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ROBUST NEURAL NETWORK CONTROL OF ROBOTIC MANIPULATORS VIA SWITCHING STRATEGY

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In this paper, a robust neural network control scheme for the switching dynamical model of the robotic manipulators has been addressed. Radial basis function (RBF) neural networks are employed to approximate unknown functions of robotic manipulators and a compensation controller is designed to enhance system robustness. The weight update law of the robotic manipulator is based on switched multiple Lyapunov function method and the periodically switching law which is suitable for practical implementation is constructed. The proposed control scheme can guarantee that the resulting closed-loop switched system is asymptotically Lyapunov stable and the tracking error performance of the control system is well reached. Finally, a simulation example of two-link robotic manipulators is shown to illustrate the effectiveness of the proposed control method.

Keywords: robotic manipulators,switching control strategy, RBF neural networks, multiple Lyapunov function

Classification: 03C65, 20G40

1. INTRODUCTION

The tracking control problem for robotic manipulator systems has drawn extensive attention and many meaningful progresses have been made [1, 4, 6, 9, 15, 17]. Until now, there are many kinds of approaches for the trajectory tracking problem of robotic manipulators, e. g., the adaptive control [4, 6, 15], sliding mode control [17] and robust control [1, 3, 6, 8, 10, 14, 15, 20]. As we know, a few results which are of practical interest on control synthesis of switched nonlinear systems have been introduced [3, 10, 14, 16, 19]. Therefore, how to use the robust tracking control scheme for the switching model of a serial n-joint robotic manipulator remains a challenging topic in the field of control. Among several robust control methods, robust neural compensation control scheme are widely accepted as the powerful control methods and the popular robust strategies for solving the tracking control problem [9, 10, 15]. When the state of a system occurs with a large change, the model of the system should be changed. Thus the presence of different states exists different model. Then, how to guarantee the system with different models in normal operation, it is necessary to choose a switching dynamic model for

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robotic manipulators to achieve the desired control effect. As we know, robotic manipulator systems are the classical switching systems. So far, however, only a few results have been reported on this topic.

In this paper, a robust neural network tracking control scheme has been addressed for the switching dynamical model of the robotic manipulators. The principal contribution described here are: (i) RBF neural networks are used as an approximator tool for modeling unknown nonlinear functions of an n-link robotic manipulators and a periodically switching signal is given; (ii) a compensation controller is introduced to enhance the robustness and keep bounded in the control system. Through switched multiple Lyapunov function method [10, 16], it is proved that the resulting close-loop switched system is asymtopically stable and UUB such that the link position of robot system follows the any given bounded desired output signal. A simulation example is provided to illustrate the effectiveness and the feasibility of the proposed method.

The organization of this paper is as follows. In Section 2, the robust tracking control problem of the n-link robotic manipulators is introduced. In Section 3, the robust neural network control scheme for the switching dynamical model of the robotic manipulators is presented based on RBF NNs. A numerical example is treated to illustrate the effectiveness by means of the proposed control scheme in Section 4. A conclusion is then followed in Section 5.

2. PROBLEM FORMULATION

In this paper, the switching dynamical model of an n-link robotic manipulators can be written as [1, 4, 6, 9]:

$$D_{\sigma(t),0}(q)\ddot{q} + C_{\sigma(t),0}(q,\dot{q})\dot{q} + G_{\sigma(t),0}(q) + \tau_{d,\sigma(t)}(q,\dot{q},t) = \tau(t). \tag{1}$$

Where, $D_{\sigma(t),0}(q) = D_{\sigma(t)}(q) + \Delta D_{\sigma(t)}(q)$, $C_{\sigma(t),0}(q,\dot{q}) = C_{\sigma(t)}(q,\dot{q}) + \Delta C_{\sigma(t)}(q,\dot{q})$, $G_{\sigma(t),0}(q) = G_{\sigma(t)}(q) + \Delta G_{\sigma(t)}(q)$. $q \in R^n$, $\dot{q} \in R^n$, $\ddot{q} \in R^n$ are the generalized link position, velocity, and acceleration, respectively. $\tau(t) \in R^n$ is the applied torque input vector, and $\tau_{d,\sigma(t)}(q,\dot{q},t)$ is the external disturbance. $D_{\sigma(t)}(q) \in R^{n\times n}$ is the symmetric positive definite manipulator inertia matrix, $C_{\sigma(t)}(q,\dot{q}) \in R^n$ is the matrix of centripetal and Coriolis torques, and $G_{\sigma(t)}(q) \in R^n$ stands for the vector of gravitational torques due to the gravity. Furthermore, $D_{\sigma(t),0}(q), C_{\sigma(t),0}(q,\dot{q}), G_{\sigma(t),0}(q)$ and $\Delta D_{\sigma(t)}(q), \Delta C_{\sigma(t)}(q,\dot{q}), \Delta G_{\sigma(t)}(q)$ denote the nominal parts and the corresponding uncertain parts, respectively. $\sigma(t) : [0,\infty) \to \Xi := \{1, 2, \cdots, N\}$ is a piecewise constant function called switching signal (or law), which takes values in the compact set Ξ. In general, the robotic manipulator dynamics still has the following properties and assumptions [6, 9]:

Property 1. The inertia matrix can be upper and lower bounded by the following inequalities: $m_{\sigma(t),1} < \|D_{\sigma(t)}(q)\| < m_{\sigma(t),2}$. Where $m_{\sigma(t),1}$ and $m_{\sigma(t),2}$ are positive constants.

Property 2. The matrix $C_{\sigma(t)}(q,\dot{q})$ and $D_{\sigma(t)}(q)$ the time derivative of the inertia matrix satisfies: $x^T[\dot{D}_{\sigma(t)}(q) - 2C_{\sigma(t)}(q,\dot{q})]x = 0 \quad \forall\, x, q, \dot{q} \in R^n$.

Property 3. The matrix $C_{\sigma(t)}(q,\dot{q})$ satisfies: $C_{\sigma(t)}(q,y)x = C_{\sigma(t)}(q,x)y \ \ \forall\, x,q,y \in R^n$.

Assumption 1. The external disturbance $\tau_d(q,\dot{q},t)$ is assumed in the following:

$$\|\tau_d(q,\dot{q},t)\| \le \tau_{d0} + \tau_{d1}\|q\| + \tau_{d2}\|\dot{q}\| + \tau_{d3}\|q\|^2 + \tau_{d4}\|\dot{q}\|^2. \tag{2}$$

Where, $\tau_{di} > 0 \quad (i = 0,1,2,3,4)$ are all constants.

Assumption 2. The desired output signal q_d and its time derivatives up to the nth order are continuous and bounded. The tracking errors are defined as follows: $e = q - q_d$, $\dot{e} = \dot{q} - \dot{q}_d, \cdots, e^{(n-1)} = q^{(n-1)} - q_d^{(n-1)}$, also $e^{(n)} = q^{(n)} - q_d^{(n)}$. Where, $q_d, \dot{q}_d, \ddot{q}_d$ represent the desired link position, velocity, and acceleration, respectively.

3. ROBUST NEURAL NETWORK CONTROLLER DESIGN VIA SWITCHING STRATEGY

In this paper, the control objective is to design a robust neural network controller design via switching strategy, such that the link position q of system (1) follows the any given bounded desired output signal q_d. If the robot modeling is perfect and there are no external disturbances, then according to the computed torque method, the control objective can be well obtained [9, 10, 15]. However, in practical application systems, the perfect robot model is difficult to obtain and the disturbances are always present in practice, so it's difficult to implement the control algorithm.

Using the feedback linearization technique, the computed torque controller is designed as:

$$\tau_t = D_{i,0}(q)[\ddot{q}_d - K_v\dot{e} - K_p e] + C_{i,0}(q,\dot{q})\dot{q} + G_{i,0}(q). \tag{3}$$

Where, K_v and K_p are proportional and derivative constant matrices, respectively, which are positive definite matrices.

Substituting (3) into (1), we get:

$$\ddot{e} + K_v\dot{e} + K_p e = f_i. \tag{4}$$

Where, $f_i = D_{i,0}^{-1}(q)[\triangle D_i(q)(\ddot{q}_d - K_v\dot{e} - K_p e) + \triangle C_i(q,\dot{q})\dot{q} + \triangle G_i(q) - \tau_{di}(q,\dot{q},t)]$.

Then, the (4) can be expressed to the closed loop system expressed in state-space form as:

$$\dot{x}_i = Ax + Bf_i(x). \tag{5}$$

Where, $A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Lemma 1. (Liberzon [10], Du et al. [3]) If $f(x) \in L_2 \cap L_\infty$, and $\dot{f}(x) \in L_\infty$, we have $\lim\limits_{t\to\infty} f(x) = 0$.

As nonlinear function of the state variable $f_i(x)$ includes uncertainties of the robot dynamics, it is unknown a priori. Due to its great approximation ability, RBF NNs will be employed in this paper to identify the function $f_i(x)$ of $(q,\dot{q})$ as follows [1, 5, 11]:

$$f_i(x) = W_f^T \xi_{fi} + \delta_i \tag{6}$$

where, W_f and $\xi_{fi}(X) \to R_p$ denote the vectors of weight values and Gaussian basis function, respectively. And δ_i is the reconstruction approximate error of RBF NNs.

So we have:

$$\|\delta_i\| \leq \|f_i(x)\| - \|W_f^T \xi_{fi}\|. \tag{7}$$

According the assumption in [6, 9],

$$\|f_i(x)\| \leq \mu = \omega_1 + \omega_2\|x\| + \omega_3\|x\|^2. \tag{8}$$

Where, $\omega_1, \omega_2, \omega_3$ are the limit parameters of uncertainties of robot dynamics.

From (7) and (8), we obtain:

$$\|\delta_i\| \leq \mu + \|W_f\| \cdot \|\xi_{fi}\|. \tag{9}$$

Where, ω_i is the limit parameter of the upper bound for the unknown uncertainty of the robotic manipulators.

Then, the estimate value of $f(x)$ is defined as :

$$\hat{f}_i(x) = \hat{W}_f^T \xi_{f_i}. \tag{10}$$

Where, $\hat{W}_f$ are the estimate values of weights vector W_f, so weights vector error of RBF NNs has the forms:

$$\tilde{W}_f = W_f - \hat{W}_f. \tag{11}$$

Then, the control law for the developed robust neural network controller via switching strategy is assumed to take in the following form:

$$\tau(t) = D_{i,0}^{-1}(q)[\ddot{q}_d - K_v\dot{e} - K_p e] + C_{i,0}(q,\dot{q})\dot{q} + G_{i,0}(q) - \hat{f}_i + u_h. \tag{12}$$

Where, u_h is the robust control law for compensating the approximation error of RBF NNs.

Then the closed-loop dynamics equation of the robotic manipulator can be described by:

$$\dot{x} = Ax + B[\tilde{W}_f^T \xi_{fi} + \delta_i + (I - D_i^{-1})\hat{W}_f^T \xi_{fi} + D_i^{-1} u_h. \tag{13}$$

Also, from (7) – (9) and (13), we can get the following inequality:

$$\|\delta_i + (I - D_i^{-1})\hat{W}_f^T \xi_{fi}\| \leq \mu + \|W_f\| \cdot \|\xi_{fi}\| + \|(I - D_i^{-1})\| \cdot \|\hat{W}_f^T \xi_{fi}\|. \tag{14}$$

So we can define:

$$\begin{cases} K = [\omega_1 \;\; \omega_2 \;\; \omega_3 \;\; \|W_f\| \;\; \|(I - D_i^{-1})\| \;]^T \\ \Phi = [1 \;\; \|x\| \;\; \|x\|^2 \;\; \|\xi_{fi}\| \;\; \|\hat{W}_f^T \xi_{fi}\| \;]^T. \end{cases} \tag{15}$$

Similarly, the (7) can be expressed in the following:

$$\|\delta_i + (I - D_i^{-1})\hat{W}_f^T \xi_{fi}\| \leq K^T \Phi \tag{16}$$

For the switching signal $\sigma(t)$, a switching sequence is given by [7, 13, 14]:

$$\Sigma := \{(i_0, t_0), (i_1, t_1), \cdots, (i_k, t_k), \cdots, | i_k \in \Xi, k \in N\} \tag{17}$$

where, (i_k, t_k) denotes that the i_kth subsystem is switched on at t_k, and the i_{k+1}th subsystem is switched off at t_{k+1}. Where, t_0 is the initial time, $t_k > 0$ is the kth switching time. When $t \in [t_k, t_{k+1})$, the trajectory of the switched nonlinear system (1) is produced by the i_{k+1}th subsystem, defining $\Delta t_k = t_k - t_{k-1}$ as dwell time of the i_kth subsystem in a period. Given the switching sequence, the periodically switching signal is constructed as follows [19]:

$$\sigma(t) = i, \quad if \;\; t \in \left[lT + \sum_{j=0}^{i-1} t_j, lT + \sum_{j=0}^{i} t_j \right) \tag{18}$$

where, $l = \{0, 1, 2, \cdots\}$, $i \in \Xi$, $t_0 = 0$, and,

$$T = \sum_{j=0}^{i} t_j \;\text{ is the period of the switching sequence.}$$

Assumption 3. (Liberzon [10], Long and Fei [11], Salas et al. [12]) For $t \in (t_{k-1}, t_k] \in \Omega_m \,(m \in \Xi)$ and $t \in [t_k, t_{k+1}) \in \Omega_{m+1} \,(m \in \Xi)$, there is a constant $\rho \geq 0$ such that

$$|\varsigma(t_{k+1})| \leq \rho |\varsigma(t_k)|. \tag{19}$$

Where, in this paper we assume $\rho = 1$.

Lemma 3. (Lewis et al. [9], Long and Fei [11]) For the closed loop dynamics equation (14), if there exists matrices $P_i = P_i^T > 0, \in Q_i = Q_i^T > 0$, the necessary and sufficient conditions that the resulting closed-loop system is stable satisfy the condition: matrices P_i are the positive definite solutions to the following Lyapunov equation:

$$A^T P_i + P_i A = -Q_i. \tag{20}$$

Also, the compensation controller u_h is to suppress the effect of the signal of the unknown terms $(I - D_i^{-1})\hat{W}_f^T \xi_{fi}$. The form of u_h can be described by:

$$u_h = \frac{-\alpha B^T P x |\hat{K}^T \Phi|^2}{\gamma(\hat{K}^T \Phi \|x^T P B\| + \beta)}. \tag{21}$$

Where, $\gamma > 0$ is a small constant. α, β are defined as following respectively: $\alpha \geq \|I - D_i^{-1}\|$, $\beta = \lambda_{\min}(D_i^{-1})$.

Also, for $i, j \in \Xi$, the following matrix inequality is designed [4, 10, 11]:

$$\begin{bmatrix} -P_i & (\Pi_{i,j} + I)^T P_j \\ P_j(\Pi_{i,j} + I) & -P_j \end{bmatrix} \leq 0. \tag{22}$$

Where, $\Pi_{i,j}$ are known $n \times n$ constant matrices. In general, when $\Pi_{i,i} = 0$ means that there is no switched jump when a subsystem is remaining active.

Also, the weight update law of the robotic manipulator is designed as:

$$\begin{cases} \dot{\hat{W}}_f = \eta_1 \xi_{fi} x^T P B - \vartheta_1 \hat{W}_f \\ \dot{\hat{K}} = \eta_2 \|x^T P B\| \Phi - \vartheta_2 \hat{K}. \end{cases} \tag{23}$$

Where, $\eta_1, \eta_2, \vartheta_1, \vartheta_2$ are all the positive constants.

Theorem 1. Consider the switching model of nonlinear robotic manipulator system (1). Given the control law (12) and (21) with the periodically switching signal (18) and the weight update law (23), it's guaranteed that the resulting closed-loop switched robot system is asymptotically Lyapunov stable and the position tracking error performance can be well obtained.

Proof. Define a switched multiple Lyapunov function candidate to analyze the stability of system (1) as:

$$V=\sum_{i=1}^{n}\theta_i(t)x^TPx+\frac{1}{\eta_1}tr(\tilde{W}_f^T\tilde{W}_f)+\frac{1}{\eta_2}(\tilde{K}^T\tilde{K}) \tag{24}$$

where, the characteristic function:

$$\theta_i(t)=\begin{cases}1 & t\in\Omega_i\\ 0 & t\notin\Omega_i\end{cases},\quad \Omega_i=\{t|\text{ the } i\text{th system is active at time instant t}\}.$$

From the switched Lyapunov function candidate $V(t)$ (24) and properties of the robotic manipulator, the $V(t)$ satisfied the condition in the following:

$$\frac{1}{2}\lambda_{\min}(Q_m)\|x\|^2+\frac{1}{2\eta_1}\|\tilde{W}_f\|_F^2+\frac{1}{2\eta_2}\|\tilde{K}\|^2\le V\le\frac{1}{2}\lambda_{\max}(Q_m)\|x\|^2+\frac{1}{2\eta_1}\|\tilde{W}_f\|_F^2+\frac{1}{2\eta_2}\|\tilde{K}\|^2. \tag{25}$$

For $t\in(t_{k-1},t_k]\in\Omega_m(m\in\Xi)$ and $t\in(t_k,t_{k+1}]\in\Omega_{m+1}$, from (22) and (24), we have:

$$\begin{aligned}\Delta V(t) &= \Delta V(t_{k+1})-\Delta V(t_k)\\ &= x^T(t_{k+1})P_{m+1}x(t_{k+1})-x^T(t_k)P_mx(t_k)\\ &= x^T(t_k)[(\Pi_{m,m+1}+I)^TP_{m+1}(\Pi_{m,m+1}+I)-P_m]x(t_k)<0.\end{aligned} \tag{26}$$

For all $T\in[t_k,t_{k+1}]\in\Omega_{m+1}$, taking the time derivative of V, and using (3), (12), (21), (23) and (25), we have:

$$\begin{aligned}\dot{V} &= \dot{x}^TP_mx+x^TP_m\dot{x}+\frac{2}{\eta_1}tr(\dot{\tilde{W}}_f^T\tilde{W}_f)+\frac{2}{\eta_2}(\dot{\tilde{K}}^T\tilde{K})\\ &= -x^TQ_mx+2x^TP_mB(\tilde{W}_f{}^T\xi_{fm}+\delta_m)+2x^TP_mB(I-D_m^{-1})\hat{W}_f{}^T\xi_{fm}\\ &\quad +2x^TP_mBD_m^{-1}u_h+\frac{2}{\eta_1}tr(\dot{\tilde{W}}_f^T\tilde{W}_f)+\frac{2}{\eta_2}(\dot{\tilde{K}}^T\tilde{K})\\ &\le -x^TQ_mx+\frac{2\vartheta_1}{\eta_1}tr(\tilde{W}_f{}^T\hat{W}_f)+\frac{2\vartheta_2}{\eta_2}(\tilde{K}^T\hat{K})+2\|x^TP_mx\|\\ &\qquad\cdot|K^T\Phi|+2x^TP_mBD_m^{-1}u_h\\ &\le -\lambda_{\min}(Q_m)\|x\|^2+\frac{2\vartheta_1}{\eta_1}tr(\tilde{W}_f{}^T\hat{W}_f)+\frac{2\vartheta_2}{\eta_2}(\tilde{K}^T\hat{K})+2\|x^TP_mx\|\\ &\qquad\cdot|K^T\Phi|+2x^TP_mBD_m^{-1}u_h\\ &\le -\lambda_{\min}(Q_m)\|x\|^2+\frac{2\vartheta_1}{\eta_1}\|\tilde{W}_f\|_F^2+\frac{2\vartheta_2}{\eta_2}\|\tilde{K}\|^2+2\|x^TP_mx\|\end{aligned}$$

$$
\begin{aligned}
&\cdot|K^T\Phi| + 2x^T P_m B D_m^{-1} u_h \\
\leq\ & -\lambda_{\min}(Q_m)\|x\|^2 + \frac{2\vartheta_1}{\eta_1}\|\tilde{W}_f\|_F^2 + \frac{2\vartheta_2}{\eta_2}\|\tilde{K}\|^2 + 2\phi.
\end{aligned} \tag{27}
$$

Then, we define $\phi = 2\|x^T Q_m x\| \cdot |K^T\Phi| + 2x^T P_m B D_m^{-1} u_h$, $\lambda = \min\left(\frac{\lambda_{\min}(Q_m)}{\lambda_{\min}(P_m)}, \vartheta_1, \vartheta_2\right)$, and we can obtains

$$\dot{V} \leq \phi - 2\lambda V. \tag{28}$$

Integrating the above inequality from 0 to t yields:

$$V(t) \leq (V(0) - \frac{\phi}{\lambda})e^{-\lambda t} + \frac{\phi}{\lambda}. \tag{29}$$

Using Barbalat's lemma, from (25) and (29), it can be shown that $\|x\|$, $\|W_f\|$ and $\|K\|$ are uniformly ultimately bounded [7, 10, 13]. So far, the proof of Theorem 1 has been completed. □

4. SIMULATION RESULTS

In this section, a simulation study is conducted to demonstrate the performance of our control scheme. A simple two-link of freedom robotic manipulator has been used in the simulation. The switching model for this robotic manipulator can be described as follows:

$$
\Sigma_1 : \begin{cases}
D_1(q) = \begin{bmatrix} 9 + 2\theta_1\cos(q_2) & 5 + 2q_2\cos(q_2) \\ 5 + 2q_2\cos(q_2) & 5 \end{bmatrix} \\
C_1(q,\dot{q}) = \begin{bmatrix} -6\dot{q}_2\sin(q_2) & -6(\dot{q}_1 + \dot{q}_2)sin(q_2) \\ 6\dot{q}_1\sin(q_2) & 0\cdot\dot{q}_1 \end{bmatrix} \\
G_1(q) = \begin{bmatrix} 7\cos(q_1) + 6\cos(q_1+q_2) \\ 6\cos(q_1+q_2) \end{bmatrix} \\
\tau_{d1} = 0.3\sin(t)
\end{cases} \tag{30}
$$

$$
\Sigma_2 : \begin{cases}
D_1(q) = \begin{bmatrix} 7 + 2\theta_2\cos(q_2) & 7 + 2q_2\cos(q_2) \\ 7 + 2q_2\cos(q_2) & 7 \end{bmatrix} \\
C_2(q,\dot{q}) = \begin{bmatrix} -3\dot{q}_2\sin(q_2) & -3(\dot{q}_1 + \dot{q}_2)sin(q_2) \\ 5\dot{q}_1\sin(q_2) & 0\cdot\dot{q}_1 \end{bmatrix} \\
G_2(q) = \begin{bmatrix} 11\cos(q_1) + 6\cos(q_1+q_2) \\ 6\cos(q_1+q_2) \end{bmatrix} \\
\tau_{d2} = 0.2\cos(t).
\end{cases} \tag{31}
$$

The design of control objective is that the link position $q = [q_1\ q_2]^T$ follows the desired output signal $q_d = [q_{1d}\ \ q_{2d}]^T = [0.5\sin(t) + \cos(t)\ \ 0.5\sin(t) - \cos(t)]^T$. In terms of the design procedures in Section 3, the proper control parameters are designed as follows: $\Delta D_i = 0.3D_i$, $\Delta C_i = 0.3C_i$, $\Delta G_i = 0.3G_i$, $\eta_1 = 16, \eta_2 = 18, \vartheta_1 = 0.36, \vartheta_2 = 0.28, T = 2$. The gains are chosen as $K_p = 18$ and $K_v = 11$. Choosing that the parameter matrices Q_1 and Q_2 are taken as diagonal matrices with diagonal elements 5 and 9 respectively. The initial values of state vectors is $q_0 = [-0.9\ 0.6\ \ -0.5\ 0.5]^T$. The initial weights values of RBF NNs are chosen randomly between 0 and 1, and the number of hidden units for the RBF NNs is taken as 25.

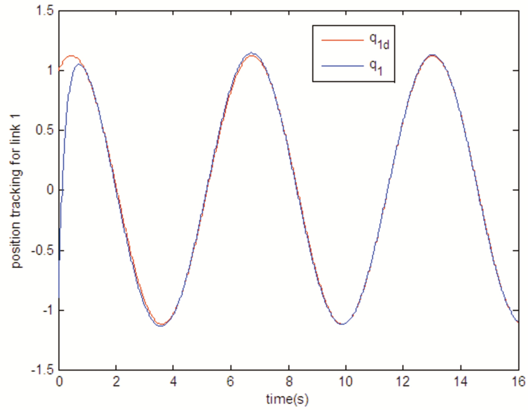

Fig. 1. Position tracking performance of link 1.

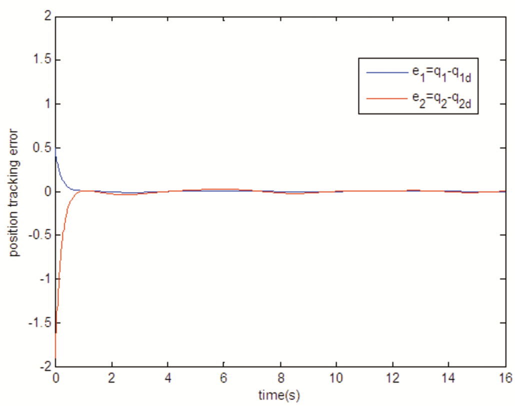

Fig. 2. Position tracking performance of link 2.

With the proposed control approach, simulation results are shown in Figures 1 − 2. Figure 1 − 2 denote the position tracking performance of the two-link robotic manipulator, and Figure 3 denotes the tracking error performance. It has been clearly observed from the above figures that, the proposed robust neural switching controller effectively attenuate the effects of uncertainties of robotic manipulators and the tracking errors converge to small values. Meanwhile, since the normal model of the uncertain system has been taken into account, the system can achieve good tracking performance.

Then, to investigate the better improved control performance of the proposed control method, the PD control has been employed for comparison. The PD control law for the same switching model of robotic manipulator can be written as: $u = K_{Pd}\dot{e} + K_{Dd}e$. The parameter matrices K_{Pd} and K_{Dd} are taken as diagonal matrices with diagonal elements

6 and 8, respectively. The desired reference signal $q_d = [q_{1d}\ q_{2d}]^T$ is the same as the above control method. Then in the following simulation, the tracking error performance of the two-link robotic manipulators is shown in Figure 4. Therefore, from the comparison and analysis of the two control methods, it is clearly investigated that this paper proposed control method gives better smooth results, fast adjustment time and little tracking errors. This shows that RBF neural network based control scheme can quickly deal with the switching dynamical models better than the PD control algorithm.

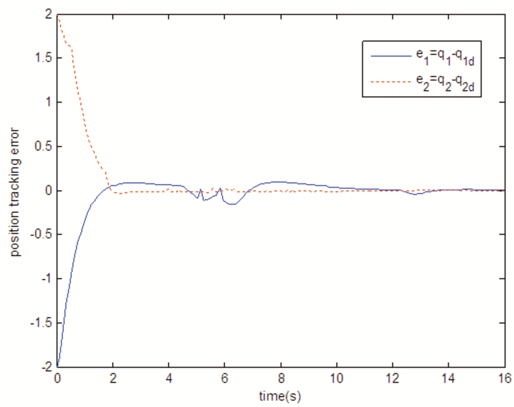

Fig. 3. Position tracking error performance with proposed control.

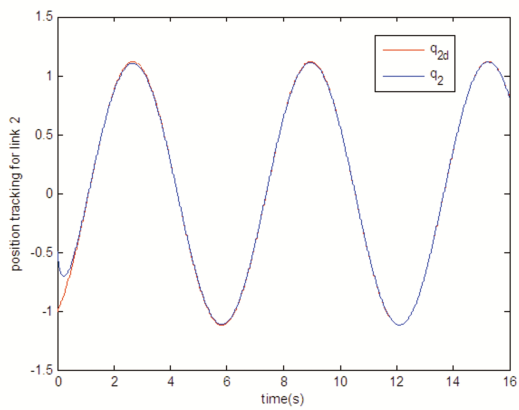

Fig. 4. Position tracking error performance with PD control.

Remark 1. In this paper, the proposed robust neural network control is running in continuous time and the final implementation will be definitely running in discrete time. So it is very important to choose the sampling period. The bigger parameter of the sampling period is, the worse the achievable control accuracy is. Also, the smaller parameter of the sampling period is, the better the achievable control accuracy is. However, if the parameter sampling period is too small, the rise time of control systems will be too long. Thus, in the simulation, the design parameter of sampling period should be adjusted carefully for achieving satisfied tracking control performance.

Remark 2. In this paper, we have presented the main contributions of the results in this paper over some existing ones. Compared with the works in [1, 2, 18, 8, 12, 20], the outstanding feature of the algorithms proposed in this paper is that: (i) Combining with the multiple Lyapunov function approach and the periodically switching method, the novel design of the robust adaptive neural switching controller is first discussed for the switching dynamical model of the robotic manipulators; (ii) The robust compensation controller is introduced to enhance the robustness and keep bounded in the control system; (iii) With the proposed control scheme of the robotic manipulator in this paper, it is quickly to reach the trajectory objective such that both system tracking stability and error convergence can be guaranteed in the closed-loop system. Also, it's well known that robust neural network control scheme design remains open for the switching dynamical model of the robotic manipulators. So we can choose another switching signal or design another robust controller for this control system. Therefore, some better improvements will be achieved further.

Remark 3. Meanwhile, the main advantages of the results in this paper have been clearly demonstrated by comparing it with the robust switching control strategy in [20]. Consequently, from the control point of view, the proposed control scheme presented in this paper provides a better effective mechanism to cope with disturbances and/or systems with uncertainties. On the other hand, from the practical application point of view, the proposed designed control parameters should be adjusted carefully for achieving suitable transient performance and control action. Also, further works are still under investigation to apply the proposed method to the more general switching dynamical model of a n-link robotic manipulators with kinematics and dynamics uncertainties.

5. CONCLUSIONS

In this paper, we have presented the robust neural network control of robotic manipulators with periodically switching method. RBF NNs are employed to approximate unknown functions and design a robust compensation controller to enhance system robustness and stabilization. The weights of RBF NNs updated law has been derived from the multiple Lyapunov function approach. It's proved that both system tracking stability and error convergence can be guaranteed in the closed-loop system. Finally, simulation results for a two-link robotic manipulators show the satisfactory position tracking performance.

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