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OBSERVER DESIGN FOR A CLASS OF NONLINEAR SYSTEM IN CASCADE WITH COUNTER-CONVECTING TRANSPORT DYNAMICS

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Observer design for ODE-PDE cascades is studied where the finite-dimension ODE is a globally Lipschitz nonlinear system, while the PDE part is a pair of counter-convecting transport dynamics. One major difficulty is that the state observation only rely on the PDE state at the terminal boundary, the connection point between the ODE and the PDE blocs is not accessible to measure. Combining the backstepping infinite-dimensional transformation with the high gain observer technology, the state of the ODE subsystem and the state of the pair of counter-convecting transport dynamics are estimated. It is shown that the observer error is asymptotically stable. A numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: nonlinear systems, observer design, backstepping, counter-convecting transport dynamics

Classification: 93Cxx, 93Dxx

1. INTRODUCTION

This paper investigates state observation of cascade systems including a nonlinear ordinary differential equation (ODE) subsystem followed with a partial differential equation (PDE) subsystem. The objective is to recover the finite-dimension state of the ODE part and the infinite-dimension state of the PDE part. Observer design for ODE-PDE cascades is motivated by problems faced in the oil and gas industry. A representative engineering application in which PDE dynamics are cascaded with a nonlinear ODE is oil drilling [13]. The torsional dynamics of an oil drillstring are modeled as a wave PDE (that describes the dynamics of the angular displacement of the drillstring) coupled with a nonlinear ODE that describes the dynamics of the bottom angular velocity of the drill bit, and the output of petroleum pipe line can be measured. Once nonlinear ODE and PDE cascades are systematically addressed, it is reasonable to ask a question whether an observer for nonlinear ODE and PDE cascades can be designed.

In the last decades, the problems of nonlinear system observability and observer design has been dealt with for systems that can be described by ODEs. Several types of

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observers have been proposed for several classes of nonlinear systems, including the high-gain observer [17], sliding-mode observer [8], Luenberger-like observers [1]. Backstepping method for the first-order hyperbolic PDEs and application to systems with delays was in [14]. The first-order hyperbolic PDEs model a variety of physical systems. Specifically, 2×2 systems of first order hyperbolic linear PDEs model processes such as open channels [7], transmission lines [6], gas flow pipelines [10] or road traffic models [9]. They also have some resemblances with systems that model the gas-liquid flow in oil production pipes [15].

Stabilization of a system of $n+1$ coupled first-order hyperbolic linear PDEs with a single boundary input was dealt with in [16]. The problem of boundary stabilization for a quasilinear 2×2 system of first-order hyperbolic PDEs was considered in [5]. Stabilizing control design for a broad class of nonlinear PDEs was in [18]. The convergence of the feedback laws design in [18] was proved, and the stability properties of the closed-loop system were established in [19]. Backstepping method was used to compensate a class of nonlinear systems under wave actuator dynamics with moving boundary in [3]. An explicit feedback law that compensates the discrete-time nonlinear systems actuated through transport dynamics was designed in [2]. Using the transport PDE to express the delay, stability analysis is conducted with infinite-dimensional backstepping transformations and by constructing a Lyapunov functional in [4]. Observer design and output feedback stabilization for nonlinear multivariable systems with diffusion PDE-governed sensor dynamics was studied in [20]. Compensating a string PDE in the actuation or sensing path of an unstable linear ODE can be found in [11]. A boundary observer has been developed for a cascade involving a linear ODE and a heat PDE equation in [12]. However, in the authors' knowledge, observer design for a class of nonlinear systems in cascade with counter-convecting transport dynamics doesn't appear.

In this paper, the ideas of [11, 12] are extended to ODE-PDE cascades where the finite-dimension subsystem is a class of nonlinear systems, while the PDE part is a pair of counter-convecting transport dynamics. The aim is to accurately estimate the state of the ODE subsystem and the state of the pair of counter-convecting transport dynamics. One major difficulty is that the state observation only relies on the PDE state at the terminal boundary, and the connection point between the ODE and the PDE blocs is not accessible to be measured. The proposed observer, which combines the backstepping infinite-dimensional transformation with the high gain observer, provides state estimates of both subsystems. It is shown that the observer error is asymptotically stable. The proposed observer design method can be applied in the oil and gas industry.

The paper is organized as follows: In Section 2, system description is given. In Section 3 we present observer design and convergence analysis. An illustrative example is given in Section 4. Some conclusions are drawn in Section 5.

For an n-vector, the norm $|\cdot|$ denotes the usual Euclidean norm. The argument of the functions and of the functionals will be omitted or simplified whenever no confusion can arise from the context. For example, one may denote a function $f(X(t))$ by simply $f(X)$.

2. SYSTEM DESCRIPTION

Consider the nonlinear ODE system in cascade with counter-convecting transport dynamics in the sensing path given by

$$\begin{aligned} Y(t) &= \eta(D,t) && (1)\\ \zeta_t(x,t) &= \zeta_x(x,t) && (2)\\ \eta_t(x,t) &= -\eta_x(x,t) && (3)\\ \eta(0,t) &= \zeta(0,t) && (4)\\ \zeta(D,t) &= CX(t) && (5)\\ \dot{X}(t) &= AX(t)+BU(t)+f(X(t)) && (6) \end{aligned}$$

where $X \in R^n$ is the ODE state, $U \in R$ is the input to the entire system, and $\zeta(x,t), \eta(x,t)$ are scalar functions, they represent the states of the sensor governed by transport PDEs equation, the PDE is defined on the interval $[0, D]$, D is a real scalar, and Y is available observation.

We assume that $f : R^n \to R^n$ is globally Lipschitz, that is, there exists $k > 0$ such that

$$|f(X_1) - f(X_2)| \le k|X_1 - X_2| \tag{7}$$

for any $X_1, X_2 \in R^n$.

The objective in this paper is to design an observer to estimate the state vector $X(t)$, and the state variables $\zeta(x,t), \eta(x,t)$.

3. OBSERVER DESIGN AND CONVERGENCE ANALYSIS

We are seeking an observer of the form

$$\begin{aligned} \hat{\zeta}_t(x,t) &= \hat{\zeta}_x(x,t)+\alpha(x)(Y(t)-\hat{\eta}(D,t)) && (8)\\ \hat{\eta}_t(x,t) &= -\hat{\eta}_x(x,t)+\beta(x)(Y(t)-\hat{\eta}(D,t)) && (9)\\ \hat{\eta}(0,t) &= \hat{\zeta}(0,t) && (10)\\ \hat{\zeta}(D,t) &= C\hat{X}(t) && (11)\\ \dot{\hat{X}}(t) &= A\hat{X}(t)+BU(t)+f(\hat{X}(t))+\Upsilon(Y(t)-\hat{\eta}(D,t)) && (12) \end{aligned}$$

where the functions $\alpha(x) \in R, \beta(x) \in R$, and the vector $\Upsilon \in R^{n\times 1}$ are to be determined later.

Let

$$\begin{aligned} \tilde{\zeta}(x,t) &= \zeta(x,t)-\hat{\zeta}(x,t) && (13)\\ \tilde{\eta}(x,t) &= \eta(x,t)-\hat{\eta}(x,t) && (14)\\ \tilde{X}(t) &= X(t)-\hat{X}(t). && (15) \end{aligned}$$

By the system equations (1) – (6), and the observer equations (8) – (12), we have

$$\begin{aligned}
\tilde{\zeta}_t(x,t) &= \tilde{\zeta}_x(x,t) - \alpha(x)\tilde{\eta}(D,t) && (16)\\
\tilde{\eta}_t(x,t) &= -\tilde{\eta}_x(x,t) - \beta(x)\tilde{\eta}(D,t) && (17)\\
\tilde{\eta}(0,t) &= \tilde{\zeta}(0,t) && (18)\\
\tilde{\zeta}(D,t) &= C\tilde{X}(t) && (19)\\
\dot{\tilde{X}}(t) &= A\tilde{X}(t) + f(X(t)) - f(\hat{X}(t)) - \Upsilon\tilde{\eta}(D,t). && (20)
\end{aligned}$$

We consider the transformation

$$\begin{aligned}
\tilde{\omega}(x,t) &= \tilde{\zeta}(x,t) - \Gamma(x)\tilde{X}(t) && (21)\\
\tilde{\varpi}(x,t) &= \tilde{\eta}(x,t) - G(x)\tilde{X}(t) && (22)
\end{aligned}$$

and try to find $\Gamma(x) \in R^{1\times n}, G(x) \in R^{1\times n}$, along with $\alpha(x), \beta(x), \Upsilon$, that convert (16) – (20) into the following system

$$\begin{aligned}
\tilde{\omega}_t(x,t) &= \tilde{\omega}_x(x,t) - \Gamma(x)(f(X) - f(\hat{X})) && (23)\\
\tilde{\varpi}_t(x,t) &= -\tilde{\varpi}_x(x,t) - G(x)(f(X) - f(\hat{X})) && (24)\\
\tilde{\varpi}(0,t) &= \tilde{\omega}(0,t) && (25)\\
\tilde{\omega}(D,t) &= 0 && (26)\\
\dot{\tilde{X}}(t) &= (A - \Upsilon G(D))\tilde{X}(t) + f(X(t)) - f(\hat{X}(t)) - \Upsilon\tilde{\varpi}(D,t). && (27)
\end{aligned}$$

By matching the system (16) – (20) and (23) – (27), we obtain the conditions

$$\begin{aligned}
\Gamma^{'}(x) &= \Gamma(x)A && (28)\\
\alpha(x) &= \Gamma(x)\Upsilon && (29)\\
G^{'}(x) &= -G(x)A && (30)\\
\beta(x) &= G(x)\Upsilon && (31)\\
G(0) &= \Gamma(0) && (32)\\
\Gamma(D) &= C. && (33)
\end{aligned}$$

Solving Eqs. (28) to (33), we have

$$\begin{aligned}
\Gamma(x) &= Ce^{A(x-D)} && (34)\\
G(x) &= Ce^{-A(x+D)} && (35)\\
\alpha(x) &= Ce^{A(x-D)}\Upsilon && (36)\\
\beta(x) &= Ce^{-A(x+D)}\Upsilon. && (37)
\end{aligned}$$

So the functions $\alpha(x)$, $\beta(x)$ for the observer (8) – (12) have been determined. Next, we choose

$$\Upsilon = e^{2AD}L \qquad (38)$$

where L will be determined later. Since A and e^{2AD} can be exchanged, using e^{2AD} as a similarity transformation, then we get $A - \Upsilon G(D) = e^{2AD}(A - LC)e^{-2AD}$.

Theorem 3.1. Suppose that system (1)−(6) satisfies the condition (7) with constant k, and e^{2AD} is non-singular, and the observer holds the form of (8)−(12). Then the observer error dynamics is asymptotically stable if there exist matrices $P > 0$, L and positive scalars $a > 0, \theta > 0$ such that the following linear matrix inequality is feasible:

$$\begin{pmatrix} (e^{-2AD})^T M e^{-2AD} + (\lambda_{\max}(P)\rho_1 + a\rho_2 + \theta\rho_3)I & -(e^{-2AD})^T PL & 0 \\ -L^T P e^{-2AD} & -\frac{\theta}{2} & 0 \\ 0 & 0 & -\frac{a-\theta(D+1)}{2} \end{pmatrix} < 0 \tag{39}$$

where

$$\begin{aligned} M &= P(A-LC) + (A-LC)^T P, \\ \rho_1 &= 2ke^{-4D\lambda_{\min}(A)}, \\ \rho_2 &= k^2 D(D+1)^2 \lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\}, \\ \rho_3 &= k^2 D(D+1)^2 \lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)}\}. \end{aligned} \tag{40}$$

P r o o f. With a Lyapunov function

$$V = \tilde{X}^T (e^{-2AD})^T P e^{-2AD} \tilde{X} + \frac{a}{2}\int_0^D (1+x)\tilde{\omega}(x,t)^2 \,\mathrm{d}x + \frac{\theta}{2}\int_0^D (D+1-x)\tilde{\varpi}(x,t)^2 \,\mathrm{d}x \tag{41}$$

where $P = P^T > 0$ is a positive definite matrix, and $a > 0, \theta > 0$. Using (23), (24), time derivation of (41) yields,

$$\begin{aligned} \dot{V} &= \dot{\tilde{X}}^T (e^{-2AD})^T P e^{-2AD} \tilde{X} + \tilde{X}^T (e^{-2AD})^T P e^{-2AD} \dot{\tilde{X}} \\ &\quad + a\int_0^D (1+x)\tilde{\omega}(x,t)\tilde{\omega}_t(x,t)\,\mathrm{d}x + \theta \int_0^D (D+1-x)\tilde{\varpi}(x,t)\tilde{\varpi}_t(x,t)\,\mathrm{d}x \\ &= \tilde{X}^T (e^{-2AD})^T (P(A-LC) + (A-LC)^T P) e^{-2AD}\tilde{X} \\ &\quad + 2\tilde{X}^T (e^{-2AD})^T P e^{-2AD}(f(X) - f(\hat{X})) - 2\tilde{X}^T (e^{-2AD})^T PL\tilde{\varpi}(D,t) \\ &\quad + a\int_0^D (1+x)\tilde{\omega}(x,t)(\tilde{\omega}_x(x,t) - Ce^{A(x-D)}(f(X) - f(\hat{X})))\,\mathrm{d}x \\ &\quad + \theta\int_0^D (D+1-x)\tilde{\varpi}(x,t)(-\tilde{\varpi}_x(x,t) - Ce^{-A(x+D)}(f(X) - f(\hat{X})))\,\mathrm{d}x. \end{aligned} \tag{42}$$

Let us compute the second term in (42) first, and in virtue of (7), we have

$$\begin{aligned} |2\tilde{X}^T (e^{-2AD})^T P e^{-2AD}(f(X) - f(\hat{X}))| &\le 2|\tilde{X}|\|(e^{-2AD})\|^2\|P\||(f(X) - f(\hat{X}))| \\ &\le 2k\lambda_{\max}(P)e^{-4D\lambda_{\min}(A)}|\tilde{X}|^2. \end{aligned} \tag{43}$$

To compute the fourth term, noting that (26), for any $b > 0$, we have

$$
\begin{aligned}
& a\int_0^D (1+x)\tilde{\omega}(x,t)(\tilde{\omega}_x(x,t) - Ce^{A(x-D)}(f(X)-f(\hat{X})))\,\mathrm{d}x \\
& = -\tfrac{a}{2}\tilde{\omega}(0,t)^2 - \tfrac{a}{2}\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x - a\int_0^D (1+x)\tilde{\omega}(x,t)Ce^{A(x-D)}(f(X)-f(\hat{X}))\,\mathrm{d}x \\
& \le -\tfrac{a}{2}\tilde{\omega}(0,t)^2 - \tfrac{a}{2}\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x \\
& \quad +\tfrac{ab(1+D)^2}{2}\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x + \frac{a}{2b}\int_0^D |Ce^{A(x-D)}(f(X)-f(\hat{X}))|^2\,\mathrm{d}x \\
& \le -\tfrac{a}{2}\tilde{\omega}(0,t)^2 - \tfrac{a(1-b(1+D)^2)}{2}\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x \\
& \quad +\tfrac{a}{2b}k^2 D\lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2
\end{aligned} \tag{44}
$$

where the first inequality is obtained using the quadratic completion, while the second inequality is drawn in virtue of (7), and for any $g > 0$, the last term in (42) is

$$
\begin{aligned}
& \theta\int_0^D (D+1-x)\tilde{\varpi}(x,t)(-\tilde{\varpi}_x(x,t) - Ce^{-A(x+D)}(f(X)-f(\hat{X})))\,\mathrm{d}x \\
& = -\tfrac{\theta}{2}\tilde{\varpi}(D,t)^2 + \tfrac{\theta(D+1)}{2}\tilde{\varpi}(0,t)^2 - \tfrac{\theta}{2}\int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x \\
& \quad -\theta\int_0^D (D+1-x)\tilde{\varpi}(x,t)Ce^{A(-x-D)}(f(X)-f(\hat{X}))\,\mathrm{d}x \\
& \le -\tfrac{\theta}{2}\tilde{\varpi}(D,t)^2 + \tfrac{\theta(D+1)}{2}\tilde{\varpi}(0,t)^2 - \tfrac{\theta(1-g(D+1)^2)}{2}\int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x \\
& \quad +\tfrac{\theta}{2g}\int_0^D |Ce^{A(-x-D)}(f(X)-f(\hat{X}))|^2\,\mathrm{d}x \\
& \le -\tfrac{\theta}{2}\tilde{\varpi}(D,t)^2 + \tfrac{\theta(D+1)}{2}\tilde{\varpi}(0,t)^2 - \tfrac{\theta(1-g(D+1)^2)}{2}\int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x \\
& \quad +\tfrac{\theta}{2g}k^2 D\lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2.
\end{aligned} \tag{45}
$$

By (42)–(45), one has

$$
\begin{aligned}
\dot{V} \le\ & \tilde{X}^T(e^{-2AD})^T(P(A-LC)+(A-LC)^TP)e^{-2AD}\tilde{X} \\
& +2k\lambda_{\max}(P)e^{-4D\lambda_{\min}(A)}|\tilde{X}|^2 \\
& +\frac{a}{2b}k^2 D\lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2 \\
& +\frac{\theta}{2g}k^2 D\lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2 \\
& -2\tilde{X}^T(e^{-2AD})^T PL\tilde{\varpi}(D,t) \\
& -\frac{\theta}{2}\tilde{\varpi}(D,t)^2 - \frac{a-\theta(D+1)}{2}\tilde{\omega}(0,t)^2 - \frac{a(1-b(1+D)^2)}{2}\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x \\
& -\frac{\theta(1-g(D+1)^2)}{2}\int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x.
\end{aligned} \tag{46}
$$

Choosing $b = \frac{1}{2(D+1)^2}$, $g = \frac{1}{2(D+1)^2}$, so we have

$$\begin{aligned}\dot{V} \leq\ & \tilde{X}^T(e^{-2AD})^T(P(A-LC)+(A-LC)^TP)e^{-2AD}\tilde{X} \\ & +2k\lambda_{\max}(P)e^{-4D\lambda_{\min}(A)}|\tilde{X}|^2 \\ & +a(D+1)^2k^2D\lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2 \\ & +\theta(D+1)^2k^2D\lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2 \\ & -2\tilde{X}^T(e^{-2AD})^TPL\tilde{\varpi}(D,t) \\ & -\frac{\theta}{2}\tilde{\varpi}(D,t)^2 - \frac{a-\theta(D+1)}{2}\tilde{\omega}(0,t)^2 \end{aligned} \tag{47}$$

and in view of (39), (40), we get

$$\dot{V} < 0 \tag{48}$$

when $[X, \tilde{\varpi}(D,t), \tilde{\omega}(0,t)] \neq 0$. Thus system (23) – (27) is asymptotical stable.

In what follows, we prove the norms for system (16) – (20) are equivalent to those of system (23) – (27). By (21), (34), and (22), (35), we have

$$\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x \leq 2\int_0^D \tilde{\zeta}(x,t)^2\,\mathrm{d}x + 2D\lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2, \tag{49}$$

$$\int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x \leq 2\int_0^D \tilde{\eta}(x,t)^2\,\mathrm{d}x + 2D\lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)}\}|\tilde{X}|^2, \tag{50}$$

so

$$\begin{aligned} |\tilde{X}|^2 + \int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x + \int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x & \\ \leq (1 + 2D\lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\} & \\ +2D\lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)})|\tilde{X}|^2 & \\ +2\int_0^D \tilde{\zeta}(x,t)^2\,\mathrm{d}x + 2\int_0^D \tilde{\eta}(x,t)^2\,\mathrm{d}x. & \end{aligned} \tag{51}$$

On the other hand, we get

$$\begin{aligned} |\tilde{X}|^2 + \int_0^D \tilde{\zeta}(x,t)^2\,\mathrm{d}x + \int_0^D \tilde{\eta}(x,t)^2\,\mathrm{d}x & \\ \leq (1 + 2D\lambda_{\max}(CC^T)\max\{1, e^{-2D\lambda_{\min}(A)}\} & \\ +2D\lambda_{\max}(CC^T)\max\{e^{-4D\lambda_{\min}(A)}, e^{-2D\lambda_{\min}(A)})|\tilde{X}|^2 & \\ +2\int_0^D \tilde{\omega}(x,t)^2\,\mathrm{d}x + 2\int_0^D \tilde{\varpi}(x,t)^2\,\mathrm{d}x. & \end{aligned} \tag{52}$$

From (51) and (52), we know that the norms for system (16) – (20) are equivalent to those of system (23) – (27). Thus the observer error dynamics (16) – (20) is asymptotically stable. □

Remark. By (39), we get $(e^{-2AD})^T M e^{-2AD} + (\lambda_{\max}(P)\rho_1 + a\rho_2 + \theta\rho_3)I < 0$. It means $(e^{-2AD})^T [P(A-LC) + (A-LC)^T P] e^{-2AD} < -(\lambda_{\max}(P)\rho_1 + a\rho_2 + \theta\rho_3)I$ since e^{-2AD} is nonsingular and $M = P(A-LC) + (A-LC)^T P$. Then $A - LC$ is a Hurwitz matrix by Lyapunov Theorem. Since A and e^{2AD} can be exchanged, using e^{2AD} as a similarity transformation for the matrix $A - \Upsilon G(D) = e^{2AD}(A-LC)e^{-2AD}$, the matrices $A - LC$ and $A - \Upsilon G(D)$ have the same eigenvalues, so the matrix $A - \Upsilon G(D)$ is Hurwitz.

4. EXAMPLE

Consider a third order nonlinear system in cascade with counter-convecting transport dynamics in the sensing path as follows

$$\begin{aligned} Y(t) &= \eta(D,t) && (53)\\ \zeta_t(x,t) &= \zeta_x(x,t) && (54)\\ \eta_t(x,t) &= -\eta_x(x,t) && (55)\\ \eta(0,t) &= \zeta(0,t) && (56)\\ \zeta(D,t) &= X_1(t) && (57)\\ \dot{X}_1(t) &= \cos(t)X_2(t) + \sin(X_3^2(t)) && (58)\\ \dot{X}_2(t) &= X_3(t) && (59)\\ \dot{X}_3(t) &= -X_3(t) + U(t). && (60) \end{aligned}$$

We have

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad f(X(t)) = \begin{pmatrix} \cos(t)X_2(t) + \sin(X_3^2(t)) \\ X_2(t) \\ 0 \end{pmatrix}$$

where $X = (X_1 \quad X_2 \quad X_3)^T$.

It is easy to know that the conditions of Theorem 1 holds. Using Theorem 1, we have

$$a = 0.2332, \ \theta = 0.0862,$$

$$P = \begin{pmatrix} 0.0734 & 0 & 0 \\ 0 & 0.1366 & 2.7846 \\ 0 & 2.7846 & 130.5910 \end{pmatrix},$$

$$L = \begin{pmatrix} 0.58 & 0 & 0 \end{pmatrix}^T.$$

So we have

$$e^{A(x-D)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-0.1(x-D)} & (x-D)e^{-0.1(x-D)} \\ 0 & 0 & e^{-0.1(x-D)} \end{pmatrix},$$

$$e^{-A(x+D)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{0.1(x+D)} & -(x+D)e^{0.1(x+D)} \\ 0 & 0 & e^{0.1(x+D)} \end{pmatrix},$$

$$e^{A(2D)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-0.2D} & 2De^{-0.2D} \\ 0 & 0 & e^{-0.2D} \end{pmatrix}.$$

The observer gains are as follows

$$\begin{aligned} \Upsilon &= e^{2AD}L = \begin{pmatrix} 0.58 \\ 0 \\ 0 \end{pmatrix}, \\ \alpha(x) &= Ce^{A(x-D)}\Upsilon = 0.58, \\ \beta(x) &= Ce^{-A(x+D)}\Upsilon = 0.58. \end{aligned}$$

Then the observer is obtained as

$$\hat{\zeta}_t(x,t) = \hat{\zeta}_x(x,t) + 0.58(Y(t) - \hat{\eta}(D,t)) \tag{61}$$

$$\hat{\eta}_t(x,t) = -\hat{\eta}_x(x,t) + 0.58(Y(t) - \hat{\eta}(D,t)) \tag{62}$$

$$\hat{\eta}(0,t) = \hat{\zeta}(0,t) \tag{63}$$

$$\hat{\zeta}(D,t) = \hat{X}_1(t) \tag{64}$$

$$\dot{\hat{X}}_1(t) = \cos(t)\hat{X}_2(t) + \sin(\hat{X}_3^2(t)) + 0.58(Y(t) - \hat{\eta}(D,t)) \tag{65}$$

$$\dot{\hat{X}}_2(t) = \hat{X}_3(t) \tag{66}$$

$$\dot{\hat{X}}_3(t) = -\hat{X}_3(t) + U(t). \tag{67}$$

In simulation $D = 1$, Figure 1 shows the system trajectories for X_1, X_2 along with their estimates, respectively. Figure 2 shows the system trajectory X_3 along with it's estimate, and the control law, respectively. The trajectories of PDE states along with their estimates are displayed in Figure 3 and Figure 4. The simulation results verify the effectiveness of the proposed design.

5. CONCLUSION

We introduce and solve observer design problem for a class ODEs in cascade with counter-convecting transport PDEs. By combining the backstepping infinite-dimensional transformation with the high gain observer technology, the state vector of the ODE subsystem and the state of the pair of counter-convecting transport dynamics are estimated. It is shown that the observer error is asymptotically stable. A numerical example is given to illustrate the effectiveness of the proposed method.

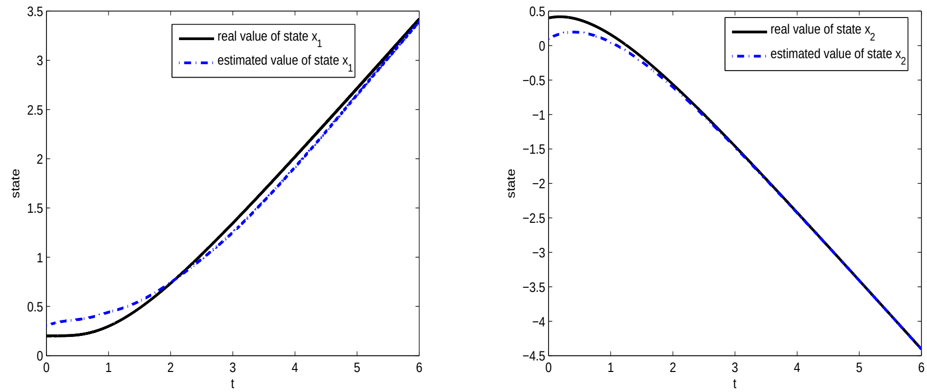

Fig. 1. Time response of the states X_1, X_2 of system along with their estimates $\hat{X}_1$, $\hat{X}_2$ for initial condition as $X_1(0) = 0.2,\ X_2(0) = 0.4,\ X_3(0) = 0.2$ and $\hat{X}_1(0) = 0.3,\ \hat{X}_2(0) = 0.1,\ \hat{X}_3(0) = 0.5$, and $\zeta(x, 0) = 0.3$, $\eta(x, 0) = 0.4$, $\hat{\zeta}(x, 0) = 0.5$, $\hat{\eta}(x, 0) = 0.2$, for all $x \in [0, 1]$.

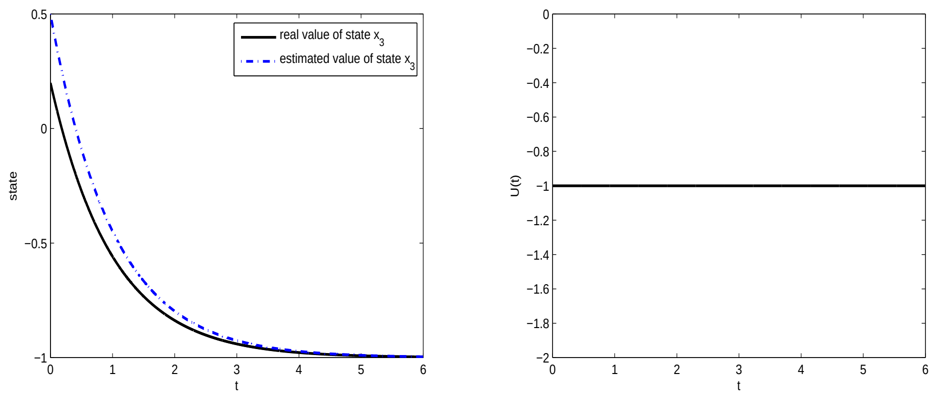

Fig. 2. Time response of the state X_3 of system along with it's estimate $\hat{X}_3$ and the control law $U(t)$.

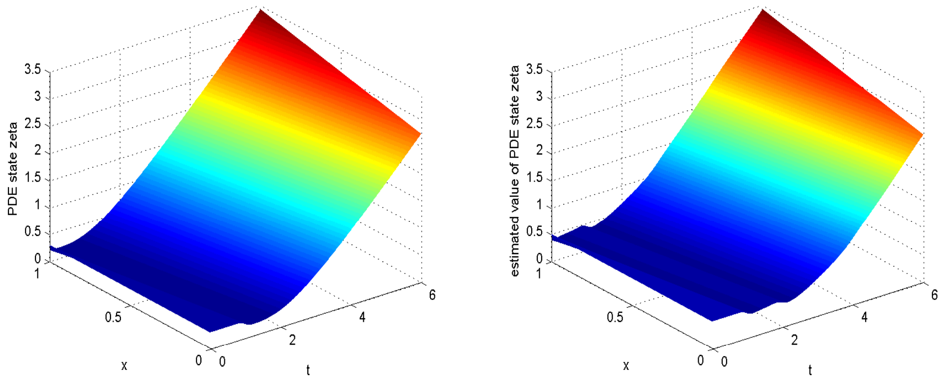

Fig. 3. Time response of the PDE state ζ along with it's estimate.

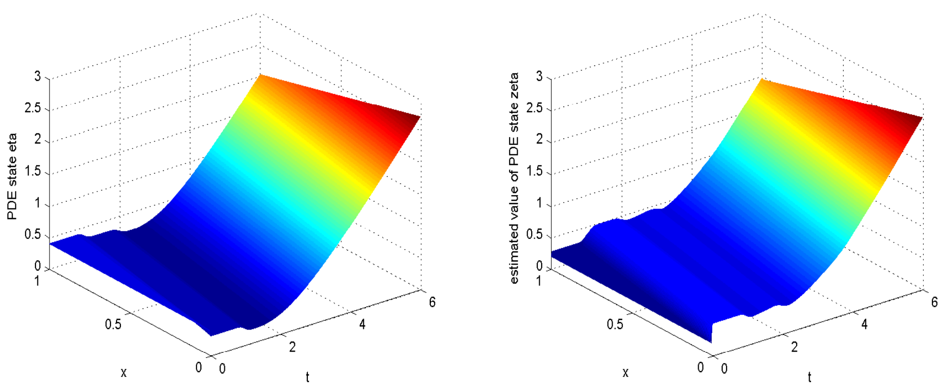

Fig. 4. Time response of the PDE state η along with it's estimate.

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