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EVENT-TRIGGERED OBSERVER-BASED TRACKING CONTROL FOR LEADER-FOLLOWER MULTI-AGENT SYSTEMS

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This paper considers the consensus tracking problem for a class of leader-follower multi-agent systems via event-triggered observer-based control. In our set-up, only a subset of the followers can obtain some relative information on the leader. Assume that the leader's control input is unknown for the followers. In order to track such a leader, we design two novel event-triggered observer-based control strategies, one centralized and the other distributed. One can prove that under the proposed control strategies, the tracking problem can be solved if the communication graph of the agents is connected. Furthermore, the Zeno-behavior of triggering time sequences can be avoided. Finally, a numerical example is given to illustrate the effectiveness of the obtained theoretical results.

Keywords: multi-agent systems, event-triggered control, leader-follower, observer

Classification: 93C35, 93C15

1. INTRODUCTION

With the rapid development of communication technology, digital sensors and microprocessors, multi-agent systems have attracted considerable attention from various fields. A critical issue for multi-agent systems is how the dynamic characteristic of the agents and the topology structure of the systems affect the collective behaviors, such as consensus [5], formation [16], flocking [10]. Leader-follower consensus is one of the most interesting behaviors which requires all followers will track the leader eventually. This issue has received the most attention and related results can be found in [4, 6, 11, 19] and the references therein, to name a few.

Traditional control strategies include continuous control [17] and sampling control [15]. Continuous controller needs to be updated continuously, which may lead to large communication load. For the sake of reducing the update frequency, sampling control strategy is often considered, in which the controller is updated in a constant period T, independently from the state of the system. Periodic sampling control can save resources to some extent. However, it seems "inefficient" since it preforms regularly regardless of fluctuation of the states.

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In practice, the agents are often equipped with a small embedded micro processor which has limited storage resources and communication bandwidth. These limitations have resulted in a novel interest in event-based control for multi-agent systems, in which the control is executed only when necessary rather than periodically. Event-triggered control is becoming a hot field in control theory in recent years. For example, an effective event-triggered control strategy is introduced in [14]. The control actuation times are determined by an event-triggering function which depends on the measurement error. When the error reaches the designed threshold, the event is triggered. In [1], both the centralized and distributed event-triggered controls for single integrator model are discussed. A new combinational measuring method is proposed in [2], in which the measurement error of each agent not only depends on its own state, but also those of its neighbors. In [7], the problem of leader-follower tracking control with communication delays is studied. The velocity of the leader is unknown for all the followers and an observer-based consensus tracking control is considered. Recently, several event-triggered control laws have been proposed for second-order multi-agent systems in such as [8, 13, 20, 21]. On the basis of [7, 8] considers the leader-follower tracking problem for a second-order dynamic model with an active leader. An observer-based control is designed and then is applied to the event-triggered tracking problem. Yan et al. develop a consensus control under a weighted directed network topology [21]. Furthermore, the event-triggered control for linear systems is also considered [9, 12, 23, 24, 25, 26], such as the event-triggered control for nonlinear systems [9], the event-based dynamic output feedback [23], the event-triggered control and observer for the tracking problem [24], the event-based control with time delay [26].

Following our previous work [18], in this paper, we consider the tracking problem for a class of leader-follower multi-agent systems. Different from the existing results, in this paper, the control input $u_l = A_l x_l + b_l$ of the leader is not known to all the followers and A_l can be of any form, which allows the state variables of the leader coupled. With different A_l that is known to the followers, the leader can move along different trajectories. The contributions of this work are three-fold: 1) To track the leader, a distributed observer is first constructed for each follower to estimate the control input of the leader, based on which a distributed feedback controller is designed. 2) Compared with our previous work [18] on which this paper is based, significant improvement is that this paper focuses on the event-triggered control. Both the observer and the controller for each follower are in an event-triggered way in the sense that they are updated when the triggering condition is satisfied. Both the centralized and distributed event-triggered feedback controllers are designed for each follower, respectively. By using the proposed event-triggered observer-based controller, the tracking problem can be solved if the underlying adjacency graph of the system is connected. 3) It is further shown that the Zeno phenomena can be avoided in the event-triggered scheme. One can prove that two adjacent inter-execution instants have a positive lower bound.

The remainder of this paper is organized as follows. Section 2 introduces some concepts on algebraic graph theory and the system model considered in this paper. The centralized event-triggered control design and convergence analysis are presented in Section 3, and then Section 4 shows the distributed event-triggered control. An example is given in Section 5 to validate the theoretical results. Section 6 is the conclusions.

2. PROBLEM STATEMENT

This paper considers the tracking problem of multi-agent systems with N followers and one leader moving on the plane. The topology relationship of the followers and the leader can be described by a graph $\bar{\mathcal{G}}$. We assume the information transfer between followers is undirected (denoted by an undirected graph $\mathcal{G}$), while the agents in graph $\mathcal{G}$ is connected to the leader by directed edges, that is, the information can only be transferred from leader to followers (refer to Figure 1).

The undirected graph $\mathcal{G} = (V, \varepsilon, A)$ consists of a vertex set $V = \{v_1, v_2, \ldots, v_N\}$ denoting N agents and an edge set $\varepsilon \subseteq V \times V$. Two nodes i and j are adjacent or neighbors if $(i, j) \in \varepsilon$. The graph $\mathcal{G}$ is called connected if there is a path between any two nodes in $\mathcal{G}$. The neighbors set of agent i at time t is denoted by $\mathcal{N}_i(t) = \{j \in V | (i, j) \in \varepsilon\}$. $A = (a_{ij})_{N\times N}$ is the weighted adjacency matrix of the undirected graph $\mathcal{G}$, where $a_{ii} = 0$ and $a_{ij} = a_{ji} \geq 0$. $a_{ij} > 0$ if and only if $(i, j) \in \varepsilon$. $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ is the degree matrix of graph $\mathcal{G}$, where $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ is the degree of agent i. The Laplacian matrix L with respect to graph $\mathcal{G}$ is defined as

$$L = D - A \tag{1}$$

The graph $\bar{\mathcal{G}}$ is said to be connected if at least one agent in each component of graph $\mathcal{G}$ is connected to the leader by a directed edge [6]. Note that graph $\bar{\mathcal{G}}$ is connected does not mean graph $\mathcal{G}$ is also connected. Define $\Delta = \mathrm{diag}\{\alpha_1, \ldots, \alpha_N\}$ as the leader adjacency matrix associated with graph $\bar{\mathcal{G}}$, where $\alpha_i > 0$ if the leader is a neighbor of agent i (that is, there is a directed edge from agent i to the leader) and otherwise, $\alpha_i = 0$.

Denote $\mathbf{1} = [1, 1, \ldots, 1]^T \in \mathbb{R}^N$. The following two lemmas are very useful in the theoretical analysis.

Lemma 2.1. (Godsil and Royle [3]) For an undirected graph, L is a symmetric and positive semi-definite matrix. Since each row sum of L is zero, L has a zero eigenvalue associated with the eigenvector $\mathbf{1}$, i. e., $L\mathbf{1} = \mathbf{0}$. Moreover, if the graph is connected, L has only one zero eigenvalue and all the other eigenvalues are positive.

Lemma 2.2. (Hong et al. [6]) If graph $\bar{\mathcal{G}}$ is connected, then the symmetric matrix $H = L + \Delta$ associated with $\bar{\mathcal{G}}$ is positive definite.

In this paper, the dynamics of each follower is

$$\dot{x}_i = u_i, \quad i = 1, \ldots, N, \tag{2}$$

where $x_i \in \mathbb{R}^2$ is the state and $u_i \in \mathbb{R}^2$ is the control input of the ith follower, $i = 1, \ldots, N$.

The dynamics of the leader is

$$\dot{x}_l = u_l, \tag{3}$$

where $x_l = [x_{1l}, x_{2l}]^T, u_l \in \mathbb{R}^2$ are the state and control input, respectively. For simplicity, we choose

$$u_l = A_l x_l + b_l, \tag{4}$$

where $A_l \in \mathbb{R}^{2\times 2}, b_l = [b_{1l}, b_{2l}]^T \in \mathbb{R}^{2\times 1}$ can be any matrix such that the leader can move along different trajectories.

To track the leader, u_i is designed only by the relative information between itself and its neighbors. In our problem, u_l is not known to any follower even if it is connected to the leader during the tracking process. In this case, we have to estimate $u_l(t)$ for each follower. An observer-based tracking control is designed by using their neighbors' information. The estimated value of $u_l(t)$ by follower i is denoted by $\hat{u}_l^i(t) = A_l \hat{x}_l^i(t) + b_l, i = 1, \ldots, N$.

Remark 2.1. In some sense, A_l and b_l denote the shape and the translation of the leader's trajectory, respectively. We assume that A_l and b_l are known to each agent, which means each follower may know the shape of the leader's trajectory, but not the initial value. In fact, it is necessary to know the shape of the target's trajectory in the tracking problem.

In this paper, the control is constructed based on event-triggering strategies. For system (2)−(3), both centralized and distributed event-triggered control protocols are investigated in the sequel.

3. CENTRALIZED EVENT-TRIGGERED CONTROL APPROACH

This section is devoted to solve the tracking problem with a centralized event-triggered control strategy for system (2). In this case, all the agents update their control simultaneously. Assume that the time sequence of event triggering is $t_0 = 0, t_1, \ldots, t_k, \ldots$, and t_k denotes the kth event instant of the system.

In event-triggered control, actuation times are determined by the event-triggering condition which depends on the measurement error. In this section, we consider the combinational measurement

$$q_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t)) + \alpha_i(x_i(t) - x_l(t)). \tag{5}$$

At time t_k, agent i measures the information of $q_i(t)$ and will keep it unchanged until the next time t_{k+1} comes. Therefore, in the interval $[t_k, t_{k+1})$ the measurement of agent i is $q_i(t_k)$. Based on (5), the measurement error is defined as

$$e_i(t) = q_i(t_k) - q_i(t), \qquad t \in [t_k, t_{k+1}). \tag{6}$$

With the stack vectors $e(t) = [e_1(t), \ldots, e_N(t)]^T$ and $x(t) = [x_1(t), \ldots, x_N(t)]^T$, the event-triggering condition is defined as $f(e(t), x(t)) = 0$, where the trigger function $f(e, x)$ will be determined later. As soon as the measurement error reaches the designed value, an event for the system is triggered. At the same time, each agent will send their states to their neighbors and the controller is updated.

Based on the measurement of $q_i(t_k)$, for $t \in [t_k, t_{k+1})$, we propose the event-based control input for agent i as follows

$$u_i(t) = -Kq_i(t_k) + A_l \hat{x}_l^i(t) + b_l, \tag{7}$$

and the observer

$$\dot{\hat{x}}_l^i(t) = -K_l q_i(t_k) + A_l \hat{x}_l^i(t) + b_l, \tag{8}$$

where the gain matrices $K, K_l \in \mathbb{R}^{2\times 2}$ need to be determined later. $\hat{x}_l^i(t)$ is the estimate value of $x_l(t)$ by agent i, $i = 1, \dots, N$.

Define the variables $\bar{x}_i(t) = x_i(t) - x_l(t)$ and $\tilde{x}_i(t) = \hat{x}_l^i(t) - x_l(t)$, denoting the tracking error and estimation error of agent i, $i = 1, \dots, N$, respectively. The control aim is to find appropriate gain matrices K and K_l such that the dynamic tracking problem is solved and at the same time all $\hat{u}_l^i$ will converge to u_l, that is,

$$\bar{x}_i \to 0, \quad \hat{u}_l^i - u_l = A_l \tilde{x}_i \to 0, \ i = 1, \dots, N. \tag{9}$$

Associate with (6), (7) and (8), we have the following error closed-loop system

$$\begin{cases} \dot{\bar{x}}_i(t) = A_l \tilde{x}_i(t) - K[q_i(t) + e_i(t)] \\ \dot{\tilde{x}}_i(t) = A_l \tilde{x}_i(t) - K_l[q_i(t) + e_i(t)]. \end{cases} \tag{10}$$

By defining the stack vectors

$$\bar{x}(t) = [\bar{x}_1(t), \dots, \bar{x}_N(t)]^T, \quad \tilde{x}(t) = [\tilde{x}_1(t), \dots, \tilde{x}_N(t)]^T, \tag{11}$$

we have

$$\begin{cases} \dot{\bar{x}}(t) = (I_N \otimes A_l)\tilde{x}(t) - (H \otimes K)\bar{x}(t) - (I_N \otimes K)e(t) \\ \dot{\tilde{x}}(t) = (I_N \otimes A_l)\tilde{x}(t) - (H \otimes K_l)\bar{x}(t) - (I_N \otimes K_l)e(t), \end{cases} \tag{12}$$

where $H = L + \Delta$, L is the Laplacian matrix of graph $\mathcal{G}$ defined in (1) and Δ is the leader adjacency matrix of graph $\bar{\mathcal{G}}$ defined in Section 2. $\otimes$ is the Kronecker product.

Since H is symmetric for the undirected graph $\mathcal{G}$, there is an orthogonal matrix T such that

$$THT^T = \Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\} \tag{13}$$

is diagonal, where λ_i, $i = 1, \dots, N$, are the eigenvalues of H. By Lemma 2.2, if graph $\bar{\mathcal{G}}$ is connected, all $\lambda_i > 0$.

By the orthogonal transformation

$$\hat{\bar{x}} = (T \otimes I_2)\bar{x}, \quad \hat{\tilde{x}} = (T \otimes I_2)\tilde{x}, \quad \hat{e} = (T \otimes I_2)e, \tag{14}$$

the system (12) can be rewritten as

$$\begin{cases} \dot{\hat{\bar{x}}}(t) = (I_N \otimes A_l)\hat{\tilde{x}}(t) - (\Lambda \otimes K)\hat{\bar{x}}(t) - (I_N \otimes K)\hat{e}(t) \\ \dot{\hat{\tilde{x}}}(t) = (I_N \otimes A_l)\hat{\tilde{x}}(t) - (\Lambda \otimes K_l)\hat{\bar{x}}(t) - (I_N \otimes K_l)\hat{e}(t), \end{cases} \tag{15}$$

or the matrix style

$$\begin{bmatrix} \dot{\hat{\bar{x}}} \\ \dot{\hat{\tilde{x}}} \end{bmatrix} = \begin{bmatrix} -\Lambda \otimes K & I_N \otimes A_l \\ -\Lambda \otimes K_l & I_N \otimes A_l \end{bmatrix} \begin{bmatrix} \hat{\bar{x}} \\ \hat{\tilde{x}} \end{bmatrix} - \begin{bmatrix} I_N \otimes K \\ I_N \otimes K_l \end{bmatrix} \hat{e}. \tag{16}$$

The corresponding decoupled subsystems are

$$\begin{bmatrix} \dot{\hat{\bar{x}}}_i \\ \dot{\hat{\tilde{x}}}_i \end{bmatrix} = \begin{bmatrix} -\lambda_i K & A_l \\ -\lambda_i K_l & A_l \end{bmatrix} \begin{bmatrix} \hat{\bar{x}}_i \\ \hat{\tilde{x}}_i \end{bmatrix} - \begin{bmatrix} K \\ K_l \end{bmatrix} \hat{e}_i, \qquad i = 1, \dots, N. \tag{17}$$

Denote $\lambda_{\min} = \min\{\lambda_i, i = 1, \dots, N\}$ the smallest eigenvalue of the matrix H, then by Lemma 2.2, if the adjacency graph $\bar{\mathcal{G}}$ is connected, $\lambda_{\min} > 0$. The special norm of a matrix $A \in \mathbb{R}^{m\times n}$ is defined as $||A|| = \max_{1\le i\le n} \sqrt{\lambda_i(A^TA)}$, where $\lambda_i(A^TA)$ are the eigenvalues of A^TA. Then we have the following main result.

Theorem 3.1. Consider the leader-follower multi-agent systems (2) and (3). Assume the adjacency graph $\bar{\mathcal{G}}$ is connected. Choose $K_l = K + A_l^T$ and $K = \frac{\delta}{\lambda_{\min}} I_2$, where $\delta \ge 4\|A_l\| + 1$. Then the dynamic consensus tracking problem can be solved and at the same time all $\hat{u}_l^i$ converge to u_l asymptotically by the event-triggered control law (7) and the observer (8) with the following event-triggering condition

$$\|\hat{e}\| \le \frac{\lambda_{\min}}{2\sigma}\|\hat{\bar{x}}_\lambda\|^2, \tag{18}$$

where $\hat{e}$ is defined in (14), $\sigma = \max\{\|\hat{\bar{x}}\|, \|\hat{\tilde{x}}\|\}$ and $\|\hat{\bar{x}}_\lambda\| = \min\{\|\hat{\bar{x}}_i\|, i = 1, \dots, N\}$. Furthermore, the event times can be written as

$$t_0 = 0, \qquad t_{k+1} = \inf\{t \mid t > t_k, \|\hat{e}\| \ge \frac{\lambda_{\min}}{2\sigma}\|\hat{\bar{x}}_\lambda\|^2\}, \quad k \in \mathbb{N}.$$

Proof. By the above analysis, one needs to prove that $\hat{\bar{x}}_i \to 0$ and $A_l\hat{\tilde{x}}_i \to 0$.

Consider the system (16) (or (17)), construct the candidate Lyapunov function $V = \sum_{i=1}^N V_i$ with

$$V_i = \hat{\bar{x}}_i^T\hat{\bar{x}}_i + \frac{1}{\lambda_i}(\hat{\bar{x}}_i - \hat{\tilde{x}}_i)^T(\hat{\bar{x}}_i - \hat{\tilde{x}}_i).$$

Then V is positive definite and the time derivative of V_i along each subsystem (17) is

$$\begin{aligned} \dot{V}_i =& \hat{\bar{x}}_i^T(-2\lambda_i K - 2K + 2K_l)\hat{\bar{x}}_i \\ &+ \hat{\bar{x}}_i^T(2A_l + 2K^T - 2K_l^T)\hat{\tilde{x}}_i \\ &+ \hat{\bar{x}}_i^T(-2K - \frac{2}{\lambda_i}K + \frac{2}{\lambda_i}K_l)\hat{e}_i + \hat{\tilde{x}}_i^T\left(\frac{2}{\lambda_i}K - \frac{2}{\lambda_i}K_l\right)\hat{e}_i. \end{aligned}$$

By choosing $K_l = K + A_l^T$, we get

$$\begin{aligned} \dot{V}_i =& \hat{\bar{x}}_i^T(-2\lambda_i K + 2A_l^T)\hat{\bar{x}}_i + \hat{\bar{x}}_i^T\left(-2K + \frac{2}{\lambda_i}A_l^T\right)\hat{e}_i \\ &+ \hat{\tilde{x}}_i^T(-\frac{2}{\lambda_i}A_l^T)\hat{e}_i \\ \le& \hat{\bar{x}}_i^T(-2\lambda_i K)\hat{\bar{x}}_i + 2\|A_l\|\hat{\bar{x}}_i^T\hat{\bar{x}}_i + 2\|\hat{\bar{x}}_i\|\|K\|\|\hat{e}_i\| \\ &+ \frac{2}{\lambda_i}\|\hat{\bar{x}}_i\|\|A_l\|\|\hat{e}_i\| + \frac{2}{\lambda_i}\|\hat{\tilde{x}}_i\|\|A_l\|\|\hat{e}_i\|. \end{aligned}$$

By enforcing the triggering condition (18), one has

$$\|\hat{e}_i\| \leq \|\hat{e}\| \leq \frac{\lambda_{\min}}{2\sigma_i}\hat{\bar{x}}_i^T\hat{\bar{x}}_i, \tag{19}$$

where $\sigma_i = \max\{\|\hat{\bar{x}}_i\|, \|\hat{\bar{x}}_i\|\}$, and

$$\begin{aligned}\dot{V}_i \leq &\hat{\bar{x}}_i^T(-2\lambda_i K)\hat{\bar{x}}_i + 2\|A_l\|\hat{\bar{x}}_i^T\hat{\bar{x}}_i \\ &+ \lambda_{\min}\|K\|\hat{\bar{x}}_i^T\hat{\bar{x}}_i + 2\|A_l\|\hat{\bar{x}}_i^T\hat{\bar{x}}_i.\end{aligned} \tag{20}$$

Choose $K = \frac{\delta}{\lambda_{\min}}I_2$ with $\delta \geq 4\|A_l\| + 1$, then inequality (20) further becomes

$$\begin{aligned}\dot{V}_i &\leq \hat{\bar{x}}_i^T(\frac{-2\lambda_i\delta}{\lambda_{\min}}I_2)\hat{\bar{x}}_i + 4\|A_l\|\hat{\bar{x}}_i^T\hat{\bar{x}}_i + \delta\hat{\bar{x}}_i^T\hat{\bar{x}}_i \\ &\leq -(\delta - 4\|A_l\|)\hat{\bar{x}}_i^T\hat{\bar{x}}_i \\ &\leq -\hat{\bar{x}}_i^T\hat{\bar{x}}_i.\end{aligned}$$

Hence,

$$\dot{V} = \sum_{i=1}^{N}\dot{V}_i \leq -\|\hat{\bar{x}}\|^2 \leq 0.$$

By the LaSalle's invariance principle, the solution of the system (16) will converge to the largest invariant set contained in the set $\{\hat{\bar{x}}, \hat{\bar{x}} \in \mathbb{R}^{2N} \mid \dot{V} = 0\}$. From $\dot{V} = 0$, we have $\hat{\bar{x}} = 0$ and then $\bar{x} = 0$ from (14). By the definition of the measurement error (6), one has $e = 0$ and $\hat{e} = 0$. Associated with the system (16), we can obtain that the largest invariant set of the set $\{\hat{\bar{x}}, \hat{\bar{x}} \in \mathbb{R}^{2N} \mid \dot{V} = 0\}$ is $\{\hat{\bar{x}}, \hat{\bar{x}} \in \mathbb{R}^{2N} \mid \hat{\bar{x}} = 0, (I_N \otimes \ A_l)\hat{\bar{x}} = 0\}$, which implies the conclusion. □

Under the above control protocol, the inter-event times $\{t_{k+1} - t_k,\ k \in \mathbb{N}\}$ are lower bounded by a positive number. This guarantees that there is no Zeno phenomenon. It is proved in the following theorem.

Theorem 3.2. Under the condition of Theorem 3.1, in the centralized event-triggered control (7), there is no Zeno behavior. That is, the inter-event times are lower bounded by a positive time $\tau \geq \frac{\mu}{a^2+ab\mu} > 0$, where $\mu = \frac{\lambda_{\min}}{2} > 0$ since the graph $\bar{\mathcal{G}}$ is connected, $a = \max\{\lambda_{\max}, \|A_l\|M + \|\Lambda \otimes K\|\}$ with $M > 0$ and $b = \max\{1, \|K\|\}$, $\lambda_{\min}$ and $\lambda_{\max}$ are the smallest and the largest eigenvalues of the matrix H, respectively.

P r o o f. By the definition (6), the stack vector is $e(t) = q(t_k) - q(t),\ t \in [t_k, t_{k+1})$, then $e(t_k) = 0$. From the triggering condition (18), one has $\|\hat{e}\| \leq \frac{\lambda_{\min}}{2}\|\hat{\bar{x}}\|$. Therefore, the range of $\frac{\|\hat{e}(t)\|}{\|\hat{\bar{x}}(t)\|}$ between two adjacent events is from 0 to $\mu = \frac{\lambda_{min}}{2}$.

Let $y(t)=\frac{\|\hat{e}(t)\|}{\|\hat{\tilde{x}}(t)\|}$, similar to [22], the time derivative of y satisfies

$$\begin{aligned}\dot{y}=&(\frac{\|\hat{e}\|}{\|\hat{\tilde{x}}\|})'=(\frac{(\hat{e}^T\hat{e})^{\frac{1}{2}}}{(\hat{\tilde{x}}^T\hat{\tilde{x}})^{\frac{1}{2}}})'\\=&\frac{(\hat{e}^T\hat{e})^{-\frac{1}{2}}\dot{\hat{e}}^T\hat{e}\|\hat{\tilde{x}}\|-(\hat{\tilde{x}}^T\hat{\tilde{x}})^{-\frac{1}{2}}\dot{\hat{\tilde{x}}}^T\hat{\tilde{x}}\|\hat{e}\|}{(\|\hat{\tilde{x}}\|)^2}\\=&\frac{\|\hat{e}\|^{-1}\dot{\hat{e}}^T\hat{e}\|\hat{\tilde{x}}\|-\|\hat{\tilde{x}}\|^{-1}\dot{\hat{\tilde{x}}}^T\hat{\tilde{x}}\|\hat{e}\|}{(\|\hat{\tilde{x}}\|)^2}\\=&\frac{\dot{\hat{e}}^T\hat{e}}{\|\hat{e}\|\|\hat{\tilde{x}}\|}-y\frac{\dot{\hat{\tilde{x}}}^T\hat{\tilde{x}}}{(\|\hat{\tilde{x}}\|)^2}\\\le&\frac{\|\dot{\hat{e}}\|\|\hat{e}^T\|}{\|\hat{e}\|\|\hat{\tilde{x}}\|}+y\frac{\|\dot{\hat{\tilde{x}}}\|\|\hat{\tilde{x}}^T\|}{(\|\hat{\tilde{x}}\|)^2}\\=&\frac{\|\dot{\hat{e}}\|}{\|\hat{\tilde{x}}\|}+y\frac{\|\dot{\hat{\tilde{x}}}\|}{\|\hat{\tilde{x}}\|}.\end{aligned}$$

By the definition $e(t)=q(t_k)-q(t)$ and $\hat{e}(t)=(T\otimes I_2)e(t)$, one has

$$\begin{aligned}\hat{e}(t)&=(T\otimes I_2)e=(T\otimes I_2)(q(t_k)-q(t)),\\\dot{\hat{e}}(t)&=-(T\otimes I_2)\dot{q}(t)=-(T\otimes I_2)(H\otimes I_2)\dot{\tilde{x}}(t)\\&=-(T\otimes I_2)(H\otimes I_2)(T^{-1}\otimes I_2)\dot{\hat{\tilde{x}}}(t)\\&=-(\Lambda\otimes I_2)\dot{\hat{\tilde{x}}}(t).\end{aligned}$$

Then, $\|\dot{\hat{e}}\|\le\lambda_{\max}\|\dot{\hat{\tilde{x}}}\|$ and $\frac{\|\dot{\hat{e}}\|}{\|\hat{\tilde{x}}\|}\le\lambda_{\max}\frac{\|\dot{\hat{\tilde{x}}}\|}{\|\hat{\tilde{x}}\|}$. From (15), we get

$$\|\dot{\hat{\tilde{x}}}\|\le\|A_l\|\|\hat{\tilde{x}}\|+\|\Lambda\otimes K\|\|\hat{\tilde{x}}\|+\|K\|\|\hat{e}\|.$$

From the above inequality, it is easy to see that $\|\hat{\tilde{x}}\|$ can not converge to 0 unless $\|\dot{\hat{\tilde{x}}}\|$ converges to 0. Therefore, there exists a finite positive number $M>0$ such that $\frac{\|\dot{\hat{\tilde{x}}}\|}{\|\hat{\tilde{x}}\|}<M$. We have

$$\begin{aligned}\dot{y}&\le(\lambda_{\max}+y)(\|A_l\|M+\|\Lambda\otimes K\|+\|K\|y)\\&\le(a+by)^2,\end{aligned}$$

where $a=\max\{\lambda_{\max},\|A_l\|M+\|\Lambda\otimes K\|\}$ and $b=\max\{1,\|K\|\}$. Then the solution $y(t)$ with the initial condition $y(0)=0$ satisfies $y(t)\le\phi(t,\phi_0)$, where $\phi(t,\phi_0)$ is the solution of

$$\dot{\phi}(t)=(a+b\phi(t))^2. \tag{21}$$

Note that the minimal time τ between two adjacent events is given by the time it takes for $y(t)$ to evolve from the value 0 to μ. Then the time τ is no smaller than the time τ_ϕ for $\phi(t)$ evolving from 0 to μ. The solution $\phi(t)$ of (21) with the initial condition $\phi(0)=0$ is given by $\phi(t)=\frac{-a^2t}{-1+abt}$. Therefore, $\tau_\phi=\frac{\mu}{a^2+ab\mu}>0$ and $\tau\ge\tau_\phi>0$, which shows that the time interval between two adjacent events is lower bounded. □

4. DISTRIBUTED EVENT-TRIGGERED CONTROL APPROACH

In the centralized event-triggered control, the triggering condition depends on all the agents' states and the triggering times for all agents are the same, which obviously causes unnecessary communication cost. In this section, we consider the distributed event-triggered control approach where the event-triggered instants are different for each agent. The distributed event-triggering strategy assigns each agent to update its own control input. The sequence of event-triggered instants for agent i is denoted by $\{t_0^i, t_1^i, \ldots\}$, and t_k^i denotes the kth event time of agent i. The measurement error for agent i is given by

$$e_i(t) = x_i(t_k^i) - x_i(t), \quad t \in [t_k^i, t_{k+1}^i). \tag{22}$$

The distributed control input of agent i is designed as

$$u_i(t) = -K\left[\sum_{j\in N_i} a_{ij}(x_i(t_k^i) - x_j(t_{k'(t)}^j)) + \alpha_i(x_i(t_k^i) - x_l(t))\right] + A_l\hat{x}_l^i(t) + b_l, \quad t \in [t_k^i, t_{k+1}^i), \tag{23}$$

where $k'(t) = argmin_{l\in\mathbb{N}:t\geq t_l^j}\{t - t_l^j\}$. For each $t \in [t_k^i, t_{k+1}^i)$, $t_{k'(t)}^j$ is the latest event-triggered instant of agent j. Thus we have $e_j(t) = x_j(t_{k'(t)}^j) - x_j(t)$, $t \in [t_k^i, t_{k+1}^i)$. That is, the control law for each agent depends on itself and its neighbors' latest event-triggering instant. And at the same time the distributed observer is designed as

$$\dot{\hat{x}}_l^i(t) = -K_l[\sum_{j\in N_i} a_{ij}(x_i(t_k^i) - x_j(t_{k'(t)}^j)) + \alpha_i(x_i(t_k^i) - x_l(t))] + A_l\hat{x}_l^i(t) + b_l, \quad t \in [t_k^i, t_{k+1}^i). \tag{24}$$

Applying the measurement error (22), the control law (23) and the observer (24) can be rewritten as

$$u_i(t) = -K\left[\sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t) + e_i(t) - e_j(t)) + \alpha_i(x_i(t) + e_i(t) - x_l(t))\right] + A_l\hat{x}_l^i(t) + b_l \tag{25}$$

and

$$\dot{\hat{x}}_l^i(t) = -K_l\left[\sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t) + e_i(t) - e_j(t)) + \alpha_i(x_i(t) + e_i(t) - x_l(t))\right] + A_l\hat{x}_l^i(t) + b_l. \tag{26}$$

Set $\bar{x}_i(t) = x_i(t) - x_l(t)$ and $\tilde{x}_i(t) = \tilde{x}_l^i(t) - x_l(t)$. Associating with (25) and (26), we have the following error closed-loop system

$$\begin{cases} \dot{\bar{x}}(t) = -(H\otimes K)\bar{x}(t) - (H\otimes K)e(t) + (I_N\otimes A_l)\tilde{x}(t) \\ \dot{\tilde{x}}(t) = -(H\otimes K_l)\bar{x}(t) - (H\otimes K_l)e(t) + (I_N\otimes A_l)\tilde{x}(t). \end{cases} \tag{27}$$

By the same orthogonal transformation as (14),

$$\hat{\bar{x}} = (T \otimes I_2)\bar{x}, \quad \hat{\tilde{x}} = (T \otimes I_2)\tilde{x}, \quad \hat{e} = (T \otimes I_2)e,$$

the system (27) can be rewritten as follows

$$\begin{cases} \dot{\hat{\bar{x}}}(t) = -(\Lambda \otimes K)\hat{\bar{x}}(t) - (\Lambda \otimes K)\hat{e}(t) + (I_N \otimes A_l)\hat{\tilde{x}}(t) \\ \dot{\hat{\tilde{x}}}(t) = -(\Lambda \otimes K_l)\hat{\bar{x}}(t) - (\Lambda \otimes K_l)\hat{e}(t) + (I_N \otimes A_l)\hat{\tilde{x}}(t), \end{cases} \tag{28}$$

that is,

$$\begin{bmatrix} \dot{\hat{\bar{x}}} \\ \dot{\hat{\tilde{x}}} \end{bmatrix} = \begin{bmatrix} -\Lambda \otimes K & I_N \otimes A_l \\ -\Lambda \otimes K_l & I_N \otimes A_l \end{bmatrix} \begin{bmatrix} \hat{\bar{x}} \\ \hat{\tilde{x}} \end{bmatrix} - \begin{bmatrix} \Lambda \otimes K \\ \Lambda \otimes K_l \end{bmatrix} \hat{e}. \tag{29}$$

The corresponding decoupled subsystems are

$$\begin{bmatrix} \dot{\hat{\bar{x}}}_i \\ \dot{\hat{\tilde{x}}}_i \end{bmatrix} = \begin{bmatrix} -\lambda_i K & A_l \\ -\lambda_i K_l & A_l \end{bmatrix} \begin{bmatrix} \hat{\bar{x}}_i \\ \hat{\tilde{x}}_i \end{bmatrix} - \begin{bmatrix} \lambda_i K \\ \lambda_i K_l \end{bmatrix} \hat{e}_i, \qquad i = 1, \ldots, N. \tag{30}$$

Then we have

Theorem 4.1. For the leader-follower multi-agent systems (2) and (3), assume the adjacency graph $\bar{\mathcal{G}}$ is connected. Then by choosing $K_l = K + A_l^T$ and $K = \delta I_2$ with $\delta \geq \frac{1}{\lambda_{\min}}(4\|A_l\| + 1)$, the tracking problem is solved and at the same time all $\hat{u}_l^i$ converge to u_l asymptotically with the distributed event-triggered control law (25) and the observer (26). In addition, the event-triggering condition is designed as

$$\|\hat{e}_i\| \leq \frac{\|\hat{\bar{x}}_i\|^2}{2\sigma_i}, \quad i = 1, \ldots, N, \tag{31}$$

where $\hat{e}_i$ and $\hat{\bar{x}}_i$ are the components of vector $\hat{e}$ and $\hat{\bar{x}}$ defined in (14), respectively. $\sigma_i = \max\{\|\hat{\bar{x}}_i\|, \|\hat{\tilde{x}}_i\|\}$.

Proof. Take a Lyapunov function candidate $V = \sum_{i=1}^N V_i$ with

$$V_i = \hat{\bar{x}}_i^T \hat{\bar{x}}_i + \frac{1}{\lambda_i}(\hat{\bar{x}}_i - \hat{\tilde{x}}_i)^T(\hat{\bar{x}}_i - \hat{\tilde{x}}_i).$$

The derivative of V_i along each subsystem (30) is

$$\begin{aligned} \dot{V}_i =& \hat{\bar{x}}_i^T(-2\lambda_i K - 2K + 2K_l)\hat{\bar{x}}_i \\ &+ \hat{\bar{x}}_i^T(2A_l + 2K^T - 2K_l^T)\hat{\tilde{x}}_i \\ &+ \hat{\bar{x}}_i^T(-2\lambda_i K - 2K + 2K_l)\hat{e}_i \\ &+ \hat{\tilde{x}}_i^T(2K - 2K_l)\hat{e}_i. \end{aligned}$$

Choose $K_l = K + A_l^T$, then

$$\begin{aligned} \dot{V}_i =& \hat{\bar{x}}_i^T(-2\lambda_i K + 2A_l^T)\hat{\bar{x}}_i \\ &+ \hat{\bar{x}}_i^T(-2\lambda_i K + 2A_l^T)\hat{e}_i \\ &+ \hat{\tilde{x}}_i^T(-2A_l^T)\hat{e}_i. \end{aligned}$$

By enforcing the triggering condition (31) and choosing $K = \delta I_2$ with $\delta \geq \frac{1}{\lambda_{\min}}(4\|A_l\| + 1)$, one has

$$\begin{aligned}\dot{V}_i \leq& \hat{\tilde{x}}_i^T(-2\lambda_i K)\hat{\tilde{x}}_i + \lambda_i\|K\|\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i + 4\|A_l\|\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i \\ =& -2\lambda_i\delta\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i + \lambda_i\delta\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i + 4\|A_l\|\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i \\ \leq& -(\lambda_{\min}\delta - 4\|A_l\|)\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i \\ \leq& -\hat{\tilde{x}}_i^T\hat{\tilde{x}}_i,\end{aligned}$$

which implies

$$\dot{V} = \sum_{i=1}^{N} V_i \leq -\|\hat{\tilde{x}}\|^2 \leq 0.$$

By the LaSalle's invariance principle, similar to the analysis of Theorem 3.1. the solution of the system converge to the largest invariant set contained in $\{\hat{\tilde{x}}, \dot{\hat{\tilde{x}}} \in \mathbb{R}^{2N} \mid \dot{V} = 0\}$. From $\dot{V} = 0$, we have $\hat{\tilde{x}} = 0$. Associated with the system (29), we can obtain the largest invariant set of the set $\{\hat{\tilde{x}}, \dot{\hat{\tilde{x}}} \in \mathbb{R}^{2N} \mid \dot{V} = 0\}$ is $\{\hat{\tilde{x}}, \dot{\hat{\tilde{x}}} \in \mathbb{R}^{2N} \mid \hat{\tilde{x}} = 0, (I_N \otimes A_l)\dot{\hat{\tilde{x}}} = 0\}$, which implies the conclusion. □

Remark 4.1. From the proofs of Theorem 3.1 and Theorem 4.1, one can see that the results of Theorem 3.1 and Theorem 4.1 are also true for any dimension of $A_l \in \mathbb{R}^{n\times n}$ and $b_l \in \mathbb{R}^n$.

Similar to Theorem 3.2, we further show that there exists a positive lower bound of the inter-event times $\{t_{k+1}^i - t_k^i,\ k \in \mathbb{N}\}$.

Theorem 4.2. In Theorem 4.1 with the distributed event-triggered control (23), the inter-event times are lower bounded by a positive number $\tau \geq \frac{\mu}{a^2+ab\mu} > 0$, where $\mu = \frac{1}{2\eta\sqrt{N}}$ with $\eta > 1$, N is the number of agents, $a = \max\{1, \|A_l\|M + \|\Lambda \otimes K\|\}$, $b = \max\{1, \|\Lambda \otimes K\|\}$ for $M > 0$.

P r o o f. Considering the triggering condition (31), one has $\frac{\|\hat{e}_i\|}{\|\hat{\tilde{x}}_i\|} \leq \frac{1}{2}$. Note that there exists a finite value $\eta > 1$ such that $\|\hat{\tilde{x}}\| \leq \eta\sqrt{N}\|\hat{\tilde{x}}_i\|$ for any i, then $\frac{\|\hat{e}_i\|}{\|\hat{\tilde{x}}_i\|} \leq \frac{\eta\sqrt{N}\|\hat{e}\|}{\|\hat{\tilde{x}}\|}$. Hence, the time of $\frac{\|\hat{e}_i\|}{\|\hat{\tilde{x}}_i\|}$ reaches $\frac{1}{2}$ from 0 is longer than the time of $\frac{\eta\sqrt{N}\|\hat{e}\|}{\|\hat{\tilde{x}}\|}$. Denote τ' is the time of $\frac{\|\hat{e}\|}{\|\hat{\tilde{x}}\|}$ grows from 0 to $\mu = \frac{1}{2\eta\sqrt{N}}$. Let $z(t) = \frac{\|\hat{e}(t)\|}{\|\hat{\tilde{x}}(t)\|}$, the time derivative of z satisfies

$$\dot{z} \leq \frac{\|\dot{\hat{e}}\|}{\|\hat{\tilde{x}}\|} + z\frac{\|\dot{\hat{\tilde{x}}}\|}{\|\hat{\tilde{x}}\|}.$$

Since $e_i(t) = x_i(t_k^i) - x_i(t)$, one has $\dot{e}_i(t) = -\dot{x}_i(t)$, and the stack style is $\dot{\hat{e}} = (T \otimes I_2)\dot{e} = -(T \otimes I_2)\dot{x}$. Similar to the proof of Theorem 3.2, we get

$$\begin{aligned}\dot{z} \leq& (1+z)(\|A_l\|M + \|\Lambda \otimes K\| + \|\Lambda \otimes K\|z) \\ \leq& (a+bz)^2,\end{aligned}$$

where $\frac{\|\tilde{x}\|}{\|\bar{x}\|} < M$ for $M > 0$, $a = \max\{1, \|A_l\|M + \|\Lambda \otimes K\|\}$, $b = \max\{1, \|\Lambda \otimes K\|\}$. Therefore, $\phi(\tau, 0) = \frac{a^2\tau}{1-ab\tau}$, from which we get the lower bound $\tau' \geq \frac{\mu}{a^2+ab\mu} > 0$. It is clear that the time interval $\tau \geq \tau' > 0$ between two adjacent events is lower bounded.

□

5. SIMULATION EXAMPLE

In this section, we give an example to show the effectiveness of the proposed event-triggered control.

Example 5.1. Consider the tracking problem of multi-agent systems with four followers and one leader satisfying (2) and (3) with $A_l = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $b_l = 0$. The communication topology between the agents is shown in Figure 1.

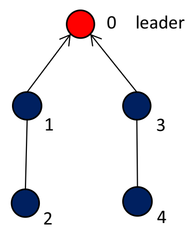

Fig. 1. The adjacency graph $\bar{\mathcal{G}}$.

The adjacency matrix A and the degree matrix D of the adjacency graph $\mathcal{G}$ (without leader) are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the Laplacian matrix and the leader adjacency matrix are

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \Delta = \mathrm{diag}\{1, 0, 1, 0\}.$$

Firstly, for the centralized event-triggered control, choose $K_l = K + A_l^T$ and $K = 13.0890 I_2$ in Theorem 3.1, then by the feedback control (7) and the observer (8) with the event triggering condition (18), the tracking problem is solved. The simulation results are shown in Figure 2 and Figure 3 with the initial values $x_l(0) = [1, 2]^T, x_1(0) = [2, 3]^T, x_2(0) = [6, 4]^T, x_3(0) = [5, -1]^T, x_4(0) = [-1, 1]^T$. The tracking errors of the followers are shown in Figure 2. All the followers can track the leader eventually. Figure 3 presents the estimation errors. The estimated value $\hat{x}_l^i$ by agent i will converge to x_l.

In addition, the time instants when the event is triggered for all agents are shown in Figure 4. In Figure 5, the blue line shows the evolution of $\|\hat{e}\|$, which stays below the threshold given by the triggering condition (18). The red line represents the function $\frac{\lambda_{\min}}{2\sigma}\|\hat{\tilde{x}}_\lambda\|^2$ on the right side of (18).

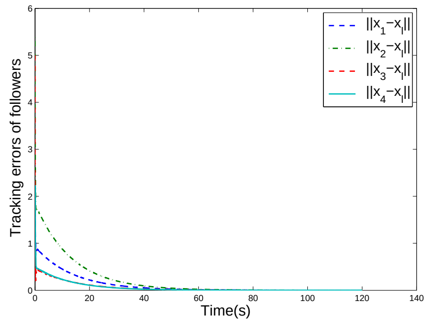

Fig. 2. The tracking errors of followers with centralized event-triggered control.

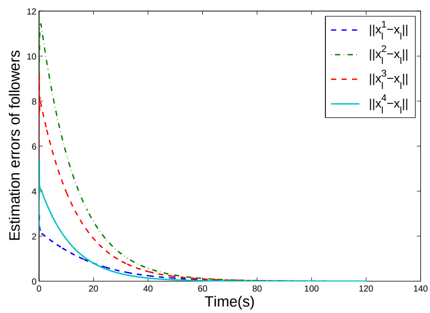

Fig. 3. The estimation errors of followers with centralized event-triggered control-

Next we validate the distributed event-triggered control. Take $K_l = K + A_l^T$ and $K = 14.3979I_2$ in Theorem 4.1, then by the feedback control (25) and the observer (26) with the event triggering condition (31), the tracking problem can also be solved. The tracking errors and the estimation errors of the followers are shown in Figure 6 and Figure 7 with the initial values $x_l(0) = [2,2]^T, x_1(0) = [2,-12]^T, x_2(0) = [8,-2]^T, x_3(0) = [-4,1]^T, x_4(0) = [6,3]^T$, respectively. The simulation result of event time instants for each agent is shown in Figure 8. Figure 9 shows the evolution of $\|\hat{e}_i\|$ in the triggering condition (31) for $i = 1,2,3,4$ (denoted by the blue line). In these figures, an event is generated when the error $\|\hat{e}_i\|$ reaches the threshold $\frac{\|\hat{\tilde{x}}_i\|^2}{2\sigma_i}$, and thereafter the error $\|\hat{e}_i\|$ is reset to zero immediately.

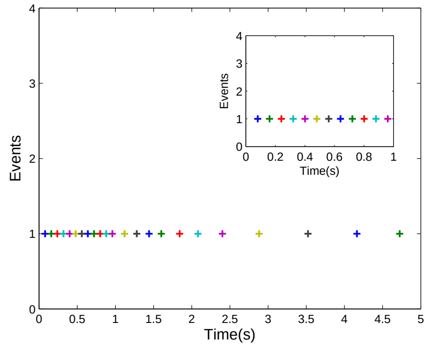

Fig. 4. Event instants for the centralized event-triggered control.

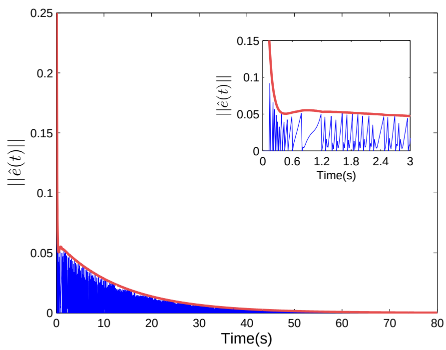

Fig. 5. Evolution of error for the centralized event-triggered control.

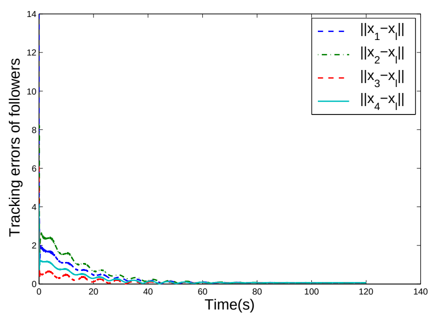

Fig. 6. The tracking errors of followers with distributed event-triggered control.

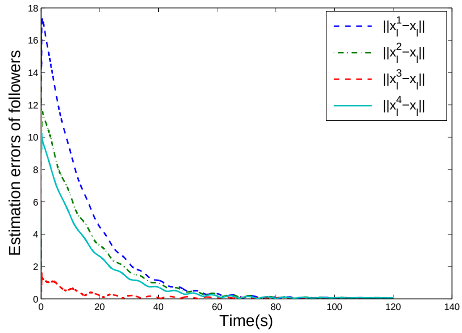

Fig. 7. The estimation errors of followers with distributed event-triggered control.

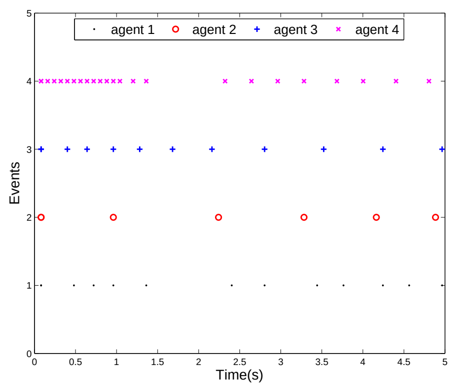

Fig. 8. Event instants for each agent under the distributed event-triggered control.

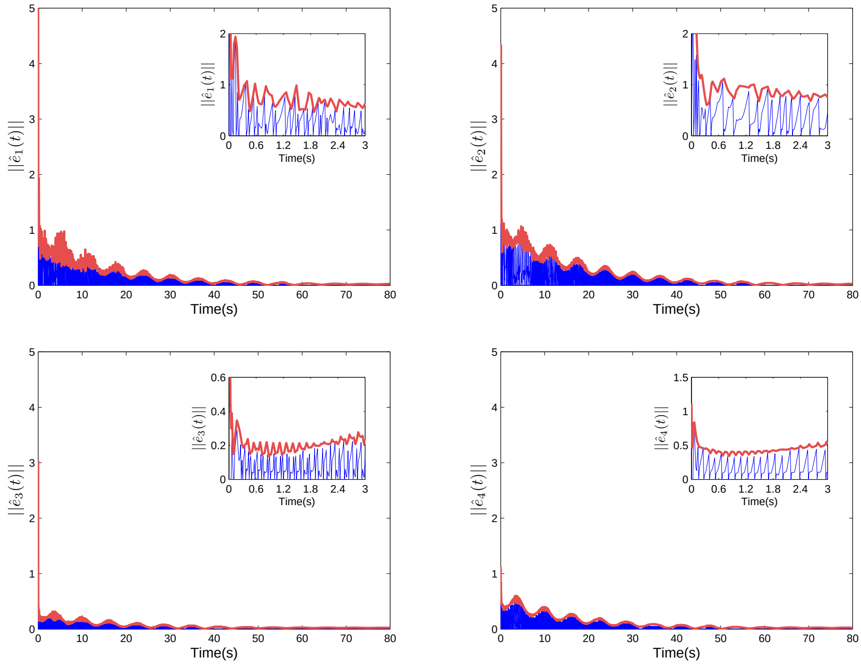

Fig. 9. Evolution of error for four agents under the distributed event-triggered control.

6. CONCLUSION

In this paper, the consensus tracking control for leader-follower multi-agent systems was considered. The leader's control input is assumed unknown to the followers even if they are connected to the leader. We first constructed a neighbor-based observer for each follower to estimate the control input of the leader, and then designed a distributed feedback controller. Both the observer and the controller are event-triggered. We designed both the centralized and distributed event-triggered feedback control for each follower. By applying the proposed event-triggered observer-based controller, the tracking problem can be solved if the underlying communication graph of the system is connected. It was also shown that the Zeno phenomena can be avoided in the event-triggered scheme. A simulation example was presented to illustrate the effectiveness of the proposed control law and the observer.

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