KYBERNETIKA — VOLUME *50* (2014), NUMBER 1, PAGES 46–65

RELAXED STABILITY CONDITIONS FOR INTERVAL TYPE-2 FUZZY-MODEL-BASED CONTROL SYSTEMS

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This paper proposes new stability conditions for interval type-2 fuzzy-model-based (FMB) control systems. The type-1 T-S fuzzy model has been widely studied because it can represent a wide class of nonlinear systems. Many favorable results for type-1 T-S fuzzy model have been reported. However, most of conclusions for type-1 T-S fuzzy model can not be applied to nonlinear systems subject to parameter uncertainties. In fact, Most of the practical applications are subject to parameters uncertainties. To address above problem, an interval type-2 T-S fuzzy model has been proposed to approximate nonlinear systems subject to parameter uncertainties, and stability conditions for interval type-2 FMB control systems have also been presented in the form of linear matrix inequalities (LMIs). The aim of this paper is to relax the existing stability conditions. The new stability conditions in terms of LMIs are derived to guarantee the stability of interval type-2 FMB control systems. The theoretical poof is given to show the proposed conditions reduce the conservativeness in stability analysis. Several numerical examples are also provided to illustrate the effectiveness of the proposed conditions.

Keywords: interval type-2 fuzzy set, interval type-2 T-S fuzzy system, linear matrix inequalities, stability analysis

Classification: 93E12, 62A10

1. INTRODUCTION

Nonlinear system control has important applications in real life [23, 24]. Type-1 T-S fuzzy model provides a powerful tool for modeling complex nonlinear systems. Based on parallel-distributed-compensation (PDC) control scheme, type-1 FMB control systems have been proposed. Many stability results for type-1 FMB control systems have been presented by using Lyapunov stability theory. Tanaka and his colleagues did a pioneering work on the stability analysis of type-1 T-S fuzzy systems, and the basic stability condition for ensuring stability of type-1 T-S fuzzy systems was given in [19]. To reduce the conservatism in stability analysis of type-1 FMB control systems, many valuable stability conditions for type-1 FMB control systems were obtained in [3, 6, 13, 14, 16, 20, 21, 25]. It is seen that the common quadratic Lyapunov function is used to investigate the stability of type-1 FMB control systems in [3, 6, 13, 14, 16, 20, 21, 25]. However, it is recognized that the common quadratic Lyapunov function is independent of membership functions,

DOI: 10.14736/kyb-2014-1-0046

which may lead to conservativeness. To further reduce the conservatism in stability analysis, several non- quadratic Lyapunov functions have been proposed, such as piecewise Lyapunov functions [4, 5, 22] and fuzzy Lyapunov functions [1, 2, 8, 12, 15, 17, 18].

It is noted that parameter uncertainties were not considered in the aforementioned stability results. The grades of membership of the T-S fuzzy systems may become uncertain in value if the original nonlinear plants have uncertain parameters. Hence, the stability results obtained by PDC technique would vanish. Of course, the aforementioned type-1 stability results cannot also applied in this situation. However, Most of the practical applications are subject to parameters uncertainties. Consequently, a type-1 non-PDC fuzzy controller was proposed to handle these systems subject to parameters uncertainties, and the stability conditions in the form of LMIs were also derived in [7]. Another solution is to utilize interval type-2 T-S fuzzy systems to represent the nonlinear systems subject to parameters uncertainties [11]. The interval type-2 fuzzy controller was also designed to stabilize the interval type-2 T-S fuzzy systems in [11]. The stability conditions for interval type-2 FMB control systems were presented in the form of LMIs. It is proved that the interval type-2 T-S fuzzy systems are more suitable to handle nonlinear systems subject to parameters uncertainties because these parameters uncertainties can be captured by the footprint of uncertainty (FOU) of interval type-2 fuzzy sets. As a result, less conservative results may be obtained because the information of the lower and upper membership functions can be applied in stability analysis. Furthermore, the stability conditions of interval type-2 fuzzy control systems under imperfect premise matching were presented in [10].

In this paper, new stability conditions for interval type-2 FMB control systems are presented. It is proved that the proposed conditions include those of [11] as a particular case. This paper is organized as follows. The interval type-2 T-S fuzzy model and the interval type-2 fuzzy controller proposed in [11] are reviewed briefly in Section 2. New stability conditions for interval type-2 FMB control systems are derived in Section 3. Several numerical examples used to illustrate the effectiveness of the proposed conditions are presented in Section 4. The last section concludes this paper.

2. INTERVAL TTYPE-2 FMB CONTROL SYSTEMS

In this section, the interval type-2 T-S fuzzy model which can represent a class of nonlinear plants subject to parameters uncertainties is recalled.

2.1. interval type-2 T-S fuzzy model

The interval type-2 T-S fuzzy model was proposed in [11]. It can be described by a set of fuzzy IF-THEN rules:

Rule i: if $f_1(x(t))$ is $\widetilde{M_1^i}$ and,…, and $f_\psi(x(t))$ is $\widetilde{M_\psi^i}$, then

$$\dot{x}(t) = \mathbf{A}_i x(t) + \mathbf{B}_i u(t) \tag{1}$$

where $\widetilde{M_a^i}$ is an interval type-2 fuzzy set, $a = 1, 2, \ldots, \psi$; $i = 1, 2, \ldots, p$; ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are known constant matrices; $x(t) \in \Re^{n\times 1}$ is the system state vector; and $x(t) \in \Re^{m\times 1}$ is the input vector. The firing strength of the th rule is the following interval sets:

$$\widetilde{\omega}_i(x(t)) = [\omega_i^L(x(t)), \omega_i^U(x(t))]$$

where

$$\omega_i^L(x(t)) = \underline{u}_{\widetilde{M_1^i}}(f_1(x(t))) \times \underline{u}_{\widetilde{M_2^i}}(f_2(x(t))) \times \cdots \times \underline{u}_{\widetilde{M_\psi^i}}(f_\psi(x(t))),$$
$$\omega_i^U(x(t)) = \bar{u}_{\widetilde{M_1^i}}(f_1(x(t))) \times \bar{u}_{\widetilde{M_2^i}}(f_2(x(t))) \times \cdots \times \bar{u}_{\widetilde{M_\psi^i}}(f_\psi(x(t)))$$

in which $\underline{u}_{\widetilde{M_a^i}}(f_a(x(t))) \in [0,1]$ and $\bar{u}_{\widetilde{M_a^i}}(f_a(x(t))) \in [0,1]$ denote the lower and upper grades of membership, respectively.

The inferred interval type-2 T-S fuzzy model is defined as

$$\dot{x}(t) = \sum_{i=1}^{p} \omega_i(x(t))(\mathbf{A}_i x(t) + \mathbf{B}_i u(t)) \tag{2}$$

where

$$\omega_i(x(t)) = \omega_i^L(x(t))\underline{v}_i(x(t)) + \omega_i^U(x(t))\bar{v}_i(x(t))$$
$$\sum_{i=1}^{p} \omega_i(x(t)) = 1 \tag{3}$$

in which $\underline{v}_i(x(t)) \in [0,1]$ and $\bar{v}_i(x(t)) \in [0,1]$ are nonlinear functions and satisfy $\underline{v}_i(x(t)) + \bar{v}_i(x(t)) = 1$ for all i.

2.2. interval type-2 fuzzy controller

The interval type-2 fuzzy controller proposed in [11] can be represented by the following format: Rule j: if $f_1(x(t))$ is $\widetilde{M_1^j}$ and, ..., and $f_\psi(x(t))$ is $\widetilde{M_\psi^j}$ then

$$u(t) = \mathbf{G}_j x(t) \tag{4}$$

where $G_j \in \Re^{m\times n},\ j = 1, 2, \ldots, p$ are the feedback gains to be determined. The final output of the interval type-2 fuzzy controller is defined as

$$u(t) = \sum_{j=1}^{p} (\underline{\omega}_j(x(t)) + \bar{\omega}_j(x(t)))\mathbf{G}_j x(t) \tag{5}$$

where

$$\underline{\omega}_j(x(t)) = \frac{\omega_j^L(x(t))}{\sum_{k=1}^{p}(\omega_k^L(x(t)) + \omega_k^U(x(t)))}, \quad \bar{\omega}_j(x(t)) = \frac{\omega_j^U(x(t))}{\sum_{k=1}^{p}(\omega_k^L(x(t)) + \omega_k^U(x(t)))}$$

$$\sum_{j=1}^{p} (\omega_j(x(t)) + \bar{\omega}_j(x(t))) = 1. \tag{6}$$

In the following analysis, $\omega_i(x(t)), \underline{\omega}_i(x(t))$ and $\bar{\omega}_i(x(t))$ are denoted as ω_i , $\underline{\omega}_i$ and $\bar{\omega}_i$, respectively, for simplicity. Thus, the interval type-2 FMB control system formed by (2) and (5) is shown as follows:

$$\begin{aligned}\dot{x}(t) =& \sum_{i=1}^{p} \omega_i \left(\mathbf{A}_i x(t) + \mathbf{B}_i \sum_{j=1}^{p} (\underline{\omega}_j + \bar{\omega}_j) \mathbf{G}_j x(t) \right) \\ =& \sum_{i=1}^{p} \sum_{j=1}^{p} \omega_i (\underline{\omega}_j + \bar{\omega}_j) \left(\mathbf{A}_\mathrm{i} + \mathbf{B}_\mathrm{i} \mathbf{G}_\mathrm{j} \right) x(t). \end{aligned} \tag{7}$$

3. MAIN RESULTS

In this section, new stability conditions for interval type-2 FMB control systems are presented in the form of LMIs. To investigate the system stability of (7), the following Lyapunov function candidate is considered:

$$V(t) = x(t)^T \mathbf{P} x(t) \tag{8}$$

where $\mathbf{P} = \mathbf{P}^T \in \Re^{n \times n} > 0$. From (7) and (8), we have

$$\begin{aligned}\dot{V}(t) =& \dot{x}(t)^T \mathbf{P} x(t) + x(t)^T \mathbf{P} \dot{x}(t) \\ =& \sum_{i=1}^{p} \sum_{j=1}^{p} \omega_i (\underline{\omega}_j + \bar{\omega}_j) x(t)^T \Big((\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j)^T \mathbf{P} + \mathbf{P} (\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j) \Big) x(t). \end{aligned} \tag{9}$$

Denote $\mathbf{M} = \mathbf{P}^{-1}, z(t) = \mathbf{M}^{-1} x(t)$ and $\mathbf{G}_j = \mathbf{N}_j \mathbf{M}^{-1}$, where $\mathbf{N}_j \in \Re^{m \times n}$ for all j. From (9), we have

$$\begin{aligned}\dot{V}(t) =& \sum_{i=1}^{p} \sum_{j=1}^{p} \omega_i (\underline{\omega}_j + \bar{\omega}_j) z(t)^T \mathbf{Q}_{ij} z(t) \\ =& z(t)^T \Pi z(t) \end{aligned} \tag{10}$$

where $\Pi = \sum_{i=1}^{p} \sum_{j=1}^{p} \omega_i (\underline{\omega}_j + \bar{\omega}_j) Q_{ij}$ and $\mathbf{Q}_{ij} = \mathbf{A}_i \mathbf{M} + \mathbf{M} \mathbf{A}_i^T + \mathbf{B}_i \mathbf{N}_j + \mathbf{N}_j^T \mathbf{B}_i^T$.

It is seen from (10) that $\Pi < 0$ implies $\dot{V}(t) < 0$. The shape information of membership functions was introduced in [23] to reduce the conservativeness. This information is still used in the following analysis. Based on the property of the membership functions that $\sum_{i=1}^{p} \omega_i = \sum_{j=1}^{p} (\underline{\omega}_j + \bar{\omega}_j) = 1$, it follows that $\sum_{i=1}^{p} (\omega_i - \underline{\omega}_i - \bar{\omega}_i) = 0$.

Thus,

$$\begin{aligned}\Xi =& \sum_{i=1}^{p} \sum_{j=1}^{p} (\omega_i - \underline{\omega}_i - \bar{\omega}_i) \times (\omega_j (\mathbf{C}_j + \mathbf{C}_j^T) + \underline{\omega}_j (\mathbf{D}_j + \mathbf{D}_j^T) + \bar{\omega}_j (\mathbf{E}_j + \mathbf{E}_j^T)) \\ =& 0 \end{aligned} \tag{11}$$

where $\mathbf{C}_j, \mathbf{D}_j, \mathbf{E}_j \in \Re^{n \times n}$ are the slack matrices to reduce the conservativeness.

$$-\omega_i + \rho_{i1} \underline{\omega}_i + \sigma_{i1} \bar{\omega}_i + \gamma_{i1} \geq 0 \text{ for all } i, x(t) \text{ and system parameters,}$$

$$\omega_i - \rho_{i2}\underline{\omega}_i - \sigma_{i2}\bar{\omega}_i + \gamma_{i2} \geq 0 \text{ for all } i, x(t) \text{ and system parameters,}$$

where $\rho_{i1}, \sigma_{i1}, \gamma_{i1}, \rho_{i2}, \sigma_{i2}$ and γ_{i2} are scalars to be determined. Defining $\mathbf{R}_{ij} + \mathbf{R}_{ij}^T \geq 0$, where $\mathbf{R}_{ij} = \mathbf{R}_{ji}{}^T \in \Re^{n\times n}$. Based on the above two assumptions, the following inequality holds:

$$\Phi = \sum_{i=1}^{p}\sum_{j=1}^{p}(-\omega_i + \rho_{i1}\omega_i + \sigma_{i1}\bar{\omega}_i + \gamma_{i1}) \times (\omega_j - \rho_{j2}\omega_j - \sigma_{j2}\bar{\omega}_j + \gamma_{j2})(\mathbf{R}_{ij} + \mathbf{R}_{ij}^T) \geq 0. \quad (12)$$

Thus, it follows that $\Pi \leq \Pi + \Xi + \Phi$ from (10), (11) and (12). Therefore,

$$\begin{aligned}
\Pi &\leq \Pi + \Xi + \Phi \\
&= -\sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i\omega_j\Bigg(\mathbf{R}_{ij} + \mathbf{R}_{ij}^T + \sum_{k=1}^{p}\gamma_{k2}(\mathbf{R}_{ik} + \mathbf{R}_{ik}^T) - \sum_{k=1}^{p}\gamma_{k1}(\mathbf{R}_{kj} + \mathbf{R}_{kj}^T) \\
&\quad - \sum_{k=1}^{p}\sum_{l=1}^{p}\gamma_{k1}\gamma_{l2}(\mathbf{R}_{kl} + \mathbf{R}_{kl}{}^T) - \mathbf{C}_j - \mathbf{C}_j{}^T\Bigg) \\
&\quad + \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i\underline{\omega}_j\Bigg(\rho_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}^T) + \rho_{j1}(\mathbf{R}_{ji} + \mathbf{R}_{ji}^T) - \sum_{k=1}^{p}\rho_{j2}\gamma_{k1}(\mathbf{R}_{kj} + \mathbf{R}_{kj}^T) \\
&\quad + \sum_{k=1}^{p}\gamma_{k2}\rho_{j1}(\mathbf{R}_{jk} + \mathbf{R}_{jk}{}^T) + \mathbf{Q}_{ij} + \mathbf{D}_j + \mathbf{D}_j{}^T - \mathbf{C}_i - \mathbf{C}_i{}^T\Bigg) \\
&\quad + \sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i\bar{\omega}_j\Bigg(\sigma_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \sigma_{j1}(\mathbf{R}_{ji} + \mathbf{R}_{ji}{}^T) - \sum_{k=1}^{p}\sigma_{j2}\gamma_{k1}(\mathbf{R}_{kj} + \mathbf{R}_{kj}{}^T) \\
&\quad + \sum_{k=1}^{p}\gamma_{k2}\sigma_{j1}(\mathbf{R}_{jk} + \mathbf{R}_{jk}{}^T) + \mathbf{Q}_{ij} + \mathbf{E}_j + \mathbf{E}_j{}^T - \mathbf{C}_i - \mathbf{C}_i{}^T\Bigg) \\
&\quad - \sum_{i=1}^{p}\sum_{j=1}^{p}\underline{\omega}_i\underline{\omega}_j\left(\rho_{i1}\rho_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \mathbf{D}_j + \mathbf{D}_j{}^T\right) \\
&\quad - \sum_{i=1}^{p}\sum_{j=1}^{p}\underline{\omega}_i\bar{\omega}_j\left(\rho_{i1}\sigma_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \sigma_{j1}\rho_{i2}(\mathbf{R}_{ji} + \mathbf{R}_{ji}{}^T) + \mathbf{E}_j + \mathbf{E}_j{}^T + \mathbf{D}_i + \mathbf{D}_i{}^T\right) \\
&\quad - \sum_{i=1}^{p}\sum_{j=1}^{p}\bar{\omega}_i\bar{\omega}_j\left(\sigma_{i1}\sigma_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \mathbf{E}_j + \mathbf{E}_j{}^T\right). \qquad (13)
\end{aligned}$$

For simplicity, some special matrices are introduced to represent the complex matrices of (13). Define $\Psi_{ij}^{uv} \in \Re^{n\times n}$, $i, j = 1, 2, \dots, p$, $u, v = 1, 2, \dots, 3$. Let

$$\begin{aligned}
\Psi_{ij}^{11} = -&\Bigg(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T + \sum_{k=1}^{p}\gamma_{k2}(\mathbf{R}_{ik} + \mathbf{R}_{ik}{}^T) - \sum_{k=1}^{p}\gamma_{k1}(\mathbf{R}_{kj} + \mathbf{R}_{kj}{}^T) \qquad (14) \\
&- \sum_{k=1}^{p}\sum_{l=1}^{p}\gamma_{k1}\gamma_{l2}(\mathbf{R}_{kl} + \mathbf{R}_{kl}{}^T) - \mathbf{C}_j - \mathbf{C}_j{}^T\Bigg\}, \quad i, j = 1, 2, \dots, p
\end{aligned}$$

$$\Psi_{ij}^{12} + (\Psi_{ij}^{12})^T = \Bigg(\rho_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \rho_{j1}(\mathbf{R}_{ji} + \mathbf{R}_{ji}{}^T) - \sum_{k=1}^{p} \rho_{j2}\gamma_{k1}(\mathbf{R}_{kj} + \mathbf{R}_{kj}{}^T) + \sum_{k=1}^{p} \gamma_{k2}\rho_{j1}(\mathbf{R}_{jk} + \mathbf{R}_{jk}{}^T) + \mathbf{Q}_{ij} + \mathbf{D}_j + \mathbf{D}_j{}^T - \mathbf{C}_i - \mathbf{C}_i{}^T\Bigg), \quad i,j = 1,2,\ldots,p \tag{15}$$

$$\Psi_{ij}^{13} + (\Psi_{ij}^{13})^T = \Bigg(\sigma_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \sigma_{j1}(\mathbf{R}_{ji} + \mathbf{R}_{ji}{}^T) - \sum_{k=1}^{p} \sigma_{j2}\gamma_{k1}(\mathbf{R}_{kj} + \mathbf{R}_{kj}{}^T) + \sum_{k=1}^{p} \gamma_{k2}\sigma_{j1}(\mathbf{R}_{jk} + \mathbf{R}_{jk}{}^T) + \mathbf{Q}_{ij} + \mathbf{E}_j + \mathbf{E}_j{}^T - \mathbf{C}_i - \mathbf{C}_i{}^T\Bigg), \quad i,j = 1,2,\ldots,p \tag{16}$$

$$\Psi_{ij}^{22} = -\left(\rho_{i1}\rho_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \mathbf{D}_j + \mathbf{D}_j{}^T\right), \ i,j = 1,2,\ldots,p \tag{17}$$

$$\Psi_{ij}^{23} + (\Psi_{ij}^{23})^T = -\Big(\rho_{i1}\sigma_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \sigma_{j1}\rho_{i2}(\mathbf{R}_{ji} + \mathbf{R}_{ji}{}^T) + \mathbf{E}_j + \mathbf{E}_j{}^T + \mathbf{D}_i + \mathbf{D}_i{}^T\Big), \ i,j = 1,2,\ldots,p \tag{18}$$

$$\Psi_{ij}^{33} = -\left(\sigma_{i1}\sigma_{j2}(\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T) + \mathbf{E}_j + \mathbf{E}_j{}^T\right), \ i,j = 1,2,\ldots,p. \tag{19}$$

From (14) to (19), (13) can be rewritten as:

$$\begin{aligned}\Pi &\leqslant \Pi + \Xi + \Phi \\ &= \sum_{i=1}^{p}\sum_{j=1}^{p} \omega_i\omega_j(\Psi_{ij}^{11}) + \sum_{i=1}^{p}\sum_{j=1}^{p} \omega_i\underline{\omega}_j(\Psi_{ij}^{12} + (\Psi_{ij}^{12})^T) + \sum_{i=1}^{p}\sum_{j=1}^{p} \omega_i\bar{\omega}_j(\Psi_{ij}^{13} + (\Psi_{ij}^{13})^T) \\ &+ \sum_{i=1}^{p}\sum_{j=1}^{p} \underline{\omega}_i\underline{\omega}_j(\Psi_{ij}^{22}) + \sum_{i=1}^{p}\sum_{j=1}^{p} \underline{\omega}_i\bar{\omega}_j(\Psi_{ij}^{23} + (\Psi_{ij}^{23})^T) + \sum_{i=1}^{p}\sum_{j=1}^{p} \bar{\omega}_i\bar{\omega}_j(\Psi_{ij}^{33}).\end{aligned} \tag{20}$$

It can be seen that (20) has been presented in [11]. New stability results for interval type-2 FMB control systems are derived on the basis of (20) in the following analysis.

Theorem 3.1. The interval type-2 FMB control system (7) is asymptotically stable if there exist predefined scalars of $\rho_{i1}, \sigma_{i1}, \gamma_{i1}, \rho_{i2}, \sigma_{i2}$ and γ_{i2} such that $-\omega_i + \rho_{i1}\underline{\omega}_i + \sigma_{i1}\bar{\omega}_i + \gamma_{i1} \geq 0$ and $\omega_i - \rho_{i2}\underline{\omega}_i - \sigma_{i2}\bar{\omega}_i + \gamma_{i2} \geq 0$ are satisfied, and there exist matrices $\mathbf{C}_j, \mathbf{D}_j, \mathbf{E}_j \in \Re^{m\times n},\ j = 1,\ldots,p$; $\mathbf{N}_j \in \Re^{m\times n},\ j = 1,\ldots,p$; $\mathbf{M} = \mathbf{M}^T \in \Re^{n\times n}$; $\mathbf{R}_{ij} = \mathbf{R}_{ji}{}^T \in \Re^{n\times n},\ i,j = 1,\cdots,p$; $\mathbf{S}_{jii}^l = (\mathbf{S}_{iij}^l)^T, \mathbf{S}_{iji}^l = (\mathbf{S}_{iji}^l)^T, \mathbf{Y}_{jii}^l = (\mathbf{Y}_{iij}^l)^T, \mathbf{Y}_{iji}^l = (\mathbf{Y}_{iji}^l)^T, \mathbf{Z}_{jii}^l = (\mathbf{Z}_{iij}^l)^T, \mathbf{Z}_{iji}^l = (\mathbf{Z}_{iji}^l)^T \in \Re^{n\times n}, i = 1,\ldots,p, j \neq i, j = 1,\ldots,p, l =$

$1,2,3$; $\mathbf{S}^l_{iii} = (\mathbf{S}^l_{iii})^T, \mathbf{Y}^l_{iii} = (\mathbf{Y}^l_{iii})^T, \mathbf{Z}^l_{iii} = (\mathbf{Z}^l_{iii})^T \in \Re^{n\times n}, i = 1,\dots,p, l = 1,2,3$; $\mathbf{S}^l_{jik} = (\mathbf{S}^l_{kij})^T, \mathbf{S}^l_{ijk} = (\mathbf{S}^l_{kji})^T, \mathbf{S}^l_{ikj} = (\mathbf{S}^l_{jki})^T, \mathbf{Y}^l_{jik} = (\mathbf{Y}^l_{kij})^T, \mathbf{Y}^l_{ijk} = (\mathbf{Y}^l_{kji})^T, \mathbf{Y}^l_{ikj} = (\mathbf{Y}^l_{jki})^T, \mathbf{Z}^l_{jik} = (\mathbf{Z}^l_{kij})^T, \mathbf{Z}^l_{ijk} = (\mathbf{Z}^l_{kji})^T, \mathbf{Z}^l_{ikj} = (\mathbf{Z}^l_{jki})^T, i = 1,\dots,p-1, j = i+1,\dots,p-1, k = j+1,\dots,p, l = 1,2,3$; $\mathbf{T}^l_{ijk}, \mathbf{U}^l_{ijk}, \mathbf{W}^l_{ijk},\ i,j,k = 1,2,\dots,p; 1 = 1,2,3$ such that the following LMIs are satisfied: $\mathbf{R}_{ij} + \mathbf{R}_{ij}{}^T \geq 0,\ i,j = 1,2,\dots P$; $\mathbf{M} > 0$; and

$$\Psi^{11}_{ii} < \mathbf{S}^1_{iii},\ i = 1,\dots,p \tag{21}$$

$$\Psi^{22}_{ii} < \mathbf{S}^2_{iii},\ i = 1,\dots,p \tag{22}$$

$$\Psi^{33}_{ii} < \mathbf{S}^3_{iii},\ i = 1,\dots,p \tag{23}$$

$$\Psi^{11}_{ii} + \Psi^{11}_{ij} + \Psi^{11}_{ji} \leq \mathbf{S}^1_{iji} + \mathbf{S}^1_{iij} + (\mathbf{S}^1_{iij})^T, i = 1,\dots,p, j \neq i, j = 1,\dots,p \tag{24}$$

$$\Psi^{11}_{ii} + \Psi^{12}_{ij} + (\Psi^{12}_{ji})^T \leq \mathbf{S}^1_{iji} + \mathbf{T}^1_{iij} + (\mathbf{T}^1_{iij})^T, i = 1,\dots,p, j = 1,\dots,p \tag{25}$$

$$\Psi^{11}_{ii} + \Psi^{13}_{ij} + (\Psi^{13}_{ji})^T \leq \mathbf{S}^3_{iji} + \mathbf{U}^1_{iij} + (\mathbf{U}^1_{iij})^T, i = 1,\dots,p, j = 1,\dots,p \tag{26}$$

$$\Psi^{22}_{ii} + \Psi^{12}_{ji} + (\Psi^{12}_{ji})^T \leq \mathbf{Y}^1_{iji} + \mathbf{T}^2_{jii} + (\mathbf{T}^2_{jii})^T, i = 1,\dots,p, j = 1,\dots,p \tag{27}$$

$$\Psi^{22}_{ii} + \Psi^{22}_{ij} + \Psi^{22}_{ji} \leq \mathbf{Y}^2_{iji} + \mathbf{Y}^2_{iij} + (\mathbf{Y}^2_{iij})^T, i = 1,\dots,p, j \neq i, j = 1,\dots,p \tag{28}$$

$$\Psi^{22}_{ii} + \Psi^{23}_{ij} + (\Psi^{23}_{ij})^T \leq \mathbf{Y}^3_{iji} + \mathbf{W}^2_{iij} + (\mathbf{W}^2_{iij})^T, i = 1,\dots,p, j = 1,\dots,p \tag{29}$$

$$\Psi^{33}_{ii} + \Psi^{13}_{ji} + (\Psi^{13}_{ji})^T \leq \mathbf{Z}^1_{iji} + \mathbf{U}^3_{jii} + (\mathbf{U}^3_{jii})^T, i = 1,\dots,p, j = 1,\dots,p \tag{30}$$

$$\Psi^{33}_{ii} + \Psi^{23}_{ji} + (\Psi^{23}_{ji})^T \leq \mathbf{Z}^2_{iji} + \mathbf{W}^3_{jii} + (\mathbf{W}^3_{jii})^T, i = 1,\dots,p, j = 1,\dots,p \tag{31}$$

$$\Psi^{33}_{ii} + \Psi^{33}_{ij} + \Psi^{33}_{ji} \leq \mathbf{Z}^3_{iji} + \mathbf{Z}^3_{iij} + (\mathbf{Z}^3_{iij})^T, i = 1,\dots,p, j \neq i, j = 1,\dots,p \tag{32}$$

$$\begin{aligned}&\Psi^{11}_{ij} + \Psi^{11}_{ji} + \Psi^{11}_{ik} + \Psi^{11}_{ki} + \Psi^{11}_{jk} + \Psi^{11}_{kj}\\ &\quad \leq \mathbf{S}^1_{jik} + (\mathbf{S}^1_{jik})^T + \mathbf{S}^1_{ijk} + (\mathbf{S}^1_{ijk})^T + \mathbf{S}^1_{ikj} + (\mathbf{S}^1_{ikj})^T,\\ &\quad\ i = 1,\dots,p-2, j = i+1,\dots,p-1, k = j+1,\dots,p\end{aligned} \tag{33}$$

$$\begin{aligned}&\Psi^{11}_{ij} + \Psi^{11}_{ji} + \Psi^{12}_{ik} + (\Psi^{12}_{ik})^T + \Psi^{12}_{jk} + (\Psi^{12}_{jk})^T\\ &\quad \leq \mathbf{S}^2_{ikj} + (\mathbf{S}^2_{ikj})^T + \mathbf{T}^1_{ijk} + (\mathbf{T}^1_{ijk})^T + \mathbf{T}^1_{jik} + (\mathbf{T}^1_{jik})^T\\ &\quad\ i = 1,\dots,p-1,\ j = i+1,\dots,p,\ k = 1,\dots,p\end{aligned} \tag{34}$$

$$\begin{aligned}&\Psi^{11}_{ij} + \Psi^{11}_{ji} + \Psi^{13}_{ik} + (\Psi^{13}_{ik})^T + \Psi^{13}_{jk} + (\Psi^{13}_{jk})^T\\ &\quad \leq \mathbf{S}^3_{ikj} + (\mathbf{S}^3_{ikj})^T + \mathbf{U}^1_{ijk} + (\mathbf{U}^1_{ijk})^T + \mathbf{U}^1_{jik} + (\mathbf{U}^1_{jik})^T\\ &\quad\ i = 1,\dots,p-1,\ j = i+1,\dots,p,\ k = 1,\dots,p\end{aligned} \tag{35}$$

$$\begin{aligned}&\Psi^{22}_{jk} + \Psi^{22}_{kj} + \Psi^{12}_{ij} + (\Psi^{12}_{ij})^T + \Psi^{12}_{ik} + (\Psi^{12}_{ik})^T\\ &\quad \leq \mathbf{Y}^1_{jik} + (\mathbf{Y}^1_{jik})^T + \mathbf{T}^2_{ijk} + (\mathbf{T}^2_{ijk})^T + \mathbf{T}^2_{ikj} + (\mathbf{T}^2_{ikj})^T\\ &\quad\ i = 1,\dots,p,\ j = 1,\dots,p-1,\ k = j+1,\dots,p\end{aligned} \tag{36}$$

$$\begin{aligned}&\Psi^{12}_{ij} + (\Psi^{12}_{ij})^T + \Psi^{13}_{ik} + (\Psi^{13}_{ik})^T + \Psi^{23}_{jk} + (\Psi^{23}_{jk})^T\\ &\quad \leq \mathbf{W}^1_{jik} + (\mathbf{W}^1_{jik})^T + \mathbf{U}^2_{ijk} + (\mathbf{U}^2_{ijk})^T + \mathbf{T}^3_{ikj} + (T^3_{ikj})^T\\ &\quad\ i = 1,\dots,p,\ j = 1,\dots,p,\ k = 1,\dots,p\end{aligned} \tag{37}$$

$$\begin{aligned}&\Psi^{33}_{jk} + \Psi^{33}_{kj} + \Psi^{13}_{ij} + (\Psi^{13}_{ij})^T + \Psi^{13}_{ik} + (\Psi^{13}_{ik})^T\\ &\quad \leq \mathbf{Z}^1_{jik} + (\mathbf{Z}^1_{jik})^T + \mathbf{U}^3_{ijk} + (\mathbf{U}^3_{ijk})^T + \mathbf{U}^3_{ikj} + (\mathbf{U}^3_{ikj})^T\end{aligned}$$

$$i=1,\dots,p,\ j=1,\dots,p-1,\ k=j+1,\dots,p \tag{38}$$

$$\begin{aligned}\Psi_{ij}^{22}+\Psi_{ji}^{22}+\Psi_{ik}^{22}&+\Psi_{ki}^{22}+\Psi_{jk}^{22}+\Psi_{kj}^{22}\\ &\le \mathbf{Y}_{jik}^{2}+(\mathbf{Y}_{jik}^{2})^T+\mathbf{Y}_{ijk}^{2}+(\mathbf{Y}_{ijk}^{2})^T+\mathbf{Y}_{ikj}^{2}+(\mathbf{Y}_{ikj}^{2})^T\\ &\quad i=1,\dots,p-2,\ j=i+1,\dots,p-1,\ k=j+1,\dots,p\end{aligned} \tag{39}$$

$$\begin{aligned}\Psi_{ij}^{22}+\Psi_{ji}^{22}+\Psi_{ik}^{23}&+(\Psi_{ik}^{23})^T+\Psi_{jk}^{23}+(\Psi_{jk}^{23})^T\\ &\le \mathbf{Y}_{ikj}^{3}+(\mathbf{Y}_{ikj}^{3})^T+\mathbf{W}_{jik}^{2}+(\mathbf{W}_{jik}^{2})^T+\mathbf{W}_{ijk}^{2}+(\mathbf{W}_{ijk}^{2})^T\\ &\quad i=1,\dots,p-1,\ j=i+1,\dots,p,\ k=1,\dots,p\end{aligned} \tag{40}$$

$$\begin{aligned}\Psi_{jk}^{33}+\Psi_{kj}^{33}+\Psi_{ij}^{23}&+(\Psi_{ij}^{23})^T+\Psi_{ik}^{23}+(\Psi_{ik}^{23})^T\\ &\le \mathbf{Z}_{jik}^{2}+(\mathbf{Z}_{jik}^{2})^T+\mathbf{W}_{ijk}^{3}+(\mathbf{W}_{ijk}^{3})^T+\mathbf{W}_{ikj}^{3}+(\mathbf{W}_{ikj}^{3})^T\\ &\quad i=1,\dots,p,\ j=1,\dots,p-1,\ k=j+1,\dots,p\end{aligned} \tag{41}$$

$$\begin{aligned}\Psi_{ij}^{33}+\Psi_{ji}^{33}+\Psi_{ik}^{33}&+\Psi_{ki}^{33}+\Psi_{jk}^{33}+\Psi_{kj}^{33}\\ &\le \mathbf{Z}_{jik}^{3}+(\mathbf{Z}_{jik}^{3})^T+\mathbf{Z}_{ijk}^{3}+(\mathbf{Z}_{ijk}^{3})^T+\mathbf{Z}_{ikj}^{3}+(\mathbf{Z}_{ikj}^{3})^T\\ &\quad i=1,\dots,p-2,\ j=i+1,\dots,p-1,\ k=j+1,\dots,p\end{aligned} \tag{42}$$

$$\begin{bmatrix}\mathbf{S}_i^l & \mathbf{T}_i^l & \mathbf{U}_i^l\\ (\mathbf{T}_i^l)^T & \mathbf{Y}_i^l & \mathbf{W}_i^l\\ (\mathbf{U}_i^l)^T & (\mathbf{W}_i^l)^T & \mathbf{Z}_i^l\end{bmatrix}<0,\ i=1,\dots,p,\ l=1,2,3 \tag{43}$$

where

$$\mathbf{S}_i^l=\begin{bmatrix}\mathbf{S}_{1i1}^l & \mathbf{S}_{1i2}^l & \cdots & \mathbf{S}_{1ip}^l\\ \mathbf{S}_{2i1}^l & \mathbf{S}_{2i2}^l & \cdots & \mathbf{S}_{2ip}^l\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{S}_{pi1}^l & \mathbf{S}_{pi2}^l & \cdots & \mathbf{S}_{pip}^l\end{bmatrix},\quad \mathbf{Y}_i^l=\begin{bmatrix}\mathbf{Y}_{1i1}^l & \mathbf{Y}_{1i2}^l & \cdots & \mathbf{Y}_{1ip}^l\\ \mathbf{Y}_{2i1}^l & \mathbf{Y}_{2i2}^l & \cdots & \mathbf{Y}_{2ip}^l\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{Y}_{pi1}^l & \mathbf{Y}_{pi2}^l & \cdots & \mathbf{Y}_{pip}^l\end{bmatrix},$$

$$\mathbf{Z}_i^l=\begin{bmatrix}\mathbf{Z}_{1i1}^l & \mathbf{Z}_{1i2}^l & \cdots & \mathbf{Z}_{1ip}^l\\ \mathbf{Z}_{2i1}^l & \mathbf{Z}_{2i2}^l & \cdots & \mathbf{Z}_{2ip}^l\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{Z}_{pi1}^l & \mathbf{Z}_{pi2}^l & \cdots & \mathbf{Z}_{pip}^l\end{bmatrix},\quad \mathbf{T}_i^l=\begin{bmatrix}\mathbf{T}_{1i1}^l & \mathbf{T}_{1i2}^l & \cdots & \mathbf{T}_{1ip}^l\\ \mathbf{T}_{2i1}^l & \mathbf{T}_{2i2}^l & \cdots & \mathbf{T}_{2ip}^l\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{T}_{pi1}^l & \mathbf{T}_{pi2}^l & \cdots & \mathbf{T}_{pip}^l\end{bmatrix},$$

$$\mathbf{U}_i^l=\begin{bmatrix}\mathbf{U}_{1i1}^l & \mathbf{U}_{1i2}^l & \cdots & \mathbf{U}_{1ip}^l\\ \mathbf{U}_{2i1}^l & \mathbf{U}_{2i2}^l & \cdots & \mathbf{U}_{2ip}^l\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{U}_{pi1}^l & \mathbf{U}_{pi2}^l & \cdots & \mathbf{U}_{pip}^l\end{bmatrix},\quad \mathbf{W}_i^l=\begin{bmatrix}\mathbf{W}_{1i1}^l & \mathbf{W}_{1i2}^l & \cdots & \mathbf{W}_{1ip}^l\\ \mathbf{W}_{2i1}^l & \mathbf{W}_{2i2}^l & \cdots & \mathbf{W}_{2ip}^l\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{W}_{pi1}^l & \mathbf{W}_{pi2}^l & \cdots & \mathbf{W}_{pip}^l\end{bmatrix}.$$

The feedback gains are defined as: $\mathbf{G}_j=\mathbf{N}_j\mathbf{M}^{-1},\ j=1,2,\dots,p$.

Proof. Based on the property of the membership functions that $\sum_{i=1}^p \omega_i=\sum_{j=1}^p(\underline{\omega}_j+\bar{\omega}_j)=1$, it is clear that $\frac{1}{2}\sum_{i=1}^p(\omega_i+\underline{\omega}_i+\bar{\omega}_i)=1$. From (20), we have

$$\Pi \le \frac{1}{2}\sum_{k=1}^p(\omega_k+\underline{\omega}_k+\bar{\omega}_k)\times\Big\{\sum_{i=1}^p\sum_{j=1}^p\omega_i\omega_j(\Psi_{ij}^{11})+\sum_{i=1}^p\sum_{j=1}^p\omega_i\underline{\omega}_j(\Psi_{ij}^{12}+(\Psi_{ij}^{12})^T)$$

$$
\begin{aligned}
&+\sum_{i=1}^{p}\sum_{j=1}^{p}\omega_i\bar{\omega}_j(\Psi_{ij}^{13}+(\Psi_{ij}^{13})^T)+\sum_{i=1}^{p}\sum_{j=1}^{p}\underline{\omega}_i\underline{\omega}_j(\Psi_{ij}^{22})+\sum_{i=1}^{p}\sum_{j=1}^{p}\underline{\omega}_i\bar{\omega}_j(\Psi_{ij}^{23}+(\Psi_{ij}^{23})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}\bar{\omega}_i\bar{\omega}_j(\Psi_{ij}^{33})\Bigg\}\\
=&\frac{1}{2}\left\{\sum_{i=1}^{p}{\omega_i}^3\Psi_{ii}^{11}+\sum_{i=1}^{p}{\underline{\omega}_i}^3\Psi_{ii}^{22}+\sum_{i=1}^{p}\bar{\omega}_i^3\Psi_{ii}^{33}\right\}+\frac{1}{2}\Bigg\{\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{p}{\omega_i}^2\omega_j(\Psi_{ii}^{11}+\Psi_{ij}^{11}+\Psi_{ji}^{11})\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}{\omega_i}^2\underline{\omega}_j(\Psi_{ii}^{11}+\Psi_{ij}^{12}+(\Psi_{ij}^{12})^T)+\sum_{i=1}^{p}\sum_{j=1}^{p}{\omega_i}^2\bar{\omega}_j(\Psi_{ii}^{11}+\Psi_{ij}^{13}+(\Psi_{ij}^{13})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}{\underline{\omega}_i}^2\omega_j(\Psi_{ii}^{22}+\Psi_{ji}^{12}+(\Psi_{ji}^{12})^T)+\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{p}{\underline{\omega}_i}^2\underline{\omega}_j(\Psi_{ii}^{22}+\Psi_{ij}^{22}+\Psi_{ji}^{22})\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}{\underline{\omega}_i}^2\bar{\omega}_j(\Psi_{ii}^{22}+\Psi_{ij}^{23}+(\Psi_{ij}^{23})^T)+\sum_{i=1}^{p}\sum_{j=1}^{p}\bar{\omega}_i^2\omega_j(\Psi_{ii}^{33}+\Psi_{ji}^{13}+(\Psi_{ji}^{13})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}\bar{\omega}_i^2\underline{\omega}_j(\Psi_{ii}^{33}+\Psi_{ji}^{23}+(\Psi_{ji}^{23})^T)+\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{p}\bar{\omega}_i^2\bar{\omega}_j(\Psi_{ii}^{33}+\Psi_{ij}^{33}+\Psi_{ji}^{33})\Bigg\}\\
&+\frac{1}{2}\Bigg\{\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}\sum_{k=j+1}^{p}\omega_i\omega_j\omega_k(\Psi_{ij}^{11}+\Psi_{ji}^{11}+\Psi_{ik}^{11}+\Psi_{ki}^{11}+\Psi_{jk}^{11}+\Psi_{kj}^{11})\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{p}\omega_i\omega_j\underline{\omega}_k(\Psi_{ij}^{11}+\Psi_{ji}^{11}+\Psi_{ik}^{12}+(\Psi_{ik}^{12})^T+\Psi_{jk}^{12}+(\Psi_{jk}^{12})^T)\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{p}\omega_i\omega_j\bar{\omega}_k(\Psi_{ij}^{11}+\Psi_{ji}^{11}+\Psi_{ik}^{13}+(\Psi_{ik}^{13})^T+\Psi_{jk}^{13}+(\Psi_{jk}^{13})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\omega_i\underline{\omega}_j\underline{\omega}_k(\Psi_{jk}^{22}+\Psi_{kj}^{22}+\Psi_{ij}^{12}+(\Psi_{ij}^{12})^T+\Psi_{ik}^{12}+(\Psi_{ik}^{12})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}\omega_i\underline{\omega}_j\bar{\omega}_k(\Psi_{ij}^{12}+(\Psi_{ij}^{12})^T+\Psi_{ik}^{13}+(\Psi_{ik}^{13})^T+\Psi_{jk}^{23}+(\Psi_{jk}^{23})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\omega_i\bar{\omega}_j\bar{\omega}_k(\Psi_{jk}^{33}+\Psi_{kj}^{33}+\Psi_{ij}^{13}+(\Psi_{ij}^{13})^T+\Psi_{ik}^{13}+(\Psi_{ik}^{13})^T)\\
&+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}\sum_{k=j+1}^{p}\underline{\omega}_i\underline{\omega}_j\underline{\omega}_k(\Psi_{ij}^{22}+\Psi_{ji}^{22}+\Psi_{ik}^{22}+\Psi_{ki}^{22}+\Psi_{jk}^{22}+\Psi_{kj}^{22})\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{p}\underline{\omega}_i\underline{\omega}_j\bar{\omega}_k(\Psi_{ij}^{22}+\Psi_{ji}^{22}+\Psi_{ik}^{23}+(\Psi_{ik}^{23})^T+\Psi_{jk}^{23}+(\Psi_{jk}^{23})^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\underline{\omega}_i\bar{\omega}_j\bar{\omega}_k(\Psi_{jk}^{33}+\Psi_{kj}^{33}+\Psi_{ij}^{23}+(\Psi_{ij}^{23})^T+\Psi_{ik}^{23}+(\Psi_{ik}^{23})^T)
\end{aligned}
$$

$$+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}\sum_{k=j+1}^{p}\bar{\omega}_i\bar{\omega}_j\bar{\omega}_k(\Psi_{ij}^{33}+\Psi_{ji}^{33}+\Psi_{ik}^{33}+\Psi_{ki}^{33}+\Psi_{jk}^{33}+\Psi_{kj}^{33})\Bigg\}. \tag{44}$$

From (21) to (42), (44) can be represented as:

$$\begin{aligned}
\Pi <&\frac{1}{2}\left\{\sum_{i=1}^{p}{\omega_i}^3\mathbf{S}_{iii}^1+\sum_{i=1}^{p}{\underline{\omega}_i}^3\mathbf{Y}_{iii}^2+\sum_{i=1}^{p}\bar{\omega}_i^3\mathbf{Z}_{iii}^3\right\}+\frac{1}{2}\Bigg\{\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{p}{\omega_i}^2\omega_j(\mathbf{S}_{iji}^1+\mathbf{S}_{iij}^1+(\mathbf{S}_{iij}^1)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}{\omega_i}^2\underline{\omega}_j(\mathbf{S}_{iji}^2+\mathbf{T}_{iij}^1+(\mathbf{T}_{iij}^1)^T)+\sum_{i=1}^{p}\sum_{j=1}^{p}{\omega_i}^2\bar{\omega}_j(\mathbf{S}_{iji}^3+\mathbf{U}_{iij}^1+(\mathbf{U}_{iij}^1)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}{\underline{\omega}_i}^2\omega_j(\mathbf{Y}_{iji}^1+\mathbf{T}_{jii}^2+(\mathbf{T}_{jii}^2)^T)+\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{p}{\underline{\omega}_i}^2\underline{\omega}_j(\mathbf{Y}_{iji}^2+\mathbf{Y}_{iij}^2+(\mathbf{Y}_{iij}^2)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}{\underline{\omega}_i}^2\bar{\omega}_j(\mathbf{Y}_{iji}^3+\mathbf{W}_{iij}^2+(\mathbf{W}_{iij}^2)^T)+\sum_{i=1}^{p}\sum_{j=1}^{p}\bar{\omega}_i^2\omega_j(\mathbf{Z}_{iji}^1+\mathbf{U}_{jii}^3+(\mathbf{U}_{jii}^3)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}\bar{\omega}_i^2\underline{\omega}_j(\mathbf{Z}_{iji}^2+\mathbf{W}_{jii}^3+(\mathbf{W}_{jii}^3)^T)+\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{p}\bar{\omega}_i^2\bar{\omega}_j(\mathbf{Z}_{iji}^3+\mathbf{Z}_{iij}^3+(\mathbf{Z}_{iij}^3)^T)\Bigg\}\\
&+\frac{1}{2}\Bigg\{\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}\sum_{k=j+1}^{p}\omega_i\omega_j\omega_k(\mathbf{S}_{jik}^1+(\mathbf{S}_{jik}^1)^T+\mathbf{S}_{ijk}^1+(\mathbf{S}_{ijk}^1)^T+\mathbf{S}_{ikj}^1+(\mathbf{S}_{ikj}^1)^T)\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{p}\omega_i\omega_j\underline{\omega}_k(\mathbf{S}_{ikj}^2+(\mathbf{S}_{ikj}^2)^T+\mathbf{T}_{ijk}^1+(\mathbf{T}_{ijk}^1)^T+\mathbf{T}_{jik}^1+(\mathbf{T}_{jik}^1)^T)\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{p}\omega_i\omega_j\bar{\omega}_k(\mathbf{S}_{ikj}^3+(\mathbf{S}_{ikj}^3)^T+\mathbf{U}_{ijk}^1+(\mathbf{U}_{ijk}^1)^T+\mathbf{U}_{jik}^1+(\mathbf{U}_{jik}^1)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\omega_i\underline{\omega}_j\underline{\omega}_k(\mathbf{Y}_{jik}^1+(\mathbf{Y}_{jik}^1)^T+\mathbf{T}_{ijk}^2+(\mathbf{T}_{ijk}^2)^T+\mathbf{T}_{ikj}^2+(\mathbf{T}_{ikj}^2)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}\omega_i\underline{\omega}_j\bar{\omega}_k(\mathbf{W}_{jik}^1+(\mathbf{W}_{jik}^1)^T+\mathbf{U}_{ijk}^2+(\mathbf{U}_{ijk}^2)^T+\mathbf{T}_{ikj}^3+(\mathbf{T}_{ikj}^3)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\omega_i\bar{\omega}_j\bar{\omega}_k(\mathbf{Z}_{jik}^1+(\mathbf{Z}_{jik}^1)^T+\mathbf{U}_{ijk}^3+(\mathbf{U}_{ijk}^3)^T+\mathbf{U}_{ikj}^3+(\mathbf{U}_{ikj}^3)^T)\\
&+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}\sum_{k=j+1}^{p}\underline{\omega}_i\underline{\omega}_j\underline{\omega}_k(\mathbf{Y}_{jik}^2+(\mathbf{Y}_{jik}^2)^T+\mathbf{Y}_{ijk}^2+(\mathbf{Y}_{ijk}^2)^T+\mathbf{Y}_{ikj}^2+(\mathbf{Y}_{ikj}^2)^T)\\
&+\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{p}\underline{\omega}_i\underline{\omega}_j\bar{\omega}_k(\mathbf{Y}_{ikj}^3+(\mathbf{Y}_{ikj}^3)^T+\mathbf{W}_{jik}^2+(\mathbf{W}_{jik}^2)^T+\mathbf{W}_{ijk}^2+(\mathbf{W}_{ijk}^2)^T)\\
&+\sum_{i=1}^{p}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\underline{\omega}_i\bar{\omega}_j\bar{\omega}_k(\mathbf{Z}_{jik}^2+(\mathbf{Z}_{jik}^2)^T+\mathbf{W}_{ijk}^3+(\mathbf{W}_{ijk}^3)^T+\mathbf{W}_{ikj}^3+(\mathbf{W}_{ikj}^3)^T)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}\sum_{k=j+1}^{p}\bar{\omega}_i\bar{\omega}_j\bar{\omega}_k(\mathbf{Z}^3_{jik}+(\mathbf{Z}^3_{jik})^T+\mathbf{Z}^3_{ijk}+(\mathbf{Z}^3_{ijk})^T+\mathbf{Z}^3_{ikj}+(\mathbf{Z}^3_{ikj})^T)\Bigg\}\\
=&\frac{1}{2}\omega_1 r^T\begin{bmatrix}\mathbf{S}^1_1 & \mathbf{T}^1_1 & \mathbf{U}^1_1\\ (\mathbf{T}^1_1)^T & \mathbf{Y}^1_1 & \mathbf{W}^1_1\\ (\mathbf{U}^1_1)^T & (\mathbf{W}^1_1)^T & \mathbf{Z}^1_1\end{bmatrix}r+\cdots+\frac{1}{2}\omega_p r^T\begin{bmatrix}\mathbf{S}^1_p & \mathbf{T}^1_p & \mathbf{U}^1_p\\ (\mathbf{T}^1_p)^T & \mathbf{Y}^1_p & \mathbf{W}^1_p\\ (\mathbf{U}^1_p)^T & (\mathbf{W}^1_p)^T & \mathbf{Z}^1_p\end{bmatrix}r\\
&+\frac{1}{2}\underline{\omega}_1 r^T\begin{bmatrix}\mathbf{S}^2_1 & \mathbf{T}^2_1 & \mathbf{U}^2_1\\ (\mathbf{T}^2_1)^T & \mathbf{Y}^2_1 & \mathbf{W}^2_1\\ (\mathbf{U}^2_1)^T & (\mathbf{W}^2_1)^T & \mathbf{Z}^2_1\end{bmatrix}r+\cdots+\frac{1}{2}\underline{\omega}_p r^T\begin{bmatrix}\mathbf{S}^2_p & \mathbf{T}^2_p & \mathbf{U}^2_p\\ (\mathbf{T}^2_p)^T & \mathbf{Y}^2_p & \mathbf{W}^2_p\\ (\mathbf{U}^2_p)^T & (\mathbf{W}^2_p)^T & \mathbf{Z}^2_p\end{bmatrix}r\\
&+\frac{1}{2}\bar{\omega}_1 r^T\begin{bmatrix}\mathbf{S}^3_1 & \mathbf{T}^3_1 & \mathbf{U}^3_1\\ (\mathbf{T}^3_1)^T & \mathbf{Y}^3_1 & \mathbf{W}^3_1\\ (\mathbf{U}^3_1)^T & (\mathbf{W}^3_1)^T & \mathbf{Z}^3_1\end{bmatrix}r+\cdots+\frac{1}{2}\bar{\omega}_p r^T\begin{bmatrix}\mathbf{S}^3_p & \mathbf{T}^3_p & \mathbf{U}^3_p\\ (\mathbf{T}^3_p)^T & \mathbf{Y}^3_p & \mathbf{W}^3_p\\ (\mathbf{U}^3_p)^T & (\mathbf{W}^3_p)^T & \mathbf{Z}^3_p\end{bmatrix}r\\
=&r^T\left(\frac{1}{2}\sum_{i=1}^{p}\omega_i\begin{bmatrix}\mathbf{S}^1_i & \mathbf{T}^1_i & \mathbf{U}^1_i\\ (\mathbf{T}^1_i)^T & \mathbf{Y}^1_i & \mathbf{W}^1_i\\ (\mathbf{U}^1_i)^T & (\mathbf{W}^1_i)^T & \mathbf{Z}^1_i\end{bmatrix}\right)r+r^T\left(\frac{1}{2}\sum_{i=1}^{p}\underline{\omega}_i\begin{bmatrix}\mathbf{S}^2_i & \mathbf{T}^2_i & \mathbf{U}^2_i\\ (\mathbf{T}^2_i)^T & \mathbf{Y}^2_i & \mathbf{W}^2_i\\ (\mathbf{U}^2_i)^T & (\mathbf{W}^2_i)^T & \mathbf{Z}^2_i\end{bmatrix}\right)r\\
&+r^T\left(\frac{1}{2}\sum_{i=1}^{p}\bar{\omega}_i\begin{bmatrix}\mathbf{S}^3_i & \mathbf{T}^3_i & \mathbf{U}^3_i\\ (\mathbf{T}^3_i)^T & \mathbf{Y}^3_i & \mathbf{W}^3_i\\ (\mathbf{U}^3_i)^T & (\mathbf{W}^3_i)^T & \mathbf{Z}^3_i\end{bmatrix}\right)r
\end{aligned}
\tag{45}
$$

where $r=[\omega_1 I\ \ \ldots\ \ \omega_p I\ \ \underline{\omega}_1 I\ \ \ldots\ \ \underline{\omega}_p I\ \ \bar{\omega}_1 I\ \ \ldots\ \ \bar{\omega}_p I]$. Thus, the conditions (43) implies $\Xi < 0$. The whole proof is completed. □

Remark 1. The conservativeness of stability analysis results can be reduced by introducing several slack matrices. However, too many variables would increase computation burden. Thus, it is very important to develop fast algorithm to reduce the computation time in future work.

To illustrate that the conditions of Theorem 3.1 reduce the conservativeness in stability analysis, the results of [11] are firstly recalled with some modified notations in Lemma 3.2. The theoretical poof is also given to show that the conditions of Theorem 3.1 obtain more relaxed results than those of Lemma 3.2.

Lemma 3.2. (Lam and Seneviratne [11]) The interval type-2 FMB control system of (7) is asymptotically stable if there exist predefined scalars of $\rho_{i1},\sigma_{i1},\gamma_{i1},\rho_{i2},\sigma_{i2}$, and γ_{i1} such that $-\omega_i+\rho_{i1}\underline{\omega}_i+\sigma_{i1}\bar{\omega}_i+\gamma_{i1}\geq 0$ and $\omega_i-\rho_{i2}\underline{\omega}_i-\sigma_{i2}\bar{\omega}_i+\gamma_{i2}\geq 0$ are satisfied, and there exist matrices of $\mathbf{C}_j,\mathbf{D}_j,\mathbf{E}_j\in Re,\ j=1,\ldots p$; $\mathbf{M}=\mathbf{M}^T\in\Re^{n\times n}$; $\mathbf{R}_{ij}={\mathbf{R}_{ji}}^T,\mathbf{S}_{ij}={\mathbf{S}_{ji}}^T,\mathbf{Y}_{ij}={\mathbf{Y}_{ji}}^T,\mathbf{Z}_{ij}={\mathbf{Z}_{ji}}^T,\mathbf{T}_{ij},\mathbf{T}_{ij},\mathbf{W}_{ij}\in\Re,\ i,j=1,\ldots,p$ such that the following LMIs are satisfied: $\mathbf{R}_{ij}+{\mathbf{R}_{ij}}^T\geq 0,\ i,j=1,2,\ldots.p;\mathbf{M}>0$;

$$\Psi^{11}_{ii}<\mathbf{S}_{ii},\ i=1,2,\ldots,p. \tag{46}$$

$$\Psi^{11}_{ij}+\Psi^{11}_{ji}\leq\mathbf{S}_{ij}+(S_{ij})^T,\ j=1,2,\ldots,p,i<j. \tag{47}$$

$$\Psi^{12}_{ij}+(\Psi^{12}_{ij})^T\leq\mathbf{T}_{ij}+(\mathbf{T}_{ij})^T,\ i=1,2,\ldots,p. \tag{48}$$

$$\Psi^{13}_{ij}+(\Psi^{13}_{ij})^T\leq\mathbf{U}_{ij}+(\mathbf{U}_{ij})^T,\ i=1,2,\ldots,p. \tag{49}$$

$$\Psi_{ii}^{22} < \mathbf{Y}_{ii}, i = 1, 2, \ldots, p. \tag{50}$$

$$\Psi_{ij}^{22} + \Psi_{ji}^{22} \leq \mathbf{Y}_{ij} + (\mathbf{Y}_{ij})^T,\ i = 1, 2, \ldots, p. \tag{51}$$

$$\Psi_{ij}^{23} + (\Psi_{ij}^{23})^T \leq \mathbf{W}_{ij} + (\mathbf{W}_{ij})^T,\ i, j = 1, 2, \ldots, p. \tag{52}$$

$$\Psi_{ii}^{33} < \mathbf{Z}_{ii}, i = 1, 2, \ldots, p. \tag{53}$$

$$\Psi_{ij}^{33} + \Psi_{ji}^{33} \leq \mathbf{Z}_{ij} + (\mathbf{Z}_{ij})^T,\ j = 1, 2, \ldots, p, i < j. \tag{54}$$

$$\begin{bmatrix} \mathbf{S} & \mathbf{T} & \mathbf{U} \\ \mathbf{T}^T & \mathbf{Y} & \mathbf{W} \\ \mathbf{U}^T & \mathbf{W}^T & \mathbf{Z} \end{bmatrix} < 0,\ i = 1, 2, \ldots, p. \tag{55}$$

Where

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} & \cdots & \mathbf{S}_{1p} \\ \mathbf{S}_{21} & \mathbf{S}_{22} & \cdots & \mathbf{S}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{p1} & \mathbf{S}_{p2} & \cdots & \mathbf{S}_{pp} \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{1p} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \cdots & \mathbf{Y}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{p1} & \mathbf{Y}_{p2} & \cdots & \mathbf{Y}_{pp} \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1p} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{p1} & \mathbf{Z}_{p2} & \cdots & \mathbf{Z}_{pp} \end{bmatrix},$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} & \cdots & \mathbf{T}_{1p} \\ \mathbf{T}_{21} & \mathbf{T}_{22} & \cdots & \mathbf{T}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{T}_{p1} & \mathbf{T}_{p2} & \cdots & \mathbf{T}_{pp} \end{bmatrix}, \mathbf{U} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} & \cdots & \mathbf{U}_{1p} \\ \mathbf{U}_{21} & \mathbf{U}_{22} & \cdots & \mathbf{U}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}_{p1} & \mathbf{U}_{p2} & \cdots & \mathbf{U}_{pp} \end{bmatrix}, \mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \cdots & \mathbf{W}_{1p} \\ \mathbf{W}_{21} & \mathbf{W}_{22} & \cdots & \mathbf{W}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{W}_{p1} & \mathbf{W}_{p2} & \cdots & \mathbf{W}_{pp} \end{bmatrix}.$$

The feedback gains are defined as $\mathbf{G}_j = \mathbf{N}_j \mathbf{M}^{-1},\ j = 1, 2, \ldots, p$.

It is proved that Theorem 3.1 always offers more relaxed results than Lemma 3.2 in the next Theorem.

Theorem 3.3. The set of solutions to LMIs in Lemma 3.2 is a subset of solutions to LMIs in Theorem 3.1.

Proof. Assume $\mathbf{S}_{ij} = \mathbf{S}_{ji}{}^T, \mathbf{Y}_{ij} = \mathbf{Y}_{ji}{}^T, \mathbf{Z}_{ij} = \mathbf{Z}_{ji}{}^T, \mathbf{T}_{ij} \in \mathbf{U}_{ji}$, and $\mathbf{W}_{ij} \in \Re^{n\times n}$ is a set of solutions to Lemma 3.2, and suppose some variables in Theorem 3.1 are chosen particularly by:

$$\mathbf{S}_{jii}^l = \mathbf{S}_{ji}, \mathbf{S}_{iji}^l = \mathbf{S}_{ii},\ i = 1, \ldots, p, j \neq i,\ j = 1, \ldots, p,\ l = 1, 2, 3 \tag{56}$$

$$\mathbf{S}_{iii}^l = \mathbf{S}_{ii}, i = 1, \ldots, p,\ l = 1, 2, 3 \tag{57}$$

$$\mathbf{S}_{jik}^l = \mathbf{S}_{jk}, \mathbf{S}_{ijk}^l = \mathbf{S}_{ik}, \mathbf{S}_{ikj}^l = \mathbf{S}_{ij},\ i = 1, \ldots, p-2,\ j = i+1, \ldots, p-1,\ k = j+1, \ldots, p,\ l = 1, 2, 3 \tag{58}$$

$$\mathbf{Y}_{jii}^l = \mathbf{Y}_{ji}, \mathbf{Y}_{iji}^l = \mathbf{Y}_{ii},\ i = 1, \ldots, p, j \neq i,\ j = 1, \ldots, p, l = 1, 2, 3 \tag{59}$$

$$\mathbf{Y}_{iii}^l = \mathbf{Y}_{ii}, i = 1, \ldots, p,\ l = 1, 2, 3 \tag{60}$$

$$\mathbf{Y}_{jik}^l = \mathbf{Y}_{jk}, \mathbf{Y}_{ijk}^l = \mathbf{Y}_{ik}, \mathbf{Y}_{ikj}^l = \mathbf{Y}_{ij},\ i = 1, \ldots, p-2,\ j = i+1, \ldots, p-1,\ k = j+1, \ldots, p,\ l = 1, 2, 3 \tag{61}$$

$$\mathbf{Z}_{jii}^l = \mathbf{Z}_{ji}, \mathbf{Z}_{iji}^l = \mathbf{Z}_{ii},\ i = 1, \ldots, p,\ j \neq i, j = 1, \ldots, p, l = 1, 2, 3 \tag{62}$$

$$\mathbf{Z}_{iii}^{l} = \mathbf{Z}_{ii}, i = 1, \ldots, p,\ l = 1, 2, 3 \tag{63}$$

$$\mathbf{Z}_{jik}^{l} = \mathbf{Z}_{jk}, \mathbf{Z}_{ijk}^{l} = \mathbf{Z}_{ik}, \mathbf{Z}_{ikj}^{l} = \mathbf{Z}_{ij},\ i = 1, \ldots, p-2,\ j = i+1, \ldots, p-1,\ k = j+1, \ldots, p,\ l = 1, 2, 3 \tag{64}$$

$$\mathbf{T}_{ijk}^{l} = \mathbf{T}_{ik},\ i, j, k = 1, \ldots .p,\ l = 1, 2, 3 \tag{65}$$

$$\mathbf{U}_{ijk}^{l} = \mathbf{U}_{ik},\ i, j, k = 1, \ldots .p,\ l = 1, 2, 3 \tag{66}$$

$$\mathbf{W}_{ijk}^{l} = \mathbf{W}_{ik},\ i, j, k = 1, \ldots .p, l = 1, 2, 3. \tag{67}$$

With the particular choice of (57), (60) and (63), the inequalities (21), (22) and (23) coincide with (46), (50) and (53), respectively.

In Lemma 3.2, (47) can be equivalently written as

$$\Psi_{ij}^{11} + \Psi_{ji}^{11} \le \mathbf{S}_{ij} + \mathbf{S}_{ji},\ i = 1, \ldots, p, j \ne i,\ j = 1, \ldots, p. \tag{68}$$

Summing (68) and (46), we have

$$\Psi_{ii}^{11} + \Psi_{ij}^{11} + \Psi_{ji}^{11} < \mathbf{S}_{ii} + \mathbf{S}_{ij} + \mathbf{S}_{ji},\ i = 1, \ldots, p, j \ne i,\ j = 1, \ldots, p. \tag{69}$$

With the particular choice of (56), the inequality (69) can be represented as

$$\Psi_{ii}^{11} + \Psi_{ij}^{11} + \Psi_{ji}^{11} < \mathbf{S}_{iji}^{1} + \mathbf{S}_{iij}^{1} + (\mathbf{S}_{iij}^{1})^T,\ i = 1, \ldots, p, j \ne i,\ j = 1, \ldots, p. \tag{70}$$

It is clear that the feasibility of (46) and (47) implies that of (70). On the other hand, the solutions to (70) are the subset of solutions to (24). Thus, the solutions to (46) and (47) are the subset of solutions to (24).

Similarly, it can be derived that the solutions to (46) and (48) − (54) are also the subset of solutions to (25) − (32).

Select $1 \le i < j < k \le p$. From (47), the following inequalities hold:

$$\Psi_{ij}^{11} + \Psi_{ji}^{11} \le \mathbf{S}_{ij} + (\mathbf{S}_{ij})^T \tag{71}$$

$$\Psi_{ik}^{11} + \Psi_{ki}^{11} \le \mathbf{S}_{ik} + (\mathbf{S}_{ik})^T \tag{72}$$

$$\Psi_{jk}^{11} + \Psi_{kj}^{11} \le \mathbf{S}_{jk} + (\mathbf{S}_{jk})^T. \tag{73}$$

Summing (71), (72) and (73), we have

$$\Psi_{ij}^{11} + \Psi_{ji}^{11} + \Psi_{ik}^{11} + \Psi_{ki}^{11} + \Psi_{jk}^{11} + \Psi_{kj}^{11} \le \mathbf{S}_{ij} + (\mathbf{S}_{ij})^T + \mathbf{S}_{ik} + (\mathbf{S}_{ik})^T + \mathbf{S}_{jk} + (\mathbf{S}_{jk})^T. \tag{74}$$

Obviously, (74) is equal to (33) with the particular choice of (58). It is sure that the feasibility of (47) implies that of (33).

Similarly, it can be checked that the feasibility of (47) − (49), (51), (52) and (54) implies that of (34) − (42).

With the particular choice of (56) − (67), the inequality (43) is reduced as (55).

Thus, we can conclude that the set of solutions to (46) − (55) is a subset of solutions to (21) − (43). The whole proof is completed. □

It can be seen form Theorem 3.3 that Theorem 3.1 obtains more relaxed results than Lemma 3.2.

4. SIMULATION EXAMPLE

In this section, several simulation examples are introduced to illustrate the effectiveness of the proposed conditions.

Example 4.1. Consider an interval type-2 T-S fuzzy system with the following rules:

Rule 1: if $x_1(t)$ is $\widetilde{M}_1$, then $\dot{x}(t) = \mathbf{A}_1 x(t) + \mathbf{B}_1 u(t)$;

Rule 2: if $x_1(t)$ is $\widetilde{M}_2$, then $\dot{x}(t) = \mathbf{A}_2 x(t) + \mathbf{B}_2 u(t)$;

where

$$x_1(t) \in [-10, 10], \mathbf{A}_1 = \begin{bmatrix} 2-a & -1.5 \\ 3+b & 2 \end{bmatrix}, \mathbf{A}_2 \begin{bmatrix} 2+a & -10+b \\ 1 & -2 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 \\ b-6 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

It is assumed that $10.40 \leq a \leq 10.60, 5.92 \leq b \leq 6.00$. The control rules are as follows:

Rule 1: if $x_1(t)$ is $\widetilde{M}_1$, then $u(t) = \mathbf{G}_1 x(t)$;

Rule 2: if $x_1(t)$ is $\widetilde{M}_2$, then $u(t) = \mathbf{G}_2 x(t)$.

The lower and upper membership functions are listed as follows:

$$\omega_1^L(x_1(t)) = 0.25 + 0.25 \times e^{-((x-5)^2/2)}, \quad \omega_1^U(x_1(t)) = 0.25 + 0.25 \times e^{-((x-5)^2/8)},$$
$$\omega_2^L(x_1(t)) = 1 - \omega_1^U(x_1(t)), \quad \omega_2^U(x_1(t)) = 1 - \omega_1^L(x_1(t)).$$

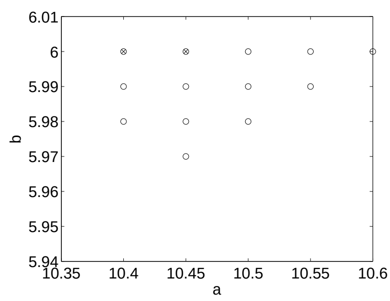


Fig. 1: Stability region for Theorem 3.1 and Lemma 3.2.

The same scalars satisfying the assumptions are adopted for both Theorem 3.1 and Lemma 1 to make an unbiased comparison. It can be shown that the assumptions hold with $\rho_{i1} = \sigma_{i1} = 2, \rho_{i2} = \sigma_{i2} = 0.1, \gamma_{i1} = -0.1$ and $\gamma_{i2} = -0.1$, for $i = 1, 2$. By employing Lemma 3.2, the stability region is shown in Figure 1 indicated by crosses. Based on the conditions in Theorem 3.1, the stability region is shown in Figure 1 indicated by open circles. It can be found from Figure 1 that Theorem 3.1 provides a larger stability region than Lemma 3.2. Hence, the proposed conditions obtain more relaxed results.

Example 4.2. Consider the following nonlinear system:

$$\dot{x}_1(t) = \sin(x_2(t)) + (\mathbf{M}_p {x_1}^2(t) + 1)u(t)$$
$$\dot{x}_1(t) = \sin(x_2(t)) + (\mathbf{M}_p {x_1}^2(t) + 1)u(t)$$

where $x_1(t) \in [-a, a], x_2(t) \in [-b, b], M_p \in [1, 2], a = 1$ and $b = 1$. M_p is regarded as parameter uncertainty. The above nonlinear system subject to parameter uncertainty can be represented by the following interval type-2 T-S fuzzy model:

Rule 1: if $x_1(t)$ is $\widetilde{M}_1^1$ and $x_2(t)$ is $\widetilde{M}_2^1$, then $\dot{x}(t) = \mathbf{A}_1 x(t) + \mathbf{B}_1 u(t)$;
Rule 2: if $x_1(t)$ is $\widetilde{M}_1^1$ and $x_2(t)$ is $\widetilde{M}_2^2$, then $\dot{x}(t) = \mathbf{A}_2 x(t) + \mathbf{B}_2 u(t)$;
Rule 3: if $x_1(t)$ is $\widetilde{M}_1^2$ and $x_2(t)$ is $\widetilde{M}_2^1$, then $\dot{x}(t) = \mathbf{A}_3 x(t) + \mathbf{B}_3 u(t)$;
Rule 4: if $x_1(t)$ is $\widetilde{M}_1^2$ and $x_2(t)$ is $\widetilde{M}_2^2$, then $\dot{x}(t) = \mathbf{A}_4 x(t) + \mathbf{B}_4 u(t)$;

where $\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 1 & a^2 \end{bmatrix}$, $\mathbf{A}_2 = \begin{bmatrix} 0 & \frac{\sin(b)}{b} \\ 1 & a^2 \end{bmatrix}$, $\mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{A}_4 = \begin{bmatrix} 0 & \frac{\sin(b)}{b} \\ 1 & 0 \end{bmatrix}$, $\mathbf{B}_1 = \begin{bmatrix} 1 + a^2 \\ 0 \end{bmatrix}$, $\mathbf{B}_2 = \begin{bmatrix} 1 + a^2 \\ 0 \end{bmatrix}$, $\mathbf{B}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{B}_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The firing strengths for each rule are defined as:

$$\omega_1^U(x(t)) = \begin{cases} 2 \times \frac{{x_1}^2}{a^2} \times \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))} & x_2 \neq 0 \\ 2 \times \frac{x_1^2}{a^2} & x_2 = 0, \end{cases}$$

$$\omega_1^L(x(t)) = \begin{cases} \frac{{x_1}^2}{a^2} \times \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))} & x_2 \neq 0 \\ \frac{x_1^2}{a^2} & x_2 = 0, \end{cases}$$

$$\omega_2^U(x(t)) = \begin{cases} 2 \times \frac{{x_1}^2}{a^2} \times (1 - \frac{{x_1}^2}{a^2} \times \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))}) & x_2 = 0 \\ 0 & x_2 \neq 0, \end{cases}$$

$$\omega_2^L(x(t)) = \begin{cases} \frac{{x_1}^2}{a^2} \times (1 - \frac{{x_1}^2}{a^2} \times \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))}) & x_2 = 0 \\ 0 & x_2 = 0, \end{cases}$$

$$\omega_3^U(x(t)) = \begin{cases} (1 - \frac{{x_1}^2}{a^2}) \times \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))} & x_2 \neq 0 \\ \left(1 - \frac{x_1^2}{a^2}\right) & x_2 = 0, \end{cases}$$

$$\omega_3^L(x(t)) = \begin{cases} (1 - 2 \times \frac{{x_1}^2}{a^2}) \times \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))} & x_2 \neq 0 \\ (1 - 2 \times \frac{x_1^2}{a^2}) & x_2 = 0, \end{cases}$$

$$\omega_4^U(x(t)) = \begin{cases} (1 - \frac{{x_1}^2}{a^2}) \times (1 - \frac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))}) & x_2 \neq 0 \\ 0 & x_2 = 0, \end{cases}$$

$$\omega_4^L(x(t)) = \begin{cases} \Big(1 - 2 \times \dfrac{{x_1}^2}{a^2}) \times (1 - \dfrac{b\sin(x_2) - x_2\sin(b)}{x_2(b - \sin(b))}\Big) & x_2 \neq 0 \\ 0 & x_2 = 0. \end{cases}$$

To stabilize the original system, the interval type-2 fuzzy controller is designed as follows:

Rule 1: if $x_1(t)$ is $\widetilde{M}_1^1$ and $x_2(t)$ is $\widetilde{M}_2^1$, then $u(t) = \mathbf{G}_1 x(t)$;
Rule 2: if $x_1(t)$ is $\widetilde{M}_1^1$ and $x_2(t)$ is $\widetilde{M}_2^2$, then $u(t) = \mathbf{G}_2 x(t)$;
Rule 3: if $x_1(t)$ is $\widetilde{M}_1^2$ and $x_2(t)$ is $\widetilde{M}_2^1$, then $u(t) = \mathbf{G}_3 x(t)$;
Rule 4: if $x_1(t)$ is $\widetilde{M}_1^2$ and $x_2(t)$ is $\widetilde{M}_2^2$, then $u(t) = \mathbf{G}_4 x(t)$.

It is noted that $\rho_{i1} = \sigma_{i1} = 2, \rho_{i2} = \sigma_{i2} = 0.01, \gamma_{i1} = -0.01$ and $\gamma_{i2} = -0.1,\ i = 1,2,3,4$ satisfy the assumptions in Theorem 3.1. Based on the Theorem 3.1, the control-gain matrices are computed as follows:

$$\mathbf{G}_1 = \begin{bmatrix} -74.663 & -151.35 \end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix} -72.835 & -147.58 \end{bmatrix};$$
$$\mathbf{G}_1 = \begin{bmatrix} -97.029 & -197.08 \end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix} -99.495 & -202.07 \end{bmatrix}.$$

The system-state responses of the closed-loop system with $M_p = 1$ under the initial states of $x_2(0) = -0.5$ and $x_2(0) = -0.5$ are shown in Figure 2. Figure 3 shows the system-state responses of the closed-loop system with $M_p = 2$ under initial states of $x_2(0) = -0.5$ and $x_2(0) = -0.5$. It can be seen from Figures 2 and 3 that the proposed conditions are effective for different parameter values.

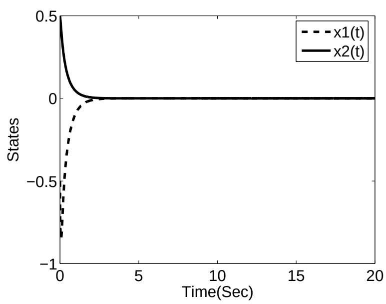


Fig. 2: State responses with $M_p = 1$.

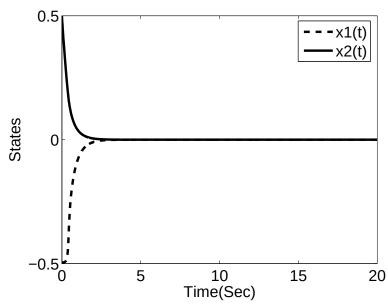


Fig. 3: State responses with $M_p = 2$.

Example 4.3. Consider the inverted pendulum subject to parameter uncertainties. The dynamic equation is described as:

$$\ddot{\theta}(t) = \frac{g\sin(\theta(t)) - am_pL\dot{\theta}(t)^2\sin(2\theta(t))/2 - a\cos(\theta(t))u(t)}{4L/3 - am_pL\cos^2(\theta(t))}$$

where $g = 9.8, m_p \in [2,3]$ and $m_c \in [8,16]$ are regarded as the parameter uncertainties, $a = 1/(m_c + m_p), 2L = 1, \theta(t)$ is the angular displacement of the pendulum, and $\mu(t)$ is the force applied to the cart.

Rule i: if $x_1(t)$ is $\widetilde{M}_1^i$ and $x_2(t)$ is $\widetilde{M}_2^i$, then $\dot{x}(t) = \mathbf{A}_i x(t) + \mathbf{B}_i u(t), i = 1,2,3,4.$
where $x(t) = \begin{bmatrix}\theta(t) & \dot{\theta}(t)\end{bmatrix}^T, \mathbf{A}_1 = \mathbf{A}_2 = \begin{bmatrix}0 & 1\\ f_{1_{\min}} & 0\end{bmatrix}, \mathbf{A}_3 = \mathbf{A}_4 = \begin{bmatrix}0 & 1\\ f_{1_{\max}} & 0\end{bmatrix}, \mathbf{B}_1 = \mathbf{B}_3 = \begin{bmatrix}0\\ f_{2_{\min}}\end{bmatrix}, \mathbf{B}_2 = \mathbf{B}_4 = \begin{bmatrix}0\\ f_{2_{\max}}\end{bmatrix}, f_{1_{\min}} = 10.0078, f_{1_{\max}} = 18.4800, f_{2_{\min}} = -0.1765, f_{2_{\max}} = -0.0261.$

It is assumed that the inverted pendulum works in the operating domain $x_1(t) = \theta(t) \in [-\frac{5}{12}\pi, \frac{5}{12}\pi]$ and $x_2(t) = \dot{\theta}(t) \in [-5, 5]$ in this example. The lower and upper membership functions are defined as follows:

$$\underline{u}_{\widetilde{M}_1^1}(x_1(t)) = 1 - e^{\frac{-x_1(t)^2}{1.2}}; \underline{u}_{\widetilde{M}_1^2}(x_1(t)) = 1 - e^{\frac{-x_1(t)^2}{1.2}}; \underline{u}_{\widetilde{M}_1^3}(x_1(t)) = 0.23e^{\frac{-x_1(t)^2}{0.25}};$$
$$\underline{u}_{\widetilde{M}_1^4}(x_1(t)) = 0.23e^{\frac{-x_1(t)^2}{0.25}}; \underline{u}_{\widetilde{M}_2^1}(x_1(t)) = 0.5e^{\frac{-x_1(t)^2}{0.25}}; \underline{u}_{\widetilde{M}_2^2}(x_1(t)) = 1 - e^{\frac{-x_1(t)^2}{1.5}};$$
$$\underline{u}_{\widetilde{M}_2^3}(x_1(t)) = 0.5e^{\frac{-x_1(t)^2}{0.25}}; \underline{u}_{\widetilde{M}_2^4}(x_1(t)) = 1 - e^{\frac{-x_1(t)^2}{1.5}}; \bar{u}_{\widetilde{M}_1^1}(x_1(t)) = 1 - 0.23e^{\frac{-x_1(t)^2}{0.25}};$$
$$\bar{u}_{\widetilde{M}_1^2}(x_1(t)) = 1 - 0.23e^{\frac{-x_1(t)^2}{0.25}}; \bar{u}_{\widetilde{M}_1^3}(x_1(t)) = e^{\frac{-x_1(t)^2}{1.2}}; \bar{u}_{\widetilde{M}_1^4}(x_1(t)) = e^{\frac{-x_1(t)^2}{1.2}};$$
$$\bar{u}_{\widetilde{M}_2^1}(x_1(t)) = e^{\frac{-x_1(t)^2}{1.5}}; \bar{u}_{\widetilde{M}_2^2}(x_1(t)) = 1 - 0.5e^{\frac{-x_1(t)^2}{0.25}}; \bar{u}_{\widetilde{M}_2^3}(x_1(t)) = e^{\frac{-x_1(t)^2}{1.5}};$$
$$\bar{u}_{\widetilde{M}_2^4}(x_1(t)) = 1 - 0.5e^{\frac{-x_1(t)^2}{0.25}}.$$

To stabilize the inverted pendulum subject to parameter uncertainties, the interval type-2 fuzzy controller is proposed with the following four rules:

Rule j: if $x_1(t)$ is $\widetilde{M}_1^j$ and $x_1(t)$ is $\widetilde{M}_2^j$, then $u(t) = \mathbf{G}_j x(t),\ j = 1,2,3,4.$

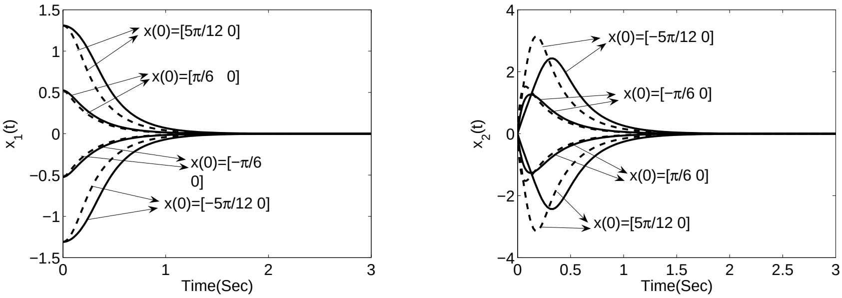
It is noted that $\rho_{ik} = 1, \sigma_{ik} = 0.01, \gamma_{11} = 0.456, \gamma_{21} = 0.27, \gamma_{31} = 0.565, \gamma_{41} = 0.195, \gamma_{i2} = 0.001,\ i = 1,2,3,4; k = 1,2,$ satisfy the assumptions in Theorem 3.1. Using the stability conditions in Theorem 3.1, the feedback-gain matrices are computed as follows:

$$\mathbf{G}_1 = \begin{bmatrix}950.71 & 244.57\end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix}954.17 & 245.57\end{bmatrix};$$
$$\mathbf{G}_1 = \begin{bmatrix}950.61 & 244.48\end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix}955.36 & 245.77\end{bmatrix}.$$

Figure 4 shows the states responses of the closed-loop systems with different initial states and different values of system parameters. It can be seen from Figure 4 that the proposed conditions are efficient for the inverted pendulum subject to different values of system parameters.

5. CONCLUSIONS

New LMI-based stability conditions for interval type-2 FMB control systems have been proposed in this paper. These proposed conditions have relaxed the existing results by using right-hand-side slack variables technique. The theoretical proof have illustrated that the obtained results have less conservativeness. Several simulation examples have



(a) $x_1(t)$ (b) $x_2(t)$

Fig. 4: State responses. Solid lines: $m_p = m_{p_{\max}}, m_c = m_{c_{\max}}$; Dotted lines: $m_p = m_{p_{\min}}, m_c = m_{c_{\min}}$.

also demonstrated the effectiveness of the proposed conditions. However, it should be noted that too many slack variables may increase computation burden. Thus, it is very important to develop fast algorithm to reduce the computation time and further work will focus on this issue.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (51177137, 61134001) and the Fundamental Research Funds for the Central Universities (SWJTU11CX034).

(Received March 18, 2013)

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