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REMARKS ON EFFECT-TRIBES

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We show that an effect tribe of fuzzy sets $\mathcal{T} \subseteq [0,1]^X$ with the property that every $f \in \mathcal{T}$ is $\mathcal{B}_0(\mathcal{T})$-measurable, where $\mathcal{B}_0(\mathcal{T})$ is the family of subsets of X whose characteristic functions are central elements in $\mathcal{T}$, is a tribe. Moreover, a monotone σ-complete effect algebra with RDP with a Loomis–Sikorski representation $(X, \mathcal{T}, h)$, where the tribe $\mathcal{T}$ has the property that every $f \in \mathcal{T}$ is $\mathcal{B}_0(\mathcal{T})$-measurable, is a σ-MV-algebra.

Keywords: effect-tribe, tribe, monotone σ-complete effect algebra, Riesz decomposition property, MV-algebra

Classification: 81P10, 81P15

1. INTRODUCTION

Effect algebras were introduced in [8] as an algebraic generalization of the set of all quantum effects, that is, self-adjoint operators between the zero and identity operator on a Hilbert space. The set of quantum effects plays an important role in the theory of quantum measurements, as the most general mathematical model of quantum measurements, positive operator valued measures (POVMs), have ranges in the set of quantum effects.

It turned out that the class of effect algebras contains, as special subclasses, many algebraic structures used so far in the foundations of quantum mechanics as models for quantum logics: the class of orthoalgebras, orthomodular posets and orthomodular lattices. In addition, MV-algebras - the algebraic bases for many-valued logic - are also a special subclass of effect algebras.

Important examples of effect algebras may be obtained in the following way. Let (G, G^+) be a (additively written) partially ordered abelian group with positive cone G^+. For any $a \in G^+$, let $\Gamma(G, a)$ be the interval between the zero element 0 and a in G^+. For $x, y \in \Gamma(G, a)$ define that $x \oplus y$ exists iff $x + y \leq a$, and then put $x \oplus y = x + y$. Then $(\Gamma(G, a), \oplus, 0, a)$ is an effect algebra, which is called an *interval effect algebra*. The effect algebra of quantum effects is an interval effect algebra (in the group of all bounded self-adjoint operators on a Hilbert space). Moreover, every MV-algebra and every effect algebra with RDP is an interval effect algebra (cf. [12] and [15]).

In the analogue of the Loomis–Sikorski theorem for monotone σ-complete effect algebras with RDP [1], an important role is played by *effect tribes*, that is, effect algebras

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of $[0,1]$-valued functions with effect operations defined pointwise, which are closed with respect to pointwise limits of ascending sequences of functions. The importance of effect tribes is in the fact, that every monotone σ-complete effect algebra with RDP is an epimorphic image of some effect tribe with RDP. We note that in the analogue of the Loomis–Sikorski theorem for σ-MV-algebras, the role of effect tribes is played by tribes [6, 13]. We recall that a *tribe* on $X \neq \emptyset$ is a collection $\mathcal{T}$ of functions from $[0,1]^X$ such that (i) $1 \in \mathcal{T}$, (ii) if $f \in \mathcal{T}$ then $1 - f \in \mathcal{T}$ and (iii) if $\{f_n\}_n$ is a sequence from $\mathcal{T}$, then $\min(\sum_{n=1}^{\infty} f_n, 1) \in \mathcal{T}$. A tribe is always a σ-complete MV-algebra.

Let $\mathcal{T}$ be an effect tribe of $[0,1]$-valued functions on a nonempty set X. Let $\mathcal{B}_0(\mathcal{T})$ denote the family of those subsets A of X whose characteristic functions χ_A are central elements of $\mathcal{T}$. Then $\mathcal{B}_0(\mathcal{T})$ is a σ-algebra of sets.

If $\mathcal{T}$ is a tribe, then by the well-known Butnariu and Klement theorem [2], all functions in $\mathcal{T}$ are $\mathcal{B}_0(\mathcal{T})$-measurable, and for every σ-additive state on $\mathcal{T}$, we have $s(f) = \int_X f(x)\mu_s(\mathrm{d}x)$, $f \in \mathcal{T}$, where μ_s is the probability measure on $\mathcal{B}_0(\mathcal{T})$ obtained by $\mu_s(A) = s(\chi_A)$. Moreover, the central elements of $\mathcal{T}$ coincide with characteristic functions contained in $\mathcal{T}$.

In this remark, we show first, that if $\mathcal{T}$ is an effect tribe such that every $f \in \mathcal{T}$ is $\mathcal{B}_0(\mathcal{T})$-measurable, then $\mathcal{T}$ is a lattice, and consequently, a tribe. Second, we prove that if M is a monotone σ-complete effect algebra with RDP such that the effect tribe $\mathcal{T}$ in its canonical Loomis–Sikorski representation has the property that every $f \in \mathcal{T}$ is $\mathcal{B}_0(\mathcal{T})$-measurable, then M is a σ-MV-algebra.

We note that effect tribes $\mathcal{T}$ with the property that every $f \in \mathcal{T}$ is $\mathcal{B}_0(\mathcal{T})$-measurable have been treated in [3, Theorem 4.4] and in [4, Theorems 4.1, 4.3, 4.5]. From what we have proved in the present paper, it follows that these results can be also obtained from known results for tribes [2] and σ-MV-algebras [11, 14].

2. EFFECT-TRIBES

Definition 2.1. (Foulis and Bennett [8]) An *effect algebra* is a partial algebra $(E; \oplus, 0, 1)$ where E is a nonempty set, $\oplus$ is a partial binary operation and $0, 1$ are constants, such that

(E1) $a \oplus b = b \oplus a$ (commutativity),

(E2) $(a \oplus b) \oplus c = a \oplus (b \oplus c)$ (associativity),

(E3) to every $a \in E$ there is $a' \in E$ with $a \oplus a' = 1$,

(E4) if $a \oplus 1$ is defined, then $a = 0$.

If $a \oplus b$ exists, we say that a and b are orthogonal ($a \perp b$). We may also define a partial order on an effect algebra by $a \leq b$ iff there exists an element c such that $a \oplus c = b$. The element c, if it exists, is uniquely defined, and we put $c := b \ominus a$. The operation $\oplus$ can be extended to the sums $a_1 \oplus a_2 \oplus \cdots \oplus a_n$ for suitable finite sequences of elements by induction.

We say that an effect algebra E satisfies *Riesz decomposition property* (*RDP* for short), if $a \leq b \oplus c$ implies there exist $b_1, c_1 \in E$ such that $b_1 \leq b, c_1 \leq c$ and $a = b_1 \oplus c_1$. Equivalently, if $x_1 \oplus x_2 = y_1 \oplus y_2$ implies that there exist four elements $c_{11}, c_{12}, c_{21}, c_{22}$

such that $x_1 = c_{11} \oplus c_{12}$, $x_2 = c_{21} \oplus c_{22}$, and $y_1 = c_{11} \oplus c_{21}$, $y_2 = c_{12} \oplus c_{22}$. By [15, Theorem 1.7.17], every effect algebra with RDP is of the form $\Gamma(G, u) := \{x \in G : 0 \le g \le u\}$, where G is a unital interpolation group with strong unit u [10].

An effect algebra E is *monotone σ-complete*, if every ascending (descending) sequence $\{a_n\}_n$ has a supremum (infimum) in E.

Definition 2.2. An algebra $(A, \boxplus, ', 0)$ with a binary operation $\boxplus$, a unary operation $'$ and a special element 0 is called an *MV-algebra* if it satisfies the following conditions for all $x, y, z \in A$:

(MV1) $x \boxplus y = y \boxplus x$,

(MV2) $(x \boxplus y) \boxplus z = x \boxplus (y \boxplus z)$,

(MV3) $x \boxplus 0 = 0$,

(MV4) $x'' = x$,

(MV5) $x \boxplus 0' = 0'$,

(MV6) $(x' \boxplus y)' \boxplus y = (y' \boxplus x)' \boxplus x$.

We recall that an effect algebra is an MV-algebra iff it is a lattice and has RDP [7, Theorems 1.8.23, 1.8.25]. Equivalently, an effect algebra is an MV-algebra iff it is a lattice and every two elements a, b are compatible, that is, the equality $(a \vee b) \ominus b = a \ominus (a \wedge b)$ is satisfied [7, Theorem 1.8.12].

Definition 2.3. An *effect-tribe* on a set $X \neq \emptyset$ is any system $\mathcal{T} \subseteq [0,1]^X$ such that

(i) $1 \in \mathcal{T}$,

(ii) if $f \in \mathcal{T}$, then $1 - f \in \mathcal{T}$,

(iii) if $f, g \in \mathcal{T}$ and $f \le 1 - g$, then $f + g \in \mathcal{T}$,

(iv) for any sequence $\{f_n\}_n$ of elements of $\mathcal{T}$ such that $f_n \nearrow f$ pointwise, then $f \in \mathcal{T}$.

Thus every effect-tribe is a monotone σ-complete effect algebra of fuzzy sets.

Definition 2.4. A *tribe* on a set $X \neq \emptyset$ is a collection $\mathcal{T}$ of functions from $[0,1]^X$ such that

(i) $1 \in \mathcal{T}$,

(ii) if $f \in \mathcal{T}$, then $1 - f \in \mathcal{T}$,

(iii) if $\{f_n\}_n$ is a sequence from $\mathcal{T}$, then $\min\{\sum_{n=1}^{\infty} f_n, 1\} \in \mathcal{T}$.

A tribe is in fact a monotone σ-complete MV-algebra.

Definition 2.5. (Greechie et al. [9]) We say that an element z of an effect algebra E is *central*, if

(i) z and z' are principal elements (element x is principal, if $a, b \leq x$ and $a \perp b$ imply $a \oplus b \leq x$),

(ii) for every $a \in E$ there are a_1, a_2 such that $a_1 \leq z$, $a_2 \leq z'$ and $a = a_1 \oplus a_2$.

When z is a central element of E, then every $a \in E$ can be written as $a = (a \wedge z) \oplus (a \wedge z')$. The set of central elements of an effect algebra E is a Boolean subalgebra of E and central elements are also called Boolean. If E has additionally RDP, the set of central elements of E is the same as the set of sharp elements of E (element $a \in E$ is sharp, if $a \wedge a' = 0$) [5].

The following definition is analogous to [10, Definition p. 131].

Definition 2.6. We say that an effect algebra E satisfies *general comparability* property (GC), if for all $x, y \in E$ there is a central element $e \in E$ such that $x \wedge e \leq y \wedge e$ and $x \wedge e' \geq y \wedge e'$.

We will denote the set of central elements of an effect tribe $\mathcal{T}$ by $B(\mathcal{T})$. Notice that central elements are characteristic functions and by $B_0(\mathcal{T})$ we will denote the corresponding sets:

$$B_0(\mathcal{T}) := \{A \subseteq X : \chi_A \in B(\mathcal{T})\}.$$

Notice, that $B_0(\mathcal{T})$ is σ-algebra of sets.

Lemma 2.7. Effect tribe $\mathcal{T}$, which is a lattice, is a tribe.

P r o o f. We prove (iii) from Definition 2.4. So let us take a sequence $\{f_n\}_n \in \mathcal{T}$ and let $\{g_n\}_n$ be the sequence made from $\{f_n\}_n$ by:

$$g_1 = f_1$$
$$g_2 = (1 - g_1) \wedge f_2$$
$$\dots$$
$$g_n = (1 - (g_1 + g_2 + \dots + g_{n-1})) \wedge f_n.$$

While $\mathcal{T}$ is a lattice, elements of $\{g_n\}_n$ are all in $\mathcal{T}$. And if we set $\boxplus_{i=1}^n f_i := \min(\sum_{i=1}^n f_i, 1)$, then

$$\sum_{i=1}^n g_i = \boxplus_{i=1}^n f_i = \min\left(\sum_{i=1}^n f_i, 1\right).$$

Indeed, if $n = 2$, we get

$$f_1 + f_2 < 1 \Rightarrow f_2 < 1 - f_1 \quad \text{and} \quad f_1 + f_2 = f_1 \boxplus f_2 < 1$$
$$\text{and} \quad g_1 + g_2 = f_1 + (1 - f_1) \wedge f_2 = f_1 + f_2 = f_1 \boxplus f_2$$

or

$$f_1 + f_2 \geq 1 \Rightarrow f_2 \geq 1 - f_1 \quad \text{and} \quad f_1 \boxplus f_2 = 1$$
$$\text{and} \quad g_1 + g_2 = f_1 + (1 - f_1) = 1 = f_1 \boxplus f_2$$

and by induction we show $\sum_{i=1}^n g_i = \boxplus_{i=1}^n f_i$ for every n. Thus $\min(\sum_{i=1}^\infty f_i, 1) = \boxplus_{i=1}^\infty f_i = \sum_{i=1}^\infty g_i$ as a limit of a nondecresing sequence $\{\boxplus_{i=1}^n f_n\}_n$ is in $\mathcal{T}$ by Definition 2.3 (iv). □

Remark. Alternatively, it is easy to see that if $\mathcal{T}$ is a lattice effect tribe, then for all $f, g \in \mathcal{T}$, the equality $(f \vee g) - g = f - (f \wedge g)$ holds (pointwise).

Lemma 2.8. Let $\mathcal{T}$ be an effect tribe satisfying (GC) property. Then $\mathcal{T}$ is a lattice.

Proof. Let $f, g \in \mathcal{T}$, while we assume (GC) property, we have a central element e such that

$$f \wedge e \leq g \wedge e \qquad \text{and} \qquad g \wedge e' \leq f \wedge e'.$$

We show that $f \wedge g = f \wedge e \oplus g \wedge e'$. At first, $f \wedge e \oplus g \wedge e' \leq 1 = e \oplus e'$ and thus is in $\mathcal{T}$. Also, $f \wedge e \oplus g \wedge e' \leq f \wedge e \oplus f \wedge e' = f$ and $f \wedge e \oplus g \wedge e' \leq g \wedge e \oplus g \wedge e' = g$ by (GC). So, let h be another element $h \leq f, g$. Then we compute

$$h = h \wedge e \oplus h \wedge e' \leq f \wedge e \oplus f \wedge e' = f$$

which implies $h \wedge e \leq f \wedge e$ and $h \wedge e' \leq f \wedge e'$; similarly

$$h = h \wedge e \oplus h \wedge e' \leq g \wedge e \oplus g \wedge e' = g$$

getting $h \wedge e \leq g \wedge e$ and $h \wedge e' \leq g \wedge e'$. So altogether we obtain $h = h \wedge e \oplus h \wedge e' \leq f \wedge e \oplus g \wedge e'$. In the same way we could show the existence of suprema $f \vee g = f \wedge e' \oplus g \wedge e$. □

Proposition 2.9. Effect-tribe $\mathcal{T}$ with the property that every $f \in \mathcal{T}$ is $B_0(\mathcal{T})$-measurable, is a tribe.

Proof. We show that $\mathcal{T}$ has GC property.

Let $f, g \in \mathcal{T}$ and let $A := \{x \in X; f(x) \leq g(x)\} = \{x \in X; (g - f)(x) \geq 0\}$. Since f and g are $B_0(\mathcal{T})$-measurable we have that χ_A is a central element of $\mathcal{T}$. For all $x \in X$ we have

$$f.\chi_A(x) = f(x).\chi_A(x) = f(x) \wedge \chi_A(x) \leq g(x) \wedge \chi_A(x) = g(x).\chi_A(x)$$

where χ_A is a central element in $\mathcal{T}$. Similarly

$$f.\chi_{A^c}(x) \geq g.\chi_{A^c}(x)$$

where $A^c = \{x \in X; x \notin A\}$ is a complement of A in X. So GC holds. Now by Lemma 2.8, $\mathcal{T}$ is a lattice and by Lemma 2.7 it is tribe. □

3. MONOTONE σ-COMPLETE EFFECT ALGEBRAS

We recall that, if E and F are two monotone σ-complete effect algebras, the mapping $h : E \to F$ is a σ-homomorphism if (i) $h(a \oplus b) = h(a) \oplus h(b)$ whenever $a \oplus b$ is defined in E, (ii) $h(1) = 1$, and (iii) $h(a_n) \nearrow h(a)$ whenever $a_n \nearrow a$. We note that h need not be a homomorphism of MV-algebras, even if E and F are MV-algebras.

Definition 3.1. A *state* on an effect algebra E is a mapping $s: E \to [0,1]$ such that

(i) $s(1) = 1$,

(ii) $s(a \oplus b) = s(a) + s(b)$ whenever $a \perp b$.

A state s is called *extremal* if $s = \lambda s_1 + (1-\lambda)s_2$ for $\lambda \in (0,1)$ implies $s = s_1 = s_2$.

We denote the set of all states of E by $\mathcal{S}(E)$ and the set of all extremal states of E by $\partial_e \mathcal{S}(E)$. $\mathcal{S}(E)$ is allways a convex set and if E has RDP, $\mathcal{S}(E)$ is nonempty. We say that a net of states $\{s_\alpha\}$ on E weakly converges to a state s on E if $s_\alpha(a) \to s(a)$ for any $a \in E$. In this topology, $\mathcal{S}(E)$ is a compact Hausdorff topological space and every state on E lies in the weak closure of the convex hull of the extremal states, as follows from the Krein-Mil'man theorem. By Choquet, $\partial_e \mathcal{S}(E)$ is a Baire space (see [1]).

In [1], the following theorem was proved.

Theorem 3.2. (Loomis–Sikorski) Let E be a monotone σ-complete effect algebra with RDP. Then there exist a convex space X, an effect tribe $\mathcal{T}$ of $[0,1]$-valued functions on X, and a σ-homomorphism h from $\mathcal{T}$ onto E.

The proof of this theorem is based on the following. Let E be a monotone σ-complete effect algebra with RDP. Due to [15], $E = \Gamma(G,u)$, where $\Gamma(G,u) = \{g \in G : 0 \leq g \leq u)$ for some interpolation group (G,u) with strong unit, which in addition is monotone σ-complete. From this it follows that every element $a \in E$ can be identified with an affine continuous function $\hat{a} : \mathcal{S}(E) \to [0,1]$, $\hat{a}(s) = s(a)$. Next, for a function f, we write $N(f)$ for the support of f (the set of x, for which $f(x)$ is nonzero). Let us denote by $\mathcal{T}$ the class of functions $f : X \to [0,1]$, where $X = \mathcal{S}(E)$, with the property that for some $b \in E$, $N(f - \hat{b}) \cap \partial_e \mathcal{S}(E)$ is a meager subset of $\partial_e \mathcal{S}(E)$ (in the relative topology). We write $f \sim b$. It can be shown that $\mathcal{T}$ is an effect tribe with RDP, and the mapping $h : \mathcal{T} \to E$ defined by $h(f) = b$ iff $f \sim b$ is a surjective σ-homomorphism. The triple $(X, \mathcal{T}, h)$ is called a *representation* of M.

Theorem 3.3. Let M be a monotone σ-complete effect algebra with RDP and let $(X, \mathcal{T}, h)$ be a representation of M such that every $f \in \mathcal{T}$ is $B_0(\mathcal{T})$-measurable. Then M is a lattice, thus a σ-MV algebra.

P r o o f. First we show that h is not only an effect-algebra morphism, but it also preserves infima with central (sharp) elements (here we denote the set of central elements of M by $B_0(M)$). Let $a \in M$, $e \in B_0(M)$. Then $a = a \wedge e \oplus a \wedge e'$. From the definition of h we know that $h(\hat{a}) = a$ for every $a \in M$. Therefore we also have $a = h(\widehat{a \wedge e}) \oplus h(\widehat{a \wedge e'})$. We also know (see the proof of Theorem 3.4 in [4]) that for central elements $e \in B_0(M)$ we have $s(e) \in \{0,1\}$ for all $s \in \partial_e \mathcal{S}(M)$. Now $\widehat{a \wedge e}(s) = s(a \wedge e)$, but $s(a) = s(a \wedge e) + s(a \wedge e')$. If $s(e) = 1$, then $s(e') = 0$ and we get $s(a \wedge e') = 0$ and $s(a) = s(a \wedge e) = s(a) \wedge s(e) = \hat{a}(s) \wedge \hat{e}(s)$. If $s(e) = 0$, then $s(a \wedge e) = 0 = s(a) \wedge s(e) = \hat{a}(s) \wedge \hat{e}(s)$. So in all cases $\widehat{a \wedge e}(s) = s(a \wedge e) = \hat{a}(s) \wedge \hat{e}(s)$. Therefore $h(\widehat{a \wedge e}) = h(\hat{a} \wedge \hat{e})$ and $h(\hat{a}) \wedge h(\hat{e}) = a \wedge e = h(\widehat{a \wedge e}) = h(\hat{a} \wedge \hat{e})$.

Now we prove that M has (GC). Let $a, b \in M$ and $f_a, f_b \in \mathcal{T}$ such that $a = h(f_a), b = h(f_b)$. While by Lemma 2.9, $\mathcal{T}$ has (GC), there exists $A \in B_0(\mathcal{T})$ such that

$$f_a \wedge \chi_A \le f_b \wedge \chi_A \quad \text{and} \quad f_a \wedge \chi_{A^c} \ge f_b \wedge \chi_{A^c}.$$

Let now $e := h(\chi_A)$. Then by Theorem 3.4 ([4]), e is a sharp (therefore also central) and so by what we have already proved:

$$a \wedge e = h(f_a) \wedge h(\chi_A) = h(f_a \wedge \chi_A) \le h(f_b \wedge \chi_A) = h(f_b) \wedge h(\chi_A) = b \wedge e$$

and similarly $a \wedge e' \ge b \wedge e'$. So M indeed has (GC) and thus is a lattice and σ-MV algebra (see Theorem 5.1. in [3]). □

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