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EXPONENTIAL H_∞ FILTER DESIGN FOR STOCHASTIC MARKOVIAN JUMP SYSTEMS WITH BOTH DISCRETE AND DISTRIBUTED TIME-VARYING DELAYS

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This paper is concerned with the exponential H_∞ filter design problem for stochastic Markovian jump systems with time-varying delays, where the time-varying delays include not only discrete delays but also distributed delays. First of all, by choosing a modified Lyapunov–Krasovskii functional and employing the property of conditional mathematical expectation, a novel delay-dependent approach is developed to deal with the mean-square exponential stability problem and H_∞ control problem. Then, a mean-square exponentially stable and Markovian jump filter is designed such that the filtering error system is mean-square exponentially stable and the H_∞ performance of estimation error can be ensured. Besides, the derivative of discrete time-varying delay $h(t)$ satisfies $\dot{h}(t) \leq \eta$ and simultaneously the decay rate β can be finite positive value without equation constraint. Finally, a numerical example is provided to illustrate the effectiveness of the proposed design approach.

Keywords: stochastic systems, distributed time-varying delay, H_∞ filter, linear matrix inequality

Classification: 93E03, 93B36

1. INTRODUCTION

During the past decades, the stability analysis and synthesis of stochastic Markovian jump systems with time delays have become very important research areas and considerable attention has been paid to these two subjects. The motivation for investigating this class of systems arises from the following three aspects. The first one is that time delay, which is a primary source of instability and performance degradation in a dynamical system [11], is frequently encountered in engineering, biology, economy and other areas [12]. The second one is that in real-time systems, the signal transmission is usually a noisy process brought on by random fluctuations from probabilistic causes. Therefore, stochastic modeling has been of vital importance in many branches of science such as biology, economics and engineering applications. The abrupt phenomena such as random failures, repairs of the components, sudden environment changes and so on in practical systems is the last one. Usually, Markov chain is used to model this class of systems. In [23], the

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author explains that jump systems have been emerging as an effective framework for various control problems in different fields such as target tracking, manufacturing processes, and fault-tolerant control systems. Many interesting results on stochastic systems with time delays have been presented, e. g., [2, 5, 14, 15, 21, 22, 24, 28, 29, 38, 39, 40, 41, 42, 43] and references therein.

On the other hand, as pointed out in [18, 30], when the number of summands in a system equation is increased and the differences between neighboring argument values are decreased, the delay phenomena may not be simply considered as delays in the velocity terms and/or discrete delays in the states. This kind of delay phenomena is usually modeled as distributed time delays. Moreover, there are important practical applications for systems including distributed time delays, for example in the modeling of feeding systems and combustion chambers in a liquid monopropellant rocket motor with pressure-feeding, as documented in [6, 12, 25, 33]. Hence, the theoretical and practical demand of taking distributed time delays into account is highly desirable. Until recently, the stability analysis and synthesis problems for the systems with distributed time delays have received considerable attention. A great number of results on the above two problems have been reported. In terms of stability analysis, a discretized Lypunov functional method to study the stability of systems with distributed delays is used in [9, 10]. However, this method is complicated and is difficult to be extended to the synthesis problems. When the discrete time delays are also included, [13, 32] and [3] investigate the asymptotical stability problem and the exponential stability problem for the neutral systems with both discrete and distributed delays by using a model transformation approach, respectively. In order to further reduce the conservatism of the existed stability criteria, by constructing a modified Lyapunov–Krasovskii functional and employing free weighting matrices, [18] studies the asymptotical stability of the neutral systems with discrete and distributed delays. For stochastic systems, the exponential stability criteria are obtained for stochastic neural networks with discrete and distributed delays in [26, 44]. Taking Makrovian jump into account, in [1, 20], the authors consider the stability problem for the stochastic Markovian jump neutral networks with both discrete and distributed delays.

As far as the synthesis problem for the systems with distributed delays, the robust H_∞ control problem is considered in [31] by employing the descriptor system approach, and the H_∞ model reduction problem is addressed in [16]. The references [33, 35, 37] investigate the H_∞ filter design problem for systems with both discrete and distributed delays, in which the discrete and distributed delays are both constant time delays. Based on the above results, the H_∞ filter design approaches for systems with discrete and distributed time-varying delays are presented in [30, 36]. It can be clearly observed that the above studies focus on the deterministic systems. For stochastic systems with discrete and distributed delays, [19] and [34] consider the H_∞ control problem and the exponential output feedback controller design, respectively. In addition, for Markovian jump systems with discrete and distributed delays, a delay-dependent H_∞ control approach is developed in [27].

From the foregoing analysis, it can be seen that there are few stability analysis and synthesis results available for stochastic Markovian jump systems with both discrete and distributed time-varying delays. Moreover, when dealing with the exponential stability problem for systems with discrete and distributed time-varying delays, there is a restrictive condition, i. e., the derivative of the discrete time-varying delay is less than one [19, 20, 21, 24, 30, 34, 36, 38]. In order to remove this restriction, a weaker assumption (i. e. the derivative of time-varying delay is only bounded and not required to be less than one) is imposed, see for example [44]. However, the decay rate must satisfy one transcendental equation or inequality for obtaining exponential stability criteria [44]. To the best of the authors' knowledge, the exponential H_∞ filter design problem for stochastic Markovian jump systems with both discrete and distributed time-varying delays has not been investigated exactly.

Motivated by the above observations, in this paper, we consider the exponential H_∞ filter design problem for stochastic Markovian jump systems with both discrete and distributed time-varying delays. The purpose is to design an exponential stability H_∞ filter such that the filtering error system is mean-square exponentially stable and the L_2-induced norm from the noise signal to the estimation error is less than a prescribed level. By employing stochastic analysis technique and constructing a novel Lyap-Krasovskii functional, a new approach is proposed to obtain the delay-dependent sufficient conditions for the solvability of the H_∞ filtering problem. The decay rate is not required to satisfy one transcendental equation and simultaneously the derivative of time-varying delay is not needed to be less than one. When the sufficient condition in terms of LMIs is feasible, the explicit expression of the desired filter can be given. Finally, a numerical example is provided to verify the effectiveness of the obtained result.

2. PROBLEM FORMULATION

Consider the following stochastic Markovian jump systems with both discrete and distributed time-varying delays:

$$
\begin{aligned}
\mathrm{d}x(t) &= \Big(A_1(r_t)x(t) + A_{1d}(r_t)x(t-h(t)) + E_1(r_t)\int_{t-\tau(t)}^{t} x(s)\,\mathrm{d}s + D_1(r_t)v(t)\Big)\,\mathrm{d}t \\
&\quad + \big(B_1(r_t)x(t) + B_{1d}(r_t)x(t-h(t))\big)\,\mathrm{d}\omega(t),\ t>0 \\
\mathrm{d}y(t) &= \Big(A_2(r_t)x(t) + A_{2d}(r_t)x(t-h(t)) + E_2(r_t)\int_{t-\tau(t)}^{t} x(s)\,\mathrm{d}s + D_2(r_t)v(t)\Big)\,\mathrm{d}t \\
&\quad + \big(B_2(r_t)x(t) + B_{2d}(r_t)x(t-h(t))\big)\,\mathrm{d}\omega(t),\ t>0 \\
z(t) &= L(r_t)x(t) + L_d(r_t)x(t-h(t)),\ t>0 \\
x(t) &= \phi(t), \quad t\in[-\tau,0],
\end{aligned} \tag{1}
$$

where $x(t)\in\mathcal{R}^n$ is the state, $v(t)\in\mathcal{R}^m$ is the noise signal which is assumed to be an arbitrary signal in $\mathcal{L}_2[0,\infty)$, $\omega(t)$ is a one-dimensional Brownian motion and satisfies $\mathcal{E}\omega(t)=0, \mathcal{E}(\omega^2(t))=t$; $y(t)\in\mathcal{R}^q$ is the measurement, $z(t)\in\mathcal{R}^p$ is the signal to be estimated, $h(t)$ and $\tau(t)$ denote the time-varying discrete delay and distributed delay, respectively. They satisfy $0<h(t)\le\bar{h}, \dot{h}(t)\le\eta, 0<\tau(t)\le\bar{\tau}$, where $\bar{h},\eta,\bar{\tau}$ are constants. Let $\tau=\max\{\bar{h},\bar{\tau}\}$, $\phi(t)\in C^b_{\mathcal{F}_0}([-\tau,0];\mathcal{R}^n)$ be the initial condition. $\{r_t\}_{t\ge0}$ is a continuous-time Markov chain taking values in a finite set $\mathcal{S}=\{1,2,\dots,N\}$. Let $\Pi=\{\pi_{ij}: i,j\in\mathcal{S}\}$ be the density matrix of $\{r_t\}_{t\ge0}$. Thus, $\pi_{ij}\ge0$ for $i\ne j$ and $\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}$. Furthermore, the transition probability from mode i at time t to mode j at time $t+\triangle$ can be described as

$$P\{r_{t+\triangle}=j|r_t=i\}=\begin{cases}\pi_{ij}\triangle+o(\triangle), & i\neq j,\\ 1+\pi_{ii}\triangle+o(\triangle), & i=j,\end{cases}$$

where $\triangle>0$ and $\lim_{\triangle\to 0}(o(\triangle)/\triangle)=0$. In addition, we assume that the Markov chain $\{r_t\}_{t\geq 0}$ is independent of the Brownian motion $\omega(t)$. The matrices $A_1(r_t), A_{1d}(r_t), E_1(r_t)$, $D_1(r_t), B_1(r_t), B_{1d}(r_t), A_2(r_t), A_{2d}(r_t), E_2(r_t), D_2(r_t), B_2(r_t), B_{2d}(r_t), L(r_t)$ and $L_d(r_t)$ are known real constant matrix functions of r_t with appropriate dimensions.

In order to avoid the notations becoming too complicated, for each possible $r_t=i, i\in\mathcal{S}$, a matrix $N(r_t)$ will be denoted by N_i.

It is well known that $\{x_t, r_t\}_{t\geq 0}$ is a $C([-\tau,0];\mathcal{R}^n)\times\mathcal{S}$-valued Markov process ([24]). Its weak infinitesimal generator $\mathcal{L}$, acting on functional $V(\cdot,\cdot,\cdot):C([-\tau,0];\mathcal{R}^n)\times\mathcal{S}\times\mathcal{R}_+\to\mathcal{R}_+$, is defined by the following formula:

$$\mathcal{L}V(x_t,r_t,t)=\lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\mathcal{E}[V(x_{t+\Delta},r_{t+\Delta},t+\Delta)|x_t,r_t]-V(x_t,r_t,t)\}.$$

Now, for stochastic time-varying delay system (1) and each $i\in\mathcal{S}$, the attention of this paper is focused on the design of a mean-square exponentially stable filter with the following form:

$$\begin{cases}\mathrm{d}\hat{x}(t)=A_{fi}\hat{x}(t)\,\mathrm{d}t+B_{fi}\,\mathrm{d}y(t), & t>0\\ \hat{z}(t)=C_{fi}\hat{x}(t), \quad \hat{x}(0)=0,\end{cases}\tag{2}$$

where $\hat{x}\in\mathcal{R}^n$ and $\hat{z}\in\mathcal{R}^p$. A_{fi}, B_{fi}, C_{fi} are filter matrices with appropriate dimensions, which will be determined later.

Denote $\xi(t)=[x^T(t),\ \hat{x}^T(t)]^T$ and $e(t)=z(t)-\hat{z}(t)$. Then, from (1) and (2), for each $i\in\mathcal{S}$, we obtain the filtering error system as follows:

$$\begin{aligned}\mathrm{d}\xi(t)&=(\tilde{A}_i\xi(t)+\tilde{A}_{di}K\xi(t-h(t))+\tilde{E}_iK\int_{t-\tau(t)}^{t}\xi(s)\,\mathrm{d}s+\tilde{D}_iv(t))\,\mathrm{d}t\\ &\quad+(\tilde{B}_i\xi(t)+\tilde{B}_{di}K\xi(t-h(t))\,\mathrm{d}\omega(t),\ t>0\\ e(t)&=\tilde{L}_i\xi(t)+L_{di}K\xi(t-h(t)),\ t>0\\ \xi(t)&=\tilde{\phi}(t)=[\phi^T(t),0]^T,\ t\in[-\tau,0],\end{aligned}\tag{3}$$

where

$$\tilde{A}_i=\begin{pmatrix}A_{1i} & 0\\ B_{fi}A_{2i} & A_{fi}\end{pmatrix},\tilde{A}_{di}=\begin{pmatrix}A_{1di}\\ B_{fi}A_{2di}\end{pmatrix},\tilde{B}_i=\begin{pmatrix}B_{1i} & 0\\ B_{fi}B_{2i} & 0\end{pmatrix},K=(I\quad 0)$$

$$\tilde{B}_{di}=\begin{pmatrix}B_{1di}\\ B_{fi}B_{2di}\end{pmatrix},\tilde{E}_i=\begin{pmatrix}E_{1i}\\ B_{fi}E_{2i}\end{pmatrix},\tilde{D}_i=\begin{pmatrix}D_{1i}\\ B_{fi}D_{2i}\end{pmatrix},\tilde{L}_i=(L_i\quad -C_{fi}).$$

Based on the above discussion, the exponential H_∞ filtering problem to be addressed in this paper can be stated as follows. Given scalars $\beta>0,\gamma>0$ and the stochastic time-varying delay system (1), we will design an exponential filter in the form of (2) such that the filtering error system (3) is mean-square exponentially stable and the H_∞ performance

$$|e(t)|_{\mathcal{E}_2}<\gamma|v(t)|_2\tag{4}$$

can be guaranteed for all nonzero $v(t) \in \mathcal{L}_2[0,\infty)$ under zero-initial condition, i. e., $\xi(t) = 0$, for $t \in [-\tau, 0]$, where $|e(t)|_{\mathcal{E}_2} = \{\mathcal{E}\int_0^\infty |e(t)|^2\,\mathrm{d}t\}^{\frac{1}{2}} = \{\int_0^\infty \mathcal{E}|e(t)|^2\,\mathrm{d}t\}^{\frac{1}{2}}$, and $|\cdot|_2$ represents the usual $\mathcal{L}_2[0,\infty)$ norm, i. e., $|v(t)|_2 = \{\int_0^\infty |v(t)|^2\,\mathrm{d}t\}^{\frac{1}{2}}$.

Before investigating the main problems, we first introduce the definition of mean-square exponential stability for the filtering error system (3).

Definition 2.1. The filtering error system (3) is said to achieve mean-square exponential stability if, for $v(t) = 0$, any $\tilde{\phi}(t) \in C^b_{\mathcal{F}_0}([-\tau, 0]; \mathcal{R}^{2n})$ and initial mode $r_0 \in \mathcal{S}$, there exist constant scalars $d > 0$ and $\beta > 0$ such that:

$$\mathcal{E}\{|\xi(t, \tilde{\phi}, r_0)|^2\} \le d \sup_{-\tau\le\theta\le 0} |\tilde{\phi}(\theta)|^2 e^{-\beta t},$$

where $\xi(t, \tilde{\phi}, r_0)$ denotes the solution of system (1) at time t under the initial conditions $\tilde{\phi}(\cdot)$ and r_0, and β is called the decay rate.

3. H_∞ PERFORMANCE ANALYSIS

In this section, we first give the following lemmas that are useful in deriving the LMI-based stability criterion.

Lemma 3.1. (Gu [8]) For any constant matrix $M \in \mathbb{R}^{n\times n}, M = M^T > 0$, scalars $b > a$, and vector function $f(\cdot) : [a, b] \to \mathbb{R}^n$ such that the integrations in the following are well defined, then the following inequality holds:

$$\left\{\int_a^b f(s)\,\mathrm{d}s\right\}^T M \left\{\int_a^b f(s)\,\mathrm{d}s\right\} \le (b-a)\int_a^b f^T(s) M f(s)\,\mathrm{d}s.$$

Lemma 3.2. (Chung [4]) For any random variable $\xi \in \mathcal{F}$ satisfying $\mathcal{E}|\xi| < +\infty$ and Borel subfield $\mathcal{G} \subseteq \mathcal{F}$, there always has $\mathcal{E}(\mathcal{E}(\xi|\mathcal{G})) = \mathcal{E}\xi$.

Lemma 3.3. (Evans [17]) For any $t \in \mathcal{R}$ and $f(s) \in \mathcal{L}_2(0, t)$, we have

$$\mathcal{E}\int_0^t f(s)\,\mathrm{d}\omega(s) = 0,\ \mathcal{E}\left\{\left(\int_0^t f(s)\,\mathrm{d}\omega(s)\right)^T\left(\int_0^t f(s)\,\mathrm{d}\omega(s)\right)\right\} = \mathcal{E}\int_0^t |f(s)|^2\,\mathrm{d}s.$$

Lemma 3.4. (Gronwall [7]) Let $u(t)$ be nonnegative, continuous functions defined on $[t_0, t_1]$, and a, b denote two constants. If $u(t) \le b\int_{t_0}^t u(s)\,\mathrm{d}s + a$, then $u(t) \le ae^{b(t-t_0)}, t \in [t_0, t_1]$.

In the following, a delay-dependent and decay-rate-dependent sufficient condition which ensures the mean-square exponential stability and the H_∞ performance for the filtering error system (3) will be proposed.

Theorem 3.5. For some given scalars $\bar{h} > 0, \bar{\tau} > 0, \beta > 0$ and η, if there exist matrices $P_i > 0, Q > 0, R_1 > 0, R_2 > 0, N_{ji}$ and $M_{ji}, j = 1, 2, 3$, for each $i \in \mathcal{S}$, such that the following LMI holds:

$$\begin{pmatrix} \Omega_{1i} & \Omega_{2i} \\ \Omega_{2i}^T & \Omega_{3i} \end{pmatrix} < 0, \tag{5}$$

where

$$\Omega_{1i}=\begin{pmatrix}\Theta_{1i} & \Theta_{2i} & K^TN_{3i}^T-K^TM_{1i} & P_i\tilde{E}_i\\ * & \Theta_{3i} & -N_{3i}^T-M_{2i}+M_{3i}^T & 0\\ * & * & -M_{3i}-M_{3i}^T & 0\\ * & * & * & -\frac{1}{\bar{\tau}}R_2\end{pmatrix},$$

$\Theta_{1i}=\beta P_i+\sum_{j=1}^N\pi_{ij}P_j+P_i\tilde{A}_i+\tilde{A}_i^TP_i+K^T(Q+\frac{e^{\beta\bar{\tau}}-1}{\beta}R_2+N_{1i}+N_{1i}^T)K,$

$\Theta_{2i}=P_i\tilde{A}_{di}-K^T(N_{1i}+N_{2i}^T+M_{1i}),$

$\Theta_{3i}=[(-(1-\eta))\vee(-(1-\eta)e^{-\beta\bar{h}})]Q-N_{2i}-N_{2i}^T+M_{2i}+M_{2i}^T,$

$$\Omega_{2i}=\begin{pmatrix}-K^TN_{1i} & -K^TM_{1i} & P_i\tilde{D}_i & \tilde{A}_i^TK^TR_1 & \tilde{B}_i^TP_i & \tilde{L}_i^T\\ -N_{2i} & -M_{2i} & 0 & \tilde{A}_{di}^TK^TR_1 & \tilde{B}_{di}^TP_i & L_{di}^T\\ -N_{3i} & -M_{3i} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \tilde{E}_i^TK^TR_1 & 0 & 0\end{pmatrix},$$

$$\Omega_{3i}=\begin{pmatrix}-\frac{1}{\bar{h}}R_1 & 0 & 0 & 0 & 0 & 0\\ * & -\frac{1}{\bar{h}}R_1 & 0 & 0 & 0 & 0\\ * & * & -\gamma^2I & \tilde{D}_i^TK^TR_1 & 0 & 0\\ * & * & * & -\frac{\beta}{e^{\beta\bar{h}}-1}R_1 & 0 & 0\\ * & * & * & * & -P_i & 0\\ * & * & * & * & * & -I\end{pmatrix},$$

then the filtering error system (3) is mean-square exponentially stable with decay rate β and the H_∞ performance (4) is satisfied under zero-initial condition for all nonzero $v(t)\in\mathcal{L}_2[0,\infty)$.

P r o o f. Let

$$\varphi(t)=\tilde{A}_i\xi(t)+\tilde{A}_{di}K\xi(t-h(t))+\tilde{E}_i\int_{t-\tau(t)}^tK\xi(s)\,\mathrm{d}s,\quad \psi(t)=\tilde{B}_i\xi(t)+\tilde{B}_{di}K\xi(t-h(t)).$$

Considering the filtering error system (3) with $v(t)\equiv 0$, the first equation in (3) becomes

$$\mathrm{d}\xi(t)=\varphi(t)\,\mathrm{d}t+\psi(t)\,\mathrm{d}\omega(t),\quad t>0. \tag{6}$$

Firstly, for $t\geq\tau$, we choose a Lyapunov–Krasovskii functional candidate as follows:

$$V(\xi_t,r_t,t)=V_1(\xi_t,r_t)+e^{-\beta t}V_2(\xi_t,r_t,t)+e^{-\beta t}V_3(\xi_t,r_t,t)+e^{-\beta t}V_4(\xi_t,r_t,t), \tag{7}$$

where

$$V_1(\xi_t,r_t)=\xi^T(t)P(r_t)\xi(t),$$

$$V_2(\xi_t,r_t,t)=\int_{t-h(t)}^te^{\beta s}\xi^T(s)K^TQK\xi(s)\,\mathrm{d}s,$$

$$V_3(\xi_t,r_t,t)=\int_{-\bar{h}}^0\int_{t+\theta}^te^{\beta(s-\theta)}\varphi^T(s)K^TR_1K\varphi(s)\,\mathrm{d}s\mathrm{d}\theta,$$

$$V_4(\xi_t,r_t,t)=\int_{-\bar{\tau}}^0\int_{t+\theta}^te^{\beta(s-\theta)}\xi^T(s)K^TR_2K\xi(s)\,\mathrm{d}s\mathrm{d}\theta.$$

By Itô's formula, for each $r_t = i, i \in \mathcal{S}$, we obtain the stochastic differential of $V(\xi_t, r_t, t)$ as follows:

$$\begin{aligned} \mathrm{d}V(\xi_t, i, t) &= \mathcal{L}V(\xi_t, i, t)\,\mathrm{d}t + 2\xi^T(t)P_i\psi(t)\,\mathrm{d}\omega(t) \\ &= (\mathcal{L}V_1(\xi_t, i) + \mathcal{L}(e^{-\beta t}V_2(\xi_t, i, t)) + \mathcal{L}(e^{-\beta t}V_3(\xi_t, i, t)) \\ &\quad + \mathcal{L}(e^{-\beta t}V_4(\xi_t, i, t)))\,\mathrm{d}t + 2\xi^T(t)P_i\psi(t)\,\mathrm{d}\omega(t), \end{aligned}$$

with

$$\mathcal{L}V_1(\xi_t, i) = \sum_{j=1}^{N}\pi_{ij}\xi^T(t)P_j\xi(t) + 2\xi^T(t)P_i\varphi(t) + \psi^T(t)P_i\psi(t),$$

$$\begin{aligned} \mathcal{L}(e^{-\beta t}V_2(\xi_t, i, t)) &= \xi^T(t)K^TQK\xi(t) - (1-\dot{h}(t))e^{-\beta h(t)}\xi^T(t-h(t))K^TQK\xi(t-h(t)) \\ &\quad - \beta e^{-\beta t}V_2(\xi_t, i, t), \end{aligned}$$

$$\begin{aligned} \mathcal{L}(e^{-\beta t}V_3(\xi_t, i, t)) &= \frac{e^{\beta\bar{h}}-1}{\beta}\varphi^T(t)K^TR_1K\varphi(t) - \int_{t-\bar{h}}^{t}\varphi^T(s)K^TR_1K\varphi(s)\,\mathrm{d}s \\ &\quad - \beta e^{-\beta t}V_3(\xi_t, i, t), \end{aligned}$$

$$\begin{aligned} \mathcal{L}(e^{-\beta t}V_4(\xi_t, i, t)) &= \frac{e^{\beta\bar{\tau}}-1}{\beta}\xi^T(t)K^TR_2K\xi(t) - \int_{t-\bar{\tau}}^{t}\xi^T(s)K^TR_2K\xi(s)\,\mathrm{d}s \\ &\quad - \beta e^{-\beta t}V_4(\xi_t, i, t). \end{aligned}$$

By a simple calculation, we have

$$\begin{aligned} -(1-\dot{h}(t))e^{-\beta h(t)} &\le -(1-\eta)e^{-\beta h(t)} \\ &\le \begin{cases} -(1-\eta)e^{-\beta\bar{h}}, & \eta \le 1, \\ -(1-\eta), & \eta > 1, \end{cases} \\ &= [(-(1-\eta)) \vee (-(1-\eta)e^{-\beta\bar{h}})]. \end{aligned} \tag{8}$$

So, we obtain

$$\begin{aligned} \mathcal{L}V(\xi_t, i, t) &\le \sum_{j=1}^{N}\pi_{ij}\xi^T(t)P_j\xi(t) + 2\xi^T(t)P_i\varphi(t) + \psi^T(t)P_i\psi(t) + \xi^T(t)K^TQK\xi(t) \\ &\quad + [(-(1-\eta)) \vee (-(1-\eta)e^{-\beta\bar{h}})]\xi^T(t-h(t))K^TQK\xi(t-h(t)) \\ &\quad + \frac{e^{\beta\bar{h}}-1}{\beta}\varphi(t)^TK^TR_1K\varphi(t) - \int_{t-\bar{h}}^{t}\varphi(s)^TK^TR_1K\varphi(s)\,\mathrm{d}s \\ &\quad + \frac{e^{\beta\bar{\tau}}-1}{\beta}\xi^T(t)K^TR_2K\xi(t) - \int_{t-\bar{\tau}}^{t}\xi^T(s)K^TR_2K\xi(s)\,\mathrm{d}s \\ &\quad - \beta e^{-\beta t}V_2(\xi_t, i, t) - \beta e^{-\beta t}V_3(\xi_t, i, t) - \beta e^{-\beta t}V_4(\xi_t, i, t). \end{aligned} \tag{9}$$

It follows from Lemma 3.1 that

$$\begin{aligned} -\int_{t-\bar{h}}^{t}\varphi^T(s)K^TR_1K\varphi(s)\,\mathrm{d}s &\le -\frac{1}{\bar{h}}\left\{\int_{t-\bar{h}}^{t-h(t)}K\varphi(s)\,\mathrm{d}s\right\}^T R_1\left\{\int_{t-\bar{h}}^{t-h(t)}K\varphi(s)\,\mathrm{d}s\right\} \\ &\quad - \frac{1}{\bar{h}}\left\{\int_{t-h(t)}^{t}K\varphi(s)\,\mathrm{d}s\right\}^T R_1\left\{\int_{t-h(t)}^{t}K\varphi(s)\,\mathrm{d}s\right\}. \end{aligned} \tag{10}$$

Similarly, we also have

$$
\begin{aligned}
-\int_{t-\bar{\tau}}^{t}\xi^T(s)K^TR_2K\xi(s)\,\mathrm{d}s \quad &\le -\frac{1}{\bar{\tau}}\left\{\int_{t-\bar{\tau}}^{t-\tau(t)}K\xi(s)\,\mathrm{d}s\right\}^T R_2\left\{\int_{t-\bar{\tau}}^{t-\tau(t)}K\xi(s)\,\mathrm{d}s\right\}\\
&\quad -\frac{1}{\bar{\tau}}\left\{\int_{t-\tau(t)}^{t}K\xi(s)\,\mathrm{d}s\right\}^T R_2\left\{\int_{t-\tau(t)}^{t}K\xi(s)\,\mathrm{d}s\right\}.
\end{aligned}
\tag{11}
$$

From (6) and the Newton–Leibniz formula, one has

$$
\begin{aligned}
&(\xi^T(t)K^TN_{1i}+\xi^T(t-h(t))K^TN_{2i}+\xi^T(t-\bar{h})K^TN_{3i})K(\xi(t)\\
&-\xi(t-h(t))-\int_{t-h(t)}^{t}\varphi(s)\,\mathrm{d}s-\int_{t-h(t)}^{t}\psi(s)\,\mathrm{d}\omega(s)=0,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&(\xi^T(t)K^TM_{1i}+\xi^T(t-h(t))K^TM_{2i}+\xi^T(t-\bar{h})K^TM_{3i})K(\xi(t-h(t))\\
&-\xi(t-\bar{h})-\int_{t-\bar{h}}^{t-h(t)}\varphi(s)\,\mathrm{d}s-\int_{t-\bar{h}}^{t-h(t)}\psi(s)\,\mathrm{d}\omega(s))=0.
\end{aligned}
\tag{13}
$$

Substituting (10) – (13) into (9) yields

$$
\begin{aligned}
&\mathcal{L}V(\xi_t,i,t)+\beta V(\xi_t,i,t)\le\zeta^T(t)\Gamma_i\zeta(t)-\tfrac{1}{\bar{\tau}}\left\{\int_{t-\bar{\tau}}^{t-\tau(t)}K\xi(s)\,\mathrm{d}s\right\}^T R_2\left\{\int_{t-\bar{\tau}}^{t-\tau(t)}K\xi(s)\,\mathrm{d}s\right\}\\
&-2(\xi^T(t)K^TN_{1i}+\xi^T(t-h(t))K^TN_{2i}+\xi^T(t-\bar{h})K^TN_{3i})K\int_{t-h(t)}^{t}\psi(s)\,\mathrm{d}\omega(s)\\
&-2(\xi^T(t)K^TM_{1i}+\xi^T(t-h(t))K^TM_{2i}+\xi^T(t-\bar{h})K^TM_{3i})K\int_{t-\bar{h}}^{t-h(t)}\psi(s)\,\mathrm{d}\omega(s),
\end{aligned}
\tag{14}
$$

where

$$
\zeta(t)=\Big[\xi^T(t),(K\xi(t-h(t)))^T,(K\xi(t-\bar{h}))^T,\Big(\int_{t-\tau(t)}^{t}K\xi(s)\,\mathrm{d}s\Big)^T,\Big(\int_{t-h(t)}^{t}K\varphi(s)\,\mathrm{d}s\Big)^T,\left(\int_{t-\bar{h}}^{t-h(t)}K\varphi(s)\,\mathrm{d}s\right)^T\Big]^T,
$$

$$
\Gamma_i=\begin{pmatrix}
\tilde{\Theta}_{1i} & \tilde{\Theta}_{2i} & K^T(N_{3i}^T-M_{1i}) & P_i\tilde{E}_i+\frac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_i^TK^TR_1K\tilde{E}_i & -K^TN_{1i} & -K^TM_{1i}\\
* & \tilde{\Theta}_{3i} & -N_{3i}^T-M_{2i}+M_{3i}^T & \frac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_{di}^TK^TR_1K\tilde{E}_i & -N_{2i} & -M_{2i}\\
* & * & -M_{3i}-M_{3i}^T & 0 & -N_{3i} & -M_{3i}\\
* & * & * & \frac{e^{\beta\bar{h}}-1}{\beta}\tilde{E}_i^TK^TR_1K\tilde{E}_i & -\frac{1}{\bar{\tau}}R_2 & 0\\
* & * & * & * & -\frac{1}{\bar{h}}R_1 & 0\\
* & * & * & * & * & \frac{1}{\bar{h}}R_1
\end{pmatrix},
$$

$$
\begin{aligned}
\tilde{\Theta}_{1i}&=\Theta_{1i}+\tilde{B}_i^TP_i\tilde{B}_i+\frac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_i^TK^TR_1K\tilde{A}_i,\\
\tilde{\Theta}_{2i}&=\Theta_{2i}+\tilde{B}_i^TP_i\tilde{B}_{di}+\frac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_i^TK^TR_1K\tilde{A}_{di},\\
\tilde{\Theta}_{3i}&=\Theta_{3i}+\tilde{B}_{di}^TP_i\tilde{B}_{di}+\frac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_{di}^TK^TR_1K\tilde{A}_{di}.
\end{aligned}
$$

Clearly, it follows from (5) and Schur complement that $\Gamma_i < 0$. Hence, we have $\mathcal{E}(\mathcal{L}V(\xi_t, i, t)) + \beta\mathcal{E}V(\xi_t, i, t) < 0$, for each $i \in \mathcal{S}$. Then, from the definition of weak infinitesimal generator $\mathcal{L}$ and Lemma 3.2, it is not difficult to verify that

$$\begin{aligned}\mathcal{E}(\mathcal{L}V(\xi_t, r_t, t)) &= \lim_{\Delta\to 0^+}\frac{1}{\Delta}\mathcal{E}\{\mathcal{E}[V(\xi_{t+\Delta}, r_{t+\Delta}, t+\Delta)|\xi_t, r_t] - V(\xi_t, r_t, t)\}\\ &= \lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\mathcal{E}V(\xi_{t+\Delta}, r_{t+\Delta}, t+\Delta) - \mathcal{E}V(\xi_t, r_t, t)\}\\ &= \frac{\mathrm{d}\mathcal{E}(V(\xi_t, r_t, t))}{\mathrm{d}t}.\end{aligned}$$

Consequently, we obtain

$$\frac{\mathrm{d}\mathcal{E}V(\xi_t, r_t, t)}{\mathcal{E}V(\xi_t, r_t, t)} + \beta\mathrm{d}t < 0. \tag{15}$$

By integrating (15) from τ to t, $t > \tau$, it can be testified that

$$\ln\frac{\mathcal{E}V(\xi_t, r_t, t)}{\mathcal{E}V(\xi_\tau, r_\tau, \tau)} + \beta(t-\tau) < 0. \tag{16}$$

In addition, by taking exponent of both sides of (16), we obtain

$$\begin{aligned}\mathcal{E}V(\xi_t, r_t, t) &\le \mathcal{E}V(\xi_\tau, r_\tau, \tau)e^{-\beta(t-\tau)}\\ &= \mathcal{E}\{\xi^T(\tau)P(r_\tau)\xi(\tau) + e^{-\beta\tau}\int_{\tau-h(\tau)}^{\tau} e^{\beta s}\xi^T(s)K^TQK\xi(s)\,\mathrm{d}s\\ &\quad + e^{-\beta\tau}\int_{-\bar{h}}^{0}\int_{\tau+\theta}^{\tau} e^{\beta(s-\theta)}\varphi^T(s)K^TR_1K\varphi(s)\,\mathrm{d}s\mathrm{d}\theta\\ &\quad + e^{-\beta\tau}\int_{-\bar{\tau}}^{0}\int_{\tau+\theta}^{\tau} e^{\beta(s-\theta)}\xi(s)K^TR_2K\xi(s)\,\mathrm{d}s\mathrm{d}\theta\}e^{-\beta(t-\tau)}.\end{aligned} \tag{17}$$

Let

$$\mu_1 = \max_{i\in\mathcal{S}}\{\|\tilde{A}_i\|\},\quad \mu_2 = \max_{i\in\mathcal{S}}\{\|\tilde{B}_i\|\},\quad \mu_3 = \max_{i\in\mathcal{S}}\{\|\tilde{A}_{di}K\|\},$$

$$\mu_4 = \max_{i\in\mathcal{S}}\{\|\tilde{B}_{di}K\|\},\quad \mu_5 = \max_{i\in\mathcal{S}}\{\|\tilde{E}_iK\|\}.$$

From Lemma 3.3 and (6), for $0 < t \le \tau$, it is easy to check that

$$\begin{aligned}&\mathcal{E}|\xi(t)|^2 = \mathcal{E}\left|\xi(0) + \int_0^t \varphi(s)\,\mathrm{d}s + \int_0^t \psi(s)\,\mathrm{d}\omega(s)\right|^2\\ &\le 2\sup_{-\tau\le\theta\le 0}\left|\phi(\theta)|^2 + 3\tau\mathcal{E}\int_0^t (|\tilde{A}(r_s)\xi(s)| + |\tilde{A}_d(r_s)K\xi(s-h(s))\right|\\ &+|\tilde{E}(r_s)K\int_{s-\tau(s)}^{s}\xi(\theta)\,\mathrm{d}\theta|)^2\,\mathrm{d}s + 2\mathcal{E}\int_0^t (|\tilde{B}(r_s)\xi(s)| + |\tilde{B}_d(r_s)K\xi(s-h(s))|)^2\,\mathrm{d}s.\end{aligned} \tag{18}$$

Notice that

$$\int_0^t \left|\tilde{E}(r_s)K\int_{s-\tau(s)}^{s}\xi(\theta)\,\mathrm{d}\theta\right|^2\,\mathrm{d}s \quad \le \mu_5^2\tau\int_{-\tau}^{t}|\xi(\theta)|^2\,\mathrm{d}\theta.$$

Then, combined with (18), we have

$$\mathcal{E}|\xi(t)|^2 \le 2(1+3\tau^2\mu_3^2+2\tau\mu_4^2)\sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2 + 2\int_{-\tau}^{t}(3\tau\mu_1^2+3\tau^2\mu_5^2+2\mu_2^2)\mathcal{E}|\xi(s)|^2\,\mathrm{d}s. \tag{19}$$

Using Lemma 3.4, for $t \le \tau$, it follows from (19) that

$$\mathcal{E}|\xi(t)|^2 \le d_1 \sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2, \tag{20}$$

where $d_1 = 2(1+3\tau^2\mu_3^2+2\tau\mu_4^2)exp\{4\tau(3\tau\mu_1^2+3\tau^2\mu_5^2+2\mu_2^2)\}$.
Consequently, from (17) and (20), we have

$$\begin{aligned}\mathcal{E}V(\xi_t,r_t,t) < e^{\beta\tau}\Big\{&\max_{i\in\mathcal{S}}\{\|P_i\|\}d_1 + d_1\|K^TQK\|\frac{1-e^{-\beta\tau}}{\beta} + 2(\mu_1^2d_1+\mu_3^2+\mu_5^2\tau d_1)\\ &\times\|K^TR_1K\|\tfrac{e^{\beta\tau}-1-\tau\beta}{\beta^2} + 2(\mu_2^2d_1+\mu_4^2)\|K^TR_2K\|\frac{e^{\beta\tau}-1-\tau\beta}{\beta^2}\Big\}\sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2e^{-\beta t}.\end{aligned} \tag{21}$$

Notice that

$$\mathcal{E}V(\xi_t,r_t,t) \ge \frac{1}{\max_{i\in\mathcal{S}}\{\|P_i^{-1}\|\}}\mathcal{E}|\xi(t)|^2.$$

This, together with (21) implies for $t > \tau$

$$\mathcal{E}|\xi(t)|^2 \le \max_{i\in\mathcal{S}}\{\|P_i^{-1}\|\}e^{\beta\tau}d_2\sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2e^{-\beta t}, \tag{22}$$

where $d_2 = \max_{i\in\mathcal{S}}\{\|P_i\|\}d_1 + d_1\|K^TQK\|\frac{1-e^{-\beta\tau}}{\beta} + 2(\mu_1^2d_1+\mu_3^2+\mu_5^2\tau d_1)\|K^TR_1K\|\frac{e^{\beta\tau}-1-\tau\beta}{\beta^2}$
$+\, 2(\mu_2^2d_1+\mu_4^2)\|K^TR_2K\|\frac{e^{\beta\tau}-1-\tau\beta}{\beta^2}$.
On the other hand, for $0 < t \le \tau$, there has

$$e^{\beta t}\mathcal{E}|\xi(t)|^2 \le e^{\beta\tau}d_1\sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2.$$

It implies

$$\mathcal{E}|\xi(t)|^2 \le e^{-\beta t}e^{\beta\tau}d_1\sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2. \tag{23}$$

Evidently, for $t > 0$, taking into account (22) and (23), the following estimate holds

$$\mathcal{E}|\xi(t)|^2 \le e^{-\beta t}d\sup_{-\tau\le\theta\le 0}|\phi(\theta)|^2,$$

where $d = \max\{e^{\beta\tau}\max_{i\in\mathcal{S}}\{\|P_i^{-1}\|\}d_2, e^{\beta\tau}d_1\}$.

Then, according to Definition 2.1, the filtering error system (3) is mean-square exponentially stable with decay rate β.

Next, under the zero initial condition, i. e., $\tilde{\phi}(t) = 0$, we will prove the H_∞ performance of the filtering error system (3) is satisfied for all nonzero $v(t) \in \mathcal{L}_2[0,\infty)$. We choose another Lyapunov–Krasovskii functional candidate as follows:

$$\hat{V}(\xi_t,r_t,t) = V_1(\xi_t,r_t) + e^{-\beta t}V_2(\xi_t,r_t,t) + e^{-\beta t}\hat{V}_3(\xi_t,r_t,t) + e^{-\beta t}V_4(\xi_t,r_t,t),$$

where V_1, V_2, V_4 are defined in the line below (7),

$$\hat{V}_3(\xi_t, r_t, t) = \int_{-\bar{h}}^{0}\int_{t+\theta}^{t} e^{\beta(s-\theta)}\hat{\varphi}^T(s)K^T R_1 K\hat{\varphi}(s)\,\mathrm{d}s\mathrm{d}\theta$$

and

$$\hat{\varphi}(t) = \begin{cases} \varphi(t) + \tilde{D}(r_t)v(t) & t > 0, \\ 0 & -\tau \le t \le 0. \end{cases}$$

For any $T > 0$, let

$$J(T) = \mathcal{E}\int_0^T (e^T(t)e(t) - \gamma^2 v^T(t)v(t))\,\mathrm{d}t. \tag{24}$$

For $t \in [-\tau, 0)$, we let $v(t) = 0$. Similar to the proof of (14), using *Itô*'s formula for $r_t = i, i \in \mathcal{S}$, we have

$$\begin{aligned}
&\mathcal{L}\hat{V}(\xi_t, i, t) + \beta\hat{V}(\xi_t, i, t) \le \hat{\zeta}^T(t)\hat{\Gamma}_i\hat{\zeta}(t) - \frac{1}{\bar{\tau}}\left\{\int_{t-\bar{\tau}}^{t-\tau(t)} K\xi(s)\,\mathrm{d}s\right\}^T R_2\left\{\int_{t-\bar{\tau}}^{t-\tau(t)} K\xi(s)\,\mathrm{d}s\right\}\\
&-2(\xi^T(t)K^T N_{1i} + \xi^T(t-h(t))K^T N_{2i} + \xi^T(t-\bar{h})K^T N_{3i})K\int_{t-h(t)}^{t}\psi(s)\,\mathrm{d}\omega(s)\\
&-2(\xi^T(t)K^T M_{1i} + \xi^T(t-h(t))K^T M_{2i} + \xi^T(t-\bar{h})K^T M_{3i})K\int_{t-\bar{h}}^{t-h(t)}\psi(s)\,\mathrm{d}\omega(s),
\end{aligned}$$

where

$$\begin{aligned}
\hat{\zeta}(t) = [&\xi^T(t), (K\xi(t-h(t)))^T, (K\xi(t-\bar{h}))^T, (\textstyle\int_{t-\tau(t)}^{t} K\xi(s)\,\mathrm{d}s)^T, (\int_{t-h(t)}^{t} K\hat{\varphi}(s)\,\mathrm{d}s)^T,\\
&(\textstyle\int_{t-\bar{h}}^{t-h(t)} K\hat{\varphi}(s)\,\mathrm{d}s)^T, v^T(t)]^T,
\end{aligned}$$

$$\hat{\Gamma}_i = \begin{pmatrix} \Gamma_i & \Psi_i \\ \Psi_i^T & \frac{e^{\beta\bar{h}}-1}{\beta}\tilde{D}_i^T K^T R_1 K\tilde{D}_i \end{pmatrix},$$

$$\begin{aligned}
\Psi_i = \Big[&(P_i\tilde{D}_i + \tfrac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_i^T K^T R_1 K\tilde{D}_i)^T, (\tfrac{e^{\beta\bar{h}}-1}{\beta}\tilde{A}_{di}^T K^T R_1 K\tilde{D}_i)^T, 0,\\
&(\tfrac{e^{\beta\bar{h}}-1}{\beta}\tilde{E}_i^T K^T R_1 K\tilde{D}_i)^T, 0, 0, (\tfrac{e^{\beta\bar{h}}-1}{\beta}\tilde{D}_i^T K^T R_1 K\tilde{D}_i)^T\Big]^T.
\end{aligned}$$

Then, it can be deduced from (24) that

$$\begin{aligned}
J(T) = \mathcal{E}&\int_0^T (e^T(t)e(t) - \gamma^2 v^T(t)v(t))\,\mathrm{d}t + \mathcal{E}\int_0^T \mathcal{L}\hat{V}(\xi_t, r_t, t)\,\mathrm{d}t\\
&- \mathcal{E}\hat{V}(\xi_T, r_T, T) + \mathcal{E}\hat{V}(\xi_0, r_0, 0).
\end{aligned}$$

Note that $\mathcal{E}\hat{V}(\xi_T, r_T, T) \ge 0$ and $\mathcal{E}\hat{V}(\xi_0, r_0, 0) = 0$. This implies for $r_t = i, i \in \mathcal{S}$

$$\begin{aligned}
&J(T) \le \mathcal{E}\int_0^T (e^T(t)e(t) - \gamma^2 v^T(t)v(t))\,\mathrm{d}t + \mathcal{E}\int_0^T \mathcal{L}\hat{V}(\xi_t, r_t, t)\,\mathrm{d}t\\
&\le \mathcal{E}\int_0^T (e^T(t)e(t) - \gamma^2 v^T(t)v(t))\,\mathrm{d}t + \mathcal{E}\int_0^T \mathcal{L}\hat{V}(\xi_t, r_t, t)\,\mathrm{d}t + \mathcal{E}\int_0^T \beta\hat{V}(\xi_t, r_t, t)\,\mathrm{d}t\\
&= \mathcal{E}\int_0^T \hat{\zeta}^T(t)\Lambda(r_t)\hat{\zeta}(t)\,\mathrm{d}t,
\end{aligned}$$

where, for $r_t = i$,

$$\Lambda_i = \hat{\Gamma}_i + \begin{pmatrix} \tilde{L}_i^T \tilde{L}_i & \tilde{L}_i^T L_{di} & 0 & 0 & 0 & 0 & 0 \\ * & L_{di}^T L_{di} & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{pmatrix}.$$

Using (5) and Schur complement, we have $\Lambda_i < 0$. Hence, it follows that $|e(t)|_{\mathcal{E}_2} < \gamma |v(t)|_2$. □

Remark 3.6. During the proof of Theorem 3.5, in order to prove the mean-quare exponential stability, a novel Lyapunov–Krasovskii functional is constructed. Then, combining with the property of conditional mathematical expectation and the stochastic analysis technique, a new approach to achieving the delay-dependent mean-square exponential stability criterion is presented. This approach makes use of the knowledge of stochastic process sufficiently and is different from the approach proposed in [21], where the mean-square exponential stability problem is also solved but the property of conditional mathematical expectation has not been used. On the other hand, we also consider the exponential H_∞ filter design problem in [22]. However, Ref. [22], in which the measurement missing phenomenon is taken into account, studies the exponential H_∞ filter design problem for discrete stochastic systems, while this paper studies the exponential H_∞ filter design problem for continuous case.

Remark 3.7. To the best of the authors' knowledge, the decay rate β in many literatures is a fixed value computed by solving a transcendental equation or inequality, for example [1, 24, 29, 38, 44]. Moreover, these results can not tell us whether a system can possess a larger decay rate than the computed value. Here, the decay rate β in Theorem 1 is a constant value without the above-mentioned constraint. This will introduce more flexibility in the analysis of systems. In addition, the LMIs-based condition (5) is not only delay-dependent but also decay-rate-dependent. The delay-dependence of the sufficient condition makes the result less conservative, while the decay-rate-dependence enables that a suboptimal upper bound of the decay rate β can be computed by convex optimization algorithms. At the same time, by using the inequality (8), the restriction on η, that is, $\eta < 1$, has been removed and a more general result is obtained.

4. EXPONENTIAL H_∞ FILTER DESIGN

In this section, based on Theorem 3.5, we aim to design the filter matrices A_{fi}, B_{fi} and C_{fi} for each $i \in \mathcal{S}$.

Theorem 4.1. For given constants $\beta > 0, \gamma > 0, \eta > 0, \bar{\tau} > 0$ and $\bar{h} > 0$, the H_∞ filtering problem is solvable if there exist matrices $Q > 0, R_1 > 0, R_2 > 0, X_i > 0, Y_i > 0, V_i, Z_i, W_i$ and any appropriately dimensional matrices $M_{ji}, N_{ji}, j = 1, 2, 3$ for each

$i \in \mathcal{S}$, such that the following LMI holds:

$$\begin{pmatrix} \tilde{\Omega}_{1i} & \tilde{\Omega}_{2i} \\ \tilde{\Omega}_{2i}^T & \tilde{\Omega}_{3i} \end{pmatrix} < 0, \tag{25}$$

where

$$\tilde{\Omega}_{1i} = \begin{pmatrix} \tilde{\Theta}_{11i} & \tilde{\Theta}_{12i} & \tilde{\Theta}_{21i} & N_{3i}^T - M_{1i} & Y_i E_{1i} \\ * & \tilde{\Theta}_{13i} & \tilde{\Theta}_{22i} & 0 & (Y_i - X_i)E_{1i} + V_i E_{2i} \\ * & * & \Theta_{3i} & -N_{3i}^T - M_{2i} + M_{3i}^T & 0 \\ * & * & * & -M_{3i} - M_{3i}^T & 0 \\ * & * & * & * & -\frac{1}{\bar{\tau}} R_2 \end{pmatrix},$$

$\tilde{\Theta}_{11i} = \sum_{j=1}^N \pi_{ij} Y_j + \beta Y_i + Y_i A_{1i} + A_{1i}^T Y_i + \frac{e^{\beta\bar{\tau}} - 1}{\beta} R_2 + Q + N_{1i} + N_{1i}^T,$

$\tilde{\Theta}_{12i} = A_{1i}^T (Y_i - X_i) + A_{2i}^T V_i^T + Z_i^T,$

$\tilde{\Theta}_{13i} = \sum_{j=1}^N \pi_{ij} (X_j - Y_j) + \beta (X_i - Y_i) + Z_i + Z_i^T,$

$\tilde{\Theta}_{21i} = Y_i A_{1di} + N_{2i}^T - N_{1i} + M_{1i}, \quad \tilde{\Theta}_{22i} = (Y_i - X_i) A_{1di} + V_i A_{2di},$

$\Theta_{3i} = [(-(1-\eta)) \vee (-(1-\eta)e^{-\beta\bar{h}})]Q - N_{2i} - N_{2i}^T + M_{2i} + M_{2i}^T,$

$$\tilde{\Omega}_{2i} = \begin{pmatrix} -N_{1i} & -M_{1i} & Y_i D_{1i} & A_{1i}^T R_1 & B_{1i}^T Y_i & B_{1i}^T (Y_i - X_i) + B_{2i}^T V_i^T & L_i^T - W_i^T \\ 0 & 0 & (Y_i - X_i) D_{1i} + V_i D_{2i} & 0 & 0 & 0 & -W_i^T \\ -N_{2i} & -M_{2i} & 0 & A_{1di}^T R_1 & B_{1di}^T Y_i & B_{1di}^T (Y_i - X_i) + B_{2di}^T V_i^T & L_{di}^T \\ -N_{3i} & -M_{3i} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{1i} R_1 & 0 & 0 & 0 \end{pmatrix},$$

$$\tilde{\Omega}_{3i} = \begin{pmatrix} -\frac{1}{\bar{h}} R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\frac{1}{\bar{h}} R_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & D_{1i}^T R_1 & 0 & 0 & 0 \\ * & * & * & -\frac{\beta}{e^{\beta\bar{\tau}} - 1} R_1 & 0 & 0 & 0 \\ * & * & * & * & -Y_i & 0 & 0 \\ * & * & * & * & * & Y_i - X_i & 0 \\ * & * & * & * & * & * & -I \end{pmatrix}.$$

Moreover, for each $i \in \mathcal{S}$, a desired filter is given in the form of (2) with filter matrices as follows:

$$A_{fi} = (X_i - Y_i)^{-1} Z_i, \; B_{fi} = (X_i - Y_i)^{-1} V_i, \; C_{fi} = W_i. \tag{26}$$

Proof. Firstly, from (25), we can obtain $X_i - Y_i > 0$ by Schur complement. Consequently, $X_i - Y_i$ is invertible.

For each $i \in \mathcal{S}$, let

$$P_i = \begin{pmatrix} X_i & Y_i - X_i \\ Y_i - X_i & X_i - Y_i \end{pmatrix}. \tag{27}$$

By using Schur complement again, we have $P_i > 0$. For each $i \in \mathcal{S}$, let $\bar{Y}_i = Y_i^{-1}$. Pre- and post-multiplying (25) by $\Sigma_{1i} = diag\{\bar{Y}_i, I, I, I, I, I, I, I, I, \bar{Y}_i, I, I\}$ and Σ_{1i}^T, respectively, we have

$$\begin{pmatrix}
\hat{\Theta}_{11i} & \hat{\Theta}_{12i} & \hat{\Theta}_{21i} & \bar{Y}_i(N_{3i}^T - M_{1i}) & E_{1i} & -\bar{Y}_i N_{1i} & -\bar{Y}_i M_{1i} & D_{1i} & \bar{Y}_i A_{1i}^T R_1 & \bar{Y}_i B_{1i}^T & \bar{Y}_i(B_{1i}^T(Y_i - X_i) + B_{2i}^T V_i^T) & \bar{Y}_i(L_i^T - W_i^T) \\
* & \tilde{\Theta}_{13i} & \tilde{\Theta}_{22i} & 0 & (Y_i - X_i)E_{1i} + V_i E_{2i} & 0 & 0 & (Y_i - X_i)D_{1i} + V_i D_{2i} & 0 & 0 & 0 & -W_i^T \\
* & * & \Theta_{3i} & -N_{3i}^T - M_{2i} + M_{3i}^T & 0 & -N_{2i} & -M_{2i} & 0 & A_{1di}^T R_1 & B_{1di}^T & B_{1di}^T(Y_i - X_i) + B_{2di}^T V_i^T & L_{di}^T \\
* & * & * & -M_{3i} - M_{3i}^T & 0 & -N_{3i} & -M_{3i} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\frac{1}{\bar{\tau}} R_2 & 0 & 0 & 0 & E_{1i} R_1 & 0 & 0 & 0 \\
* & * & * & * & * & -\frac{1}{\bar{h}} R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\frac{1}{\bar{h}} R_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\gamma^2 I & D_{1i}^T R_1 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\frac{\beta}{e^{\beta\tau} - 1} R_1 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\bar{Y}_i & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & Y_i - X_i & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -I
\end{pmatrix} < 0, \tag{28}$$

where

$$\hat{\Theta}_{11i} = \sum_{j=1}^{N} \pi_{ij} \bar{Y}_i Y_j \bar{Y}_i + \beta \bar{Y}_i + A_{1i} \bar{Y}_i + \bar{Y}_i A_{1i}^T + \bar{Y}_i \left(\frac{e^{\beta\bar{\tau}} - 1}{\beta} R_2 + Q + N_{1i} + N_{1i}^T \right) \bar{Y}_i,$$

$$\hat{\Theta}_{12i} = \bar{Y}_i A_{1i}^T (Y_i - X_i) + \bar{Y}_i A_{2i}^T V_i^T + \bar{Y}_i Z_i^T,$$

$$\hat{\Theta}_{21i} = A_{1di} + \bar{Y}_i (N_{2i}^T - N_{1i} + M_{1i}).$$

If we define $\Lambda_i = \begin{pmatrix} \bar{Y}_i & 0 \\ \bar{Y}_i & I \end{pmatrix}, \forall i \in \mathcal{S}$, one can conclude from (26) and (27) that (28) can be rewritten as:

$$\begin{pmatrix}
\Lambda_i^T\Theta_{1i}\Lambda_i & \Lambda_i^T\Theta_{2i} & \Lambda_i^T K^T(N_{3i}^T - M_{1i}) & \Lambda_i^T P_i\tilde{E}_i & -\Lambda_i^T K^T N_{1i} & -\Lambda_i^T K^T M_{1i} & \Lambda_i^T P_i\tilde{D}_i & \Lambda_i^T\tilde{A}_i^T K^T R_1 & \Lambda_i^T\tilde{B}_i^T P_i\Lambda_i & \Lambda_i^T\tilde{L}_i^T \\
* & \Theta_{3i} & -N_{3i}^T - M_{2i} + M_{3i}^T & 0 & -N_{2i} & -M_{2i} & 0 & \tilde{A}_{di}^T K^T R_1 & \tilde{B}_{di}^T P_i\Lambda_i & L_{di}^T \\
* & * & -M_{3i} - M_{3i}^T & 0 & -N_{3i} & -M_{3i} & 0 & 0 & 0 & 0 \\
* & * & * & -\frac{1}{\bar{\tau}}R_2 & 0 & 0 & 0 & \tilde{E}_i^T K^T R_1 & 0 & 0 \\
* & * & * & * & -\frac{1}{\bar{h}}R_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\frac{1}{\bar{h}}R_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\gamma^2 I & \tilde{D}_i^T K^T R_1 & 0 & 0 \\
* & * & * & * & * & * & * & -\frac{\beta}{e^{\beta\bar{h}} - 1}R_1 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\Lambda_i^T P_i\Lambda_i & 0 \\
* & * & * & * & * & * & * & * & * & -I
\end{pmatrix}$$

Now, pre- and post-multiplying the above LMI by $\Sigma_{2i} = \mathrm{d}iag\{(\Lambda_i^T)^{-1}, I, I, I, I, I, I, I, (\Lambda_i^T)^{-1}, I\}$ and Σ_{2i}^T, respectively, we can obtain from Theorem 3.5 that the filter in (2) assures the solvability of H_∞ filtering problem for the filtering error system (3). □

Remark 4.2. When let $E_{1i} = E_{2i} = 0$ for each $i \in \mathcal{S}$, the stochastic time-varying delay system (1) becomes the systems investigated in [21], in which the exponential H_∞ filter design problem is also considered and the decay rate has no equation constraint. However, the derivative of time-varying delay is required to less than one. Here, this confined condition is eliminated. The sufficient criteria obtained in Theorem 3.5 and Theorem 4.1 are also valid when the derivative of discrete time-varying delay is not less than one. Hence, the results obtained in this paper are more general than the ones in [21].

5. NUMERICAL EXAMPLE

Example 5.1. In the following example, we consider the stochastic Markovian jump systems (1) with two modes, that is, $\mathcal{S} = \{1, 2\}$. The systems data are shown as follows:
In mode 1:

$$A_{11} = \begin{bmatrix} -1.5 & 0 \\ 0.3 & -1 \end{bmatrix}, A_{1d1} = \begin{bmatrix} -0.5 & -0.1 \\ 0.2 & 0.3 \end{bmatrix}, E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, D_{11} = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix},$$

$$B_{11}=\begin{bmatrix}-0.2 & 0\\ 0.1 & 0.5\end{bmatrix}, B_{1d1}=\begin{bmatrix}-0.5 & 0\\ -0.1 & 0.3\end{bmatrix}, A_{21}=[0.2\quad -0.5], A_{2d1}=[0.3\ \ 0],$$

$$E_{21}=[1\ \ 0], D_{21}=-0.5, B_{21}=[-0.5\ \ 0], B_{2d1}=[0.1\quad -0.6], L_1=[-0.3\ \ 0.5],$$

$$L_{d1}=[0.2\quad -0.3],$$

In mode 2:

$$A_{12}=\begin{bmatrix}-0.5 & 0\\ 0.1 & -0.2\end{bmatrix}, A_{1d2}=\begin{bmatrix}-0.2 & 0\\ -0.1 & -0.2\end{bmatrix}, E_{12}=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}, D_{12}=\begin{bmatrix}0.1\\ -0.4\end{bmatrix},$$

$$B_{12}=\begin{bmatrix}-0.5 & 0.5\\ 0.2 & -0.1\end{bmatrix}, B_{1d2}=\begin{bmatrix}-0.6 & 0.2\\ 0 & 1\end{bmatrix}, A_{22}=[1\quad -0.2], A_{2d2}=[-0.1\ \ 0.1],$$

$$E_{22}=[0\ \ 1], D_{22}=1, B_{22}=[-0.1\ \ 0], B_{2d2}=[-0.5\ \ 0], L_2=[0\quad -0.1],$$

$$L_{d2}=[0\quad -0.1].$$

The density matrix $\Pi=\{\pi_{ij}\}$ is given by

$$\Pi=\begin{bmatrix}-0.5 & 0.5\\ 1 & -1\end{bmatrix}.$$

The suboptimal upper bound of decay rate β can be computed by Matlab LMI toolbox. With above parameters and $\bar{h}=1.2, \bar{\tau}=0.4, \eta=1.2, \gamma=0.5$, from (25) we can obtain the suboptimal upper bound of decay rate is 0.2322. For demonstration purpose, the value of β is fixed as 0.1. By using the Matlab LMI Toolbox to solve LMI (25), a set of feasible solution is obtained as follows:

$$X_1=\begin{bmatrix}1.8552 & 0.0802\\ 0.0802 & 0.6949\end{bmatrix}, X_2=\begin{bmatrix}2.3247 & -0.1199\\ -0.1199 & 0.1579\end{bmatrix}, Y_1=\begin{bmatrix}1.5949 & 0.0893\\ 0.0893 & 0.5068\end{bmatrix},$$

$$Y_2=\begin{bmatrix}2.1482 & -0.0492\\ -0.0492 & 0.0222\end{bmatrix}, Q=\begin{bmatrix}0.0709 & 0.0044\\ 0.0044 & 0.0022\end{bmatrix}, R_1=\begin{bmatrix}1.8597 & 0.2625\\ 0.2625 & 1.4825\end{bmatrix},$$

$$R_2=\begin{bmatrix}2.0741 & 0.1336\\ 0.1336 & 0.4732\end{bmatrix}, V_1=\begin{bmatrix}-0.0062\\ -0.2303\end{bmatrix}, V_2=\begin{bmatrix}0.3306\\ -0.0719\end{bmatrix},$$

$$Z_1=\begin{bmatrix}-0.9429 & -0.0776\\ 0.4027 & -0.2003\end{bmatrix}, Z_2=\begin{bmatrix}-1.7248 & -0.1763\\ -0.2299 & -0.2747\end{bmatrix}, W_1=[0.0525\ \ 0.1373],$$

$$W_2=[0.0151\quad -0.1235], M_{11}=\begin{bmatrix}-0.0011 & -0.0004\\ -0.0001 & -0.0002\end{bmatrix}, M_{12}=\begin{bmatrix}-0.0001 & -0.0001\\ -0.0014 & -0.0014\end{bmatrix},$$

$$M_{21}=\begin{bmatrix}-1.5468 & -0.2180\\ -0.2181 & -1.2335\end{bmatrix}, M_{22}=\begin{bmatrix}-1.5472 & -0.2181\\ -0.2178 & -1.2335\end{bmatrix}, M_{31}=\begin{bmatrix}1.5473 & 0.2181\\ 0.2180 & 1.2336\end{bmatrix},$$

$$M_{32}=\begin{bmatrix}1.5470 & 0.2179\\ 0.2174 & 1.2331\end{bmatrix}, N_{11}=\begin{bmatrix}-1.5465 & -0.2179\\ -0.2190 & -1.2353\end{bmatrix}, N_{12}=\begin{bmatrix}1.9823 & -0.2670\\ 0.2432 & 1.2558\end{bmatrix},$$

$$N_{21}=\begin{bmatrix}1.5470 & 0.2180\\ 0.2185 & 1.2340\end{bmatrix}, N_{22}=\begin{bmatrix}0.9075 & 0.2972\\ -0.0329 & 0.5105\end{bmatrix}, N_{31}=\begin{bmatrix}0.0012 & 0.0004\\ 0.0008 & 0.0016\end{bmatrix},$$

$$N_{32} = \begin{bmatrix} 0.0004 & 0.0006 \\ 0.0006 & 0.0011 \end{bmatrix}.$$

So, according to Theorem 4.1, the H_∞ filtering problem is solvable. Then, from (26), the desired filter matrices can be given as:

$$A_{f1} = \begin{bmatrix} -3.5530 & -0.3361 \\ 1.9676 & -1.0813 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0673 \\ -1.2276 \end{bmatrix}, C_{f1} = [0.0525 \quad 0.1373],$$

$$A_{f2} = \begin{bmatrix} -13.2064 & -2.2864 \\ -8.5230 & -3.2146 \end{bmatrix}, B_{f2} = \begin{bmatrix} 2.0990 \\ 0.5634 \end{bmatrix}, C_{f2} = [0.0151 \quad -0.1235].$$

If we assume that the discrete time-varying delay $h(t) = 0.5\sin^2(t)$, the distributed time-varying delay $\tau(t) = 0.4\cos^2(t)$, and the noise signal $v(t) = \frac{10}{1+2t}$, which belongs to $\mathcal{L}_2[0,\infty)$, the simulation results of estimation error $e(t)$ are shown in Figures 1−2. From these two figures, we can verify that the designed filter is effective for exponentially stabilizing the filter error system (3) with decay rate 0.1.

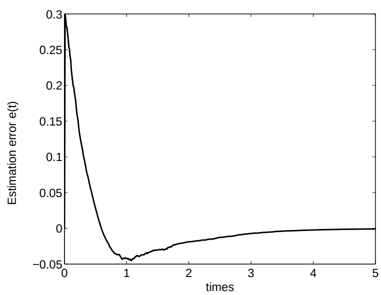

Fig. 1. The estimation error $e(t)$ at mode 1.

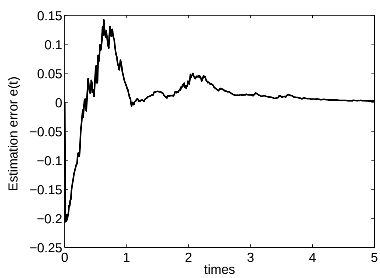

Fig. 2. The estimation error $e(t)$ at mode 2.

6. CONCLUSION

In this paper, we consider the stochastic Markovian jump systems with time-varying delays, in which the discrete and distributed time-varying delays are both included. By constructing a suitable Lyapunov–Krasovskii functional and employing the property of conditional mathematical expectation, sufficient condition that guarantees the mean-square exponential stability and the H_∞ performance satisfied for the filtering error system is established. Then, based on obtained sufficient condition, the exponential H_∞ filter is designed. There is no conventional constraint on the decay rate β and the derivative of discrete time-varying delay is only required to satisfy $\dot{h}(t) < \eta$ with any constant η. A numerical example is given to demonstrate the effectiveness of the proposed result in this paper.

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