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GLOBAL FINITE-TIME STABILIZATION FOR A CLASS OF STOCHASTIC NONLINEAR SYSTEMS BY DYNAMIC STATE FEEDBACK

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This paper addresses the problem of global finite-time stabilization by dynamic state feedback for a class of stochastic nonlinear systems. Firstly, we show a dynamic state transformation, under which the original system is transformed into a new system. Then, a state feedback controller with a dynamic gain is designed for the new system. It is shown that global finite-time stabilization in probability for a class of stochastic nonlinear system under linear growth condition can be guaranteed by appropriately choosing design parameters. Finally, a simulation example is provided to demonstrate the effectiveness of the proposed design scheme.

Keywords: stochastic nonlinear systems, dynamic state transformation, finite-time stabilization

Classification: 93E12, 62A10

1. INTRODUCTION

In this paper, we consider the problem of global finite-time stabilization for a class of stochastic nonlinear systems described by

$$\begin{aligned} dx_i &= (x_{i+1} + f_i(x))\,dt + g_i^T(x)\,dw,\ i = 1, \dots, n-1, \\ dx_n &= (u + f_n(x))\,dt + g_n^T(x)\,dw \end{aligned} \tag{1}$$

where $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n,\ u \in \mathbb{R}$ are the system states and control input, respectively. w is an s-dimensional standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$ with Ω being a sample, $\mathcal{F}$ being a σ-field, and P being a probability measure. The nonlinear functions $f_i : \mathbb{R}^n \to \mathbb{R}$ and $g_i : \mathbb{R}^n \to \mathbb{R}^s, i = 1, \dots, n$ are $\mathcal{C}^1$ functions with respect to their arguments satisfying $f_i(0) = 0, g_i(0) = 0$. The objective of this paper is to find a dynamic state feedback controller of the form

$$\dot{r} = \Gamma(r, x),\ u = v(r, x) \tag{2}$$

with continuous functions $\Gamma : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+,\ v : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ satisfying $\Gamma(0, 0) = 0,\ v(0, 0) = 0$, such that the closed-loop system $(1)-(2)$ is globally finite-time stable in probability.

Stochastic modeling has come to play an important role in the field of engineering where stochastic differential equation has been applied for the analysis and control of stochastic system. Stochastic stability describes the most important characteristic of stochastic systems and has been extensively investigated for the analysis and design of stochastic control problems. A number of works have been focused on the problem of global asymptotic stabilization in probability for stochastic systems [3, 8, 17]. Deng and Krstic [3, 6] studied the problem of global asymptotic stabilization for a class of stochastic nonlinear systems by introducing the quartic Lyapunov function. The problem of state feedback stabilization for a class of high-order stochastic nonlinear systems with stochastic inverse dynamics which are neither feedback linearizable nor affine in the control input was investigated in [17]. The works [7] and [15] focused on adaptive state feedback controller design for a class of more general stochastic systems. Li et al. [8] further discussed a class of high-order stochastic nonlinear systems without strict triangular conditions. An output tracking problem for a class of stochastic system was addressed by designing a smooth state feedback controller in [16].

In this paper, combining the theory of finite-time stability in probability [2, 18] and the dynamic state feedback technique [13, 14, 21, 22], we aim to address the problem of global finite-time state feedback stabilization for a class of stochastic nonlinear systems. Due to fast convergence and good performance on robustness and disturbance rejection [1], the finite-time stabilization problem for nonlinear systems is very important both from the practical and theoretic point of view. However, all the aforementioned works are only limited to the global asymptotic stabilization in probability. To our best knowledge, there exist few research results on global finite-time stabilization for stochastic nonlinear systems. The main contributions of this paper are summarized as follows:

(i) The design procedure in our paper becomes much simpler than the backstepping approach and other recursive design [19, 20].

(ii) Compared with [10], the dynamic gain $r(t)$ in this paper has different features: (a) the initial value of gain r satisfies $r(0) > 0$; (b) the dynamic gain $r(t)$ is not required to be non-decreasing, but to keep positive before arriving at the origin and satisfy $r(t+T) = 0,\ \forall\, t \geq 0$ where $T := \inf\{t \geq 0; r(t; r(0)) = 0\}$.

(iii) The proposed controller in our paper is linear and it is easy to implement.

The remainder of this paper is organized as follows. Some preliminary results are indicated in Section 2. Section 3 provides the main design procedure of the finite-time controller. A simulation example is included in Section 4 to demonstrate the effectiveness of the proposed design scheme. Our conclusion is in Section 5.

Notations: $\mathbb{R}_+$ denotes the set of all nonnegative real numbers. For a given vector or matrix X, X^T denotes its transpose, $\mathrm{tr}\{X\}$ is its trace when X is square. $\|X\|$ denotes the Euclidean norm of a vector X, and the Frobenius norm of matrix X with $\|X\|_\infty = \max_{1\leq i\leq n}\{\sum_{j=1}^m |X_{ij}|\}$. $\lambda_{\max}(X)$ denotes the maximum eigenvalue of symmetric real matrix X. I is an identity matrix of appropriate dimension. $\mathcal{C}^i$ denotes the set of all functions with continuous ith partial derivatives. $\mathcal{K}$ denotes the set of all functions, $\mathbb{R}_+ \to \mathbb{R}_+$, which are continuous, strictly increasing and vanishing at zero. $\mathcal{K}_\infty$ denotes the set of all functions which are of class $\mathcal{K}$ and unbounded.

2. PRELIMINARIES

In this section, we review some terminologies related to the finite-time stability and the corresponding Lyapunov stability theory for n-dimensional stochastic nonlinear system of the form:

$$dx = f(x)\,dt + g^T(x)\,dw \tag{3}$$

where $x \in \mathbb{R}^n$ is the system states, w is an s-dimensional independent standard Wiener process. The functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{s\times n}$, also called the coefficients of the equation, are Borel measurable, continuous and satisfied with $f(0) = 0, g(0) = 0$.

Definition 2.1. (Khoo et al. [5]) The trivial solution of (3) is said to be finite-time stable in probability, if the solution exists for any initial data $x_0 \in \mathbb{R}^n$, denoted by $x(t; x_0)$. Moreover, the following statements hold:

(i) Finite-time attractiveness in probability: For every initial condition $x_0 \in \mathbb{R}^n\backslash\{0\}$, the first hitting time $T_{x_0} = \inf\{t; x(t; x_0) = 0\}$, which is called the stochastic setting time, is finite almost surely, that is, $P\{T_{x_0} < \infty\} = 1$;

(ii) Stability in probability: For every pair of $\varepsilon \in (0, 1)$ and $\epsilon > 0$, there exists a $\delta = \delta(\varepsilon, \epsilon) > 0$ such that

$$P\{|x(t; x_0)| < \epsilon, \text{for all } t \geq 0\} \geq 1 - \varepsilon \tag{4}$$

whenever $|x_0| < \delta$;

(iii) The solution $x((t + T_{x_0}); x_0)$ is unique for $t \geq 0$.

Remark 2.2. The finite-time attractiveness in probability states that the trajectories of a stochastic system will reach the origin in finite time with probability one, while stability in probability means insensitivity of the trajectories to small changes in the initial condition.

Definition 2.3. (Krstić and Deng [6]) For any given $V(x) \in \mathcal{C}^2$, associated with stochastic system (3), the differential operator $\mathcal{L}$ is defined as

$$\mathcal{L}V(x) = \frac{\partial V(x)}{\partial x} f(x) + \frac{1}{2}\mathrm{tr}\Big\{g(x)\frac{\partial^2 V(x)}{\partial x^2} g^T(x)\Big\}. \tag{5}$$

Lemma 2.4. (Yin et al. [18]) If there exists a $\mathcal{C}^2$ function $V : \mathbb{R}^n \to \mathbb{R}_+$, $\mathcal{K}_\infty$ class functions μ_1 and μ_2, positive real numbers $\eta > 0$ and $0 < \rho < 1$, such that for all $x \in \mathbb{R}^n$,

$$\mu_1(|x|) \leq V(x) \leq \mu_2(|x|),\ \mathcal{L}V(x) \leq -\eta(V(x))^\rho \tag{6}$$

then the trivial solution of (3) is finite-time attractive and stable in probability.

Lemma 2.5. (Qian and Lin [11]) Let c, d be positive real numbers and $\gamma(x, y) > 0$ be a real-valued function. Then,

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}(x,y)|y|^{c+d}. \tag{7}$$

3. FINITE-TIME CONTROLLER DESIGN

In this section, we will introduce a dynamic state transformation technique to design a state feedback controller globally stabilizing the stochastic nonlinear system (1) in finite time in probability. In order to achieve this goal, we list one basic assumption, which is a common requirement once in the early literature of robust and adaptive nonlinear control. For additional details in this regard, the interested reader is referred to [9] for a comprehensive exposition.

Assumption 3.1. For $i = 1, \dots, n$, there are constants $c_1 \geq 0$ and $c_2 \geq 0$ such that

$$\begin{aligned} |f_i(x)| &\leq c_1(|x_1| + \cdots + |x_i|), \\ \|g_i(x)\| &\leq c_2(|x_1| + \cdots + |x_i|). \end{aligned} \tag{8}$$

Remark 3.2. It must be pointed out that system (1) satisfying Assumption 3.1 represents an important class of nonlinear systems similar to those reported in [12]. The condition (8) is a general linear growth condition, which can be seen as a natural generalization of the well-known feedback linearizable condition [4, 9]. To tackle the nonlinearities in system (1) and obtain a linear state feedback controller, Assumption 3.1 is a sufficient condition to illustrate system stability, which can be seen in the proof of Theorem 3.3 in details.

With the help of Assumption 3.1, we are now ready to design a finite-time state feedback controller for system (1).

Theorem 3.3. Under Assumption 3.1, there exists a dynamic gain $r(t)$ such that system (1) can be globally finite-time stable in probability by the following dynamic state feedback controller:

$$u = \begin{cases} -\sum_{i=1}^{n} \alpha_i x_i \left(k_1 + \frac{k_2}{r^{1-d}}\right)^{n-i+1}, & r \neq 0, \\ 0, & r = 0, \end{cases} \tag{9}$$

$$\dot{r} = \begin{cases} -k_0 r^d + \sum_{i=1}^{n} \frac{\lambda_i x_i^2}{r^{2-d}} \left(k_1 + \frac{k_2}{r^{1-d}}\right)^{2(n-i+1)}, & r \neq 0, \\ 0, & r = 0, \end{cases} \tag{10}$$

where $r(0) > 0$, $k_0 > 0$, $k_1 > 1$, $k_2 > 0$, $\lambda_i > 0, i = 1, \dots, n$, d is a fraction whose numerator and denominator are odd integers with $0 < d < 1$. $\alpha_i > 0, i = 1, \dots, n$ are coefficients of the Hurwitz polynomial $p(s) = s^n + \alpha_n s^{n-1} + \cdots + \alpha_2 s + \alpha_1$.

Proof. The proof is divided into two steps. Firstly, we will provide a dynamic state transformation for (1) in order to obtain a linear controller in the given set. Then, a Lyapunov function is constructed for stability analysis, which shows that the proposed controller can render the closed-loop system globally finite-time stable in probability.

Step 1: Let $T_r := \inf\{t; r(t) = 0, r(0) > 0\}$. For $0 \leq t < T_r$, we introduce the change of coordinates:

$$z_i = (k_1 + \frac{k_2}{r^{1-d}})^{n-i+1} x_i, \ i = 1, \dots, n. \tag{11}$$

With the help of (11), (9) and (10) are equivalent to

$$u = -\sum_{i=1}^{n} \alpha_i z_i, \tag{12}$$

$$\dot{r} = -k_0 r^d + \frac{z^T \Lambda z}{r^{2-d}} \tag{13}$$

where $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n), z = (z_1, \ldots, z_n)^T$. Note that the controller (12) is linear in the set $\{(z,r) : z \in R^n, r > 0\}$. Meanwhile, system (1) can be transformed into

$$\begin{aligned} dz_i =& \Big((k_1 + \frac{k_2}{r^{1-d}}) z_{i+1} + (k_1 + \frac{k_2}{r^{1-d}})^{n-i+1} f_i(x) - \frac{(1-d)k_2 \dot{r}}{r(r^{1-d}k_1 + k_2)} (n-i+1) z_i \Big)\, dt \\ &+ (k_1 + \frac{k_2}{r^{1-d}})^{n-i+1} g_i^T(x)\, dw, \ \ i = 1, \ldots, n, \end{aligned} \tag{14}$$

with $z_{i+1} := u$. Substituting (12) into (14) yields

$$dz = \Big((k_1 + \frac{k_2}{r^{1-d}}) Az + \Phi(r,x) - \frac{(1-d)k_2 \dot{r}}{r(r^{1-d}k_1 + k_2)} Dz \Big)\, dt + \Psi^T(r,x)\, dw \tag{15}$$

where

$$A = \begin{bmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & \cdots & -a_n \end{bmatrix}, \qquad D = \begin{bmatrix} n & 0 & \cdots & 0 \\ 0 & n-1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix},$$

$$\Phi(r,x) = \begin{bmatrix} (k_1 + \frac{k_2}{r^{1-d}})^n f_1(x) \\ (k_1 + \frac{k_2}{r^{1-d}})^{n-1} f_2(x) \\ \vdots \\ (k_1 + \frac{k_2}{r^{1-d}}) f_n(x) \end{bmatrix}, \qquad \Psi^T(r,x) = \begin{bmatrix} (k_1 + \frac{k_2}{r^{1-d}})^n g_1^T(x) \\ (k_1 + \frac{k_2}{r^{1-d}})^{n-1} g_2^T(x) \\ \vdots \\ (k_1 + \frac{k_2}{r^{1-d}}) g_n^T(x) \end{bmatrix}.$$

Step 2: To analysis the stability of the closed-loop system (13), (15), we construct the Lyapunov function $V(z,r) = z^T P z + \frac{r^2}{2}$. From [10], there exists a symmetric, positive definite matrix P satisfying

$$A^T P + PA \le -I \quad \text{and} \quad DP + PD \ge 0. \tag{16}$$

A direct calculation yields

$$\begin{aligned} \mathcal{L}V =& r(-k_0 r^d + \frac{z^T \Lambda z}{r^{2-d}}) + (k_1 + \frac{k_2}{r^{1-d}}) z^T (PA + A^T P) z + 2z^T P \Phi \\ &- z^T (PD + DP) z \frac{(1-d)k_2 \dot{r}}{r(k_1 r^{1-d} + k_2)} + \frac{1}{2} \mathrm{tr}\Big\{ \Psi \frac{\partial^2 V}{\partial z^2} \Psi^T \Big\} \\ \le& - k_0 r^{1+d} + \frac{\lambda_{\max}(\Lambda) \|z\|^2}{r^{1-d}} - (k_1 + \frac{k_2}{r^{1-d}}) \|z\|^2 + 2z^T P \Phi \\ &- z^T (PD + DP) z \frac{(1-d)k_2 \dot{r}}{r(k_1 r^{1-d} + k_2)} + \frac{s\sqrt{s}}{2} \Big\| \Psi \frac{\partial^2 V}{\partial z^2} \Psi^T \Big\| \end{aligned} \tag{17}$$

where the last term is obtained by using $\frac{1}{2}\mathrm{tr}\{X\} \le \frac{s}{2}\|X\|_\infty \le \frac{s\sqrt{s}}{2}\|X\|$($X$ is an s-dimension square matrix). Next, we estimate some terms on the right-hand side of (17).

From Assumption 3.1, it can be verified that for $i = 1, \dots, n$,

$$
\begin{aligned}
|\Phi_i| &\le c_1(k_1 + \frac{k_2}{r^{1-d}})^{n-i+1}(|x_1| + |x_2| + \cdots + |x_i|) \\
&\le c_1\Big(\frac{|z_1|}{|k_1 + \frac{k_2}{r^{1-d}}|^{i-1}} + \frac{|z_2|}{|k_1 + \frac{k_2}{r^{1-d}}|^{i-2}} + \cdots + |z_i|\Big) \\
&\le c_1(|z_1| + |z_2| + \cdots + |z_i|). \qquad (18)
\end{aligned}
$$

By Lemma 2.5, we can obtain

$$
\begin{aligned}
2z^TP\Phi &\le 2\|z\|\|P\|\|\Phi\| \\
&\le 2c_1\|z\|\|P\|\Big((|z_1|)^2 + (|z_1| + |z_2|)^2 + \cdots + (|z_1| + \cdots + |z_n|)^2\Big)^{1/2} \\
&\le 2c_1\|z\|\|P\|\Big(n(|z_1| + \cdots + |z_n|)^2\Big)^{1/2} \\
&\le 2c_1\|z\|\|P\|\Big(n^2(|z_1|^2 + |z_2|^2 + \cdots + |z_n|^2)\Big)^{1/2} \\
&\le 2c_1 n\|P\|\|z\|^2 = \rho_1\|z\|^2 \qquad (19)
\end{aligned}
$$

where $\rho_1 = 2c_1 n\|P\|$. Since $PD + DP \ge 0$, it implies that

$$
\begin{aligned}
&- z^T(PD + DP)z\frac{(1-d)k_2\dot{r}}{r(k_1r^{1-d} + k_2)} \\
=&- \frac{(1-d)k_2}{r(k_1r^{1-d} + k_2)}\Big(- k_0r^d + \frac{z^T\Lambda z}{r^{2-d}}\Big)z^T(PD + DP)z \\
\le&\frac{(1-d)k_2k_0}{r^{1-d}(k_1r^{1-d} + k_2)}z^T(PD + DP)z \\
\le&\frac{(1-d)k_0}{r^{1-d}}\lambda_{\max}(PD + DP)\|z\|^2 = \frac{\rho_2\|z\|^2}{r^{1-d}} \qquad (20)
\end{aligned}
$$

where $\rho_2 = (1-d)k_0\lambda_{\max}(PD + DP)$.

As for the last term on the right-hand side of (17), similar to (18) and (19), we can obtain

$$
\begin{aligned}
\frac{s\sqrt{s}}{2}\Big\|\Psi\frac{\partial^2 V}{\partial z^2}\Psi^T\Big\| &\le \frac{s\sqrt{s}}{2}\|\Psi\|\|\Psi^T\|\Big\|\frac{\partial^2 V}{\partial z^2}\Big\| \\
&\le s\sqrt{s}\|P\| \cdot \|\Psi\|^2 \\
&\le s\sqrt{s}c_2^2n^2\|P\|\|z\|^2 = \rho_3\|z\|^2 \qquad (21)
\end{aligned}
$$

where $\rho_3 = s\sqrt{s}c_2^2n^2\|P\|$. Substituting (19)−(20)−(21) into (17) yields

$$
\begin{aligned}
\mathcal{L}V \le& - k_0r^{1+d} + \frac{\lambda_{\max}(\Lambda)\|z\|^2}{r^{1-d}} - (k_1 + \frac{k_2}{r^{1-d}})\|z\|^2 + (\rho_1 + \frac{\rho_2}{r^{1-d}} + \rho_3)\|z\|^2 \\
&- \frac{k_0}{2}\|z\|^{1+d} + \frac{k_0}{2}\|z\|^{1+d}. \qquad (22)
\end{aligned}
$$

According to Lemma 2.5, we can obtain that

$$\|z\|^{1+d} = \|z\|^{1+d} \times \frac{r^{(1-d^2)/2}}{r^{(1-d^2)/2}} \leq \frac{1-d}{2} r^{1+d} + \frac{1+d}{2} \frac{\|z\|^2}{r^{1-d}}. \tag{23}$$

Substituting (23) into (22) yields

$$\begin{aligned}\mathcal{L}V \leq & -\frac{k_0}{2} r^{1+d} - \frac{k_0}{2}\|z\|^{1+d} + (\rho_1 + \rho_3 - k_1)\|z\|^2 \\ & + (\lambda_{\max}(\Lambda) + \rho_2 + \frac{k_0(1+d)}{4} - k_2)\frac{\|z\|^2}{r^{1-d}}.\end{aligned} \tag{24}$$

Choosing the parameters as

$$k_1 \geq \max\{1, \rho_1 + \rho_3\},\ k_2 \geq \max\left\{\lambda_{\max}(\Lambda) + \rho_2 + \frac{k_0(1+d)}{4}\right\} \tag{25}$$

(24) becomes

$$\mathcal{L}V \leq -\frac{k_0}{2}(r^{1+d} + \|z\|^{1+d}). \tag{26}$$

Picking $\rho = \frac{1+d}{2}$ and $\eta = \frac{k_0}{2\max\{\lambda_{\max}^{\frac{1+d}{2}}(P), 2^{-\frac{1+d}{2}}\}}$, one has

$$\begin{aligned}\mathcal{L}V + \eta V^\rho \leq & -\frac{k_0}{2}(r^{1+d} + \|z\|^{1+d}) + \eta(z^T P z + \frac{r^2}{2})^{\frac{1+d}{2}} \\ \leq & -\frac{k_0}{2}(r^{1+d} + \|z\|^{1+d}) + \frac{k_0}{2\max\{\lambda_{\max}^{\frac{1+d}{2}}(P), 2^{-\frac{1+d}{2}}\}} \\ & \times (\lambda_{\max}^{\frac{1+d}{2}}(P)\|z\|^{1+d} + 2^{-\frac{1+d}{2}} r^{1+d}) \\ \leq & \ 0.\end{aligned} \tag{27}$$

Now we claim that $r(t)$ cannot become zero until $V(z(t), r(t))$ converges to zero at time T_V. Let T_V be the first time that $V(z(t), r(t))$ converges to zero and T_r be the first time that $r(t)$ becomes zero. Clearly, T_r cannot be greater than T_V because $r(t) = 0$ when $V(z(t), r(t)) = 0$. On the other hand, T_r cannot be less than T_V either. If $T_r < T_V$, then $z(T_r) \neq 0$ since $V(z(T_r), r(T_r)) > 0$, and this implies that $\dot{r} = -k_0 r^d + \frac{z^T \Lambda z}{r^{2-d}} > 0$ for a short time period just before T_r because r is very small but positive while $z^T \Lambda z$ is strictly larger than zero. This means that $r(t)$ does not decrease, which is a contradiction. Therefore, it follows that $T_r = T_V = T$ and proves that $r(t) > 0$ for $0 \leq t < T$.

Since $r(t) > 0$ for $0 \leq t < T$, the coordinate changes (9) is valid for the period, and the analysis up to now is justified. Since,

$$x_i = \left(\frac{r^{1-d}}{k_1 r^{1-d} + k_2}\right)^{n-i+1} z_i,\ i = 1, \ldots, n \tag{28}$$

it follows that $\lim_{t \to T}(x(t), r(t)) = (0, 0)$. Then, the closed-loop system (1) – (9) – (10) is globally finite-time attractive and stable in probability.

Then, we will show that the existence and uniqueness of solutions for the closed-loop stochastic nonlinear system (1) − (9) − (10). It is obvious that, for $0 \le t < T$, $f_i(x), g_i(x),\ i = 1, \dots, n$ and u are $\mathcal{C}^1$ and therefore, the solution is unique while $r(t) > 0$. When $t \ge T$, the trivial solution $(x(t), r(t))$ is the origin, that is, $(x(t+T), r(t+T))$ is unique for $t \ge 0$. According to Definition 2.1, the closed-loop system (1) − (9) − (10) is globally finite-time stable in probability. □

Remark 3.4. As pointed in [18], it is impossible to discuss the finite-time stability in probability for those stochastic nonlinear systems whose coefficients satisfy the local Lipschitz condition. Therefore, if the trivial solution of (1) is finite-time stable in probability, there is at least one coefficient that does not satisfy the local Lipschitz condition. That is the reason why we introduce transformation (11) and (13).

4. A SIMULATION EXAMPLE

Example 4.1. Consider the following system

$$\begin{aligned} dx_1 &= (x_2 + x_1 \sin x_2)\, dt + \frac{x_1}{1 + x_2^2} dw \\ dx_2 &= (u + x_1 + x_2 \sin x_1)\, dt + \ln(1 + (x_1 + x_2)^2) \cos^2 x_1 dw \end{aligned} \tag{29}$$

where $x = (x_1, x_2)^T$, $f_1(x) = x_1 \sin x_2, f_2(x) = x_1 + x_2 \sin x_1, g_1(x) = \frac{x_1}{1+x_2^2}, g_2(x) = \ln(1 + (x_1 + x_2)^2) \cos^2 x_1$ are $\mathcal{C}^1$ and satisfied with Assumption 3.1. Using Theorem 3.3, we can explicitly construct the finite-time state feedback controller as follows

$$\begin{aligned} u &= \begin{cases} -a_1 x_1 (k_1 + \frac{k_2}{r^{1-d}})^2 - a_2 x_2 (k_1 + \frac{k_2}{r^{1-d}}), & r \ne 0, \\ 0, & r = 0, \end{cases} \\ \dot{r} &= \begin{cases} -k_0 r^d + \frac{\lambda_1 x_1^2 (k_1 + \frac{k_2}{r^{1-d}})^4}{r^{2-d}} + \frac{\lambda_2 x_2^2 (k_1 + \frac{k_2}{r^{1-d}})^2}{r^{2-d}}, & r \ne 0, \\ 0, & r = 0, \end{cases} \end{aligned} \tag{30}$$

where $k_0, k_1, k_2, a_1, a_2, \lambda_1, \lambda_2$ and d are appropriate constants. In the simulation, the parameters are chosen as $a_1 = 1,\ a_2 = 2,\ \lambda_1 = \lambda_2 = 0.001,\ d = 3/5, k_0 = 1.25,\ k_1 = 13.7$ and $k_2 = 4.4$. We choose the initial conditions $x_1(0) = 0.5, x_2(0) = -1$ and $r(0) = 4$. The numerical simulation is based on the Runge–Kutta method(Ode45) and realized by Matlab Simulink for simplicity. The stochastic noise is an independent standard Wiener process. The noise power is set to be 20 and the sampling time is set to be $0.05s$. Figure 1 and Figure 2 illustrate the responses of the closed-loop system (29) and (30), which demonstrate the effectiveness of the control scheme.

5. CONCLUSION

In this paper, we discuss the problem of global finite-time state feedback stabilization for a class of stochastic nonlinear systems. The design process includes two steps: we first introduce a dynamic state transformation to transform the original system into a new system; then, we design a state feedback controller with an appropriate choice of design parameters to render the original system globally finite-time stable in probability.

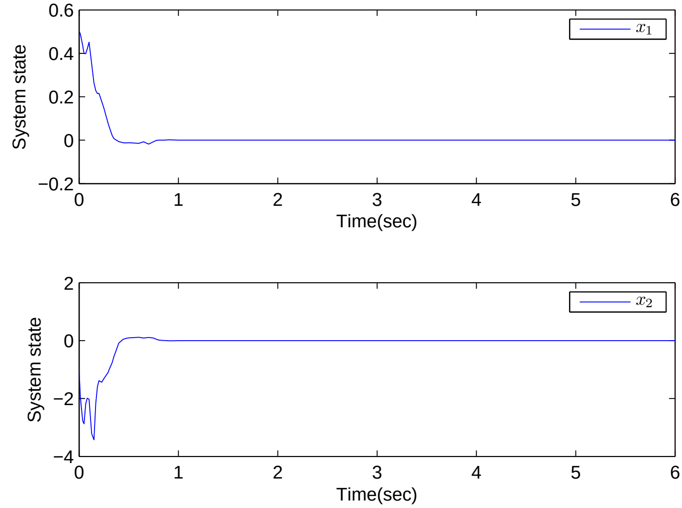

Fig. 1. Responses of the states under controller (30).

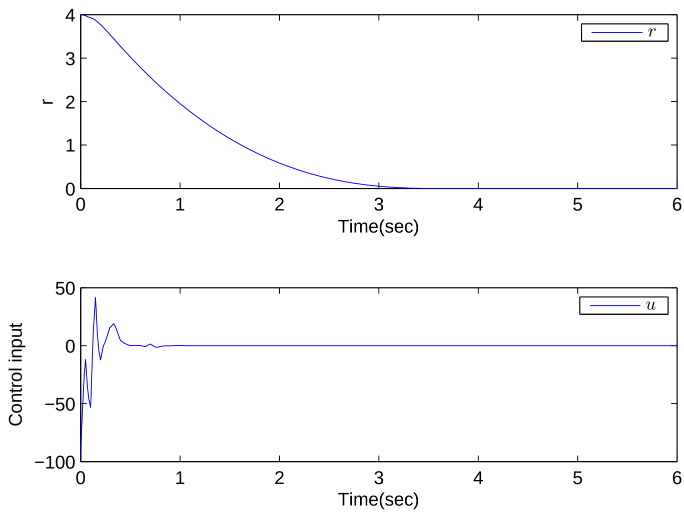

Fig. 2. Responses of the controller and dynamic gain r.

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REFERENCES

[1] S. P. Bhat and D. S. Bernstein: Finite-time stability of continuous autonomous systems. SIAM J. Control Optim. *38* (2000), 751–766.

[2] W. S. Chen and L. C. Jiao: Authors' reply to "Comments on 'finite-time stability theorem of stochastic nonlinear systems [Automatica 46(2010)2105-2108]'". Automatica *47* (2011), 1544–1545.

[3] H. Deng and M. Krstić: Stochastic nonlinear stabilizationq,– I: A backstepping design. Syst. Control Lett. *32* (1997), 143–150.

[4] A. Isidori: Nonlinear Control Systems. 3rd edition. Springer–Verlag, New York 1995.

[5] S. Y. Khoo, J. L. Yin, Z. H. Man, and X. H. Yu: Finite-time stabilization of stochastic nonlinear systems in strict-feedback form. Automatica *49* (2013), 1403–1410.

[6] M. Kristić and H. Deng: Stabilization of uncertain systems. Springer–Verlag, New York 1998.

[7] W. Q. Li, Y. W. Jing, and S. Y. Zhang: Adaptive state-feedback stabilization for a large class of high-order stochastic nonlinear systems. Automatica *47* (2011), 819–828.

[8] W. Q. Li, Y. W. Jing, S. Y. Zhang, and G. M. Dimirovski: State-feedback stabilization for high-order stochastic nonlinear systems without strict triangular conditions. In: Proc. Amer. Control Conference 2010, pp. 367–372.

[9] R. Marino and P. Tomei: Nonlinear Control Design. Prentice Hall, U.K. 1995.

[10] L. Praly and Z. P. Jiang: Linear output feedback with dynamic high gain for nonlinear systems. Systems Control Lett. *53* (2004), 107–116.

[11] C. J. Qian and W. Lin: A continuous feedback approach to global strong stabilization of nonlinear systems. IEEE Trans. Automat. Control *46* (2001), 1061–1079.

[12] C. J. Qian and W. Lin: Output feedback control of a class of nonlinear systems: a nonseparation principle paradigm. IEEE Trans. Automat. Control *47* (2002), 1710–1715.

[13] S. Seo, H. Shim, and J. Seo: Global finite-time stabilization of a nonlinear system using dynamic exponent scaling. In: Proc. 47th IEEE Conference on Decision Control 2008, pp. 3805–3810.

[14] S. Seo, H. Shim, and J. Seo: Finite-time stabilizing dynamic control of uncertain multi-input linear systems. IMA J. Math. Control Inform. *28* (2011), 525–537.

[15] J. Tian and X.J. Xie: Adaptive state-feedback stabilization for more general high-order stochastic nonlinear systems. Acta Autom. Sinica *34* (2008), 1188–1192.

[16] X. J. Xie and N. Duan: Output tracking of high-order stochastic nonlinear systems with application to benchmark mechanical system. IEEE Trans. Automat. Control *55* (2010), 1197–1202.

[17] X. J. Xie and J. Tian: State-feedback stabilization for high-order stochastic nonlinear systems with stochastic inverse dynamics. Internat. J. Robust. Nonlinear Control *17* (2007), 1343–1362.

[18] J. L. Yin, S. Y. Khoo, Z. H. Man, and X. H. Yu: Finite-time stability and instability of stochastic nonlinear systems. Automatica *47* (2011), 2671–2677.

[19] J. Y. Zhai and S. M. Fei: Global practical tracking control for a class of uncertain nonlinear systems. IET Control Theory Appl. *5* (2011), 1343–1351.

[20] J. Y. Zhai and C. J. Qian: Global control of nonlinear systems with uncertain output function using homogeneous domination approach. Internat. J. Robust Nonlinear Control *22* (2012), 1543–1561.

[21] X. Zhang and G. Feng: Global finite-time stabilisation of a class of feedforward non-linear systems. IET Control Theory Appl. *5* (2011), 1450–1457.

[22] X. Zhang, G. Feng, and Y. Sun: Finite-time stabilization by state feedback control for a class of time-varying nonlinear systems. Automatica *48* (2012), 499–504.

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