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REALIZABILITY OF PRECOMPENSATORS IN LINEAR MULTIVARIABLE SYSTEMS: A STRUCTURAL APPROACH

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In this work, given a linear multivariable system, the problem of static state feedback realization of dynamic compensators is considered. Necessary and sufficient conditions for the existence of a static state feedback that realizes the dynamic compensator (square or full column rank compensator) are stated in structural terms, i. e., in terms of the zero-pole structure of the compensator, and the eigenvalues and the row image of the controllability matrix of the compensated system. Based on these conditions a formula is presented to find the state feedback matrices realizing a given compensator. It is also shown that the static state feedback realizing the compensator is unique if and only if the closed-loop system is controllable.

Keywords: linear systems, feedback control, compensator realizability

Classification: 93C05, 93B52

1. INTRODUCTION

The design of a state feedback control law having the same effect from the input-output point of view than a certain dynamic precompensator is a problem often encountered in linear control theory. Indeed, in some problems where state feedback is used, a possible approach is to relate the solution of the problem to the issue of feedback realizability of compensators. This approach has been used in problems related to desirable input-output characteristics of the controlled system, such as decoupling [12], model matching [8], etc. Roughly speaking, a suitable dynamic compensator is found such that the compensated system has the required properties, and then the solution to the problem (the state feedback) is obtained by finding the conditions for the feedback realizability of the given compensator.

The key problem of this approach consists in finding conditions under which the desired compensator is feedback realizable. The problem of realizability of dynamic compensators has already been solved, in the sense that there exist necessary and sufficient conditions under which the compensator is realizable ([3, 4, 6, 9, 10]). The first solution to this problem was presented by Hautus and Heymann in [3] for nonsingular compensators, where the realizability conditions are stated in terms of a polynomial

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matrix description of the system. The general case of non-square full column rank compensators was considered by Kučera and Herrera [11], Herrera [4], and Castañeda and Ruiz-León [1], where different realizability conditions are presented. In fact, the conditions given in [10] are based on a polynomial matrix description of the system, the conditions in [4] are expressed in terms of a constant basis for the left kernel of a rational matrix, and in [1] the authors presented necessary structural conditions for the realizability of full column rank compensators. A necessary condition for the realizability of full column rank compensators in terms of Morse's list I_2 is presented in [5].

Although the problem of realizability of dynamic compensators has already been solved, there do not exist conditions in the general case relating the realizability of compensators to the structural properties of the system or the closed-loop system as such. The structural conditions are an important issue, since the analysis of several problems in linear control theory has as a starting point the structural information of the system, therefore a structural interpretation of the realizability conditions of a given precompensator would provide a deeper understanding of the solution to these problems, and it would help to find the solution to unsolved problem, for instance, non-regular decoupling and model matching problems.

In this paper, we present necessary and sufficient structural conditions under which a dynamic compensator can be realized via a static state feedback control law. In the case of non singular compensators, necessary and sufficient conditions are stated in terms of the zeros of the compensator and the row image of the controllability matrix of the system. Based on these conditions, a simple formula is derived to find the static state feedback matrices realizing a given compensator.

In the case of full column rank compensators, a non singular static state feedback is required to realize it. The difficulty of this problem arises from the fact that a non-regular state feedback control law can modify the structural properties of the system. Necessary and sufficient conditions for the realizability of full column rank compensators are stated in terms of the poles of the compensator, and the eigenvalues and the row image of the controllability matrix of the compensated system. A formula to find the required state feedback matrices realizing a given compensator is presented. It is also shown that the state feedback is unique if and only if the closed-loop system is controllable. If the compensated closed-loop system is non controllable, the set of static state feedbacks realizing the full column rank compensator is parameterized.

This paper is organized as follows. Preliminaries and relevant previous results are summarized in Section 2. The structural conditions of realizability for non-singular dynamic compensators are presented in Section 3. Realizability conditions for full column rank compensators in structural terms are given in Section 4 along with an illustrative example, and finally some conclusions are presented in Section 5.

2. PRELIMINARIES

Throughout the paper, $\mathbb{R}$ denotes the field of real numbers. Accordingly, $\mathbb{R}^n$ stands for the $n-$dimensional vector space over $\mathbb{R}$ and $\mathbb{R}^{m\times r}$ stands for the set of $m \times r$ matrices with entries in $\mathbb{R}$; $\mathbb{R}_p(s)$ denotes the set of proper rational functions over $\mathbb{R}$ and $\mathbb{R}_p^{m\times r}(s)$ denotes the set of $m \times r$ matrices with entries in $\mathbb{R}_p(s)$.

Let $\Sigma(A, B, C)$ represent a linear multivariable system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are, respectively, the state, input and output vectors of the system, and

$$T(s) = C(sI - A)^{-1}B \tag{2}$$

is the transfer matrix of the system.

Considering the static state feedback (F, G)

$$u(t) = Fx(t) + Gv(t), \tag{3}$$

where $F \in \mathbb{R}^{m\times n}$ and $G \in \mathbb{R}^{m\times r}$ are constant matrices, with rank $G = r$, then the closed-loop transfer matrix of the system is given by

$$T_{F,G}(s) = C(sI - A - BF)^{-1}BG. \tag{4}$$

If $r = m$ (matrix G is square and nonsingular), then (3) is said to be a regular state feedback, and if $r < m$ then it is said to be a nonregular state feedback.

After some algebraic manipulations, (4) can be expressed as

$$T_{F,G}(s) = T(s)W(s), \tag{5}$$

where

$$W(s) = [I - F(sI - A)^{-1}B]^{-1}G \tag{6}$$

is a proper rational matrix. In the case of regular state feedback, (6) is a biproper matrix, i.e. a proper rational matrix whose inverse is also proper rational. In the case of non-regular state feedback, (6) is column biproper, i.e.

$$\operatorname{rank} \lim_{s\to\infty} W(s) = r.$$

Thus, the action of a state feedback (F, G) on the system $\Sigma(A, B, C)$ can be represented in transfer function matrix form as the post-multiplication of the system transfer function matrix $T(s)$ by a proper rational matrix $W(s)$.

The converse problem, i.e. under which conditions a proper rational matrix that post-multiplies $T(s)$ can be realized using a state feedback law applied to the system, is known as feedback realizability of compensators. Then, a given proper matrix $C(s)$ (usually called *compensator* or *precompensator*) is said to be *feedback realizable* if there exists a static state feedback control law (F, G) such that

$$C(s) = [I - F(sI - A)^{-1}B]^{-1}G.$$

The realizability of compensators is used in some problems where desired characteristics can be given in terms of the transfer matrix of the closed-loop system. Indeed, the compensator realizability is a possible way to obtain necessary and sufficient conditions for some control problems where the state feedback is used, for example in decoupling where the closed-loop transfer function matrix is required to be diagonal, or in model matching where it is considered to match a desired transfer function matrix, etc. Then, given a compensator $C(s)$ such that the transfer matrix $T(s)C(s)$ is the solution to some of these problems, the next step is to determine under which conditions there exists a state feedback realizing the compensator $C(s)$.

Conditions for the static state feedback realizability of dynamic compensators have already been reported in the literature. The following results state the conditions for a proper compensator to be realizable, Lemma 2.1 for the case of square nonsingular compensators, and Lemma 2.2 for the case of full column rank compensators.

Lemma 2.1. (Hautus and Heymann [3]) Let the matrices $N_1(s)$ and $D(s)$ be a right coprime matrix fraction description of the system $\Sigma(A, B, I_n)$, and let $C(s) \in \mathbb{R}_p^{m\times m}(s)$ be a nonsingular compensator. Then $C(s)$ is feedback realizable if and only if

i) $C(s)$ is biproper, and

ii) $C^{-1}(s)D(s)$ is a polynomial matrix.

Lemma 2.2. (Herrera [4]) The full column rank compensator $C(s) \in \mathbb{R}_p^{m\times r}(s)$ is static state feedback realizable if and only if the left Kronecker indexes $\{\mu_i\}$ of the matrix

$$H(s) := \begin{bmatrix} (sI-A)^{-1}BC(s) \\ \bar{C}(s) \end{bmatrix}, \tag{7}$$

where $\bar{C}(s)$ is the strictly proper part of $C(s)$, satisfy the following two conditions:

a) For some integer q,

$$\begin{aligned} &\mu_1 = \cdots = \mu_m = \cdots = \mu_{m+q} = 0 \\ &0 < \mu_{m+q+1} \leq \cdots \leq \mu_{n+m-\varphi} \\ &\varphi := \text{rank } H(s). \end{aligned}$$

b) Among the rows corresponding to $\mu_1, \ldots, \mu_{m+q}$ in a minimal basis for the left kernel of $H(s)$, there exist m rows which have the form $[X \ E_m]$, where E_m is a $m \times m$ nonsingular constant matrix.

A way to obtain the required non regular static state feedback (F, G) realizing the full column rank compensator $C(s)$ is given by

$$F = -E_m^{-1}X, \quad G = \lim_{s\to\infty} C(s).$$

Note that the previous results stating necessary and sufficient conditions for the realizability of dynamic compensators are not related in any way to the structural properties of the system, the compensator or the compensated system. As we stated before, this is an important issue, since we believe such an interpretation would provide a deeper understanding of the existing solution to the problems in linear control theory approached by realization of compensators.

3. NON SINGULAR COMPENSATORS

In this section necessary and sufficient conditions in structural terms for the realizability of non singular compensators are presented.

Observe that for the realizability of a nonsingular compensator $C(s) \in \mathbb{R}_p^{m\times m}(s)$ a regular static state feedback (F,G) is used. Then, the basic problem is to find the conditions for the existence of a state feedback (F,G) such that

$$C(s) = [I - F(sI-A)^{-1}B]^{-1}G.$$

Note that if $C(s)$ is biproper, then $G = C_0 = \lim_{s\to\infty} C(s)$ is the constant part of $C(s)$, and $\bar{C}(s)$ is a strictly proper rational matrix, such that

$$C(s) = C_0 + \bar{C}(s).$$

Then, the problem remains to find the necessary and sufficient conditions (in our case, in structural terms) for the existence of the state feedback matrix F.

After some algebraic manipulations on (6), it is obtained that

$$F(sI-A)^{-1}B = \bar{C}(s)C^{-1}(s). \tag{8}$$

Observe that the feedback realization of $C(s)$ is equivalent to the existence of a constant solution F to (8).

Now, we have that [2]

$$(sI-A)^{-1} = \frac{1}{\det(sI-A)}\mathrm{Adj}(sI-A) \tag{9}$$

where

$$\mathrm{Adj}(sI-A) = [Is^{n-1} + (A+a_1I)s^{n-2} + \cdots + (A^{n-1} + a_1A^{n-2} + \cdots + a_{n-1}I)] \tag{10}$$

and

$$a(s) = \det(sI-A) = s^n + a_1s^{n-1} + \cdots + a_{n-1}s + a_n \tag{11}$$

is the characteristic polynomial of the system, and its roots are the eigenvalues of the system. Then, from (8) we have that

$$F\mathrm{Adj}(sI-A)B = a(s)\bar{C}(s)C^{-1}(s). \tag{12}$$

A first necessary condition for the realizability of the compensator $C(s)$ on the system $\Sigma(A,B,C)$ in structural terms is the following.

Theorem 3.1. Let $C(s)$ be a non singular compensator realizable on the system $\Sigma(A,B,C)$. Then all zeros of $C(s)$, given by the roots of the numerator polynomials in the Smith–McMillan form of $C(s)$, are eigenvalues of the system.

P r o o f. Assuming that there exists a constant matrix F satisfying (12), this implies that $a(s)C(s)C^{-1}(s)$ is a polynomial matrix. Using the Smith–McMillan form of $C(s)$, we have that there exist unimodular matrices $U_1(s)$ and $U_2(s)$, such that

$$C(s) = U_1(s)\,\mathrm{diag}\left\{\frac{\epsilon_i(s)}{\psi_i(s)}\right\}_{i=1}^{m} U_2(s),$$

where the finite zeros of $C(s)$ are the roots of the polynomials $\epsilon_i(s)$.

Then,

$$a(s)\overline{C}(s)C^{-1}(s) = a(s)[I - GC^{-1}(s)] = a(s)I - a(s)GU_2^{-1}(s)\,\mathrm{diag}\left\{\frac{\psi_i(s)}{\epsilon_i(s)}\right\}_{i=1}^{m} U_1^{-1}(s).$$

Since the above equation is polynomial, then $a(s)$ is divided by $\epsilon_i(s)$, which implies that all finite zeros of the compensator $C(s)$ are eigenvalues of the system. □

The previous condition relating the zeros of $C(s)$ to the eigenvalues of the system is a necessary condition for $C(s)$ to be realizable, but it can be shown that it is not a sufficient condition, i. e. all zeros of $C(s)$ being eigenvalues of the system does not imply that $C(s)$ is realizable.

It is considered next the problem of obtaining necessary and sufficient conditions for the realizability of the biproper compensator $C(s)$ in structural terms.

Denote

$$P(s) = a(s)\bar{C}(s)C^{-1}(s), \tag{13}$$

then, using (9), (13), and (8), we have that

$$F\mathrm{Adj}(sI - A)B = P(s). \tag{14}$$

It can be seen that $P(s)$ is a polynomial matrix of degree less than or equal to n (which is a necessary condition for (8) to have a constant solution F).

Considering that this is the case, $P(s)$ can be written as

$$P(s) = P_0 s^{n-1} + P_1 s^{n-2} + \cdots + P_{n-1}. \tag{15}$$

Substituting (15) in (14), it is obtained that

$$F\mathrm{Adj}(sI - A)B = P_0 s^{n-1} + P_1 s^{n-2} + \cdots + P_{n-1}, \tag{16}$$

and substituting (10) in (14), we have that

$$F\mathrm{Adj}(sI - A)B = F[Is^{n-1} + (A + a_1 I)s^{n-2} + \cdots + (A^{n-1} + a_1 A^{n-2} + \cdots + a_{n-1} I)]. \tag{17}$$

From (16) and (17), the following relationships are obtained for each power of s

$$\begin{array}{rcl}
s^{n-1} & \rightarrow & P_0 = FB \\
s^{n-2} & \rightarrow & P_1 = FAB + a_1 FB \\
s^{n-3} & \rightarrow & P_2 = FA^2B + a_1 FAB + a_2 FB \\
\vdots & \rightarrow & \vdots \\
s^0 & \rightarrow & P_{n-1} = FA^{n-1}B + \cdots + a_{n-1} FB.
\end{array}$$

From the above relations the equation (14) can be expressed using matrices with real elements as follows

$$F\Omega\Gamma = P, \tag{18}$$

where

$$\Omega = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} \tag{19}$$

is the controllability matrix of the system $\Sigma(A, B, C)$,

$$P = [P_0 \quad P_1 \quad \cdots \quad P_{n-1}], \tag{20}$$

and

$$\Gamma = \begin{bmatrix} I_m & a_1 I_m & a_2 I_m & \cdots & a_{n-1} I_m \\ 0 & I_m & a_1 I_m & \cdots & a_{n-2} I_m \\ 0 & 0 & I_m & \cdots & a_{n-3} I_m \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & I_m \end{bmatrix}. \tag{21}$$

Now, from the equation (18), it is obtained that

$$F\Omega = P\Gamma^{-1}. \tag{22}$$

Then, it can be seen that there exists a constant solution F to (8) if and only if $P\Gamma^{-1}$ belongs to the row image of the controllability matrix Ω. Thus, the following result is obtained.

Theorem 3.2. Let $\Sigma(A, B, C)$ be a linear multivariable system and let $C(s)$ be a biproper compensator. Then $C(s)$ is static state feedback realizable if and only if

i) the zeros of $C(s)$ are eigenvalues of the open-loop system, i. e. $P(s)$ given by (13) is a polynomial matrix with column degrees less than or equal to n;

ii) the matrix $P\Gamma^{-1}$ belongs to the row image of the controllability matrix Ω of the system, where P and Γ are respectively given by (20) and (21).

The previous result states structural conditions for the feedback realizability of biproper dynamic compensators. These conditions are structural in the sense that they relate the realizability of $C(s)$ to structural properties of the system, namely zeros of $C(s)$, system eigenvalues, and the row image of the controllability matrix of the system.

Structural conditions for the realizability of dynamic compensators in the case of single-input-single-output (SISO) systems are straightforward.

Remark 3.3. In the particular case of SISO systems, notice that condition ii) of Theorem 3.2 is always satisfied if the system is controllable. Then, a given biproper rational function $c(s)$ is state feedback realizable for a SISO system if and only if all zeros of $c(s)$ are eigenvalues of the system.

With respect to the static state feedback realizing $C(s)$, it is noted that if the system $\Sigma(A,B,C)$ is controllable, then its controllability matrix Ω has full row rank n, and it follows that $\Omega\Omega'$ is nonsingular, where Ω' is the transpose of Ω. Therefore, if the conditions of Theorem 3.2 are satisfied, the required static state feedback realizing $C(s)$ is derived from (18), and is given by

$$F = P\Gamma^{-1}\Omega'(\Omega\Omega')^{-1}, \quad G = C_0. \tag{23}$$

If the controllability matrix of the system $\Sigma(A,B,C)$ has rank $\rho < n$, then the static state feedback realizing $C(s)$ is not unique. The matrix F can be parameterized as

$$F = F_0 + LK, \tag{24}$$

where F_0 is a particular solution to (8), $K \in \mathbb{R}^{(n-\rho)\times n}$ is a basis for the left kernel of Ω, and L is any $m \times (n-\rho)$ constant matrix.

4. FULL COLUMN RANK COMPENSATORS

Obtaining structural necessary and sufficient conditions for the feedback realizability of full column rank compensators is more complicated than the previous considered case of nonsingular compensators. The main reason for this is that a non-regular state feedback needs to be applied to realize a full column rank compensator, and as it is well known, such a feedback can modify the structural information of the system $\Sigma(A,B,C)$, for instance, the controllability, infinite structure, etc.

Since the structural information of the system $\Sigma(A,B,C)$ is not invariant under non-regular state feedback, then the relevant structural information for the realizability of a full column rank compensator should be obtained from the compensated system, i. e. the system considering the application of a given compensator. Then, in finding a structural interpretation of the conditions for the static state feedback realizability of full column rank compensators, it will be used a controllable state space representation of the matrix $(sI-A)^{-1}BC(s)$. Certainly, if $C(s)$ is realizable via a non regular static state feedback (F,G) then, there exists a direct connection between the structure of the obtained controllable state space representation of the matrix $(sI-A)^{-1}BC(s)$ and the structure of the compensated closed-loop system $\Sigma(A+BF,BG,C)$.

Let $C(s) \in \mathbb{R}_p^{m\times r}(s)$ be a full column rank compensator and let $T(s)$ be the transfer matrix of the system $\Sigma(A,B,C)$. Then, the transfer matrix $T_C(s)$ of the *compensated system* is considered in this work as the one obtained when applying the compensator on the system, i. e.,

$$T_C(s) = T(s)C(s) = C(sI-A)^{-1}BC(s).$$

Let us first consider that there exists (F,G) such that

$$T_C(s) = T_{F,G}(s) = C(sI-A-BF)^{-1}BG,$$

this implies that the compensator $C(s)$ is feedback realizable, i. e.,

$$C(s) = [I - F(sI-A)^{-1}B]^{-1}G,$$

with

$$G = \lim_{s\to\infty} C(s) \tag{25}$$

a full column rank matrix.

After some algebraic manipulations, it is obtained that

$$C(s) - G = F(sI - A)^{-1}BC(s), \tag{26}$$

which can also be written as

$$C(s) - G = F(sI - A - BF)^{-1}BG. \tag{27}$$

Observe that a non regular static state feedback is used to realize this kind of compensator, since matrix G is non-square but of full column rank.

On the other hand, it can be seen that the static state feedback realization of the full column rank compensator $C(s)$ is equivalent to the existence of a constant solution F to (26).

Without loss of generality, let $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ be a controllable state space representation of order $q \le n$ of the matrix $(sI - A)^{-1}BC(s)$. Then, matrices $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are, respectively, of dimensions $(q \times q)$, $(q \times r)$ and $(n \times q)$.

Further, $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ is a controllable state space representation of $(sI - A)^{-1}BC(s)$, and since

$$T(s)C(s) = C(sI - A)^{-1}BC(s),$$

then a controllable state space representation for the transfer matrix of the compensated system $T(s)C(s)$ is given by $\Sigma(\mathcal{A},\mathcal{B},C_\Delta)$, where $C_\Delta = C\mathcal{C}$.

In order to provide the main results in this section, it is first introduced a result about the controllable state space representation $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ of the matrix $(sI_n - A)^{-1}BC(s)$ and some characteristics of matrix $\mathcal{C}$. The following result will be needed afterwards.

Theorem 4.1. Let $C(s)$ be a full column rank compensator realizable by static state feedback (F,G) on the system $\Sigma(A,B,C)$. Let $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ be a controllable state space representation of order $q \le n$ of the matrix $(sI_n - A)^{-1}BC(s)$. Then the following conditions hold:

i) $\mathcal{C}$ is a $n \times q$ matrix of full column rank.

ii) The columns of $\mathcal{C}$ form a basis for the range of the controllability matrix $C_{F,G}$ of the compensated closed-loop system $\Sigma(A + BF, BG, C)$.

Proof. Suppose that a particular state feedback (F,G) is applied to realize the action of the compensator $C(s)$ on the system $\Sigma(A,B,C)$. Suppose that the compensated closed-loop system $\Sigma(A + BF_0, BG, C)$ is not controllable and denote $C_{F,G}$ its controllability matrix. Let $\{v_1, v_2, \ldots, v_q\}$ be a basis for the range of $C_{F,G}$ and define the $n \times n$ similarity transformation matrix

$$T = \begin{bmatrix} v_1 & v_2 & \cdots & v_q & T_{n-q} \end{bmatrix},$$

where the $n \times (n-q)$ matrix T_{n-q} contains $n-q$ linear independent vectors chosen so that T is nonsingular. It is noted that a state space representation for the matrix $(sI-A)^{-1}BC(s)$ is given by $\Sigma(A+BF, BG, I_n)$, then T decomposes the system $\Sigma(A+BF, BG, I)$ into controllable-uncontrollable parts.

We have that

$$(sI-A)^{-1}BC(s) = (sI-A-BF)^{-1}BG.$$

When the similarity transformation T is applied to the system $\Sigma(A+BF, BG, I_n)$, it is obtained that

$$T^{-1}(A+BF)T = \begin{bmatrix} A_c & \Delta_{12} \\ 0 & \Delta_{nc} \end{bmatrix}, \quad T^{-1}BG = \begin{bmatrix} B_c \\ 0 \end{bmatrix}, \quad I_nT = \begin{bmatrix} \mathcal{V} & T_{n-q} \end{bmatrix},$$

where

$$\mathcal{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_q \end{bmatrix}. \tag{28}$$

Therefore, $\Sigma(A_c, B_c, \mathcal{V})$ is a controllable state space representation of $(sI-A)^{-1}BC(s)$.

Moreover

$$(sI-A)^{-1}BC(s) = \mathcal{C}(sI-\mathcal{A})^{-1}\mathcal{B},$$

then it holds that

$$(sI-A)^{-1}BC(s) = \mathcal{V}(sI-A_c)^{-1}B_c = \mathcal{C}(sI-\mathcal{A})^{-1}\mathcal{B}.$$

Now, there exists a nonsingular transformation matrix E, such that

$$E^{-1}A_cE = \mathcal{A}, \quad E^{-1}B_c = \mathcal{B}, \quad \mathcal{V}E = \mathcal{C}.$$

Since E is a non singular matrix, then $\mathcal{C} = \mathcal{V}E$ is a full column rank matrix, this proves i). Now, it can be seen that the span of $\mathcal{V}E$ and $\mathcal{C}$ is the same, therefore $\mathcal{C}$ is a basis for the range of $C_{F,G}$, which is ii). □

When it is used the state space representation $\Sigma(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of $(sI-A)^{-1}BC(s)$ in (26), it is obtained that

$$F\mathcal{C}(sI-\mathcal{A})^{-1}\mathcal{B} = C(s) - G. \tag{29}$$

We have that

$$(sI-\mathcal{A})^{-1} = \frac{1}{\det(sI-\mathcal{A})}\mathrm{Adj}(sI-\mathcal{A}) \tag{30}$$

where

$$\mathrm{Adj}(sI-\mathcal{A}) = [Is^{q-1} + (\mathcal{A}+\alpha_1 I)s^{q-2} + \cdots + (\mathcal{A}^{q-1} + \alpha_1\mathcal{A}^{q-2} + \cdots + \alpha_{q-1}I)] \tag{31}$$

and

$$\alpha(s) = \det(sI-\mathcal{A}) = s^q + \alpha_1 s^{q-1} + \cdots + \alpha_{q-1}s + \alpha_q. \tag{32}$$

When (30) is replaced in (29), it is obtained

$$F\mathcal{C}\left[\frac{\mathrm{Adj}(sI-\mathcal{A})\mathcal{B}}{\alpha(s)}\right] = C(s) - G,$$

and then

$$F\mathcal{C}\mathrm{Adj}(sI-\mathcal{A})\mathcal{B}=\alpha(s)[C(s)-G]. \tag{33}$$

Based on the above equations, it is first presented a necessary structural condition for the realizability of a given full column rank compensator.

Theorem 4.2. Let $C(s)\in\mathbb{R}_p^{m\times r}(s)$ be a full column rank compensator realizable on the system $\Sigma(A,B,C)$. Then the poles of $C(s)$ obtained from its Smith–McMillan form are eigenvalues of the system $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$.

Proof. It is supposed that there exists (F,G), such that

$$C(s)=[I-F(sI-A)^{-1}B]^{-1}G.$$

After some algebraic manipulations, it is obtained that

$$F(sI-A)^{-1}BC(s) \quad = \quad C(s)-G,$$

which also can be written as

$$F\mathcal{C}(sI-\mathcal{A})^{-1}\mathcal{B}=C(s)-G.$$

Now, the finite structure of $C(s)$ can be obtained from its Smith–McMillan form. Then, the poles of $C(s)$ are given by the roots of the denominator polynomials in the Smith–McMillan form [7] of $C(s)$, i.e.

$$C(s)=U_1(s)\begin{bmatrix}\mathrm{diag}\left\{\frac{\epsilon_i(s)}{\psi_i(s)}\right\}\\ 0\end{bmatrix}U_2(s)$$

where $U_1(s)$ and $U_2(s)$ are unimodular matrices. Then

$$F\mathcal{C}(sI-\mathcal{A})^{-1}\mathcal{B}=U_1(s)\begin{bmatrix}\mathrm{diag}\left\{\frac{\epsilon_i(s)}{\psi_i(s)}\right\}\\ 0\end{bmatrix}U_2(s)-G.$$

From this equation, it is finally obtained

$$F\mathcal{C}\mathrm{Adj}(sI-\mathcal{A})\mathcal{B} \quad = \quad U_1(s)\alpha(s)\begin{bmatrix}\mathrm{diag}\left\{\frac{\epsilon_i(s)}{\psi_i(s)}\right\}\\ 0\end{bmatrix}U_2(s)-\alpha(s)G,$$

where the eigenvalues of the system $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ are the roots of its characteristic polynomial $\alpha(s)=\det(sI-\mathcal{A})$. Since the left hand side of the previous equation is a polynomial matrix, so it should be the right hand side, which implies that $\psi_i(s)\mid\alpha(s)$, i.e. the poles of $C(s)$ are eigenvalues of the system $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$. □

The previous condition relating the poles of $C(s)$ to the eigenvalues of the system $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ is a necessary condition for $C(s)$ to be realizable, but it can be shown that such condition is not sufficient.

It is considered next the problem of obtaining necessary and sufficient conditions for the realizability of full column rank compensators in structural terms.

Define

$$Q(s) = \alpha(s)[C(s) - G], \tag{34}$$

then, from (33), it is obtained that

$$F\mathcal{C}\mathrm{Adj}(sI - \mathcal{A})\mathcal{B} = Q(s). \tag{35}$$

It is easily seen that $Q(s)$ is a polynomial matrix if and only if the poles of $C(s)$ are eigenvalues of the system $\Sigma(\mathcal{A}, \mathcal{B}, \mathcal{C})$ (see the proof of Theorem 4.2).

On the other hand, if the compensator $C(s)$ is static state feedback realizable, it can be seen that the column degrees of the left hand side of (35) are less than q, since $\mathrm{Adj}(sI - \mathcal{A})$ has column degrees less than q, denoted as $\deg_{ci} \mathrm{Adj}(sI - \mathcal{A}) < q$ and F, $\mathcal{C}$, and $\mathcal{B}$ are real matrices. This implies that the right hand side of (35) has column degrees less than q, i. e. $\deg_{ci} Q(s) < q$.

Certainly, a necessary condition for the static sate feedback realizability of the compensator $C(s)$ is that $Q(s)$, given in (34), is a polynomial matrix whose column degrees are less than q.

Suppose that $Q(s)$ is a polynomial matrix with $\deg_{ci} Q(s) < q$. Then $Q(s)$ can be written as

$$Q(s) = \alpha(s)[C(s) - G] = Q_0 s^{q-1} + Q_1 s^{q-2} + \cdots + Q_{q-1}. \tag{36}$$

Moreover, following a procedure similar to the one presented in Section 3, (35) can be expressed using matrices with real elements as follows

$$F\mathcal{C}\mathcal{R}\mathcal{T} = Q, \tag{37}$$

where

$$\mathcal{R} = \begin{bmatrix} \mathcal{B} & \mathcal{A}\mathcal{B} & \cdots & \mathcal{A}^{q-1}\mathcal{B} \end{bmatrix} \tag{38}$$

is the controllability matrix of the pair $(\mathcal{A}, \mathcal{B})$,

$$Q = [Q_0 \quad Q_1 \quad \cdots \quad Q_{q-1}], \tag{39}$$

and

$$\mathcal{T} = \begin{bmatrix} I_r & \alpha_1 I_r & \alpha_2 I_r & \cdots & \alpha_{q-1} I_r \\ 0 & I_r & \alpha_1 I_r & \cdots & \alpha_{q-2} I_r \\ 0 & 0 & I_r & \cdots & \alpha_{q-3} I_r \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & I_r \end{bmatrix}. \tag{40}$$

Now, from the equation (37), it is obtained that

$$F\mathcal{C}\mathcal{R} = Q\mathcal{T}^{-1}. \tag{41}$$

Note that the row span of $\mathcal{C}\mathcal{R}$ and $\mathcal{C}$ is the same. The reason for this is because $\mathcal{C}$, which is a full column rank matrix, only produces a linear combination of the rows of

$\mathcal{R}$. Therefore, it can be seen that there exists a constant solution F to (41) if and only if $Q\mathcal{T}^{-1}$ belongs to the row image of the controllability matrix $\mathcal{R}$. Thus, the following result is obtained.

Theorem 4.3. Let $\Sigma(A,B,C)$ be a linear multivariable system. Let $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ be a controllable state space representation of order q of the matrix $(sI-A)^{-1}BC(s)$, where $C(s)$ is a full column rank compensator. Then, there exists a non regular static state feedback realizing $C(s) \in \mathbb{R}_p^{m\times r}(s)$ if and only if

i) $Q(s)$, given in (36), is a polynomial matrix with $\deg_{ci} Q(s) < q$,

ii) the matrix $Q\mathcal{T}^{-1}$ belongs to the row image of the controllability matrix $\mathcal{R}$ of the pair $(\mathcal{A},\mathcal{B})$, where Q and $\mathcal{T}$ are respectively given by (39) and (40).

Note that any controllable state space representation of order $q \leq n$ of the matrix $(sI-A)^{-1}BC(s)$ can be used to verify the static state feedback realizability conditions for $C(s)$ given in Theorem 4.3. Indeed, let $\Sigma(\mathcal{A}_1,\mathcal{B}_1,\mathcal{C}_1)$ and $\Sigma(\mathcal{A}_2,\mathcal{B}_2,\mathcal{C}_2)$ be two different controllable state space representations of order $q \leq n$ of the matrix $(sI-A)^{-1}BC(s)$. Then, they are related by a non singular transformation $\mathcal{P}$, such that

$$\mathcal{P}^{-1}\mathcal{A}_1\mathcal{P} = \mathcal{A}_2, \quad \mathcal{P}^{-1}\mathcal{B}_1 = \mathcal{B}_2, \quad \mathcal{C}_1\mathcal{P} = \mathcal{C}_2.$$

Moreover, the controllability matrices

$$\mathcal{R}_1 = \begin{bmatrix} \mathcal{B}_1 & \mathcal{A}_1\mathcal{B}_1 & \cdots & \mathcal{A}_1^{q-1}\mathcal{B}_1 \end{bmatrix}$$

and

$$\mathcal{R}_2 = \begin{bmatrix} \mathcal{B}_2 & \mathcal{A}_2\mathcal{B}_2 & \cdots & \mathcal{A}_2^{q-1}\mathcal{B}_2 \end{bmatrix}$$

are also related by $\mathcal{P}$. That is

$$\mathcal{R}_1 = \mathcal{P}\mathcal{R}_2$$

which implies that the row image of $\mathcal{R}_1$ and $\mathcal{R}_2$ is the same.

4.1. A method to find a realizing static state feedback

In this subsection, for a given full column rank compensator $C(s)$ satisfying the conditions of Theorem 4.3, it is shown that the static state feedback realizing the action of $C(s)$ on the system is unique if and only if the rank of the controllability matrix $\mathcal{R}$ is n. Then, if (F,G) is unique, a formula to obtain the required static state feedback (F,G) is provided. Moreover, if such non regular static state feedback is not unique then, it is presented a result characterizing all the set of matrices (F,G) realizing the compensator $C(s)$.

According to Theorem 4.1, in the case that $q = n$ the state space representation $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ is observable and controllable, i.e. $\mathcal{C}$ is a non singular matrix. Thus, a formula to find the required static state feedback is given by

$$F = Q\mathcal{T}^{-1}\mathcal{R}'(\mathcal{R}\mathcal{R}')^{-1}\mathcal{C}^{-1}. \tag{42}$$

In the following, a result relating the uniqueness of (F,G) to the rank of the controllability matrix $\mathcal{R}$ is presented.

Theorem 4.4. Let $C(s) \in \mathbb{R}^{m\times r}(s)$ be a full column rank compensator realizable on the system $\Sigma(A, B, C)$, then the static state feedback (F, G) realizing $C(s)$ is unique if and only if the rank of the controllability matrix $\mathcal{R}$ of the system $\Sigma(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is n.

P r o o f. (Only if) If the controllability matrix $\mathcal{R}$ in the equation

$$F\mathcal{C}\mathcal{R} = Q\mathcal{T}^{-1}$$

is supposed of rank n, then $\mathcal{C}$ is a $n\times n$ nonsingular matrix. This implies that the matrix $\mathcal{C}\mathcal{R}$ does not have linearly dependent rows, therefore F is unique.

(If) If the state feedback F is supposed unique, this implies that the matrix $\mathcal{C}\mathcal{R}$ in the equation

$$F\mathcal{C}\mathcal{R} = Q\mathcal{T}^{-1}$$

does not have linearly dependent rows, therefore $\mathcal{C}$ is a $n \times n$ nonsingular matrix and $\mathcal{R}$ has full row rank n, i. e. the compensated closed-loop system is controllable. □

Consequently, if the controllability matrix $\mathcal{R}$ has rank $q < n$, then there exists an infinity of nonregular static state feedbacks equivalent to the action of the realizable full column rank compensator. The non-unique matrix F can be parametrized as

$$F = F_0 + LK,$$

where F_0 is a particular solution to (37), with $K \in \mathbb{R}^{(n-q)\times n}$ a basis for the left kernel of $\mathcal{C}$, and $L \in \mathbb{R}^{m\times(n-q)}$ is an arbitrary real matrix.

It has been proved in Theorem 4.1 that the columns of $\mathcal{C}$ form a basis for the range of the controllability matrix $C_{F,G}$ of the compensated closed-loop system $\Sigma(A + BF, BG, C)$. This implies that a basis for the left kernel of $\mathcal{C}$ is also a basis for the left kernel of $C_{F,G}$. Then, it is arrived to the following result.

Theorem 4.5. Let $C(s)$ be a full column rank compensator that is feedback realizable on the system $\Sigma(A, B, C)$ via a static state feedback (F, G) control law. Suppose that the compensated closed-loop system $\Sigma(A+BF, BG, C)$ is not controllable and let $q < n$ be the rank of its controllability matrix. Then there exists an infinity of non-regular static state feedback control laws (F, G) that realize $C(s)$ that are parameterized as

$$F = F_0 + LK,$$

where (F_0, G) is a particular static state feedback realizing $C(s)$, with $K \in \mathbb{R}^{(n-q)\times n}$ a basis for the left kernel of the compensated closed-loop system controllability matrix, and $L \in \mathbb{R}^{m\times(n-q)}$ is an arbitrary matrix.

The next example illustrates the main results of this section.

Example 4.6. Let the controllable system $\Sigma(A, B, C)$ be given by

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

with transfer matrix

$$T(s) = \begin{bmatrix} \frac{1}{s} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s^2} & \frac{1}{s^3} & 0 & 0 \end{bmatrix}.$$

Consider the following full column rank compensator

$$C(s) = \begin{bmatrix} \frac{s}{s+1} & 0 & 0 \\ -\frac{s}{s+1} & 0 & \frac{s^2}{(s+1)(s+3)} \\ 0 & 0 & 0 \\ 0 & \frac{s}{s+2} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

which produces

$$T(s)C(s) = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ 0 & \frac{1}{s+2} & 0 \\ 0 & 0 & \frac{1}{(s+3)(s+1)} \end{bmatrix},$$

resulting in a decoupled compensated system, i. e. the proposed compensator is such that in the compensated system the outputs are controlled independently [12].

Can the compensator $C(s)$ be realized by non regular static state feedback? To answer this, let us apply Theorem 4.3.

It follows that

$$(sI - A)^{-1}BC(s) = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ -\frac{1}{s(s+1)} & 0 & \frac{1}{(s+1)(s+3)} \\ -\frac{1}{s+1} & 0 & \frac{s}{(s+1)(s+3)} \\ \frac{1}{s(s+1)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{s+2} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

A controllable state space representation $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ of the rational matrix $(sI-A)^{-1}BC(s)$ is given by

$$\mathcal{A}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & 0 & \frac{35}{4} \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -\frac{3}{4} & 0 & -\frac{19}{4} \end{bmatrix}, \quad \mathcal{B}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}, \quad \mathcal{C}=\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ -1 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Observe that $q=5<n$ is the order of the state space representation $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ whose characteristic polynomial is given by

$$\alpha(s)=s\,(s+3)\,(s+2)\,(s+1)^2\,.$$

The controllability matrix $\mathcal{R}$ associated to $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ is

$$\mathcal{R}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & 0 & 4 & 0 & 0 & -8 & 0 & 0 & 16 & 0 \\ 0 & 0 & -\frac{1}{4} & 0 & 0 & 2 & 0 & 0 & -\frac{29}{4} & 0 & 0 & 23 & 0 & 0 & -\frac{281}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & -1 & 0 & 0 & \frac{13}{4} & 0 & 0 & -10 & 0 & 0 & \frac{121}{4} \end{bmatrix}.$$

It is obtained that

$$\begin{aligned} Q(s) &= a(s)[C(s)-G] \\ &= \begin{bmatrix} -s^4-6s^3-11s^2-6s & 0 & 0 \\ s^4+6s^3+11s^2+6s & 0 & -4s^4-15s^3-17s^2-6s \\ 0 & 0 & 0 \\ 0 & -2s^4-10s^3-14s^2-6s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

where

$$G=\lim_{s\to\infty} C(s)=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Note that $\deg_{ci} Q(s)<q$, therefore the condition $i)$ of Theorem 4.3 is satisfied.
Now, the matrix $Q\mathcal{T}^{-1}$ is given by

$$Q\mathcal{T}^{-1}=\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -4 & -1 & 0 & 13 & 1 & 0 & -40 & -1 & 0 & 121 & 1 & 0 & -364 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 4 & 0 & 0 & -8 & 0 & 0 & 16 & 0 & 0 & -32 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and it can be verified that it belongs to the row image of the controllability matrix $\mathcal{R}$. Then the full column rank compensator $C(s)$ is static state feedback realizable.

From (41), it is obtained that a particular static state feedback (F_0, G) realizing the compensator $C(s)$ is given by

$$F_0 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & -3 & -4 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Since the controllability matrix $\mathcal{R}$ has rank $q < n$, then there exists an infinity of non-regular static state feedbacks equivalent to the action of the realizable full column rank compensator that are parameterized as

$$F = F_0 + LK,$$

where $L \in \mathbb{R}^{5\times 3}$ is an arbitrary matrix, and

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

In [4], the method to obtain the required static state feedback (F, G) is based on a minimal basis for the left kernel of a rational matrix (see Lemma 2.2). Obtaining a static minimal basis for the kernel of a rational matrix is generally complicated. However, there exist computational programs, for instance Matlab, that quickly provides a minimal basis for a given rational matrix. When a computational tool is used to obtain it, generally the obtained basis has numerical errors, and this leads to an incorrect static state feedback.

In this section, it has been presented a new method to obtain a static state feedback (F, G) having the same effects that the realizable compensator $C(s)$ produces on the system $\Sigma(A, B, C)$. As conclusion, it can be said that an advantage of the method proposed in this work, with respect to the solution presented in [4], is the use of only real matrices to obtain the required matrices F and G, these real matrices are directly obtained from the controllability matrix and the characteristic polynomial of the compensated system. Thus, the numerical error problem is avoided, and it leads to the correct required static state feedback (F, G).

5. CONCLUSIONS

In this paper, we have presented a new solution for the realizability of dynamic compensators in structural terms. It has been shown that the problem has a solution if and only if a constant matrix belongs to the row image of the controllability matrix of the closed-loop system along with a method to find such realizing state feedback. The static state feedback is unique if and only if the closed-loop system is controllable. The row span of the controllability matrix of both, closed-loop system and open-loop system, is the same when a realizable biproper compensator is considered. The proposed realizability conditions are structural in the sense that the solution is related to the structural properties of the compensated system and compensator, such as the controllability matrix of the compensated system, poles of the compensator and eigenvalues of the compensated system.

As future work, we consider to tackle problems where non regular feedback is used, and relating the obtained conditions to the solution of some of these problems.

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