KYBERNETIKA — VOLUME *50* (2014), NUMBER 4, PAGES 596–615

OPTIMAL, ADAPTIVE AND SINGLE STATE FEEDBACK CONTROL FOR A 3D CHAOTIC SYSTEM WITH GOLDEN PROPORTION EQUILIBRIA

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In this paper, the problems on purposefully controlling chaos for a three-dimensional quadratic continuous autonomous chaotic system, namely the chaotic Pehlivan–Uyaroglu system are investigated. The chaotic system, has three equilibrium points and more interestingly the equilibrium points have golden proportion values, which can generate single folded attractor. We developed an optimal control design, in order to stabilize the unstable equilibrium points of this system. Furthermore, we propose Lyapunov stability to control the Pehlivan–Uyaroglu system with unknown parameters by way of a feedback control approach and a single controller. Numerical simulations are performed to demonstrate the effectiveness of the proposed control strategies.

Keywords: autonomous chaotic system, optimal control, adaptive control, single state feedback control, Pontryagin Minimum Principle

Classification: 34D20, 58E25, 93C10, 37N35

1. INTRODUCTION

Chaos control is an important topic in the nonlinear control systems and has great significance in the application of chaos. Chaos is of fundamental concern in a wide range of fields, including secure communications, optics, chemical and biological systems, and so forth [1, 2, 3, 4]. The desirability, or otherwise, of chaos depends on the particular application. Sometimes chaos effect is undesirable in practice, and it restricts the operating range of many electronic and mechanic devices. In this case, therefore, it is necessary that the chaotic behavior should be controlled, e. g. by driving the chaotic attractors to a specific region of the system or by eliminating chaos entirely through the application of suitable control laws.

Since Ott, Grebogi, and Yorke [5] firstly proposed the method of chaos control in 1990, chaos control including stabilization of unstable equilibrium points, and more generally, unstable periodic solutions, has attracted increasing attention in recent years, and lots of successful experiments have been reported. These include adaptive control, adaptive fuzzy control, sliding mode control, robust control, time-delayed feedback control, double delayed feedback control, bang-bang control, optimal control, intelligent control, etc.; see

DOI: 10.14736/kyb-2014-4-0596

[6, 7, 8, 9, 10, 11, 12, 13, 14]. It is also a challenging topic for the control of multiscroll chaotic systems. In general, compared with the single-scroll chaotic attractors, the multi-scroll chaotic attractors have much higher complexity and more adjustability. In particular, multi-scroll chaotic attractors have many specific properties and functions. For example, Lu et.al reviewed the main advances in theories, methods, implementations, and applications of multi-scroll chaos generation [15, 16].

Over the past two decades, there has been increasing interest in exploiting chaotic dynamics in engineering applications, where some attention has been focused on effectively creating chaos via simple physical systems, such as electronic circuits [17, 18]. Lately, the pursuit of designing circuits to produce chaotic attractors has become a focal point for electronics engineers, not only because of their theoretical interest, but also due to their potential real-world applications in various chaos-based technologies and information systems [19, 20]. Recently, a new three-dimensional autonomous chaotic system is presented by Pehlivan and Uyaroglu [21], with golden ratio equilibria. This system has eight terms, two quadratic nonlinearities and two parameters (a and b). The chaotic attractor equations have three equilibrium points and more interestingly the equilibrium points have golden proportion values. Dynamical properties of this new system were analyzed by means of equilibrium points, eigenvalue structures, Lyapunov exponents. The chaos generator of the new chaotic system was confirmed through a novel electronic circuit design. Module-based approach was used to chaotic circuit design. All these characteristics are very useful in many real-world applications. It is obvious that, the unknown dynamical behaviors of the strange chaotic attractors deserve further investigation and are very desirable for engineering applications such as secure communications in the near future [21]. As we know, the famous golden proportion $\tau = \frac{1+\sqrt{5}}{2}$, found often in nature. Many objects alive in the natural world that possess pentagonal symmetry, such as marine stars, inflorescences of many flowers, and phyllotaxis objects have a numerical description given by the Fibonacci numbers which are themselves based on the golden proportion. In this study, we propose a strategy for optimal control of the chaotic Pehlivan–Uyaroglu system. For this purpose, we will apply the Pontryagin Minimum Principle (PMP) [22]. Furthermore, the design of the feedback controller is achieved through an application of the optimal control and Lyapunov stability theories which guarantee the global stability of the nonlinear error system. Meanwhile, the single state feedback stabilization of the Pehlivan–Uyaroglu system is also addressed.

This paper is organized as follows. In Section 2, the preliminaries and problem description of the chaotic Pehlivan–Uyaroglu system and it's stability analysis are presented. In Section 3, the problem statement and optimal control scheme are presented for the Pehlivan–Uyaroglu system. In Section 4, an adaptive control law is designed to stabilize the chaotic system with unknown parameters. Section 5, presents a single state feedback stabilization of the Pehlivan–Uyaroglu system. In Section 6, we summarize the main results obtained in this paper.

2. THE PEHLIVAN–UYAROGLU SYSTEM AND STABILITY ANALYSIS

In this section, we discuss the equilibrium and stability of the chaotic Pehlivan–Uyaroglu system. The equations describing the dynamics of the Pehlivan–Uyaroglu system can be written as

$$\begin{aligned} \dot{x} &= y - x - az, \\ \dot{y} &= xz - x, \\ \dot{z} &= -xy - y + b, \end{aligned} \tag{1}$$

where x, y and z are system state variables and a and b are positive constant parameters. This new system is found to be chaotic in a wide parameter range and has many interesting complex dynamical behaviors. Typical parameters are $a = 2, b = 1$ or $a = 0.5, b = 1$. Chaotic attractor and phase diagrams of (1) are shown in Figures 1−4.

2.1. Dissipation

The differential coefficient of the system (1) can be obtained as

$$\nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = -1 = r, \tag{2}$$

while $F = (F_1, F_2, F_3) = (y - x - az, xz - x, -xy - y + b)$. Therefore, system (1) is dissipative.

$$\frac{\mathrm{d}F}{\mathrm{d}t} = rF \Rightarrow F = F_0 e^{-t}. \tag{3}$$

Therefore, system (1) is dissipative.

2.2. Equilibrium and stability

In the following, we consider the equilibrium of system (1). By calculation, it can be shown that when $a = 2, b = 1$ or $a = 0.5, b = 1$, the system (1) has three equilibrium points, which are respectively, described as follows

$$\begin{cases} E_1 = \left(\frac{-a-1+\sqrt{a^2-2a+1+4b}}{2}, \frac{a-1+\sqrt{a^2-2a+1+4b}}{2}, 1\right), \\ E_2 = \left(\frac{-a-1-\sqrt{a^2-2a+1+4b}}{2}, \frac{a-1-\sqrt{a^2-2a+1+4b}}{2}, 1\right), \\ E_3 = \left(0, b, \frac{b}{a}\right). \end{cases} \tag{4}$$

As the variables $x, y, z \in \mathbb{R}$, this implies that fixed point to exist, $a \neq 0$ and $a^2 - 2a + 1 + 4b > 0$. So, $(a-1)^2 + 4b > 0$, and $a \in \mathbb{R}$. When $a = 0$, the system has unbounded solutions.

The equilibrium points of the system are
$E_1 = \left(\frac{-3+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}, 1\right), E_2 = \left(\frac{-3-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}, 1\right), E_3 = \left(0, 1, \frac{1}{2}\right)$ for $a = 2$, for $b = 1$ values.
More interestingly the equilibrium points have golden proportion values as

$$E_1 = \left(-\tau^{-2}, \tau, \tau^0\right), E_2 = \left(-\tau^2, -\tau^{-1}, \tau^0\right), E_3 = \left(0, \tau^0, \frac{\tau^0}{2}\right).$$

The famous golden proportion $\tau = \frac{1+\sqrt{5}}{2}$, found often in nature. In the last years, the golden proportion has played an increasing role in modern physical research.

Proposition 2.1. The equilibrium points E_1, E_2, E_3 of system (1) with $a = 2, b = 1$ or $a = 0.5, b = 1$, are unstable.

Proof. The Jacobian matrix of the system (1) is given by

$$J = \begin{bmatrix} -1 & 1 & -a \\ z-1 & 0 & x \\ -y & -x-1 & 0 \end{bmatrix}. \tag{5}$$

The eigenvalues of the Jacobian matrix J_{E_1} are given by
$\lambda_1 = 1.565818, \lambda_2 = -2.331903, \lambda_3 = -0.233915.$ for $a = 2, b = 1$. □

It is observed that the eigenvalue λ_1 is positive. According to Lyapanov theorem the equilibrium point E_1 is unstable. By this manner, we can see that other equilibrium points E_2, E_3 are also unstable.

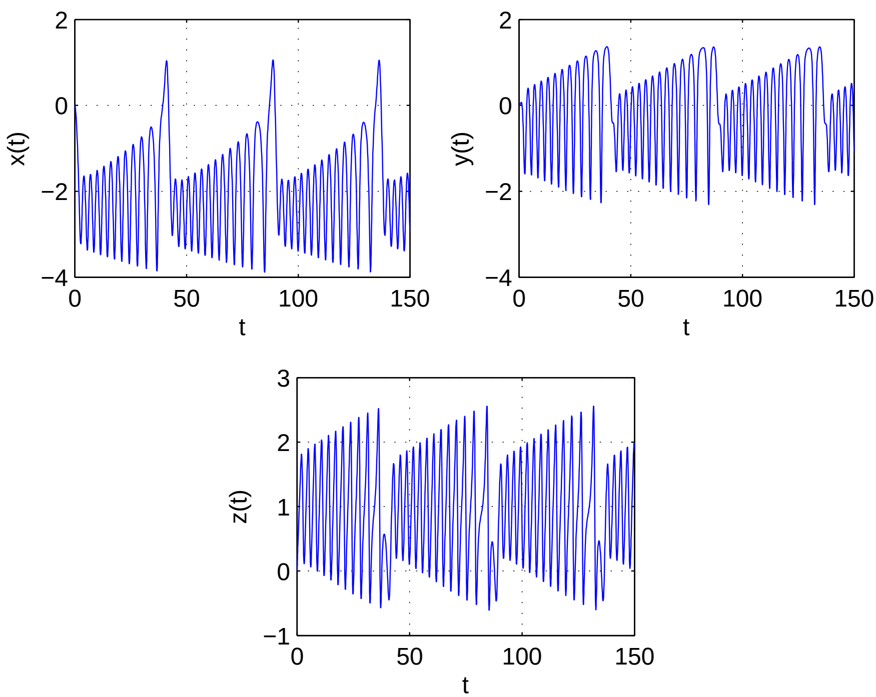

Fig. 1. Time response of the system states with $a = 2, b = 1$.

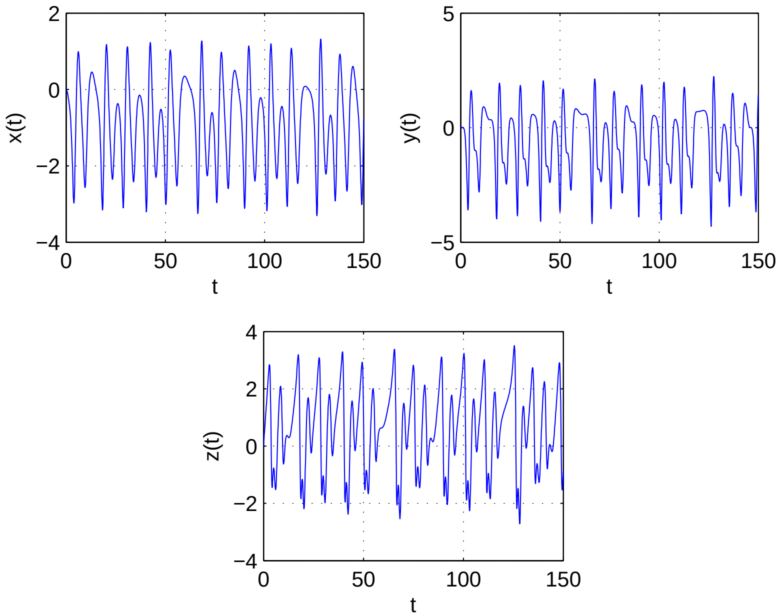

Fig. 2. Time response of the system states with $a = 0.5, b = 1$.

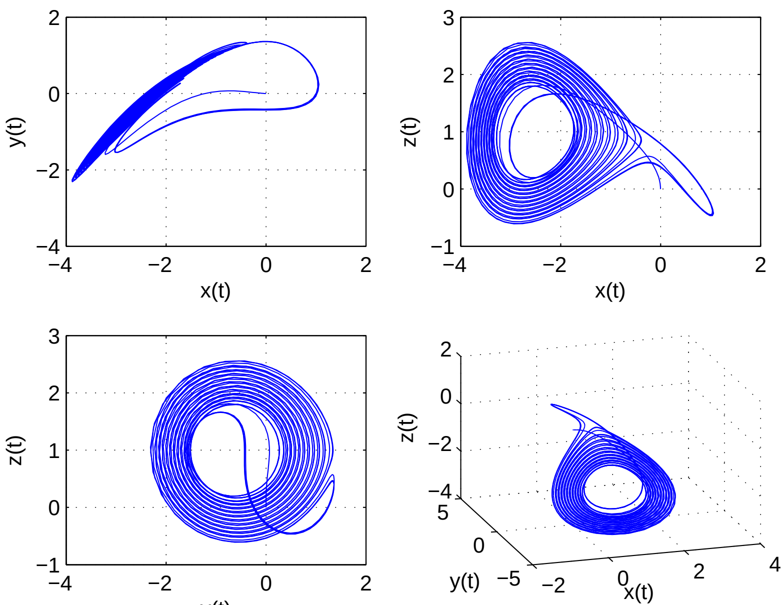

Fig. 3. Chaotic attractor of system with $a = 2, b = 1$.

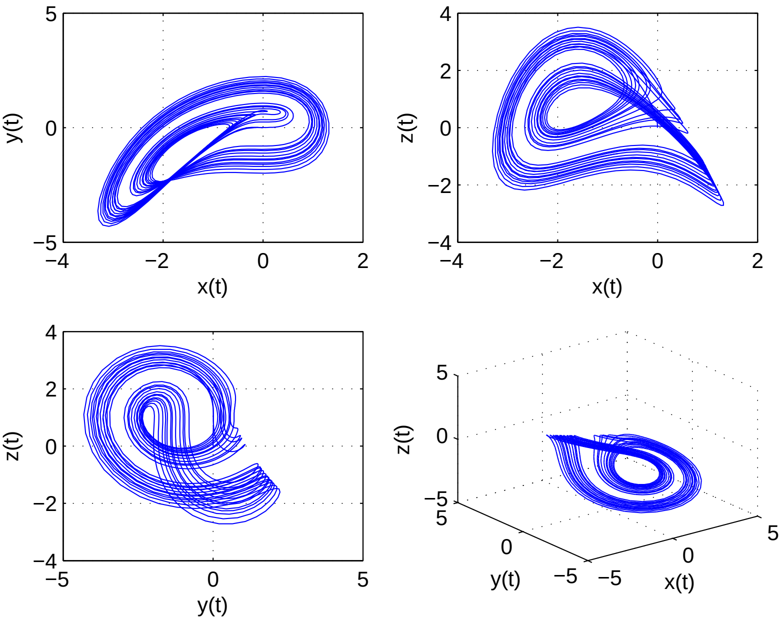

Fig. 4. Chaotic attractor of system with $a = 0.5, b = 1$.

3. OPTIMAL CONTROL OF THE PEHLIVAN–UYAROGLU CHAOTIC SYSTEM

In this section, optimal stabilization of the Pehlivan–Uyaroglu chaotic system (1) is about its steady states is discussed. First, we add the controls u_1, u_2 and u_3 to the equations in system (1):

$$\begin{cases} \dot{x} = y - x - az + u_1, \\ \dot{y} = xz - x + u_2, \\ \dot{z} = -xy - y + b + u_3, \end{cases} \tag{6}$$

where $u_j (j = 1, 2, 3)$ are control inputs which will be satisfied from the conditions of the optimal the dynamical system (1) about its equilibrium points $E_i, (i = 1, 2, 3)$ with respect to the cost function J. The proposed control strategy is designed to achieve in a given time t_f to the equilibrium point with an optimal control inputs. The initial and final conditions are

$$\begin{cases} x(0) = x_0, x(t_f) = \overline{x}, \\ y(0) = y_0, y(t_f) = \overline{y}, \\ z(0) = z0, z(t_f) = \overline{z}, \end{cases} \tag{7}$$

where $(\overline{x}, \overline{y}, \overline{z})$ denote the coordinates of the equilibrium points $E_1 - E_3$.

3.1. Designed of the optimal controller

The objective functional to be minimized is defined as

$$J = \frac{1}{2}\int_0^{t_f} \sum_{i=1}^{3} (\alpha_i(\phi_i - \overline{\phi}_i)^2 + \beta_i {u_i}^2)\,\mathrm{d}t, \tag{8}$$

where $\alpha_i, \beta_i, (i = 1, 2, 3)$ are positive constants, $\phi_1 = x, \phi_2 = y, \phi_3 = z$, and $\overline{\phi}_1 = \overline{x}, \overline{\phi}_2 = \overline{y}, \overline{\phi}_3 = \overline{z}$. It is note that, the cost function is a positive definite function of the variables ϕ_i, and u_i, $i = 1, \dots, 3$. In particular, we will derive the fundamental nonlinear Two-Point Boundary Value Problem (TPBVP) arising in PMP. The corresponding Hamiltonian function will be

$$\begin{aligned} H = &-\frac{1}{2}\left[\alpha_1(x-\overline{x})^2 + \alpha_2(y-\overline{y})^2 + \alpha_3(z-\overline{z})^2 + \beta_1{u_1}^2 + \beta_2{u_2}^2 + \beta_3{u_3}^2\right] \\ &+ \lambda_1[y - x - az + u_1] + \lambda_2[xz - x + u_2] \\ &+ \lambda_3[-xy - y + b + u_3]. \end{aligned} \tag{9}$$

Where, $\lambda_i, (i = 1, 2, 3)$ are co-state variables. According to the Pontryagin Minimum Principle, we obtain the Hamiltonian equations :

$$\begin{aligned} \dot{\lambda}_1 &= -\frac{\partial H}{\partial x}, \\ \dot{\lambda}_2 &= -\frac{\partial H}{\partial y}, \\ \dot{\lambda}_3 &= -\frac{\partial H}{\partial z}. \end{aligned} \tag{10}$$

Substituting (9) into (10), the co-state equations can be derived in the form:

$$\begin{cases} \dot{\lambda}_1 = \alpha_1(x-\overline{x}) + \lambda_1 - \lambda_2 z + \lambda_2 + \lambda_3 y, \\ \dot{\lambda}_2 = \alpha_2(y-\overline{y}) - \lambda_1 + \lambda_3 x + \lambda_3, \\ \dot{\lambda}_3 = \alpha_3(z-\overline{z}) + a\lambda_1 - \lambda_2 x. \end{cases} \tag{11}$$

The optimal control functions that have to be used are determined from the conditions $\frac{\partial H}{\partial u_i} = 0, (i = 1, 2, 3)$. Hence, we get

$${u_i}^* = \frac{\lambda_i}{\beta_i}, \quad (i = 1, 2, 3). \tag{12}$$

Substituting from (12) into (6) we get the nonlinear controlled state equations:

$$\begin{cases} \dot{x} = y - x - az + \frac{\lambda_1}{\beta_1}, \\ \dot{y} = xz - x + \frac{\lambda_2}{\beta_2}, \\ \dot{z} = -xy - y + b + \frac{\lambda_3}{\beta_3}. \end{cases} \tag{13}$$

This system of nonlinear differential equations in addition to (11) form a complete system to solve the optimal control of Pehlivan–Uyaroglu chaotic system. This system has the following boundary conditions

$$\begin{cases} x(0) = x_0, & x(t_f) = \overline{x}, \\ y(0) = y_0, & y(t_f) = \overline{y}, \\ z(0) = z_0, & z(t_f) = \overline{z}, \\ \lambda_i(t_f) = 0, & i = 1, 2, 3. \end{cases} \tag{14}$$

Then, by solving the nonlinear systems (11) and (13) with the boundary conditions of (14), we obtain the optimal control law and the optimal state trajectory.

3.2. Analysis and numerical simulation

In this subsection to demonstrate and verify the effectiveness of the theoretical analysis, we solve the systems (13) and (14). In the following numerical simulations, the MATLAB's `bvp4c` in-built solver is used to solve the systems. The initial values and system parameters are selected as $x(0) = 50$, $y(0) = 0$, $z(0) = 0$, $a = 2$, $b = 1$ and $a = 0.5$, $b = 1$ in all simulations so that new autonomous chaotic system exhibits a chaotic behavior if no control is applied. Also, the positive constants in cost function J, are chosen $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\alpha_3 = 0.1$, $\beta_1 = 5$, $\beta_2 = 5$, $\beta_3 = 5$. The behaviors of the states (x, y, z) of the controlled new autonomous chaotic system (1) with time are displayed in Figures 5 – 10.

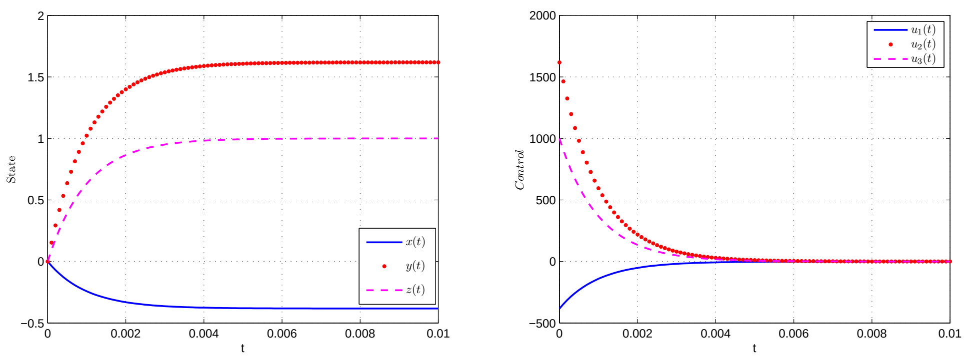

Fig. 5. The stabilized behavior of state and control functions for equilibrium point E_1 $(a = 2, b = 1)$.

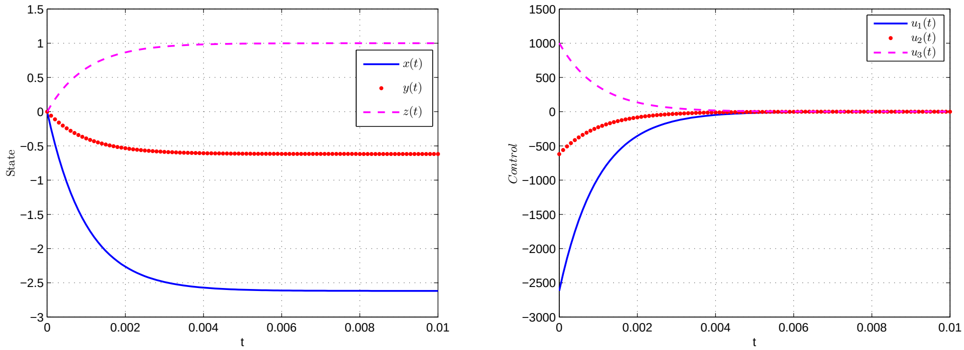

Fig. 6. The stabilized behavior of state and control functions for equilibrium point E_2 $(a = 2, b = 1)$.

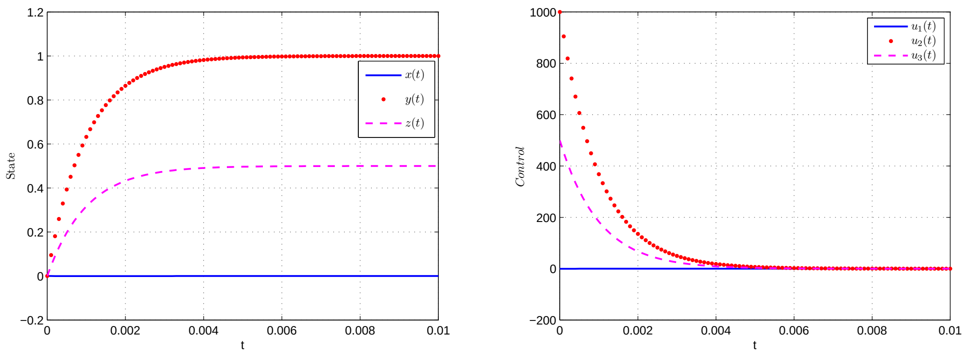

Fig. 7. The stabilized behavior of state and control functions for equilibrium point E_3 $(a = 2, b = 1)$.

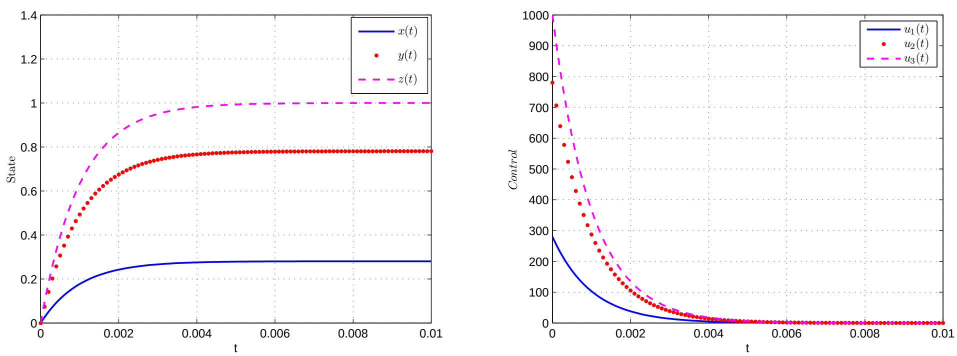

Fig. 8. The stabilized behavior of state and control functions for equilibrium point E_1 $(a = 0.5, b = 1)$.

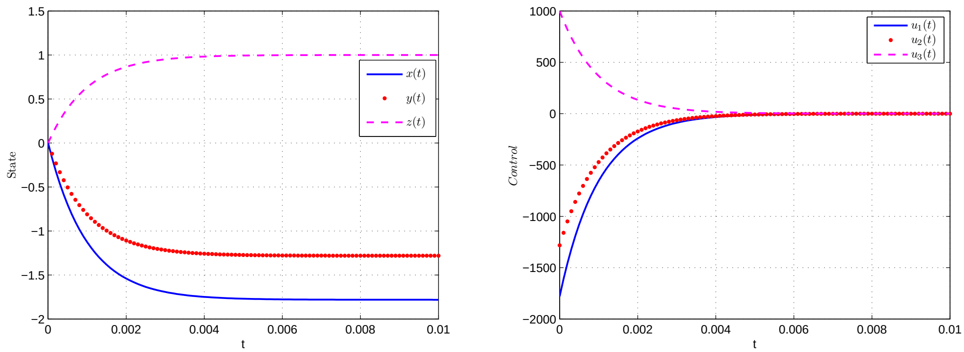

Fig. 9. The stabilized behavior of state and control functions for equilibrium point E_2 $(a = 0.5, b = 1)$.

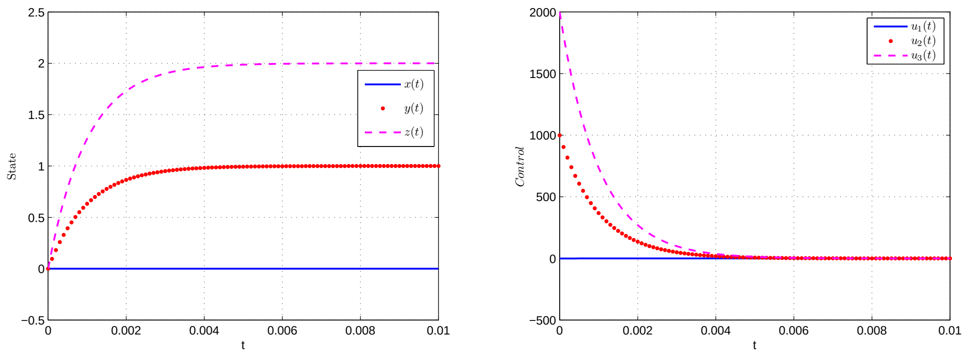

Fig. 10. The stabilized behavior of state and control functions for equilibrium point E_3 $(a = 0.5, b = 1)$.

4. ADAPTIVE CONTROL OF THE PEHLIVAN–UYAROGLU CHAOTIC SYSTEM

In this section, we obtain new results for the adaptive control of the Pehlivan–Uyaroglu chaotic system based on the Lyapunov stability theory and from the conditions of the asymptotic stability of this system about its steady states.

4.1. Designed of the adaptive controller

Let us assume that we have the controlled coupled system in the following form

$$\begin{cases} \dot{x} = y - x - az + v_1, \\ \dot{y} = xz - x + v_2, \\ \dot{z} = -xy - y + b + v_3, \end{cases} \tag{15}$$

where x, y, z are the states of the system, a, b are unknown parameters of the system, and v_1, v_2, v_3 are the adaptive controllers to be designed.

Theorem 4.1. The novel chaotic system (15) with unknown system parameters is globally and exponentially stabilized for all initial states by the adaptive control law:

$$\begin{aligned} v_1 &= -y + x + a_1 z - k_1(x - \overline{x}), \\ v_2 &= -xz + x - k_2(y - \overline{y}), \\ v_3 &= xy + y - b - k_3(z - \overline{z}), \end{aligned} \tag{16}$$

and the following parameter estimation update law

$$\begin{aligned} \dot{a_1} &= -z(x - \overline{x}) + k_4(a - a_1), \\ \dot{b_1} &= k_5(b - b_1), \end{aligned} \tag{17}$$

where a_1, b_1 are estimate values of uncertain parameters a, b and $k_i (i = 1, \dots, 5)$ are positive constants, respectively.

Proof. Substituting (16) into (15), we get the closed-loop system as

$$\begin{cases} \dot{x} = -(a - a_1)z - k_1(x - \overline{x}), \\ \dot{y} = -k_2(y - \overline{y}), \\ \dot{z} = -k_3(z - \overline{z}). \end{cases} \tag{18}$$

For the derivation of the update law for adjusting the parameter estimates, the Lyapunov approach is used.
We consider the quadratic Lyapunov function

$$V(x, y, z, \tilde{a}, \tilde{b}) = \frac{1}{2}\left((x - \overline{x})^2 + (y - \overline{y})^2 + (z - \overline{z})^2 + \tilde{a}^2 + \tilde{b}^2\right) \tag{19}$$

where the variables $\tilde{a} = a - a_1, \tilde{b} = b - b_1$.
Taking time derivative of the Lyapunov function V along the trajectories of (18), we obtain

$$\begin{aligned} \dot{V} &= (x - \overline{x})\dot{x} + (y - \overline{y})\dot{y} + (z - \overline{z})\dot{z} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} \\ &= -k_1(x - \overline{x})^2 - k_2(y - \overline{y})^2 - k_3(z - \overline{z})^2 + (a - a_1)\left(-z(x - \overline{x}) - \dot{a}_1\right) \\ &\quad -(b - b_1)\dot{b}_1. \end{aligned} \tag{20}$$

Substituting (17) into (20), the time derivative of the Lyapunov function becomes

$$\dot{V} = -k_1(x - \overline{x})^2 - k_2(y - \overline{y})^2 - k_3(z - \overline{z})^2 - k_4(a - \overline{a})^2 - k_5(b - \overline{b})^2 < 0, \tag{21}$$

The Lyapunov function V is positive definite and its derivative $\dot{V}$ is negative definite in the neighborhood of the zero solution for system (15). According to the Lyapunov stability theory, the equilibrium solution $E(\overline{x}, \overline{y}, \overline{z})$ of the controlled system (15) is asymptotically stable, namely, the controlled system (15) can asymptotically converge to the equilibrium $E(\overline{x}, \overline{y}, \overline{z})$ with the adaptive control law (16) and the parameter estimation update law (17). This completes the proof. □

4.2. Numerical results

For the numerical simulations, we solve the controlled Pehlivan–Uyaroglu chaotic system (15) with the adaptive control law (16) and the parameter update law (17). In the following numerical simulations, the MATLAB's `ode45` in-built solver is used to solve the systems. The initial values and system parameters are selected as $x(0) = 0,\ y(0) = 0,\ z(0) = 0,\ a = 2,\ b = 1$ and $a = 0.5,\ b = 1$. For the adaptive and update laws, we take $k_i = 5$ for $i = 1, 2, \ldots, 5$.

Suppose that the initial values of the parameter estimates are chosen as $a_1(0) = 0, b_1(0) = 0$. Figures 11 − 16 show that the controlled chaotic system (15) converges to E_i $(i = 1, \ldots, 3)$ exponentially with time. Also, these figures show that the parameter estimates $a_1(t), b_1(t)$ converge to the system parameter values $a = 2,\ b = 1$ and $a = 0.5,\ b = 1$. exponentially with time.

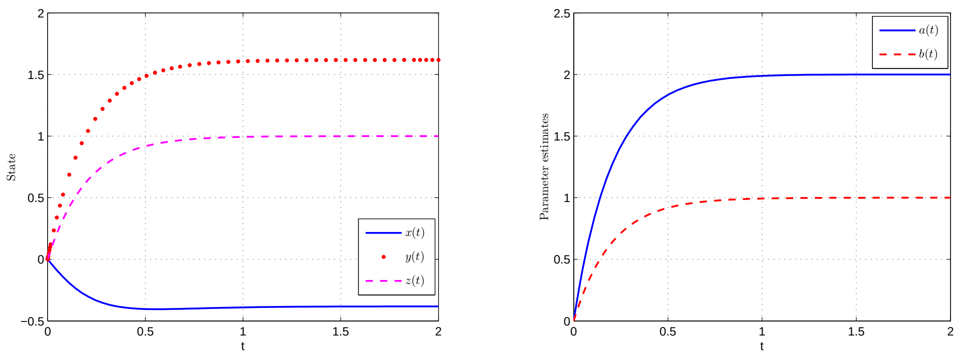

Fig. 11. Time history of the state functions and parameter estimates for equilibrium point E_1 $(a = 2, b = 1)$.

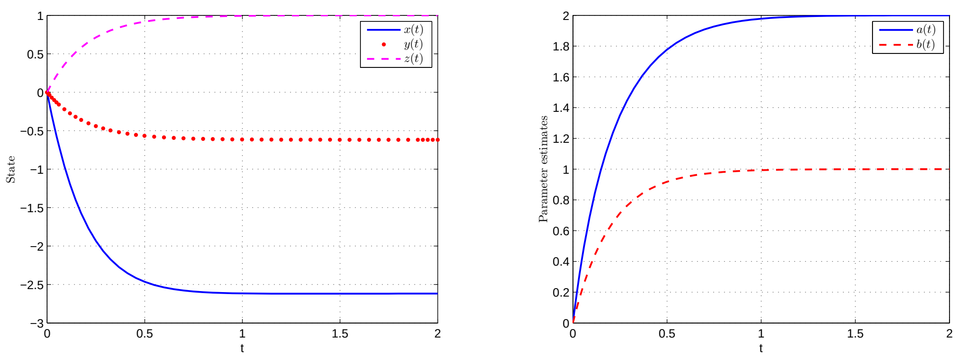

Fig. 12. Time history of the state functions and parameter estimates for equilibrium point E_2 $(a = 2, b = 1)$.

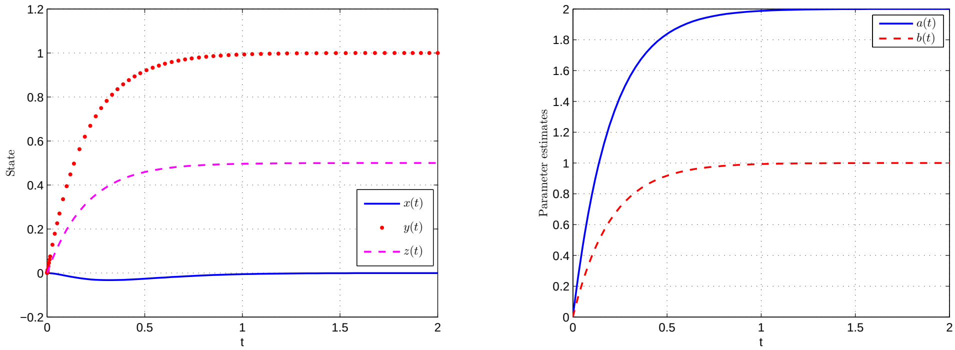

Fig. 13. Time history of the state functions and parameter estimates for equilibrium point E_3 $(a = 2, b = 1)$.

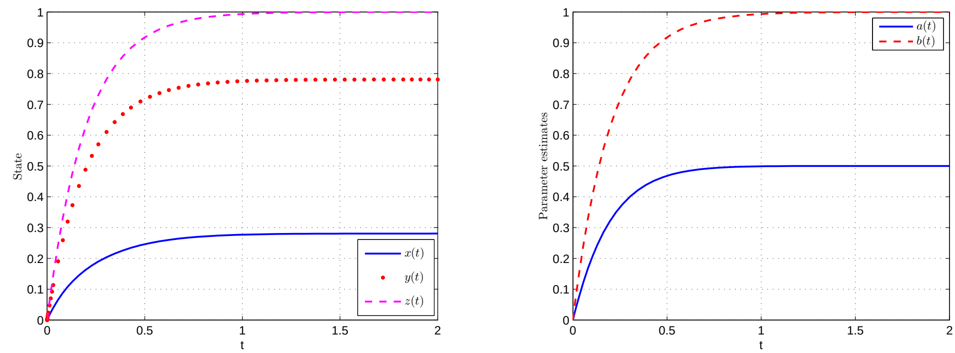

Fig. 14. Time history of the state functions and parameter estimates for equilibrium point E_1 $(a = 0.5, b = 1)$.

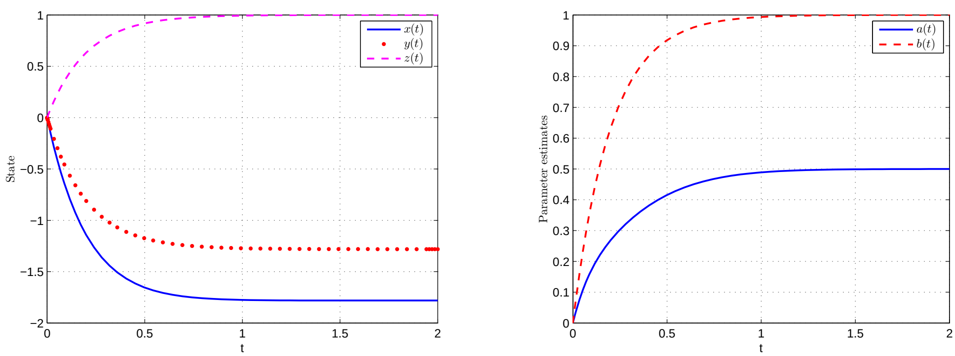

Fig. 15. Time history of the state functions and parameter estimates for equilibrium point E_2 $(a = 0.5, b = 1)$.

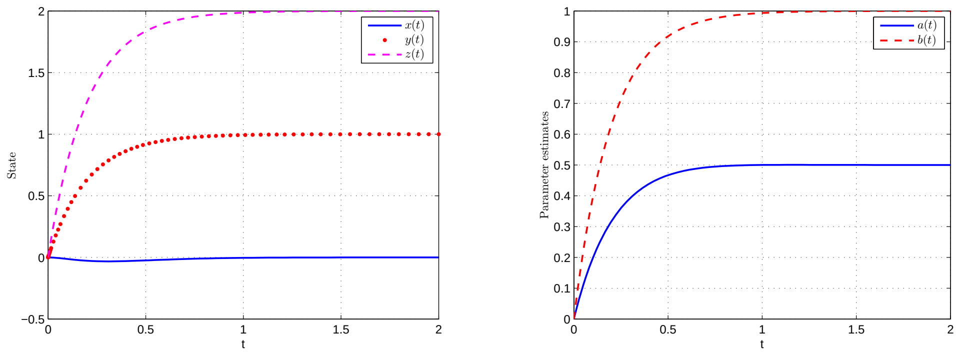

Fig. 16. Time history of the state functions and parameter estimates for equilibrium point E_3 $(a = 0.5, b = 1)$.

5. SINGLE STATE FEEDBACK STABILIZATION OF THE PEHLIVAN–UYAROGLU CHAOTIC SYSTEM

The new chaotic system is also can be stabilization by a single controller added in state y. In this section, we discuss single state feedback stabilization of the Pehlivan–Uyaroglu chaotic system. The controlled of the Pehlivan–Uyaroglu system can be written as

$$\begin{cases} \dot{x} = y - x - az, \\ \dot{y} = xz - x + u, \\ \dot{z} = -xy - y + b, \end{cases} \tag{22}$$

where u is the controller.

5.1. Designed of single state feedback controller

Let $\mathcal{O}(\bar{x}, \bar{y}, \bar{x})$ is the equilibrium of Pehlivan–Uyaroglu chaotic system (1), that is,

$$\begin{cases} 0 = -\bar{x}\bar{y} - \bar{y} + b, \\ 0 = \bar{x}\bar{z} - \bar{x}, \\ 0 = \bar{y} - \bar{x} - a\bar{z}. \end{cases} \tag{23}$$

Let us consider the following transformation:

$$\begin{cases} x_1 = z - \bar{z}, \\ x_2 = y - \bar{y}, \\ x_3 = x - \bar{x}. \end{cases} \tag{24}$$

Using this transformation, the Pehlivan–Uyaroglu chaotic system (22) can be written as follows:

$$\begin{cases} \dot{x}_1 = -(\bar{x} + 1)x_2 - \bar{y}x_3 - x_2x_3, \\ \dot{x}_2 = \bar{x}x_1 + (\bar{z} - 1)x_3 + x_1x_3 + u, \\ \dot{x}_3 = -ax_1 + x_2 - x_3. \end{cases} \tag{25}$$

Consider a output equation as follows:

$$y = x_1. \tag{26}$$

We can rewrite the system (25) – (26) with a compaction form, which is given by

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u, \\ y = h(\hat{x}) \end{cases} \tag{27}$$

where

$$\hat{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, f(\hat{x}) = \begin{bmatrix} f_1(\hat{x}) \\ f_2(\hat{x}) \\ f_3(\hat{x}) \end{bmatrix} = \begin{bmatrix} -(\bar{x}+1)x_2 - \bar{y}x_3 - x_2x_3 \\ \bar{x}x_1 + (\bar{z}-1)x_3 + x_1x_3 \\ -ax_1 + x_2 - x_3 \end{bmatrix},$$

$$g(\hat{x}) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad h(\hat{x}) = x_1.$$

By the simple calculation, we have

$$\begin{cases} L_g h(\hat{x}) = 0, \\ L_f h(\hat{x}) = -(\bar{x}+1)x_2 - \bar{y}x_3 - x_2x_3, \\ L_f^2 h(\hat{x}) = -(\bar{x}+1)[\bar{x}x_1 + (\bar{z}-1)x_3 + x_1x_3] - (\bar{y}+x_2)[-ax_1 + x_2 - x_3], \\ L_g L_f h(\hat{x}) = -(\bar{x}+1) \neq 0. \end{cases} \tag{28}$$

Then, the system (27) has a relative degree 2. Let us define the following transformation

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} h(\hat{x}) \\ L_f h(\hat{x}) \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ -(\bar{x}+1)x_2 - \bar{y}x_3 - x_2x_3 \\ x_3 \end{bmatrix}. \tag{29}$$

Therefore, the system (27) can be written as follows:

$$\begin{cases} \dot{z}_1 = z_2, \\ \dot{z}_2 = L_f^2 h(\hat{x}) + L_g L_f h(\hat{x})u, \\ \dot{z}_3 = -az_1 - \frac{z_2 + \bar{y}z_3}{(\bar{x}+1)+z_3} - z_3. \end{cases} \tag{30}$$

The zero dynamics equation of the system (30) is

$$\dot{z}_3 = -\frac{\bar{y}z_3}{(\bar{x}+1)+z_3} - z_3. \tag{31}$$

The linearization of the system (31) is

$$\dot{z}_3 = -\left(1 + \frac{\bar{y}}{(\bar{x}+1)}\right) z_3 + o(z_3). \tag{32}$$

which is asymptotically stable.

Under the action of controller

$$u = -\frac{1}{L_g L_f h(\hat{x})}(L_f^2 h(\hat{x}) + c_1 z_1 + c_2 z_2). \tag{33}$$

From (24), (33) and applying (28), it follows that

$$u = -\frac{1}{-(\bar{x}+1)}\{-(\bar{x}+1)[\bar{x}(z-\bar{z})+(\bar{z}-1)(x-\bar{x})+(z-\bar{z})(x-\bar{x})] \\ -y[-a(z-\bar{z})+(y-\bar{y})-(x-\bar{x})]+c_1(z-\bar{z}) \\ +c_2[-(\bar{x}+1)(y-\bar{y})-\bar{y}(x-\bar{x})-(y-\bar{y})(x-\bar{x})]\}. \quad (34)$$

5.2. Numerical Simulations

In this subsection, some numerical examples are given to verity the effectiveness of the proposed control scheme. We solve the controlled Pehlivan–Uyaroglu chaotic system (22) with the single controller (34). In the following numerical simulations, the MATLAB's `ode45` in-built solver is used to solve the systems. The initial values and system parameters are selected as $x(0) = 1,\ y(0) = 2,\ z(0) = -1$. Figures 17–22, show that the controlled chaotic system (22) converges to E_i $(i = 1, 2, 3)$ exponentially with time.

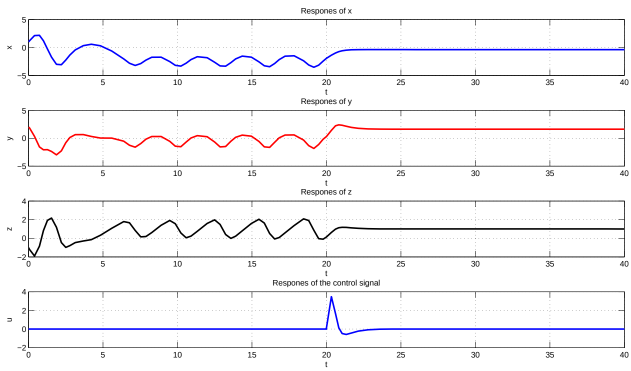

Fig. 17. The stabilized behavior of state and control functions for equilibrium point E_1 $(a = 2, b = 1)$.

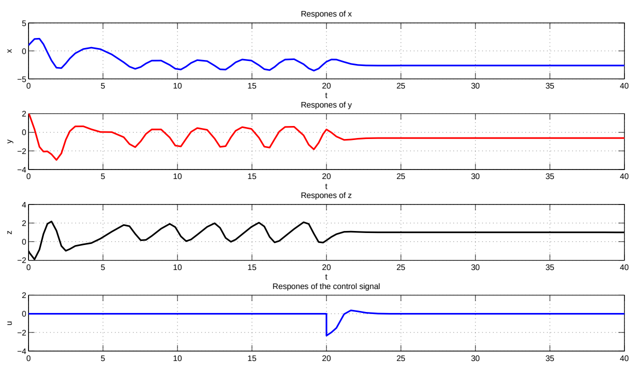

Fig. 18. The stabilized behavior of state and control functions for equilibrium point E_2 $(a = 2, b = 1)$.

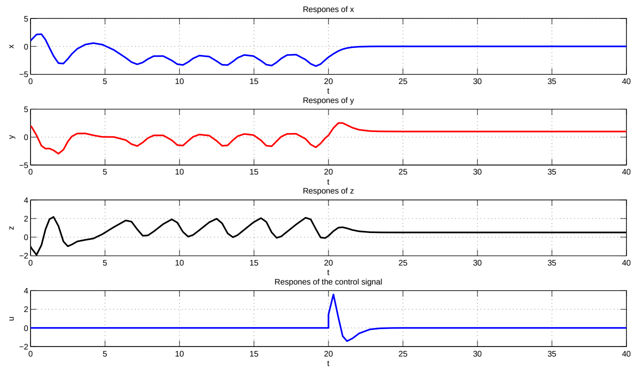

Fig. 19. The stabilized behavior of state and control functions for equilibrium point E_3 $(a = 2, b = 1)$.

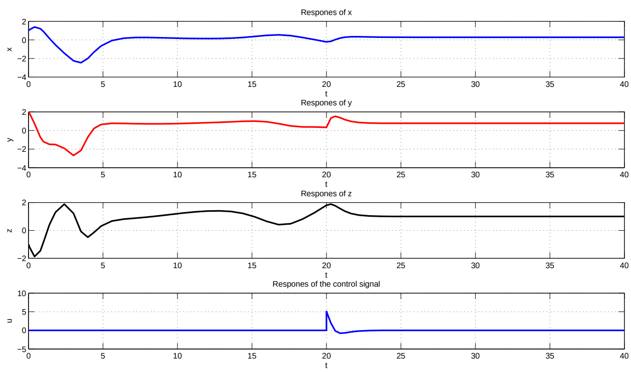

Fig. 20. The stabilized behavior of state and control functions for equilibrium point E_1 $(a = 0.5, b = 1)$.

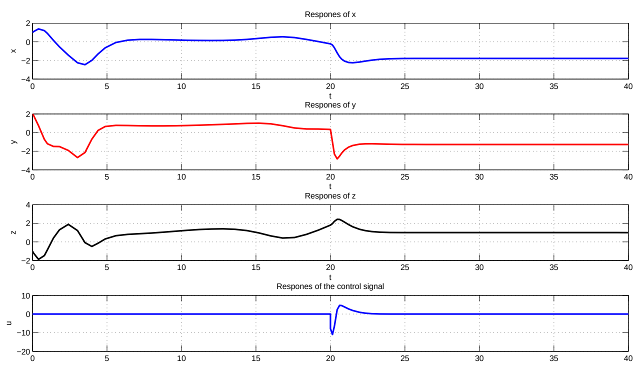

Fig. 21. The stabilized behavior of state and control functions for equilibrium point E_2 $(a = 0.5, b = 1)$.

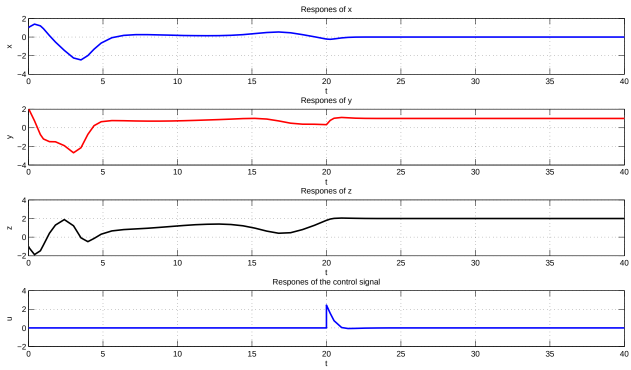

Fig. 22. The stabilized behavior of state and control functions for equilibrium point E_3 $(a = 0.5, b = 1)$.

6. CONCLUSION

We have studied the chaotic Pehlivan–Uyaroglu system. First, an optimal control law was designed for the chaotic system, based on the PMP. Then, an adaptive and feedback control law was introduced to stabilize the chaotic system with unknown parameters. Furthermore, we discuss on single state feedback stabilization of the Pehlivan–Uyaroglu system. Numerical simulations demonstrate the effectiveness of the analytical results. It is noteworthy that, the control and synchronization problem of complex networks has been a focus for many researchers in recent years [23, 24, 25, 26]. In the future, we will consider the optimal control problem of complex networks [27, 28, 29].

ACKNOWLEDGEMENT

This work was jointly supported by the Research Foundation of Department of Education of Sichuan Province (Grant Nos. 14ZA0203 and 14ZB0210), the Open Foundation of Enterprise Informatization and Internet of Things Key Laboratory of of Sichuan Province (Grant Nos. 2014WYJ01 and 2013WYY06), the Open Foundation of Artificial Intelligence Key Laboratory of Sichuan Province (Grant Nos. 2014RYY02, 2013RYJ01 and 2012RYJ01), and the National Natural Science Foundation of China (Grant No. 61203001), and Breeding Project Foundation of Sichuan University of Science & Engineering (Grant Nos. 2014PY14 and 2012KY19).

(Received July 14, 2013)

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