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OUTPUT SYNCHRONIZATION OF MULTI-AGENT PORT-HAMILTONIAN SYSTEMS WITH LINK DYNAMICS

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In this paper, the output synchronization control is considered for multi-agent port-Hamiltonian systems with link dynamics. By using Hamiltonian energy function and Casimir function comprehensively, the design method is proposed to overcome the difficulties taken by link dynamics. The Hamiltonian function is used to handle the dynamic of agent, while the Casimir function is constructed to deal with the dynamic of link. Thus the Lyapunov function is generated by modifying the Hamiltonian function of forced Hamiltonian systems. Then, the proposed approach is applied in multi-machine power systems, which are interconnected in microgrid with power frequencies as link dynamics. Finally, the simulation result demonstrates the effectiveness of the gotten method.

Keywords: multi-agent system, port-Hamiltonian system, Casimir function, link dynamics, multi-machine power system

Classification: 93C02, 94C15

1. INTRODUCTION

In recent years, multi-agent systems have become an attractive topic and gain increasing research interest, partly due to its broad applications, including sensor networks, mobile robots, and smart grids. One of the central problems for multi-agent systems is to make the outputs of all agents converge to a common output trajectory, which is usually called as output consensus or synchronization. This problem has been studied extensively and abundant results have been obtained by efforts of many researchers from various viewpoints, including passivity, potential functions, and optimization [1, 5, 6, 13, 15, 17]. Particularly, nonlinear multi-agent systems attract more and more research attention in the control society, but many problems remain to be done yet [3, 9, 21].

It is known that port-Hamiltonian systems are generalization of conventional Hamiltonian systems in the classical mechanics, which provide a framework with nice structure and clear physical meaning for nonlinear control systems. This class of systems has been well investigated in a series of paper [2, 12, 19, 23, 24]. The first important property of port-Hamiltonian systems is that the interconnection of port-Hamiltonian systems usually yields a port-Hamiltonian system. Therefore, it is easy to treat more complex

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systems consisting of these port-Hamiltonian systems [11]. The second important property of port-Hamiltonian systems is energy-based, which can be designed with the help of passivity and Casimir methods [19]. Different from passivity, the Casimir function, one of the structure properties of a class of physical systems, can be used to modify the Hamiltonian function [12]. In fact, the energy-shaping plus damping injection technique was proposed for interconnected port-Hamiltonian systems in [14]. Additionally, the impedance control was designed for a class of port-Hamiltonian systems with flow-input based on Casimir functions, with an illustrative mass-spring example in [16].

Although port-Hamiltonian systems are good at the modeling and analysis of nonlinear systems, most of the previous research still focused on single port-Hamiltonian system, and very few works studied the control problems of multi-agent port-Hamiltonian systems. For example, in [8], the output consensus problem of a class of multi-agent port-Hamiltonian system was investigated for weakly connected unbalanced graph in the fixed topology, then the result was expanded to the case of switching topologies, but the link weights were still fixed. Moreover, even in those initiating works, the link of the communication topologies were usually static without any dynamics, and the dynamic communication link got less attention. In fact, Hamiltonian systems provide an effective way to study both dynamics in the nodes and links. The first try could be found in [20], where the author proposed a geometric framework for the description of physical network dynamics as port-Hamiltonian systems on graphs, related to the internal flows. A formation problem for a network of point masses subject to Coulomb friction was analyzed in [7], where discontinuous and continuous controllers were proposed and compared for this multi-agent port-Hamiltonian system. Additionally, power generation systems are important practical example of port-Hamiltonian systems. With Hamiltonian function method, many significant achievements have been obtain for single-machine and multi-machine systems [22, 24]. Nevertheless, without discussing network graphs, these multi-machine systems are not regarded as multi-agent systems.

The objective of this paper is to investigate the output synchronization problem of multi-agent port-Hamiltonian systems. Considering there exists link dynamics in addition to the node dynamics in the multi-agent system, we propose a design method to overcome the difficulties by using Hamiltonian energy function and Casimir function. At first, the link dynamics are modeled and integrated into the port-Hamiltonian system. So the augmented system consists of the dynamics of both agents and links. Next, the Casimir function is constructed to shape the Hamiltonian energy function, which reduces the augmented system and embeds the link dynamics into the system. Then, the output synchronization can be achieved and the Lyapunov function can be generated with the modified Hamiltonian function. Furthermore, the design method is applied to the multi-machine systems in a microgrid, where the power frequency is decided by the angular velocity and regarded as the link dynamics. When the active power generated by each machine influences it, the power frequency as the link dynamics reacts on every machine. This illustrative example with simulation demonstrates the effectiveness of the design method proposed in this paper.

The main contribution of this paper includes the following three aspects. Firstly, we introduce the link dynamics into the field of multi-agent port-Hamiltonian systems and investigate the output synchronization problem of this class of systems. Different from

[20], where the whole physical network was modeled as port-Hamiltonian system and the link dynamics were expressed in port-Hamiltonian model, we propose our system with each agent represented as one port-Hamiltonian system and the dynamic links connecting these agents with the topological graph. Secondly, the proposed design method based on the Hamiltonian energy function and Casimir function overcomes the obstacles of multi-agent port-Hamiltonian systems with link dynamics: the Hamilton function is used to design the control of the agents, while the Casimir function is constructed to deal with the dynamics of the links. Thirdly, the proposed approach can be implemented in multi-machine power systems, considering that the machines in some microgrids satisfy the multi-agent port-Hamiltonian structure and the link dynamics are power frequencies flowing between machines.

The paper is organized as follows: Section 2 reviews some basic knowledge and formulates the output synchronization problem of multi-agent port-Hamiltonian systems. Section 3 introduces the link dynamics into multi-agent port-Hamiltonian systems and designs the control law by combining Hamiltonian energy function with Casimir function. Section 4 provides the practical example of multi-machine power systems in a microgrid to confirm the effectiveness of design approach. Finally, Section 5 gives the conclusion.

2. PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, we first review some concepts and basic knowledge on graph theory [4] and describe the output synchronization problem of multi-agent port-Hamiltonian systems.

2.1. Graph theory

A weighted graph consists of a vertex (node) set $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, an edge (link) set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A_N = [a_{ij}] \in \mathbb{R}^{N \times N}$. An edge (i, j) in a weighted directed graph denotes that vehicle j can obtain information from vehicle i, but not necessarily vice versa. In contrast, the pairs of nodes in a weighted undirected graph are unordered, where an edge (i, j) denotes that vehicles i and j can obtain information from one another. The weighted adjacency matrix A_N of a weighted directed graph is defined such that a_{ij} is a positive weight if $(j, i) \in \varepsilon$, while $a_{ij} = 0$ if $(j, i) \notin \varepsilon$. The weighted adjacency matrix A_N of a weighted undirected graph is defined analogously except that $a_{ij} = a_{ji}$, $\forall i \neq j$, since $(j, i) \in \varepsilon$ implies $(i, j) \in \varepsilon$.

A directed path is a sequence of edges in a directed graph of the form (i_1, i_2), $(i_2, i_3), \dots$, where $i_j \in \mathcal{V}$. An undirected path in an undirected graph is defined analogously. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all of the other nodes. An undirected graph is connected if there is an undirected path between every pair of distinct nodes.

A Laplacian matrix $\mathcal{L}_w = [l_{ij}] \in \mathbb{R}^{N \times N}$ associated with A_N can be defined as

$$l_{ii} = \sum_{j=1, j \neq i}^{N} a_{ij}, \qquad l_{ij} = -a_{ij}, \qquad i \neq j. \tag{1}$$

For an undirected graph, $\mathcal{L}_w$ is symmetric positive semidefinite. However, $\mathcal{L}_w$ for a directed graph does not have this property. In both the undirected and directed cases, 0 is an eigenvalue of $\mathcal{L}_w$ with the associated eigenvector $\mathbf{1}_N$, where $\mathbf{1}_N$ is a $p \times 1$ column vector of all ones. In the case of undirected graphs, 0 is a simple eigenvalue of $\mathcal{L}_w$ and all of the other eigenvalues are positive if and only if the undirected graph is connected. In the case of directed graphs, 0 is a simple eigenvalue of $\mathcal{L}_w$ and all of the other eigenvalues have positive real parts if and only if the directed graph has a directed spanning tree.

2.2. System and problem description

In the following, the formulation of output synchronization problem of multi-agent port-Hamiltonian systems is given.

Consider the multi-agent port-Hamiltonian (MAPH) system as follows:

$$\begin{cases} \dot{x}_i = (J_i - R_i)\nabla H_i(x_i) + G_i u_i \\ y_i = G_i^T \nabla H_i(x_i) \quad i = 1, 2, \ldots, N \end{cases} \tag{2}$$

where $x_i \in \mathbb{R}^n$ is the state, $y_i \in \mathbb{R}^m$ is the output and $u_i \in \mathbb{R}^m$ is the control input of the ith agent. $J_i = -J_i^T$ is a skew-symmetric matrix, $R_i = R_i^T \geq 0$ is a positive semi-definite matrix and $G_i \in \mathbb{R}^{n \times m}$. $H_i(x_i) : \mathbb{R}^n \to \mathbb{R}$ is called the Hamiltonian energy function, which is bounded from below; $\nabla H_i(x_i) = \frac{\partial H_i(x_i)}{\partial x_i}$.

The definition of output synchronization is proposed as follows:

Definition 2.1. Consider the MAPH system (2). The agents are said to output synchronization if

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j = 1, \ldots, N \tag{3}$$

where $\| \cdot \|$ denotes the Euclidean norm.

3. CONTROL OF MAPH SYSTEM WITH LINK DYNAMICS

In this section, the link dynamics are introduced into the MAPH system and its control problem is researched. In order to be compared with the system without link dynamics, we give its corresponding result of output synchronization first.

3.1. Case of the system without link dynamics

In absence of link dynamics, the output synchronization problem of MAPH system (2) is solvable by designing the control law based on the communication among agents, which is shown as the following theorem.

Theorem 3.1. Consider the MAPH system (2). If the communication topology has a directed spanning tree, then under the control law

$$u_i = -\sum_{j=1}^{N} a_{ij}(y_i - y_j), \quad \forall i = 1, \ldots, N \tag{4}$$

the equilibrium point of nonlinear system (2) is globally stable and the agents are output synchronization.

Proof. Consider the Hamiltonian energy function $H(x) = \sum_{i=1}^{N} H_i(x_i)$, $i = 1, \dots, N$. Along the trajectories of (2), the derivative of H is

$$\dot{H} = \sum_{i=1}^{N} \nabla^T H_i (J_i - R_i) \nabla H_i + \sum_{i=1}^{N} \nabla^T H_i G_i u_i.$$

Taking the control law (4) into (2), the closed-loop system in the vector form is

$$\begin{aligned} \dot{H} &= \nabla^T H (J - R) \nabla H + \nabla^T H G u \\ &= \nabla^T H (J - R) \nabla H + \nabla^T H G [-(\mathcal{L}_w \otimes I_m) y] \\ &= -\nabla^T H R \nabla H - y^T (\mathcal{L}_w \otimes I_m) y \leq 0. \end{aligned}$$

Thus the solution is globally stable and all output signals are bounded. Consider the set $E = \{x_i | \dot{H} = 0\} = \{x_i | \nabla^T H R \nabla H = 0, y^T (\mathcal{L}_w \otimes I_m) y = 0\}$. We have $\bar{E} = \{(y_i - y_j)^T (y_i - y_j) \equiv 0\} \supseteq E$. By using LaSalle's Invariance Principle, all bounded solutions of systems converge to E as $t \to \infty$ and the agents can reach output synchronization. □

3.2. Control of the system with link dynamics

In this part, there are two problems to be researched. The first one is to investigate whether the similar result to Theorem 3.1 can be gotten after introducing link dynamics into the system (2). The second one is to search the way to obtain the Lyapunov function from the Hamiltonian function for the MAPH system with link dynamics.

In most of previous research about multi-agent systems, the links in the graph are static. In this paper, the link dynamics will be introduced into the MAPH systems. Now, a practical example is provided to motivate the formulation of link dynamics.

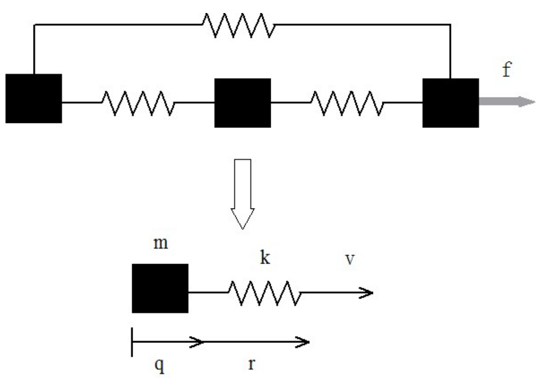

Fig. 1. Mass-spring system.

This is well-known mass-spring system in Figure 1 [16], whose equation is given as

$$m\ddot{q} = kr \tag{5}$$

where $q \in \mathbb{R}$ is the mass position, $r \in \mathbb{R}$ is the spring length, $f \in \mathbb{R}$ is the external force, the inertia $m > 0$, and the spring coefficient $k > 0$. The system is written in port-Hamiltonian form as follows:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \frac{\partial H_0(x)}{\partial x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} kr \tag{6}$$

where $p \in \mathbb{R}$ is the momentum $m\dot{q}$, and the Hamiltonian function $H_0(t) = \frac{p^2}{2m}$.

In the system, the spring is dynamic link between the masses. The model (6) only describes the dynamic of the agent (the mass), but the link dynamic is ignore. One nature idea is to add the dynamic of the link (the spring) into the model. Since the system consists of a kinematic energy storing element and a potential energy storing element, the system is rewritten in the following port-Hamiltonian system.

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \frac{\partial \bar{H}_0(x)}{\partial x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \tag{7}$$

where the control input $v \in \mathbb{R}^m$ is the velocity of the spring and the Hamiltonian function $\bar{H}_0(t) = \frac{p^2}{2m} + \frac{kr^2}{2}$.

Remark 3.2. In Theorem 3.1, the links of MAPH system are static. However, in the above example, the link (spring) is dynamic. So Theorem 3.1 does not apply to the mass-spring system. The case with link dynamics represents a few practical systems and needs getting attention. Firstly as shown in the example, the dynamics of links should be described and augmented to the dynamic equations of plant system. Then, we will shape the Hamiltonian energy function by using the Casimir function to obtain the Lyapunov function for the MAPH system with link dynamics.

With the help of mass-spring system, the link dynamics are introduced into our research work. Now, we start to discuss the case of the MAPH system with link dynamics. The control law is still taken as:

$$u_i = -\sum_{j=1}^{N} a_{ij}(y_i - y_j), \quad \forall i = 1, \ldots, N.$$

So the closed-loop system is

$$\begin{cases} \dot{x}_i = (J_i - R_i)\frac{\partial H_i}{\partial x_i} - G_i \sum_{j=1}^{N} a_{ij}(y_i - y_j) \\ \quad = (J_i - \tilde{R}_i)\frac{\partial H_i}{\partial x_i} + G_i \bar{u}_i \\ y_i = G_i^T \frac{\partial H_i}{\partial x_i} \\ \bar{u}_i = \sum_{j=1}^{N} a_{ij} y_j \end{cases} \tag{8}$$

where $\tilde{R}_i = R_i + A_i G_i G_i^T$, $A_i = \sum_{j=1}^{N} a_{ij}$.

Corresponding to $\bar{u}_i = \sum_{j=1}^{N} a_{ij} y_j$, the forced equilibrium $\bar{x}_i$ are solutions of

$$(J_i - R_i)\frac{\partial H_i}{\partial x_i} + G_i \bar{u}_i = 0. \tag{9}$$

In this way, we make the first assumption.

Assumption 1. $A_i G_i G_i^T$ does not change the forced equilibrium $\bar{x}_i$.

That is, the equilibrium of the system (8) is also at $\bar{x}_i$. This assumption can be satisfied by a few practical systems, such as multi-machine power systems [22, 24].

Considering the link dynamics, we define the augment system as follow: for $i = 1, 2, \ldots, N$,

$$\begin{cases} \dot{\xi}_i = u_{si} = y_i \\ y_{si} = \frac{\partial H_{ei}}{\partial \xi_i} = -\bar{u}_i \end{cases} \tag{10}$$

with state $\xi_i \in \mathbb{R}^m$ and the energy of augment system

$$H_{ei}(\xi_i) = -\xi_i^T \sum_{j=1}^{N} a_{ij} y_j \tag{11}$$

which is assumed to be bounded from below.

By interconnecting the original system (2) and the augment system (10) with $y_{si} = -u_i$ and $u_{si} = y_i$, the obtained equations are

$$\dot{x}_i = (J_i - \tilde{R}_i)\frac{\partial H_i}{\partial x_i} - G_i \frac{\partial H_{ei}}{\partial \xi_i} \tag{12}$$

$$\dot{\xi}_i = G_i^T \frac{\partial H_i}{\partial x_i}. \tag{13}$$

For the following design, we make the following assumption.

Assumption 2. The matrix $(J_i - \tilde{R}_i)$ is invertible.

Then, we can set

$$K_i = -(J_i - \tilde{R}_i)^{-1} G_i. \tag{14}$$

So the equation (12) and (13) can be rewritten as

$$\begin{bmatrix} \dot{x}_i \\ \dot{\xi}_i \end{bmatrix} = (\bar{J}_i - \bar{R}_i) \begin{bmatrix} \frac{\partial \bar{H}_i}{\partial x_i} \\ \frac{\partial \bar{H}_i}{\partial \xi_i} \end{bmatrix} \tag{15}$$

where $\bar{H}_i = H_i(x_i) + H_{ei}(\xi_i)$, and the structure matrices

$$\bar{J}_i \triangleq \begin{bmatrix} J_i & J_iK_i \\ -(J_iK_i)^T & J_{si} \end{bmatrix}, \qquad \bar{R}_i \triangleq \begin{bmatrix} \tilde{R}_i & \tilde{R}_iK_i \\ (\tilde{R}_iK_i)^T & R_{si} \end{bmatrix}$$

where $J_{si} \triangleq K_i^T J_i K_i$, $R_{si} \triangleq K_i^T \tilde{R}_i K_i$ with $J_{si} = -J_{si}^T$, $R_{si} = R_{si}^T$.

From the augmented system (15), we can immediately deduce that the MAPH system with link dynamics (2) is globally stable and the outputs of agents are synchronous. However, the result does not make a distinct progress than the result in Theorem 3.1, except for adding the link dynamics to the system. Next, we will search the condition under which the Hamiltonian function can qualify as the Lyapunov function. Then, the Lyapunov function of the system (15) is constructed from its Hamiltonian function $\bar{H}_i$ by using the Casimir function, which plays an important role in the control by interconnection and energy shaping methodology.

Note that there is no relation between the state of the controller and the state of the controlled system. Then, it is not clear how the controller energy has to be selected in order to solve the control problem. A possible solution can be to constrain the state of the closed-loop system (15) on a certain submanifold. So by properly selecting the Casimir function of the system (10) to shape the closed-loop energy $\bar{H}_i(x_i, \xi_i)$, the augmented system (15) can be mapped into the submanifold.

Note that K_i is a constant matrix, where $K_{i_{\alpha\beta}}$ satisfies

$$\frac{\partial K_{i_{\alpha\beta}}}{x_{i_\gamma}} = \frac{\partial K_{i_{\gamma\beta}}}{x_{i_\alpha}} = 0, \quad \alpha, \gamma \in \bar{n} \triangleq \{1, \ldots, n\}, \quad \beta \in \bar{m} \triangleq \{1, \ldots, m\}. \tag{16}$$

Then from Poincare's lemma, it is known that there exist smooth $C_{i\beta} : \mathbb{R}^n \to \mathbb{R}$, such that

$$K_{i_{\alpha\beta}} = \frac{\partial C_{i_\beta}}{\partial x_{i_\alpha}}, \quad \alpha \in \bar{n}, \quad \beta \in \bar{m}. \tag{17}$$

Hence, it immediately follows that the function

$$F_{i_\beta} = \xi_{i_\beta} - C_{i_\beta}(x_i), \quad \beta \in \bar{m} \tag{18}$$

which are constant along the trajectories of (15). Therefore, we can write

$$\frac{\mathrm{d}F_{i_\beta}}{\mathrm{d}t} = \left[-\frac{\partial^T C_{i_\beta}}{\partial x_i}, e_{i_\beta}^T \right] (\bar{J}_i - \bar{R}_i) \begin{bmatrix} \frac{\partial \bar{H}_i}{\partial x_i} \\ \frac{\partial \bar{H}_i}{\partial \xi_i} \end{bmatrix}. \tag{19}$$

Since the expression (19) is equal to zero, we can get the Casimir function

$$\xi_i = C_i(x_i) + c_i \tag{20}$$

where the constant c_i $(i = 1, \ldots, N)$ depends on the initial condition of ξ_i. Based on the Casimir function, the system (15) can be reduced by replacing ξ_i with x_i and the link dynamics are embedded into the system. Now the Hamiltonian function can be written as

$$H_{ri}(x_i) \triangleq \bar{H}_i(x_i, C_i(x_i) + c_i) = H_i(x_i) + H_{ei}(C_i(x_i) + c_i) \tag{21}$$

while the dynamics of agents and links are restricted to a submanifold given by

$$\dot{x}_i = (J_i - \tilde{R}_i)\frac{\partial H_{ri}}{\partial x_i}. \tag{22}$$

Note that by (17)

$$\frac{\partial H_{ri}}{\partial x_i} = \frac{\partial H_i}{\partial x_i} - \frac{\partial C_i}{\partial x_i}\sum_{j=1}^{N} a_{ij}y_j = \frac{\partial H_i}{\partial x_i} - K_i\sum_{j=1}^{N} a_{ij}y_j. \tag{23}$$

Hence, premultiplying by $(J_i - \tilde{R}_i)$ and using (14), we have

$$(J_i - \tilde{R}_i)\frac{\partial H_{ri}}{\partial x_i} = (J_i - \tilde{R}_i)\frac{\partial H_i}{\partial x_i} + G_i\sum_{j=1}^{N} a_{ij}y_j. \tag{24}$$

Therefore, by (9) and Assumption 1, the unique forced equilibrium $\bar{x}_i$ corresponding to $\bar{u}_i$ is an extremum of H_{ri} for $\frac{\partial H_{ri}}{\partial x_i} = 0$. Furthermore, the Hamiltonian energy function is defined by $H_r(x) = \sum_{i=1}^{N} H_{ri}(x_i)$, we have its derivative

$$\frac{\mathrm{d}}{\mathrm{d}t}H_r = \sum_{i=1}^{N}\frac{\partial^T H_{ri}}{\partial x_i}(J_i - \tilde{R}_i)\frac{\partial H_{ri}}{\partial x_i} = -\sum_{i=1}^{N}\frac{\partial^T H_{ri}}{\partial x_i}\tilde{R}_i\frac{\partial H_{ri}}{\partial x_i} \le 0. \tag{25}$$

Now let us summarize the developments above in the following main theorem.

Theorem 3.3. Consider the MAPH system with link dynamics (2) with Assumption 1 and Assumption 2, and the communication topology of $\mathcal{G}$ has a directed spanning tree. Under the control law (4), the link dynamics are presented in port-Hamiltonian form (10) and the augmented system is described as (15). Then the closed-loop system with link dynamics is globally stable and the outputs of agents are synchronous. Moreover, since K_i satisfies the condition (16), there exists locally smooth function $C_i(x_i)$ $(i = 1, \ldots, N)$ in the form of (20), the system (15) can be alternatively represented by (22), and Hamiltonian function H_{ri} (21) has extremum at $\bar{x}_i$, which is an equilibrium. Furthermore, if we can show that H_{ri} not only has extremum at $\bar{x}_i$ but even a minimum, then H_{ri} qualifies as a Lyapunov function for the system (2).

Remark 3.4. Pay attention to the expression (11). Compared with the previous work [12] for the single port-Hamiltonian system, the energy function of augment system is $-\zeta^T\bar{u}$, where ζ is the state of augment system corresponding to ξ_i and $\bar{u}$ is a constant non-zero input, which is developed to $\sum_{j\in\mathcal{N}_i} a_{ij}y_j$ in MAPH system. So our work expands the result of single port-Hamiltonian system to the case of MAPH system.

Remark 3.5. For the MAPH system with dynamic links, we can find it is composed of two parts: agents and links. Firstly, the port-Hamiltonian system describes the dynamics of agents and Hamiltonian function represents the energy of agent. Secondly, the dynamics of links are integrated into the system by augmenting design. Next, the augmented system is reduced through the construction and solution of Casimir function. So the dynamics of links are embedded into the port-Hamiltonian system. In the above procedure, we design the controller of agents based on Hamiltonian function and deal with the dynamics of links by using Casimir function. Therefore, this method is suitable for the MAPH system with link dynamics.

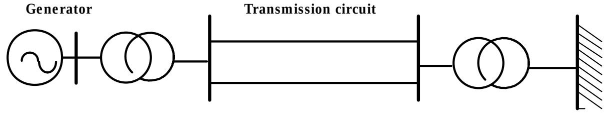

Fig. 2. Simple-machine power system.

4. APPLICATION TO MULTI-MACHINE POWER SYSTEMS

In this section, we apply our results to multi-machine problems in power systems. We start with the single-machine case and then give results for the multi-machine case.

4.1. Model of single-machine systems

Consider the transient stability of an equivalent generator supplying to an infinite bus through two transmission circuits as shown in Figure 2. For single-machine power systems [18], the system model can be written as follows:

$$\begin{cases} \dot{\delta} = \omega - \omega_0 = \Delta\omega \\ \Delta\dot{\omega} = \frac{\omega_0}{T}(P_m - P_M \sin\delta) - \frac{D}{T}\Delta\omega + \frac{\omega_0}{T}u \end{cases} \tag{26}$$

where δ is rotor angle; ω is the angular velocity of rotor, $\Delta\omega$ is its deviation and ω_0 is synchronous speed; P_m is constant mechanical input power; P_M is the maximum value of electric output power; T is inertia constant; D is damping coefficient; u is a control variable corresponding to adjustable input power.

When $\omega_0 = 1$, construct the Hamiltonian function

$$H(\delta, \Delta\omega) = \frac{1}{2}T\Delta\omega^2 + P_m(\pi - \delta) - P_M \cos\delta$$

where δ_s is the equilibrium point of rotor angle. It is rewritten in the matrix form as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} &= \begin{bmatrix} 0 & \frac{1}{T} \\ -\frac{1}{T} & -\frac{D}{T^2} \end{bmatrix} \nabla H + \begin{bmatrix} 0 \\ \frac{1}{T} \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & \frac{1}{T} \end{bmatrix} \nabla H = \Delta\omega. \end{aligned}$$

It is known that the output of a generator in power systems is the angular velocity deviation $\Delta\omega$. In a microgrid, each machine can be regarded as one agent and each link dynamics can describe the time-varying transmission energy in the network. As we know, the energy frequency is changing and determined by the active power of the whole network. In other words, each machine generates its active power, which influences its deviation $\Delta\omega$ and the frequency of the link f due to the relation $\Delta\omega = 2\pi\Delta f$. When each machine produces its active power for the requirement of constant frequency, the frequencies in the network may change due to uncertain factors. On the other hands, the active power of machines will be changed accordingly in order to synchronize all the frequencies. Thus, the frequencies can be described by the link dynamics in multi-machine power systems.

4.2. Multi-machine systems in microgrid

A microgrid is a small-scale power grid that can not only operate independently but also be in conjunction with the main electrical grid as back-up power to bolster the main grid during periods of heavy demand. Often, microgrids can involve multiple energy sources as a way of incorporating renewable power. Commonly, the microgrid is composed of its own power resources, loads and links.

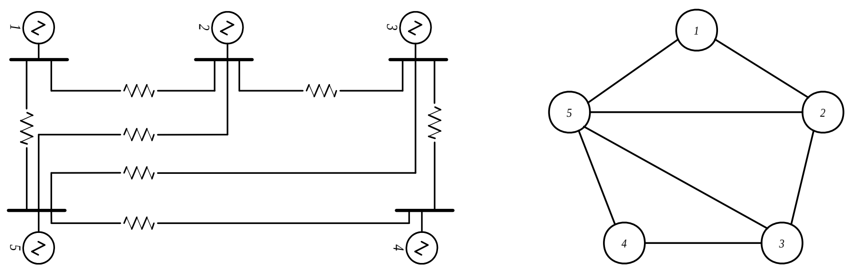

Fig. 3. Microgrid connected graph and the corresponding topological structure.

In this example, the microgrid consists of five generators and some loads, whose connection graph and the corresponding topological structure are shown in Figure 3. Suppose the microgrid works in islanding mode and the loads are fixed constants. By modifying the model of single-machine (26), the model of multi-machines [10, 18] in Figure 3 is

$$\begin{cases} \dot{\delta}_i = \Delta\omega_i \\ \Delta\dot{\omega}_i = \frac{1}{T_i}[P_{mi} - P_{Mi}\sum_{j\in\mathcal{N}_i}\sin(\delta_i - \delta_j)] - \frac{D_i}{T_i}\Delta\omega_i + \frac{1}{T_i}u_i \end{cases} \tag{27}$$

where $i = 1, \ldots, 5$ and $\mathcal{N}_i$ denotes the set of neighbors linking to the ith machine.

The model in MAPH form is

$$\begin{cases} \begin{bmatrix} \dot{\delta}_i \\ \Delta\dot{\omega}_i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{T_i} \\ -\frac{1}{T_i} & -\frac{D_i}{T_i^2} \end{bmatrix} \nabla H_i + \begin{bmatrix} 0 \\ \frac{1}{T_i} \end{bmatrix} u_i \\ y_i = \begin{bmatrix} 0 & \frac{1}{T_i} \end{bmatrix} \nabla H_i = \Delta\omega_i \\ u_i = -\sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j) \end{cases} \tag{28}$$

with $H_i(\delta_i, \Delta\omega_i) = \frac{1}{2}T_i\Delta\omega_i^2 + P_{mi}(\pi - \delta_i) - P_{Mi}\sum_{j\in\mathcal{N}_i}\cos(\delta_i - \delta_j)$ and

$$\nabla H_i = \begin{bmatrix} -P_{mi} + P_{Mi}\sum_{j\in\mathcal{N}_i}\sin(\delta_i - \delta_j) \\ T_i\Delta\omega_i \end{bmatrix}$$

Since the input is $u_i = -\sum_{j\in\mathcal{N}_i} a_{ij}y_i + \sum_{j\in\mathcal{N}_i} a_{ij}y_j$, the model (28) can be rewritten as

$$\begin{cases} \begin{bmatrix} \dot{\delta}_i \\ \Delta\dot{\omega}_i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{T_i} \\ -\frac{1}{T_i} & -\frac{D_i+A_i}{T_i^2} \end{bmatrix} \nabla H_i + \begin{bmatrix} 0 \\ \frac{1}{T_i} \end{bmatrix} \bar{u}_i \\ y_i = \begin{bmatrix} 0 & \frac{1}{T_i} \end{bmatrix} \nabla H_i = \Delta\omega_i \\ \bar{u}_i = \sum_{j\in\mathcal{N}_i} a_{ij}y_j \end{cases} \tag{29}$$

with $A_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. For the system (29), we check whether it satisfies the assumption conditions. First, the equilibrium point is $\bar{x}_i = (\delta_{ei}, \Delta\omega_{ei})$ and $\Delta\omega_{ei} = 0$, so Assumption 1 is satisfied. Second, it is obvious that $(J_i - \tilde{R}_i)$ is invertible. Therefore, the multi-machine power system can be applied with the proposed method.

For the link dynamics, we design the augment system as follow: for $i = 1, \dots, 5$,

$$\begin{cases} \dot{\xi}_i = u_{si} = y_i \\ y_{si} = \frac{\partial H_{ei}}{\partial \xi_i} = -u_i \end{cases} \tag{30}$$

where

$$H_{ei} = -\xi_i \sum_{j\in\mathcal{N}_i} a_{ij}\Delta\omega_j \tag{31}$$

which is Hamiltonian function, bounded from below. So the augmented system is expressed as the following matrix form [12, 18]:

$$\begin{bmatrix} \dot{\delta}_i \\ \Delta\dot{\omega}_i \\ \dot{\xi}_i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{T_i} & 0 \\ -\frac{1}{T_i} & -\frac{D_i+A_i}{T_i^2} & -\frac{1}{T_i} \\ 0 & \frac{1}{T_i} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}_i}{\partial \delta_i} \\ \frac{\partial \bar{H}_i}{\partial \Delta\omega_i} \\ \frac{\partial \bar{H}_i}{\partial \xi_i} \end{bmatrix} \tag{32}$$

where $\bar{H}_i(x_i, \xi_i) = H_i(x_i) + H_{ei}(\xi_i)$ and $x_i = [\delta_i, \Delta\omega_i]^T$.

Next, the Casimir function is constructed for the link dynamics in the grid, which are connected by the frequency (or angular velocity).

$$F_i(x_i, \xi_i) = \xi_i - C_i(x_i)$$

which are constant along the trajectories of (32). Therefore, we can obtain

$$\begin{aligned}\frac{\mathrm{d}F_i}{\mathrm{d}t} &= \dot{\xi}_i - \frac{\partial C_i}{\partial \delta_i}\dot{\delta}_i - \frac{\partial C_i}{\partial \Delta\omega_i}\Delta\dot{\omega}_i \\ &= \left(1 - \frac{\partial C_i}{\partial \delta_i}\right)\Delta\omega_i - \frac{\partial C_i}{\partial \Delta\omega_i}\Delta\dot{\omega}_i = 0.\end{aligned}$$

Therefore, the Casimir function is designed as $F_i = \xi_i - \delta_i = c_i$, where constant c_i depends on the initial condition of source system (28). Here it can be chosen as $c_i = -\delta_{ei}$ from $\xi_i = \delta_{ei} + c_i = 0$.

Then, taking the Casimir function into the Hamiltonian function $H_{ri}(x_i) = \bar{H}_i(x_i, (\delta_i - \delta_{ei}))$, we have

$$H_{ri}(x_i) = \frac{1}{2}T_i\Delta\omega_i^2 + P_{mi}(\pi - \delta_i) - P_{Mi}\sum_{j\in\mathcal{N}_i}\cos(\delta_i - \delta_j) - (\delta_i - \delta_{ei})\sum_{j\in\mathcal{N}_i}a_{ij}\Delta\omega_j \quad (33)$$

with

$$\begin{bmatrix}\frac{\partial H_{ri}}{\partial \delta_i} \\ \frac{\partial H_{ri}}{\partial \Delta\omega_i}\end{bmatrix} = \begin{bmatrix}-P_{mi} + P_{Mi}\sum_{j\in\mathcal{N}_i}\sin(\delta_i - \delta_j) - \sum_{j\in\mathcal{N}_i}a_{ij}\Delta\omega_j \\ T_i\Delta\omega_i\end{bmatrix}.$$

The augmented system (32) is reduced to the following matrix equation.

$$\begin{bmatrix}\dot{\delta}_i \\ \Delta\dot{\omega}_i\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{T_i} \\ -\frac{1}{T_i} & -\frac{D_i + A_i}{T_i^2}\end{bmatrix}\begin{bmatrix}\frac{\partial H_{ri}}{\partial \delta_i} \\ \frac{\partial H_{ri}}{\partial \Delta\omega_i}\end{bmatrix}. \quad (34)$$

Furthermore, in order to prove the stability of it, the Hamiltonian function of multi-machine system is $H_r = \sum_{i=1}^N H_{ri}(x_i)$. It is easy to show that

$$\begin{aligned}\dot{H}_r &= \sum_{i=1}^N \dot{H}_{ri}(x_i) \\ &= \sum_{i=1}^N \begin{bmatrix}\frac{\partial H_{ri}}{\partial \delta_i} & \frac{\partial H_{ri}}{\partial \Delta\omega_i}\end{bmatrix}\begin{bmatrix}0 & \frac{1}{T_i} \\ -\frac{1}{T_i} & -\frac{D_i + A_i}{T_i^2}\end{bmatrix}\begin{bmatrix}\frac{\partial H_{ri}}{\partial \delta_i} \\ \frac{\partial H_{ri}}{\partial \Delta\omega_i}\end{bmatrix} \\ &= -\sum_{i=1}^N \left(D_i + A_i\right)\Delta\omega_i^2 \le 0.\end{aligned}$$

Based on Theorem 3.3, we can verify that Hamiltonian function H_{ri} has an extremum at the equilibrium point $\bar{x}_i = (\delta_{ei}, 0)$. In order to show H_{ri} can qualifies as Lyapunov function, it is necessary to confirm that $\bar{x}_i$ is a minimum. Consider Hessian matrix of $H_{ri}(x_i)$, a straightforward calculation shows that

$$H_{ESS}(H_{ri}(\bar{x}_i)) = \begin{bmatrix}P_{Mi}\sum_{j\in\mathcal{N}_i}\cos(\delta_{ei} - \delta_{ej}) & 0 \\ 0 & T_i\end{bmatrix}.$$

Hence, H_{ESS} is positive provided $\sum_{j\in\mathcal{N}_i}\cos(\delta_{ei}-\delta_{ej})>0$. It means that the equilibrium of rotor angle $\delta_{ei}(i=1,\ldots,N)$ lies in the range of $0°<\delta_{ei}<90°$. Furthermore, it is noticed that this condition is sufficient, but not necessary. Hence, more systems than that satisfying this condition have a minimum in $\bar{x}_i$. Next, we can summarize the above results in the following theorem.

Theorem 4.1. Consider the microgrid composed of 5 generators as Figure 4. The dynamic equation is given as model (28), in which the transmission frequencies in grid are regarded as the link dynamics. By using Theorem 3.3, under the action of controller, the closed-loop system is globally stable and the outputs of generators can achieve synchronization. Furthermore, if the equilibrium $\bar{x}_i=(\delta_{ei},0)$ satisfies $0°<\delta_{ei}<90°(i=1,\ldots,N)$, then the Hamiltonian function H_r is qualified as a Lyapunov function for the multi-machine power systems.

4.3. Simulation Verification

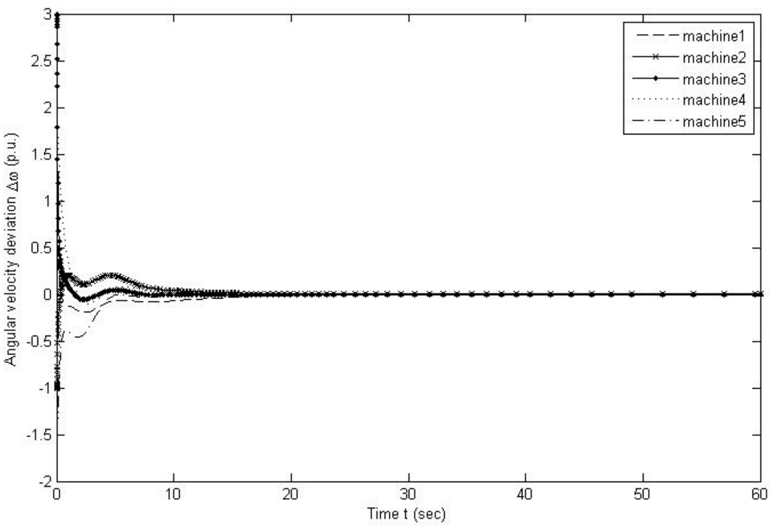

Fig. 4. Dynamic responses of multi-machine system in grid.

In order to confirm the effectiveness of design method, the simulation is done in Matlab. The result is shown in Figure 4 with the parameters of power generation systems as follows: the mechanical input power

$$(P_{m1},P_{m2},P_{m3},P_{m4},P_{m5})=(2,1,0.8,0.5,1.2);$$

the maximum electric output power

$$(P_{M1},P_{M2},P_{M3},P_{M4},P_{M5})=(1.5,1.1,0.7,0.9,2);$$

the inertia constant

$$(T_1, T_2, T_3, T_4, T_5) = (3, 2, 1, 4, 3);$$

the damping coefficient

$$(D_1, D_2, D_3, D_4, D_5) = (5, 3, 4, 2, 1).$$

Take the initial values as $\{(2,1), (-1,-1), (0,3), (-1,2), (2,-2)\}$. The weighted Laplacian of graph as Figure 3 is

$$\begin{bmatrix} 5 & -2 & 0 & 0 & -3 \\ -2 & 8 & -2 & 0 & -4 \\ 0 & -2 & 5 & -2 & -1 \\ 0 & 0 & -2 & 5 & -3 \\ -3 & -4 & -1 & -3 & 11 \end{bmatrix}$$

From the simulation, it is shown that the generators in network, starting from different initial points, can achieve output synchronization. So the controllers are effective for the multi-machine power systems. This example has proved the design method is feasible for a class of practical systems.

5. CONCLUSIONS

This paper researched the output synchronization problem of MAPH systems, where there existed link dynamics in the systems. The dynamics of links were modeled and augmented into port-Hamiltonian systems. Then, the Casimir function was formulated to shape the Hamiltonian energy function and reduce the model of MAPH systems. It was shown that the output synchronization of MAPH systems with link dynamics was achieved with the modified Hamiltonian function. The practical example of multi-machine power systems proved the validity of the design method. In the future, the saturation control problem of the class of systems can be investigated further.

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