210023. P. R. China.
e-mail: zwang@njupt.edu.cn

Yuanzhen Feng, College of Science, Nanjing University of Posts and Telecommunications, Nanjing 210023. P. R. China.
e-mail: yzfeng@njupt.edu.cn

Cong Zheng, College of Science, Nanjing University of Posts and Telecommunications, Nanjing 210023. P. R. China.
e-mail: zhengcong@njupt.edu.cn

Yanling Lu, College of Science, Nanjing University of Posts and Telecommunications, Nanjing 210023. P. R. China.
e-mail: luyl@njupt.edu.cn

Lijun Pan, Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing, 211106. P. R. China.
e-mail: 98010149@163.com