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TUBE-MPC FOR A CLASS OF UNCERTAIN CONTINUOUS NONLINEAR SYSTEMS WITH APPLICATION TO SURGE PROBLEM

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This paper presents a new robust adaptive model predictive control for a special class of continuous-time non-linear systems with uncertainty. These systems have bounded disturbances with unknown upper bound, as well as constraints on input states. An adaptive control is used in the new controller to estimate the system uncertainty. Also, to avoid the system disturbances, a H_∞ method is employed to find the appropriate gain in Tube-MPC. Finally, a surge avoidance problem in centrifugal compressors is solved to show the efficiency and effectiveness of the proposed algorithm.

Keywords: robust control, adaptive control, H_∞ method, tube-MPC, surge

Classification: 93C10, 93D09, 93C40, 93C42, 37N35

1. INTRODUCTION

Predictive controllers are widely used in control industrial systems [1, 13]. Given the fact that, most operational systems are non-linear and bounded, predictive controllers are used on the basis of linear or non-linear models with uncertainty [9, 10, 12]. One reason for this condition is that non-linear predictive control algorithms lead to non-convex, non-linear optimization problems; which the solution requires reiterative methods with extended calculation times [13]. In addition, the convergence region of this kind of algorithms is qualitative, which increases online calculation time. On the other hand, the linear model and square cost function can lead to a convex square optimization problem that can be solved easily for predictive control algorithms. However, in many systems, the nonlinear effects cannot be ignored. In these conditions, the system can be approximated by a linear model and be considered on the scope approximation error [15, 16, 17]. Predictive control has a strong advantage: it can consider constraints explicitly in the problem, but it cannot explicitly calculate the model uncertainty in the formulation. Using robust predictive control methods, uncertainty in the process model can be explicitly combined with the problem [2, 3, 11, 22]. Since, model predictive control (MPC) is a popular and effective approach to design controller in different areas

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[4, 6]. Adaptive method is used to compensate the parametric uncertainty and stabilize the controller against disturbances applied to the system [5].

Modeling the system is a major prerequisite in predictive control design. Also, the model accuracy plays an important role in controller performance. Practically, the systems have uncertainty in models, which should be considered in designing robust controller in order to guarantee the stability of the closed-loop system throughout the uncertainty scope. In this paper, a new controller is designed for a special class of non-linear systems using Tube-MPC, a model predictive controller that can control the system even when the model and the process are not matched. Then, system's uncertainty is approximated by an adaptive controller. Finally, a H_∞ method is used to find the ancillary controller gain in Tube-MPC. This leads to a controller robust to disturbances.

This paper is composed of the following sections: Section two presents some preliminaries. Section three presents the new robust adaptive Tube-MPC. Section four designs a robust adaptive Tube-MPC to stabilize the compressor system and by a simulation, it proves the efficiency of the controller. Finally, section five concludes the paper.

2. PRELIMINARIES

Consider the following continuous nonlinear system

$$\dot{x}(t) = f(x(t), u(t), w(t)). \tag{1}$$

Where $x(t) \in R^{n_x}$ shows the system states, $u(t) \in R^{n_u}$ is the control input, The signal $w(t) \in R^{n_w}$ is the disturbance or model-plant mismatch, which is unknown but bounded, and lies in a compact set,

$$W = \{w(t) \in R^{n_w} |\, \|w\| \le w_{\max}\}. \tag{2}$$

The system has the following limitations

$$x(t) \in X,\ u(t) \in U,\ \forall\, t > 0. \tag{3}$$

Where, $X \subset R^{n_x}$ is bounded and $U \subset R^{n_u}$ is compact [20]. The following lemma provides us a way to construct a robust control invariant set for the system (1).

Lemma 2.1. (Yu et al. [20]) Let $S:\ R^{n_x} \to [0, \infty)$ be a continuously differentiable function and $\alpha_1(\|x\|) < S(x) < \alpha_2(\|x\|)$, where α_1, α_2 are class k_∞ functions. Suppose $u: R \to R^{n_u}$ is chosen, and there exit $\lambda > 0$ and $\mu > 0$ such that

$$\dot{S}(x) + \lambda S(x) - \mu w^T(t)\, w(t) \le 0. \tag{4}$$

With $x \in X,\ d \in D$. Then, the system trajectory starting from $x(t_0) \in \Omega \subseteq X$, will remain in the set Ω, where

$$\Omega = \left\{ x \in R^{n_x} |\ S(x) \le \frac{\mu w_{\max}^2}{\lambda} \right\}. \tag{5}$$

Lemma 2.2. (Poursafar et al. [15]) Let M, N be real constant matrices and P be a positive matrix of compatible dimensions. Then

$$M^T PN + N^T PM \leq \varepsilon M^T PM + \varepsilon^{-1} N^T PN. \tag{6}$$

Holds for any $\varepsilon > 0$.

3. ROBUST ADAPTIVE TUBE-MPC

Consider the following continuous nonlinear system

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x)\,\theta + d(t). \tag{7}$$

Where $x(t) \in R^{n_x}$ shows the system states, $u(t) \in R^{n_u}$ is the control input, $f(x) : R^{n_x} \to R^{n_\theta}$ is the continuous nonlinear function, $\theta(t) \in R^{n_\theta}$ denotes uncertainty in the system, and $d(t) \in R^{n_x}$ shows bounded and unknown system disturbances. The disturbances are considered in the following set

$$D = \{d(t) \in R^{n_x} \mid \|d\| \leq d_{\max}\}. \tag{8}$$

The system has the following limitations

$$x(t) \in X,\ u(t) \in U,\ \forall\ t > 0. \tag{9}$$

Where, $X \subset R^{n_x}$ is bounded and $U \subset R^{n_u}$ is compact. Now, the nominal model of the system is given as

$$\dot{\overline{x}}(t) = A\overline{x}(t) + B\overline{u}(t) + f(\overline{x})\,\widehat{\theta}. \tag{10}$$

Where $\overline{x}(t) \in R^{n_x}$ shows the nominal model states, $\overline{u}(t) \in R^{n_u}$ is the control input of nominal model and $\widehat{\theta}(t) \in R^{n_\theta}$ is estimator of uncertainty in the system. By defining the cost function as

$$\begin{aligned} J\big(\overline{x},\overline{u}\big) = \int_{t_k}^{t_k+T_p} \Big(\overline{x}\big(\tau;\overline{x}(t_k),t_k\big)^T Q\overline{x}\big(\tau;\overline{x}(t_k),t_k\big) + \overline{u}\big(\tau;\overline{x}(t_k),t_k\big)^T \\ R\overline{u}\big(\tau;\overline{x}(t_k),t_k\big)\Big)\mathrm{d}\tau. \end{aligned} \tag{11}$$

Where, T_p is prediction horizon. We can solve the following problem to find $u(t)$

$$\begin{gathered} \text{minimize}\ \ J\left(\bar{x}(t_k), \bar{u}(0;\bar{x}(t_k), t_k)\right) \\ \bar{u}(0;\bar{x}(t_k), t_k) \\ \text{subject to} \\ \dot{\bar{x}}(\tau;\bar{x}(t_k), t_k) = A\bar{x}(\tau;\bar{x}(t_k), t_k) + B\bar{u}(\tau;\bar{x}(t_k), t_k) + f(\bar{x}(\tau;\bar{x}(t_k), t_k))\,\hat{\theta}(t_k) \\ \bar{x}(\tau;\bar{x}(t_k), t_k) \in \bar{X}, \tau \in [t_k, t_k + T_p] \\ \bar{u}(\tau;\bar{x}(t_k), t_k) \in \bar{U}, \tau \in [t_k, t_k + T_p]. \end{gathered} \tag{12}$$

Where $\bar{X} \subset R^{n_x}$, $\bar{U} \subset R^{n_u}$ and $\bar{u}(0;\bar{x}(t_k), t_k)$ are the given control inputs from $\bar{x}(t_k)$ state in t_k, and $\bar{x}(0;\bar{x}(t_k), t_k)$ is (10) nominal system trajectory, started from state in and control input.

Problem (12) is solved in discrete time with a sample time of δ, and the nominal control during the sample interval δ is

$$\bar{u}(\tau) = \bar{u}^*(\tau; \bar{x}^*(t_k), t_k), \tau \in [t_k, t_k + \delta]. \tag{13}$$

Where $\bar{u}^*(\tau; \bar{x}^*(t_k), t_k)$ shows the optimum solution of the optimization problem in t_k, and $\bar{x}^*(\tau; \bar{x}^*(t_k), t_k)$ is the nominal system trajectory. Should note that $\bar{u}(\tau)$ is obtained from the problem (12) and to solve the optimization problem (12), is used $\hat{\theta}(t_k)$ that also $\hat{\theta}(t_k)$ will follow the adaptive law which subsequently will be achieved in the proof process.

The overall applied control input for the actual system (7) during the sampling interval δ consequently is

$$u(\tau) = \bar{u}(\tau) + Ke(\tau), \tau \in [t_k, t_k + \delta]. \tag{14}$$

Where $K \in R^{n_u \times n_x}$ shows gain of state feedback and error is defined as

$$e(t) = x(t) - \bar{x}(t). \tag{15}$$

According to (7), (10) and (14), the dynamic error equation is given as

$$\dot{e}(t) = (A + BK)e(t) + f(x(t))\theta - f(\bar{x}(t))\hat{\theta} + d(t). \tag{16}$$

By defining $\tilde{\theta} = \theta - \hat{\theta}$, we have

$$\dot{e}(t) = (A + BK)e(t) + B_w w(t). \tag{17}$$

Where

$$g(t) = (f(x(t)) - f(\bar{x}(t)))\hat{\theta} + d(t), w(t) = \begin{bmatrix} \tilde{\theta} \\ g(t) \end{bmatrix}, B_w = \begin{bmatrix} f(x(t)) & I_{n_x \times n_x} \end{bmatrix}. \tag{18}$$

Lemma 3.1. Suppose that there exit positive definite matrix $X \in R^{n_x \times n_x}$, non-square matrix $Y \in R^{n_u \times n_x}$, and scalars $\alpha > 0,\ \beta > 0,\ \lambda > 0,\ \eta > 0,\ \epsilon = \frac{\alpha\lambda}{\eta\beta}$ and $\mu = \frac{\lambda}{\eta(1+\beta)}$ such that

$$(AX + BY)^T + AX + BY + (\alpha + \lambda)X \leq 0 \tag{19}$$

$$X \leq \frac{1}{\epsilon} I. \tag{20}$$

Then, the set $\Omega = \left\{ e \in R^{n_x} |\ S\left(e, \tilde{\theta}\right) \leq \frac{\mu w_{\max}^2}{\lambda} \right\}$ is a robust invariant set for the error system (17), where

$$S\left(e(t), \tilde{\theta}\right) = e^T(t) Pe(t) + \frac{1}{\eta}\tilde{\theta}^T\tilde{\theta} \tag{21}$$

$$P = X^{-1}, K = YX^{-1}. \tag{22}$$

P r o o f. According to Lemma 2.1, for system (17) we have

$$\dot{S}\left(e\left(t\right),\tilde{\theta}\right)+\lambda S\left(e\left(t\right),\tilde{\theta}\right)-\mu w^{T}\left(t\right)w\left(t\right)\leq 0. \tag{23}$$

Then, according to (21) we have

$$\dot{e}^{T}\left(t\right)Pe\left(t\right)+e^{T}\left(t\right)P\dot{e}\left(t\right)-\frac{1}{\eta}\left(\dot{\tilde{\theta}}^{T}\tilde{\theta}+\tilde{\theta}^{T}\dot{\tilde{\theta}}\right)+\lambda\left(e^{T}\left(t\right)Pe\left(t\right)+\frac{1}{\eta}\tilde{\theta}^{T}\tilde{\theta}\right)-\mu w^{T}\left(t\right)w\left(t\right)\leq 0. \tag{24}$$

$$\begin{gathered}e^{T}\left(t\right)\left(\left(A+BK\right)^{T}P+P\left(A+BK\right)+\lambda P\right)e\left(t\right)\\+\tfrac{\lambda}{\eta}\tilde{\theta}^{T}\tilde{\theta}+g^{T}\left(t\right)Pe\left(t\right)+e^{T}\left(t\right)Pg\left(t\right)\\-\mu\left(\tilde{\theta}^{T}\tilde{\theta}+\tilde{\theta}^{T}\left(t\right)g\left(t\right)+g^{T}\left(t\right)\tilde{\theta}\left(t\right)+g^{T}\left(t\right)g\left(t\right)\right)+\tilde{\theta}^{T}f^{T}\left(x\left(t\right)\right)Pe\left(t\right)+e^{T}\left(t\right)Pf\left(x\left(t\right)\right)\tilde{\theta}\\-\tfrac{1}{\eta}\left(\dot{\tilde{\theta}}^{T}\tilde{\theta}+\tilde{\theta}^{T}\dot{\tilde{\theta}}\right)\leq 0.\end{gathered} \tag{25}$$

By choosing

$$\dot{\hat{\theta}}=\eta f^{T}\left(x\left(t\right)\right)Pe\left(t\right) \tag{26}$$

we have

$$\begin{gathered}e^{T}\left(t\right)\left(\left(A+BK\right)^{T}P+P\left(A+BK\right)+\lambda P\right)e\left(t\right)\\+\tfrac{\lambda}{\eta}\tilde{\theta}^{T}\tilde{\theta}+g^{T}\left(t\right)Pe\left(t\right)+e^{T}\left(t\right)Pg\left(t\right)\\-\mu\left(\tilde{\theta}^{T}\tilde{\theta}+\tilde{\theta}^{T}\left(t\right)g\left(t\right)+g^{T}\left(t\right)\tilde{\theta}\left(t\right)+g^{T}\left(t\right)g\left(t\right)\right)\leq 0.\end{gathered} \tag{27}$$

According to Lemma 2.2, we have

$$\begin{gathered}g^{T}\left(t\right)Pe\left(t\right)+e^{T}\left(t\right)Pg\left(t\right)\leq\alpha e^{T}\left(t\right)Pe\left(t\right)+\alpha^{-1}g^{T}\left(t\right)Pg\left(t\right)\\\tilde{\theta}^{T}\left(t\right)g\left(t\right)+g^{T}\left(t\right)\tilde{\theta}\left(t\right)\leq\beta\tilde{\theta}^{T}\left(t\right)\tilde{\theta}\left(t\right)+\beta^{-1}g^{T}\left(t\right)g\left(t\right).\end{gathered} \tag{28}$$

By substituting (28) in (27), it is obtained that

$$\begin{gathered}e^{T}\left(t\right)\left(\left(A+BK\right)^{T}P+P\left(A+BK\right)+\left(\alpha+\lambda\right)P\right)e\left(t\right)\\+\left(\tfrac{\lambda}{\eta}-\mu\left(1+\beta\right)\right)\tilde{\theta}^{T}\tilde{\theta}+\alpha^{-1}g^{T}\left(t\right)Pg\left(t\right)-\mu\left(1+\beta^{-1}\right)g^{T}\left(t\right)g\left(t\right)\leq 0.\end{gathered} \tag{29}$$

Consider

$$P\leq\lambda_{\max}I\leq\epsilon I. \tag{30}$$

Where $\lambda_{\max}$ is the maximum eigenvalue of $P$and ϵI is the corresponding upper bound [15], then

$$\begin{gathered}e^{T}\left(t\right)\left(\left(A+BK\right)^{T}P+P\left(A+BK\right)+\left(\alpha+\lambda\right)P\right)e\left(t\right)\\+\left(\tfrac{\lambda}{\eta}-\mu\left(1+\beta\right)\right)\tilde{\theta}^{T}\tilde{\theta}+\left(\alpha^{-1}\epsilon-\mu\left(1+\beta^{-1}\right)\right)g^{T}\left(t\right)g\left(t\right)\leq 0.\end{gathered} \tag{31}$$

By choosing

$$\begin{aligned}\mu&=\tfrac{\lambda}{\eta(1+\beta)}\\\epsilon&=\tfrac{\alpha\lambda}{\eta\beta}\end{aligned} \tag{32}$$

equation (31) is reduced to

$$e^T(t)\left((A+BK)^T P + P(A+BK) + (\alpha+\lambda)P\right)e(t) \le 0. \tag{33}$$

By multiplying (19) from left and right by $diag\{P\}$ and substituting $P = X^{-1}$and $K = YX^{-1}$ we have

$$\left((A+BK)^T P + P(A+BK) + (\alpha+\lambda)P\right) \le 0. \tag{34}$$

By multiplying inequity (34) from left by $e^T(t)$ and from right by $e(t)$, (33) is obtained. Also, equation (20) is obtained from (30).

According to Lemma 2.1, there is a set $\Omega = \left\{e \in R^{n_x} |\ S\left(e, \tilde{\theta}\right) \le \frac{\mu w_{\max}^2}{\lambda}\right\}$ so that it is a robust invariant set for the system (17). □

Finally, the following controlling algorithm is employed to stabilize the system [21].

Step 0. At time t_0, set $\bar{x}(t_0) = x(t_0)$ in which $x(t_0)$ is the current state.

Step 1. At time t_k and current state $(\bar{x}(t_k),\ x(t_k))$, solve problem (12) to obtain the nominal control action $\bar{u}(t_k)$ and the actual control action $u(t_k) = \bar{u}(t_k) + Ke(t_k)$.

Step 2. Apply the control $u(t_k)$to the system (7), during the sampling interval $[t_k,\ t_{k+1}]$, where $t_{k+1} = t_k + \delta$.

Step 3. Measure the state $x(t_{k+1})$ at the next time instant t_{k+1} of the system (7) and compute the successor state $\bar{x}(t_{k+1})$ of the nominal system (10) under the nominal control $\bar{u}(t_k)$.

Step 4. Set $(\bar{x}(t_k),\ x(t_k)) = (\bar{x}(t_{k+1}),\ x(t_{k+1}))$, $t_k = t_{k+1}$, and go to step 1.

4. NUMERICAL EXAMPLE

Pure surge model of Moore and Greitzer for the centrifugal compressor are as the followings

$$\begin{aligned} \dot{\psi} &= \tfrac{1}{4B^2 l_c}\left(\phi - \phi_T(\psi) - d_\phi(t)\right) \\ \dot{\phi} &= \tfrac{1}{l_c}\left(\psi_c(\phi) - \psi + d_\psi(t)\right). \end{aligned} \tag{35}$$

Where ψ is the coefficient of increase in compressor pressure, ϕ is the coefficient of compressor's mass flow, $d_\phi(t)$ and $d_\psi(t)$ are the disturbances of flow and pressure. Also, $\phi_T(\psi)$ is the characteristic of throttle valve and is the characteristic of the compressor. is the Greitzer's parameter and shows the length of canals (ducts). Moor and Greitzer's [14] compressor characteristic is defines as

$$\psi_c(\phi) = \psi_{c0} + H\left(1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\right). \tag{36}$$

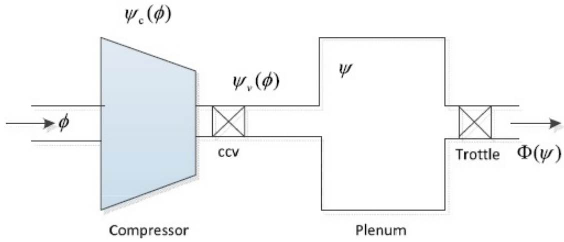

Fig. 1. The compressor system with CCV [18].

Where, ψ_{c0} is the value of characteristic curve in zero db, H is half of the height of the characteristic curve, and $W$is the half of the width of the characteristic curve. The equation for throttle valve characteristic is also derived from [7] and is as follows

$$\phi_T(\psi) = \gamma_T \sqrt{\psi}. \tag{37}$$

Where, γ_T is also the valve's yield. Figure 1 is the diagram of compression system with Close Couple Valve (CCV).

The system model equations, considering a CCV, are

$$\begin{aligned} \dot{\psi} &= \frac{1}{4B^2 l_c}\left(\phi - \phi_T(\psi) - d_\phi(t)\right) \\ \dot{\phi} &= \frac{1}{l_c}\left(\psi_c(\phi) - \psi - \psi_V(\phi) + d_\psi(t)\right). \end{aligned} \tag{38}$$

Considering $\psi_V(\phi)$ as the input for system control and $x_1 = \psi,\ x_2 = \phi$, the equations of compressor state space are

$$\begin{aligned} \dot{x}_1 &= \frac{1}{4B^2 l_c}\left(x_2 - \phi_T(x_1) - d_\phi(t)\right) \\ \dot{x}_2 &= \frac{1}{l_c}\left(\psi_c(x_2) - x_1 - u + d_\psi(t)\right). \end{aligned} \tag{39}$$

In designing a surge controller in the compressor system (39), it is assumed that the value of throttle valve, as well as the compressor characteristic, are unknown. So, the fuzzy system [19] is used to approximate the compressor characteristic. To do this, a 9-membership is used, with the following equations

$$p_j(x_2) = e^{-(x_2 - 0.1j)^2},\ j = 1, 2, \ldots, 9 \tag{40}$$

$$\psi_c(x_2) = W^{*T} P(x_2) + \Delta\psi_c(x_2) \tag{41}$$

$$W^{*T} = \left[W_1^*, \ldots, W_9^*\right] \tag{42}$$

$$P(x_2) = \left[p_1(x_2), \ldots, p_9(x_2)\right]^T. \tag{43}$$

In which $P(x_2)$ is fuzzy basis function vector, W^* shows the component vector and $\Delta\psi_c(x_2)$ satisfies $\Delta\psi_c(x_2) < \epsilon$, where $\epsilon > 0$ is real number [19]. Then the equations of compressor systems are obtained

$$\begin{aligned} \dot{x}_1 &= \tfrac{1}{4B^2 l_c}\left(x_2 - \gamma_T\sqrt{x_1} - d_\phi(t)\right) \\ \dot{x}_2 &= \tfrac{1}{l_c}\left(-x_1 - u + W^{*T}P(x_2) + \Delta\psi_c(x_2) + d_\psi(t)\right). \end{aligned} \tag{44}$$

Next, the existing constraints in the compressor system should be incorporated into the optimization problem (12). The first existing constraint is on the controlling input. Since the controlling signal has a CCV output, so we have

$$u(t) > 0. \tag{45}$$

The next constraint and limitation is that the flow has some maximum and minimum values. This constraint should also be considered.

$$-\phi_m \le \phi(t) \le \phi_{Choke}. \tag{46}$$

The open-loop optimization problem described by problem (12) is solved in discrete time with a sample time of $\delta = 0.1$ time units and prediction horizon of $T_p = 0.3$ time units.

Next, the compressor equations are rewritten according to (7) relation

$$A = \begin{bmatrix} 0 & \frac{1}{4B^2 l_c} \\ -\frac{1}{l_c} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -\frac{1}{l_c} \end{bmatrix} \tag{47}$$

$$f(x) = \begin{bmatrix} -\frac{1}{4B^2 l_c}\sqrt{x_1} & \frac{1}{l_c}P(x_2) \end{bmatrix}, \quad \theta = \begin{bmatrix} \gamma_T \\ W^* \end{bmatrix} \tag{48}$$

$$d(t) = \begin{bmatrix} \frac{1}{4B^2 l_c} d_\phi(t) \\ \frac{1}{l_c}\left(\Delta\psi_c(x_2) + d_\psi(t)\right) \end{bmatrix}. \tag{49}$$

In the objective function (11), the values of weight matrixes Q and R are selected as

$$Q = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \quad R = 1. \tag{50}$$

By choosing

$$\alpha = 10^{-3},\ \beta = 10^{-7},\ \lambda = 10^{-3}, \eta = 10^{3}, \mu = 10^{-6}, \epsilon = 10^{-2} \tag{51}$$

and solving LMI for (19) and (20) relations, P and K are obtained as

$$K = \begin{bmatrix} -0.34 & 1.7 \end{bmatrix} \tag{52}$$

$$P = \begin{bmatrix} 0.58 & 0.18 \\ 0.18 & 0.56 \end{bmatrix} \times 10^{-3}. \tag{53}$$

Finally, the presented controller is compared to an active controller [18] in order to confirm its efficiency. Values of compressor parameters used in simulation are taken from [8].

$$B = 1.8,\ l_c = 3,\ H = 0.18,\ W = 0.25,\ \psi_{c0} = 0.3. \tag{54}$$

The initial points of process were $(x_1(0), x_2(0)) = (0.6, -0.23)$ in the left side of the surge line and the initial points of model were $(x_1(0), x_2(0)) = (1, 1)$. The system is simulated in two different scenarios. The first scenario assumes no disturbances on the system. However, during the first 500 seconds, while the system is in the stability region, the throttle valve value is $\gamma_T = 0.65$. After that, the throttle valve value drops to $\gamma_T = 0.6$ and the system enters the surge area.

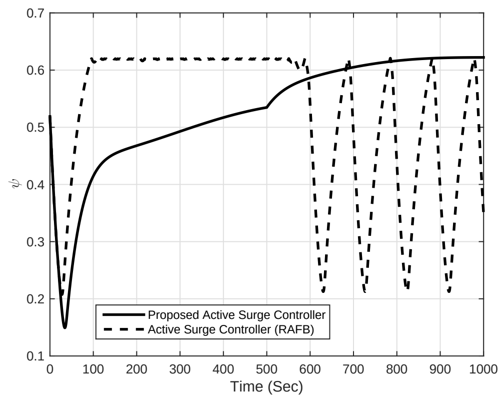

Fig. 2. Pressure of compressor.

As it can be seen in Figure 2, the controller [18] can control the system until $t = 500s$. However, in this time interval the system is in the compressor stability region. After $t = 500s$, when the value of throttle valve is reduced, the controller cannot control the compressor system anymore, which leads to surge phenomenon. On the other hand, the controller presented in this paper can stabilize the system well after $t = 500s$.

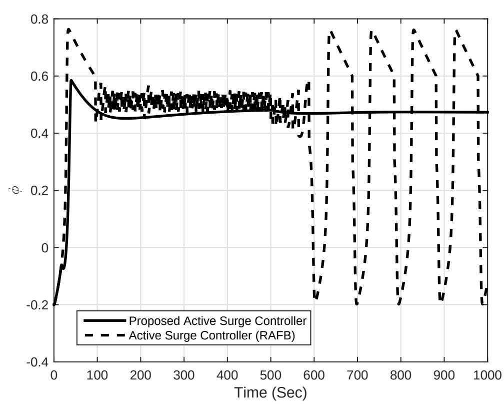

Fig. 3. Flow of compressor.

Figure 3 shows the compressor flow. It is obvious in this figure that the controller

[18] cannot control the system after a reduction in the throttle valve value. Even before this reduction, some fluctuations can be seen in the system flow. However, the controller presented in this paper can stabilize the system during the whole time interval.

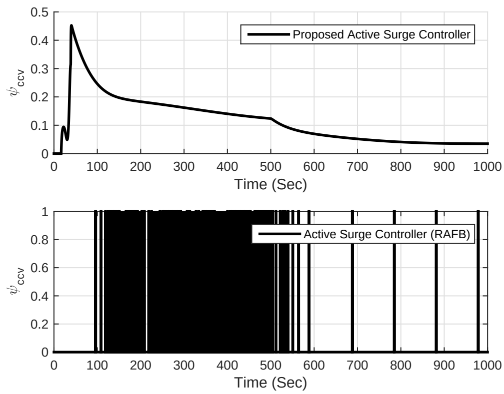

Fig. 4. Control signal.

Figure 4 shows the control signal behavior. Since this signal is the CCV output and have upper and lower bounds, it can be seen that the controller [18] fluctuates continuously during this time, which is not practical. On the other hand, the controller presented in this paper has a smooth control signal.

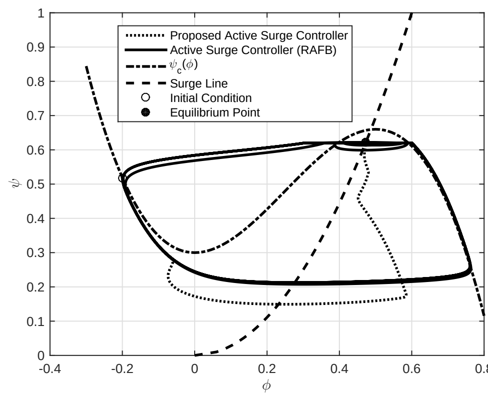

Fig. 5. Compression system trajectories.

Figure 5 presents the compressor characteristic curve. It can be seen from this figure that using controller [18] can lead to limit cycle in the system. On the other hand, the controller presented in this paper propels the system from the starting point in surge area towards the stability region.

The second scenario assumes that the throttle valve value is always $\gamma_T = 0.65$, so that the system will remain in the stability region. However, after $t = 500s$, disturbance from (55) is applied to the system.

$$\begin{aligned} d_\phi(t) &= 0.15e^{-0.015t}\cos(0.2t) \\ d_\psi(t) &= 0.1e^{-0.005t}\sin(0.3t). \end{aligned} \tag{55}$$

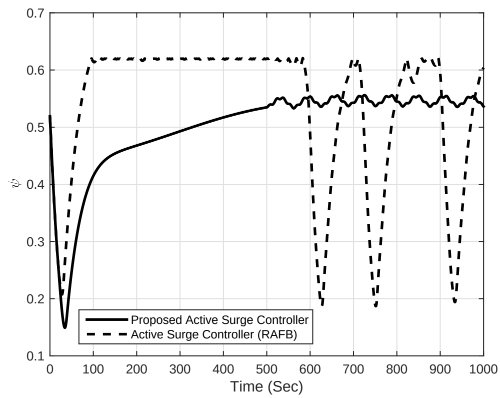

Fig. 6. Pressure of compressor.

The compressor pressure is shown in Figure 6. Despite the fact that the throttle valve keeps the system in stability region, the controller [18] cannot save the system from surge, because a disturbance is applied to the system. However, the controller presented in this paper can ward off the disturbances easily and stabilize the system.

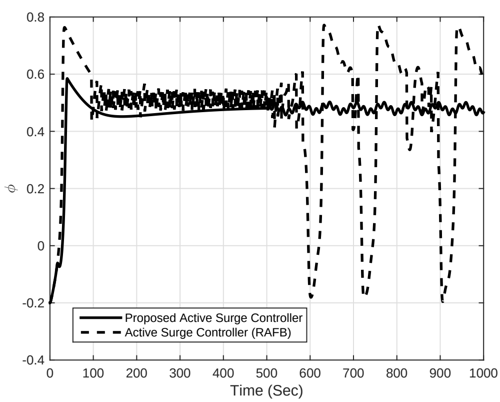

Fig. 7. Flow of compressor.

Figure 7 presents the system flow behavior. It can easily be seen that before applying disturbance, the system controlled by [18] experiences some fluctuations in the flow.

Also, after applying the disturbance, the system enters the surge area. On the other hand, the controller presented in this paper can stabilize the system before and after applying disturbances.

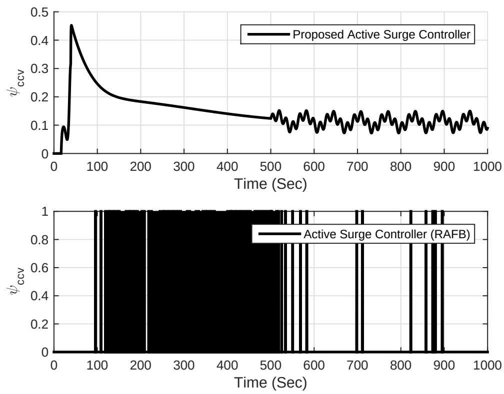

Fig. 8. Control signal.

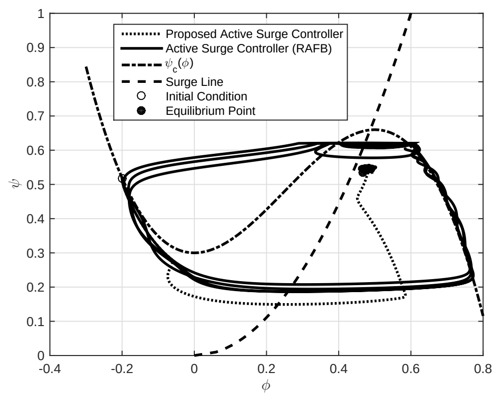

Fig. 9. Compression system trajectories.

The CCV behavior is shown in Figure 8. The weakness of the controller [18] compared to the controller presented in this paper is obvious in this figure.

Figure 9 shows the compressor characteristic curve. It can be seen in this figure that using the controller [18], the system experiences limit cycle. On the other hand, when the controller presented in this paper is employed, the system moves from surge area to the stability region.

5. CONCLUSION

This paper has presented a new robust adaptive Tube-MPC for a special class of continuous-time nonlinear systems, with uncertainty and unknown bounded disturbances. This approach was characterized by:

I) Consideration of input and state constraints;

II) Robustness of local controllers to model uncertainty and disturbances;

III) Bounded disturbances in the systems (but their upper bound is not specified).

IV) A robust invariant set for the controller; and,

V) Proven closed-loop system overall stability and convergence.

Finally, the proposed controller is used to solve a surge problem in centrifugal compressors. The results obtained from the simulation show the efficiency and robustness of this controller.

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REFERENCES

[1] R. Bindlish: Nonlinear model predictive control of an industrial polymerization process. Computers Chemical Engrg. *73* (2015), 43–48. DOI:10.1016/j.compchemeng.2014.11.001

[2] F. A. Cuzzola, J. C. Geromel, and M. Morari: An improved approach for constrained robust model predictive control. Automatica *38* (2002), 1183–1189. DOI:10.1016/s0005-1098(02)00012-2

[3] B. Ding and T. Zou: A synthesis approach for output feedback robust model predictive control based-on input-output model. J. Process Control *24* (2014), 60–72. DOI:10.1016/j.jprocont.2013.12.006

[4] M. Ghanavati and A. Chakravarthy: Demand-side energy management by use of a design-then-approximate controller for aggregated thermostatic loads. In: Amer. Control Conference (ACC), Chicago 2015. DOI:10.1109/acc.2015.7171900

[5] M. Ghanavati and A. Chakravarthy: Demand-side energy management using an adaptive control strategy for aggregate thermostatic loads. In: AIAA SciTech Forum 2015, pp. 1–7. DOI:10.2514/6.2015-0121

[6] M. Ghanavati, S. Mobayen, and V. J. Majd: A new robust model predictive control strategy for rotational inverted pendulum system. In: Int. Siberian Conference on Control and Communications (SIBCON) 2011, pp. 33–38. DOI:10.1109/sibcon.2011.6072589

[7] J. T. Gravdahl and O. Egeland: Compressor Surge and Rotating Stall: Modeling and Control. Springer–Verlag, London 1999. DOI:10.1007/978-1-4471-0827-6

[8] E. M. Greitzer: Surge and rotating stall in axial flow compressors. Part I: Theoretical compression system model. ASME J. Engrg. for Power *98* (1976), 2, 191–198. DOI:10.1115/1.3446138

[9] B. Gu and Y. P. Gupta: Control of nonlinear processes by using linear model predictive control algorithms. ISA Trans. *47* (2008), 211–216. DOI:10.1016/j.isatra.2007.12.002

[10] D. F. He, H. Huang, and Q. X. Chen: Quasi-min-max MPC for constrained nonlinear systems with guaranteed input-to-state stability. J. Franklin Inst. *351* (2014), 3405–3423. DOI:10.1016/j.jfranklin.2014.03.006

[11] M. V. Kothare, V. Balakrishnan, and M. Morari: Robust constrained model predictive control using linear matrix inequalities. Automatica *32* (1996), 1361–1379. DOI:10.1016/0005-1098(96)00063-5

[12] B. Kouvaritakis, J. A. Rossiter, and J. Schuurmans: Efficient robust predictive control. IEEE Trans. Automat. Control *45* (2000), 1545–1549. DOI:10.1109/9.871769

[13] L. Magni, G. De Nicolao, L. Magnani, and R. Scattolini: A stabilizing model based predictive control algorithm for nonlinear systems. Automatica *37* (2001), 1351–1362. DOI:10.1016/s0005-1098(01)00083-8

[14] F. K. Moore and E. M. Greitzer: A theory of post-stall transientin axial compression systems: Part I – Development of equations. ASME J. Engrg. for Gas Turbines and Power *108* (1986), 68–76. DOI:10.1115/1.3239887

[15] N. Poursafar, H. D. Taghirad, and M. Haeri: Model predictive control of nonlinear discrete time systems: a linear matrix inequality approach. IET Proc. Control Theory Appl. *4* (2010), 1922–1932. DOI:10.1049/iet-cta.2009.0650

[16] M. Razi and M. Haeri: Design of a robust model predictive controller with reduced computational complexity. ISA Trans. *53* (2014), 1754–1759. DOI:10.1016/j.isatra.2014.09.008

[17] S. Shamaghdari, S. K. Y. Nikravesh, and M. Haeri: Integrated guidance and control of elastic flight vehicle based on robust MPC. Int. J. Robust Nonlinear Control *25* (2015), 2608–2630. DOI:10.1002/rnc.3215

[18] H. Sheng, W. Huang, T. Zhang, and X. Huang: Robust Adaptive Fuzzy Control of Compressor Surge Using Backstepping. Arabian J. Science and Engrg. *39* (2014), 9301–9308. DOI:10.1007/s13369-014-1448-1

[19] L. X. Wang: Adaptive Fuzzy Systems and Control: Design and Stability Analysis. Prentice-Hall, Englewood Cliffs 1994.

[20] S. Y. Yu, C. Bohm, H. Chen, and F. Allgower: Robust model predictive control with disturbance invariant sets. In: Proc. Amer. Contr. Conf., Baltimore 2010, pp. 6262–6267. DOI:10.1109/acc.2010.5531520

[21] S. Yu, C. Maier, H. Chen, and F. Allgower: Tube MPC scheme based on robust control invariant set with application to Lipschitz nonlinear systems. Systems Control Lett. *62* (2013), 194–200. DOI:10.1016/j.sysconle.2012.11.004

[22] P. Zheng, D. Li, Y. Xi, and J. Zhang: Improved model prediction and RMPC design for LPV systems with bounded parameter changes. Automatica *49* (2013), 3695–3699. DOI:10.1016/j.automatica.2013.09.024

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