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A NEW LMI-BASED ROBUST FINITE-TIME SLIDING MODE CONTROL STRATEGY FOR A CLASS OF UNCERTAIN NONLINEAR SYSTEMS

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This paper presents a novel sliding mode controller for a class of uncertain nonlinear systems. Based on Lyapunov stability theorem and linear matrix inequality technique, a sufficient condition is derived to guarantee the global asymptotical stability of the error dynamics and a linear sliding surface is existed depending on state errors. A new reaching control law is designed to satisfy the presence of the sliding mode around the linear surface in the finite time, and its parameters are obtained in the form of LMI. This proposed method is utilized to achieve a controller capable of drawing the states onto the switching surface and sustain the switching motion. The advantage of the suggested technique is that the control scheme is independent of the order of systems model and then, it is fairly simple. Therefore, there is no complexity in the utilization of this scheme. Simulation results are provided to illustrate the effectiveness of the proposed scheme.

Keywords: robust tracking, finite-time control, sliding mode control, nonlinear system, LMI, uncertainties

Classification: 93C10, 35F20, 93D09, 37N35

1. INTRODUCTION

The practical control systems contain nonlinear and time-varying behavior with various uncertainties and external disturbances due to the linearization approximations, modeling errors and measurement errors that can make the performance differ from the nominal design [22, 23, 26, 28]. Therefore, control of such systems has attracted great research interest in the past decades [24]. Over the past years, many researchers have considered the problem of stabilizing the nonlinear systems using the state feedback control method [6, 21, 32]. Many stabilization and tracking approaches for uncertain nonlinear systems such as state-feedback control, output-feedback control, fuzzy controllers, linear matrix inequality (LMI) and H_∞ control are sensitive to uncertainties and disturbances [8]. Various control techniques have been suggested to stabilize this class of dynamical systems via several discontinuous controllers based on hybrid control, sliding mode control (SMC), and some time-varying approaches [10].

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SMC as an efficient and robust control methodology is magnificently employed for the stabilization and control of various linear and nonlinear structures such as underwater vehicles, robotic manipulators, aircraft, spacecraft, electrical motors, flexible space structures, and power network systems [33]. The major specifications of SMC control systems are the robustness in contrast to uncertainties, insensitivity to the bounded perturbations, fast response, reasonable transient performance, controller straightforward implementation, considerable computational facility, and the possibility of the stabilization control of some complex and nonlinear schemes which are difficult to be stabilized via continuous state-feedback control methods [17]. The process of this method is separated into two phases, specifically, the sliding phase and the reaching phase [24, 33]. In the first stage, a sliding (switching) surface is described which should have the property wherein the preferred performance can be achieved so that the states stay on the switching surface. In the subsequent stage, an appropriate control law is planned such that forces the states of the system to reach the switching surface in the finite time [4]. Because of the impact of switching surface on the stability and transient performance of the system, the design procedure of the switching surface is one of the chief subjects in SMC [24]. Typically, it is designed as a linear sliding surface. The stability of the sliding behavior is assured by assigning the parameters of the linear surface and is analyzed by creating an appropriate Lyapunov functional [11]. LMI has appeared as an influential computational tool in solving of the control problems due to its computational flexibility and efficiency and to treat with a large category of design problems. It helps to solve some minimization convex problems, for instance, H_∞ control [3], H_2 control [34] and guaranteed cost control [20]. An LMI procedure is a semi-definite inequality which is a linear relation in unknown variables. Due to the latest advancements in convex optimization, particular efficient algorithms exist for solving LMIs and remarkable developments have been applied in the linear control theory to solve the optimal problems with multiple constraints via LMIs [12]. The main goal of the multi-objective approach is to seek a common Lyapunov matrix that fulfills the parametric constraints determined by the design performances [1]. The common variables estimation may cause the conservatism; however, the flexibility of the control design with multiple objectives and the simplicity of synthesis in the parameter space are provided by LMIs [9, 13].

Many techniques such as pole-placement, H_∞ control, mixed H_2/H_∞ optimization, eigen-structure assignment and optimal quadratic methods have been proposed for the design of the sliding surfaces. In [19], an H_∞ disturbance attenuation-based SMC method is considered for the nonlinear stochastic systems with disturbance-dependent noises such that the controlled system is asymptotically stable in probability with a prescribed H_∞ performance. In [31], a sliding mode H_∞ controller for offshore steel jacket platform with nonlinear self-excited wave forces and external disturbances is designed to decrease the amplitudes of oscillation of the offshore platform. In [29], by intentionally introducing an appropriate time-delay into the control channel, a novel SMC-based active control for an offshore steel jacket platform with wave-induced force and parametric perturbations is proposed. In [30], the combination of SMC with H_∞ control and regional pole-placement technique is exploited for a fluid power electrohydraulic actuator system with load disturbance and external noise to derive the optimal feedback gain which is calculated in the form of LMIs. In [7], the linear-quadratic SMC technique for the remote plant control in networked environment with information transfer delay is addressed. In [27], the SMC surface design is concerned with multi-objective mixed H_2/H_∞ optimization for the perturbed nonlinear systems in the existence of matched and unmatched uncertainties and disturbances. In [14], the asymptotic boundedness of the state errors using SMC technique for a class of chaotic systems with matched and unmatched uncertainties is proved. In [16], a terminal sliding mode design approach using composite nonlinear feedback technique is provided to guarantee the finite-time boundedness of the state errors during the sliding mode. In [15], a global SMC method using nonlinear surfaces is investigated to improve the transient and steady state performance of uncertain nonlinear systems and a design method is presented to assure the exponentially convergence of the states to the origin. In this paper, an original SMC method is presented to analyze the stability, robustness and finite-time control of a class of uncertain and nonlinear systems. A design approach via LMI is employed to guarantee the asymptotically convergence of the state errors to the origin throughout the sliding phase. Furthermore, a control law is planned that guarantees the error trajectory reaches at the switching surface in a finite-time. In comparison with the previous researches which derive the asymptotic boundedness and finite-time boundedness of the state errors, the asymptotic convergence is investigated in this paper. Moreover, unlike the former researches, the resultant LMI conditions have much less pre-assumed design parameters and also, the control scheme is independent of the order of systems model.

The presentation of the paper is divided into several parts. The problem description and some assumptions are described in Section 2. Then, the stability analysis and the proposed design method are discussed in Section 3. Next, the simulation results on two numerical examples are given in Section 4 and finally, conclusions are presented in Section 5.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

Consider the uncertain nonlinear system described as:

$$\begin{aligned} \dot{x} &= Ax + (\tilde{B} + \Delta\tilde{B})u + \tilde{f}(x,u,t) \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control input, $y \in R^p$ is the output vector and $\tilde{B}$, C, A are matrices with appropriate dimensions, $\Delta\tilde{B}$ denote the time-varying uncertainties, and $\tilde{f}(x,u,t)$ is the uncertain nonlinear function. The following assumptions are considered:

Assumption 1. Pair $(A, \tilde{B})$ is stabilizable.

Assumption 2. Pair (A, C) is observable.

Assumption 3. The nonlinear function $\tilde{f}(x,u,t)$ and the time-varying uncertainties $\Delta\tilde{B}$ are assumed to be bounded.

Now, without loss of generality and considering $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, $\tilde{B} = \begin{pmatrix} 0 \\ B \end{pmatrix}$, $\Delta\tilde{B} = \begin{pmatrix} 0 \\ B \end{pmatrix}$ and $\tilde{f}(x,u,t) = \begin{pmatrix} 0 \\ f(x,u,t) \end{pmatrix}$, the system described by (1) can be transformed into regular form as:

$$\begin{aligned} \dot{x_1} &= A_{11}x_1 + A_{12}x_2, \\ \dot{x_2} &= A_{21}x_1 + A_{22}x_2 + (B + \Delta B)u + f(x,u,t), \\ y &= Cx, \end{aligned} \tag{2}$$

where $x = [x_1,\ x_2]^T$, $x_1 \in R^{n-1}$, $x_2 \in R$ are the states of the system and $A_{ij}(i,j = 1,2)$ are constant matrices.

The switching surface is defined as:

$$S(e) = \Lambda e, \tag{3}$$

where,

$$\Lambda = [F,\ g] \tag{4}$$

and $e = [e_1,\ e_2]^T$ with:

$$\begin{aligned} e_1 &= x_1 - x_{m_1}, \\ e_2 &= x_2 - x_{m_2}, \end{aligned} \tag{5}$$

and $x_m = [x_{m_1}^T, x_{m_2}^T]^T$ is the reference trajectory, F and g are gain vector and scalar value, respectively.

When the sliding condition is reached, (3) yields:

$$e_2 = -g^{-1}Fe_1. \tag{6}$$

The error dynamical system is attained from (2), (5) and (6) as:

$$\dot{e_1} = (A_{11} - A_{12}g^{-1}F)e_1 + A_{11}x_{m_1} + A_{12}x_{m_2} - \dot{x}_{m_1}. \tag{7}$$

Assumption 4. A control law u_m is existed for the reference trajectory such that:

$$\dot{x}_{m_1} = A_{11}x_{m_1} + A_{12}x_{m_2}, \tag{8}$$

where using (7) and (8), the closed-loop system becomes as:

$$\dot{e_1} = (A_{11} - A_{12}g^{-1}F)e_1. \tag{9}$$

Lemma 1. (Moulay and Peruquetti [18]) Assume that a continuous positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\alpha V(t) - \beta V(t)^{\eta} \quad \forall t \geq t_0, \;\; V(t_0) \geq 0, \tag{10}$$

where α and β are two positive coefficients, and η is a fraction of two odd positive integers with $0 < \eta < 1$. Then, for the initial time t_0, the Lyapunov function $V(t)$ approaches to the origin at least in a finite time as follows:

$$t_r = t_0 + \frac{1}{\alpha(1-\eta)} \ln\Big(\frac{\alpha V(t_0)^{(1-\eta)} + \beta}{\beta}\Big). \tag{11}$$

3. MAIN RESULTS

In the subsequent theorem, a design method via LMI is provided which guarantees the asymptotic reachability of the state errors to zero during the sliding phase.

Theorem 1. Consider the error dynamical system (7). If there exist scalar value $g > 0$, and matrices $X > 0$, Y and $W > 0$ with appropriate dimensions such that the subsequent LMI is fulfilled:

$$\begin{pmatrix} A_{11}X - A_{12}Y + XA_{11}^T - Y^T A_{12}^T & X \\ X & -W \end{pmatrix} < 0,$$

then using $P = X^{-1}$ and $F = gYX^{-1}$ in (3), the error dynamical system in (7) will be asymptotically stable.

P r o o f . Define the Lyapunov functional for system (7) as:

$$V_1(e_1) = e_1^T P e_1, \tag{12}$$

where P is a symmetric positive-definite matrix. By regarding the derivative of Lyapunov function along the system trajectory (7), one yields:

$$\begin{aligned} \dot{V}_1(e_1) &= e_1^T P \dot{e}_1 + \dot{e}_1^T P e_1, \\ &= e_1^T P(A_{11} - A_{12}g^{-1}F)e_1 + e_1^T(A_{11} - A_{12}g^{-1}F)^T P e_1. \end{aligned} \tag{13}$$

Now, supposing that the subsequent inequality is fulfilled:

$$P(A_{11} - A_{12}g^{-1}F) + (A_{11} - A_{12}g^{-1}F)^T P \leq -W^{-1}, \tag{14}$$

then (13) can be simplified as:

$$\dot{V}_1(e_1) \leq -e_1^T W^{-1} e_1 \leq -\lambda_{\min}(W^{-1}) \left\|e_1\right\|^2, \tag{15}$$

where $\lambda_{\min}(\cdot)$ signifies the minimum eigenvalue. Then, the sufficient condition for (15) is resulted as:

$$\dot{V}_1(e_1) \leq -\alpha_1 V_1(e_1), \tag{16}$$

where:

$$\alpha_1 = \frac{\lambda_{\min}(W^{-1})}{\lambda_{\max}(P)}. \tag{17}$$

Considering (17), it is apparent that α_1 is a positive scalar parameter. Considering $X = P^{-1}$, and pre- and post-multiplying X in (14) gives:

$$A_{11}X - A_{12}g^{-1}FX + XA_{11}^T - (A_{12}g^{-1}FX)^T \leq -XW^{-1}X. \tag{18}$$

Defining $Y = g^{-1}FX$, and using Schur complement on (18), the resultant LMI can be concluded. Hence, the sliding equation (7) is asymptotically stable if the LMI is feasible. □

Remark 1. It can be easily concluded from (6) that when the sliding mode $s(e) = 0$ is satisfied, the term $e_2(t)$ will also converge to the region asymptotically. Therefore, the asymptotical stability of the error dynamical system is guaranteed.

The following theorem offers a novel control law that assures the state errors arrive at the switching surface in a finite time.

Theorem 2. Consider the uncertain nonlinear system (2). Assume that the gains and are attained from Theorem 1. Using the control law:

$$u = -(gB)^{-1}(\Lambda A_{reg}x - \Lambda \dot{x}_m + Q\text{sgn}(s) + \gamma s + \sigma \text{sgn}(s)|s|^{\eta}), \tag{19}$$

with arbitrary positive coefficients σ and γ, and considering that Q is a vector which its elements are the upper bounds of the corresponding elements of Π where $\Pi = g(\Delta Bu + f(x, u.t))$, i.e., $Q \geq \Pi_{\max}$, then the state trajectories of the system (2) are forced to move from the initial conditions to the switching surface (3) in the finite time and to remain on it.

P r o o f. The positive-definite function is considered as:

$$V_2(s) = \frac{1}{2}s^T s. \tag{20}$$

Differentiating $V_2(s)$ and using (3), (4) and (5) yields:

$$\begin{aligned} \dot{V}_2(s) &= s^T \dot{s}, \\ &= s^T(F(A_{11}x_1 + A_{12}x_2) + g(A_{21}x_1 + A_{22}x_2 + (B + \Delta B)u + f(x, u, t)) \\ &\quad -F\dot{x}_{m_1} - g\dot{x}_{m_2}), \\ &= s^T(\Lambda A_{reg}x + g(B + \Delta B)u + gf(x, u, t) - \Lambda \dot{x}_m). \end{aligned} \tag{21}$$

Substituting (19) in (21), one can obtain:

$$\dot{V}_2(s) = -s^T \sigma \text{sgn}(s)|s|^{\eta} - s^T \gamma s - s^T Q\text{sgn}(s) + s^T g(\Delta Bu + f(x, u, t)). \tag{22}$$

Based on the condition $Q \geq \max(\Pi)$ follows that:

$$\begin{aligned}\dot{V}_2(s) &\leq -\lambda_{\min}(\gamma)(\|s\|)^2 - \lambda_{\min}(\sigma)(\|s\|)^{\eta+1}\\ &= -\alpha_2 V_2(s) - \beta_2 (V_2(s))^{\eta_2}, \end{aligned} \tag{23}$$

where $\alpha_2 = 2\lambda_{\min}(\gamma) > 0$, $\beta_2 = 2^{(\eta+1)/2}\lambda_{\min}(\sigma) > 0$ and $\eta_2 = (\eta+1)/2 < 1$. Now, according to the Lemma 1, the system states will reach sliding surface in finite time t_r calculated as follows:

$$t_r = \frac{1}{\alpha_2(1-\eta_2)} \ln\Big(\frac{\alpha_2 V_2(s(e(t_0)))^{1-\eta_2} + \beta_2}{\beta_2}\Big) \tag{24}$$

where $V_2(s(e(t_0))) = \frac{1}{2}s^T(e(t_0))s(e(t_0))$. This finalizes the proof. □

4. ROBUST PERFORMANCE ANALYSIS

For the control problems to have reasonable actions, the stability and performance purposes must be fulfilled [2]. The control law (19) is designed so as to guarantee the asymptotical stability in the Lyapunov sense and the performance measure in L_2 sense satisfying:

$$\int_0^T \|s\|^2 \,\mathrm{d}t \leq \varsigma^2 \int_0^T \|\Pi\|^2 \,\mathrm{d}t \tag{25}$$

for some $\varsigma > 0$, $T \geq 0$, and all $\Pi \in L_2(0,T)$. To evidence the condition (25), the subsequent inequality holds:

$$-(\varsigma\Pi - s)^T(\varsigma\Pi - s) \leq 0. \tag{26}$$

Form (26), one can obtain:

$$\|s\| - \varsigma^2\|\Pi\|^2 \leq 2\|s\|^2 - 2\varsigma s^T\Pi. \tag{27}$$

Then, it follows from (23) and (27) that:

$$\begin{aligned}\int_0^T (\|s\| - \varsigma^2\|\Pi\|^2)\,\mathrm{d}t &\leq \int_0^T 2(\|s\|^2 - \varsigma s^T\Pi)\,\mathrm{d}t,\\ &\leq \int_0^T \Big[2(\|s\|^2 - \varsigma s^T\Pi) + \dot{V}_2\Big]\,\mathrm{d}t - (V_2(T) - V_2(0)),\\ &\leq \int_0^T \Big[2(\|s\|^2 - \varsigma s^T\Pi) - \lambda_{\min}(\gamma)\|s\|^2 - \lambda_{\min}(\sigma)\|s\|^{\eta+1}\Big]\,\mathrm{d}t,\\ &\leq \int_0^T \|s\|\,\big[2\|s\| + 2\varsigma\overline{\Pi} - \lambda_{\min}(\gamma)\|s\| - \lambda_{\min}(\sigma)\|s\|^{\eta}\big]\,\mathrm{d}t, \end{aligned} \tag{28}$$

where $\overline{\Pi} = \max(\|\Pi\|)$. To guarantee the inequality (28), one requires that the parameters σ and γ be chosen such that satisfy the following condition:

$$\lambda_{\min}(\gamma) + \lambda_{\min}(\sigma)\Im^{\eta-1} \geq 2\Big(1 + \frac{\varsigma}{\Im}\overline{\Pi}\Big) \tag{29}$$

where $\Im = \max\|s\|$.

5. SIMULATION RESULTS

In this section, the planned LMI-based robust controller is employed on two numerical examples.

Example 1. Consider the uncertain nonlinear system described by (2) with [5]:

$$A_{11} = \begin{pmatrix} 2 & 0 \\ 1.75 & 0.25 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 1 \\ 0.8 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} -2 & 0 \end{pmatrix}, A_{22} = 1, \quad B = 1,$$

where $\Delta B = 0.2\sin(t)$ and $f(x,u,t) = 0.5\sin(10x_2 t)$. The uncertainty and nonlinear function have the following bounds: $|f(x,u,t)| \le 0.5$, $|\Delta B| \le 0.2$.

For simulation use, take $Q = 5$, $\gamma = 2$, $\sigma = 1$, and $\eta = \frac{3}{5}$. The sampling time is 0.01 second and the simulation run time is 10 seconds. The initial conditions are taken as: $x(0) = [1,1,1]^T$. The possible solutions of gains F and g are obtained using Matlab LMI toolbox as: $F = [10.9105, 13.2702]$, $g = 5.4424$.

Figure 1 shows the system states controlled using the control signal (20). It is shown from this figure that the states reach zero rapidly and the offered control structure is able to overcome the parameter uncertainties and system nonlinearities. The trajectory of the control signal is displayed in Figure 2. We can see that the designed robust tracking control signal has suitable amplitude and is free of high frequency oscillations. The trajectory of the switching surface is given in Figure 3. Evidently, the switching surface converges to the origin rapidly.

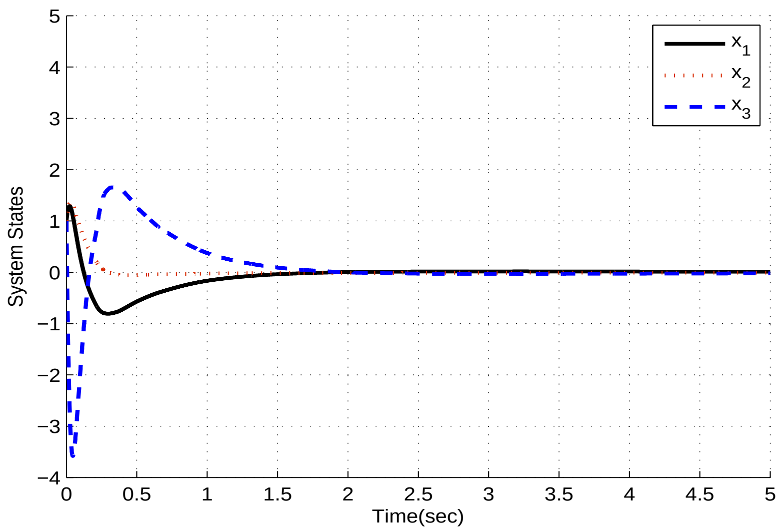

Fig. 1. Trajectory of the system states.

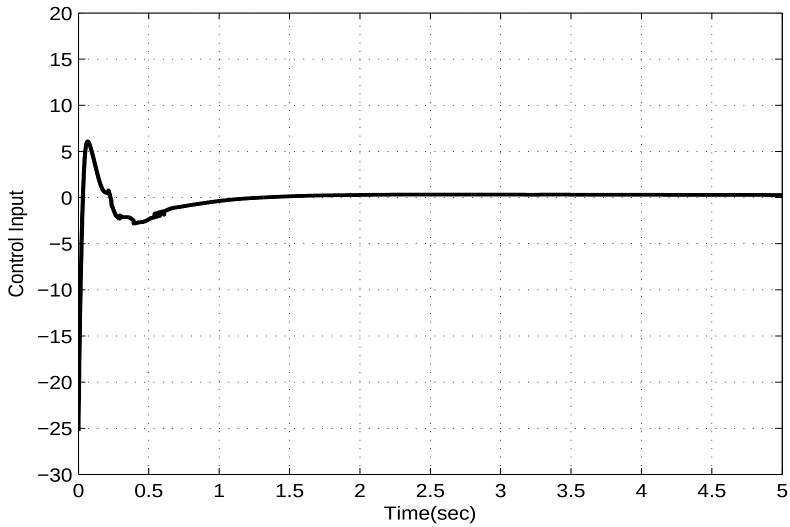

Fig. 2. Applied control input.

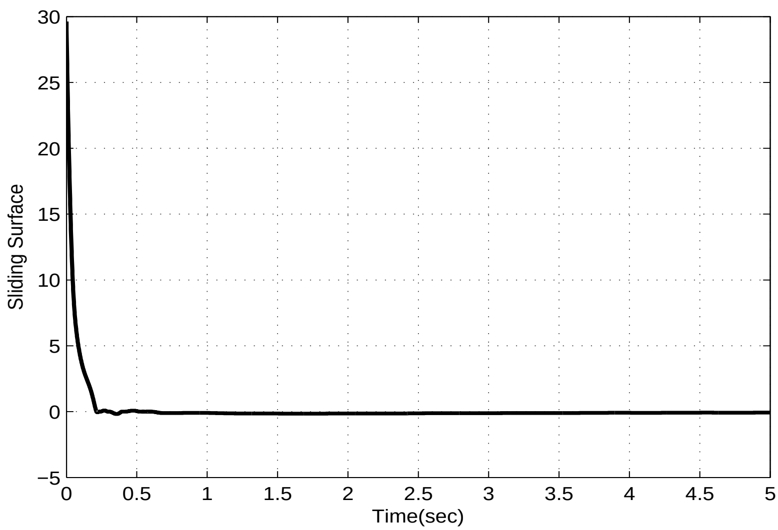

Fig. 3. Trajectory of the sliding surface.

Example 2. Consider the following fourth-order system [25]:

$$\begin{aligned}
\dot{x} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ -3.4\times 10^{-3} & -2.3 & 3.4\times 10^{+3} & 0 \\ 0 & 0 & 0 & 1 \\ 3.4\times 10^{+3} & 0 & -3.4\times 10^{+3} & -731.8 \end{pmatrix} x \\
&+ \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.6\times 10^{+3} \end{pmatrix} + \Delta B \right] u + f(x,u,t) \\
y &= [1 \;\; 0 \;\; 0 \;\; 0]\, x
\end{aligned}$$

which, when expressed in the form of equation (2), gives:

$$\begin{aligned}
A_{11} &= \begin{pmatrix} 0 & 1 & 0 \\ -3.4\times 10^{-3} & -2.3 & 3.4\times 10^{+3} \\ 0 & 0 & 0 \end{pmatrix} \\
A_{12} &= \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \\
A_{21} &= \begin{pmatrix} 3.4\times 10^{+3} & 0 & -3.4\times 10^{+3} \end{pmatrix}, \\
A_{22} &= -731.8, \quad B = 1.6\times 10^{+3},
\end{aligned}$$

and the parametric uncertainty and nonlinear function are given as:

$$\Delta B = [0 \;\; 0 \;\; 0 \; 5\cos(3t)]^T,$$

$$f(x,u,t) = [0 \;\; 0 \;\; 0 \; (-5\cos(4\pi x_4 t) - 2(1 + \mathrm{sgn}(t-0.5)))]^T.$$

The desired trajectory is $x_m = [0 \;\; 0 \;\; 0 \;\; 0]^T$. The constant parameters are selected as: $Q = 6$, $\gamma = 3$, $\sigma = 2$, and $\eta = \frac{3}{5}$. For the simulation usage, two different initial conditions are set as: $x(0) = [-25 \;\; 0 \;\; -25 \;\; 0]^T$ and $x(0) = [-5 \;\; 0 \;\; -5 \;\; 0]^T$. The solutions of the LMI are obtained using MATLAB LMI toolbox and YALMIP solver as: $F = [4.6678 \;\; 4.1576 \;\; 5.4102]$, $g = 1.0613$.

Figure 4 shows the trajectories of the output responses for different values of initial states. From Figure 4, although the nominal system has the uncertain term and nonlinear function, our proposed control law can successfully restrain the effects of uncertainties and nonlinearities sand obtain good performance. The time trajectories of the control signal and sliding surface are shown in Figures 5 and 6. Obviously, the sliding motion trends to the origin in finite time in spite of uncertainties. Due to the discontinuous part of the control law, a slight chattering exists in SMC. The simulation results indicate the feasibility and the effectiveness of the proposed method for the uncertain nonlinear control system.

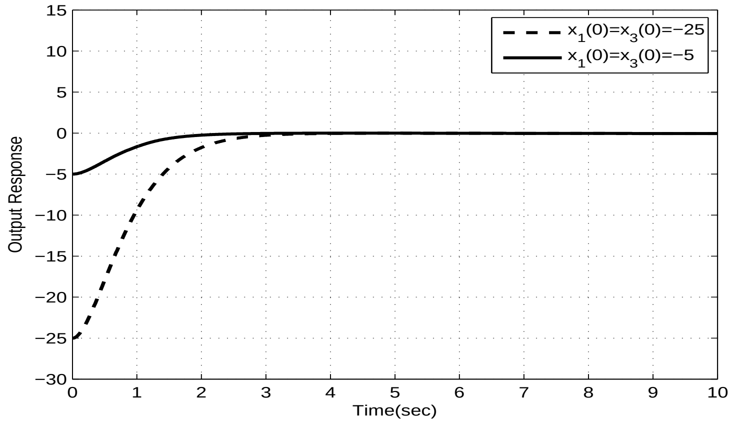

Fig. 4. Trajectory of the system output.

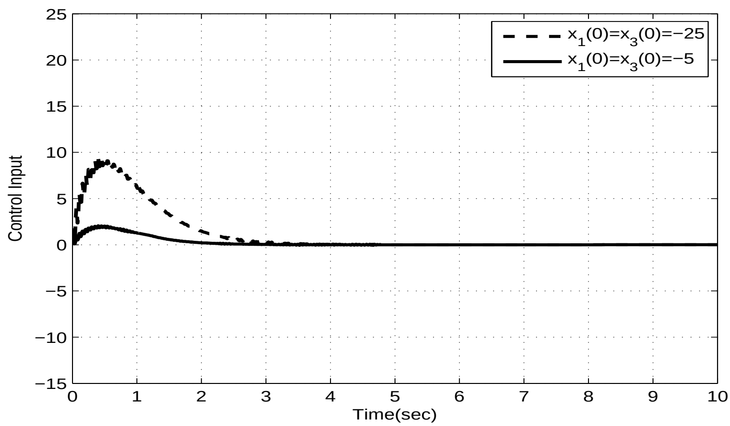

Fig. 5. Applied control input.

6. CONCLUSIONS

In this paper, a novel SMC technique is employed to control a class of uncertain and nonlinear systems. The design coefficients of the offered SMC can be specified by sufficient conditions via LMI. The resultant LMI is relatively straightforward in the computational aspect. The reaching law is suggested to guarantee the presence of the sliding behavior around the sliding surface. The stability action of the system can be proved and the system errors are asymptotically stable. It is demonstrated that this simple controller suffices to asymptotically stabilize the system to the origin. Simulations demonstrate

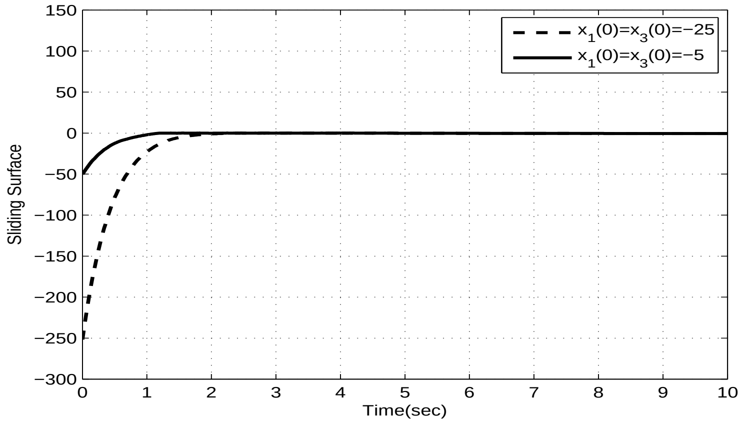

Fig. 6. Trajectory of the sliding surface.

that the planned control law has strong robustness and good control effect and achieves favorable performance for the control of uncertain nonlinear systems. Therefore, via LMI optimization, one can easily solve the robust finite-time sliding mode control design problem for several control systems which cannot be easily solved using the previous approaches.

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