KYBERNETIKA — VOLUME 49 (2013), NUMBER 4, PAGES 524–538

DISTRIBUTED OUTPUT REGULATION FOR LINEAR MULTI-AGENT SYSTEMS WITH UNKNOWN LEADERS

Xinghu Wang, Haibo Ji and Chuanrui Wang

In this paper, the distributed output regulation problem of linear multi-agent systems with parametric-uncertain leaders is considered. The existing distributed output regulation results with exactly known leader systems is not applicable. To solve the leader-following with unknown parameters in the leader dynamics, a distributed control law based on an adaptive internal model is proposed and the convergence can be proved.

Keywords: output regulation problem, multi-agent systems, internal model

Classification: 93A14, 93C10

1. INTRODUCTION

It is known that the output regulation design aims to force a plant to achieve asymptotically tracking and/or rejecting for a class of reference signals and/or disturbances with maintaining the stability of the closed-loop system, where the reference inputs and disturbances are generated by a so-called exosystem. Since 1970s, many results on this problem for both linear and nonlinear systems have been obtained [1, 7, 8]. Moreover, different adaptive control methods have been proposed for the exosystem with uncertain parameters [11, 13, 14].

Recently, as the rapid development of multi-agent systems [4, 15], distributed output regulation (DOR) has attracted much attention [5, 16, 17, 19]. In DOR formulation, the leader in the multi-agent system is regarded as an exosystem which provides reference signals for the follower agents. The information communication topology is described by a digraph. Not all the followers can access the information of the leader systems. Therefore, the results on the conventional output regulation problem for single systems fail to solve this challenging problem. In fact, the DOR problem provides a general framework for leader-follower multi-agent systems. There are two main approaches to solve the problem. One is based on distributed estimaters or observers. The research started with the tracking of active leaders, viewed as exosystems, by distributed estimation [3, 6], and later the distributed observer was provided in [16]. The other approach is based on internal model [5, 17, 18]. An internal model is a copy of the exosystem and allows parameter uncertainties in the system matrices. In [5, 17], a p-copy type internal model was constructed to achieve the solvability of the DOR for an uncertain multi-agent system. Additionally, [18] provided a design for multi-agent systems with switching topologies by using canonical internal model.

In the existing results, a precise linear model of the leader agent must be known. Based on the known leader assumption, the DOR problem of linear multi-agent systems is converted into a simultaneous eigenvalue placement problem of an augmented system composed of the nominal system and a dynamic compensator (i. e., a distributed internal model or a distributed observer). However, this approach can not handle the case when there exists some uncertain parameters in the leader system. In this paper, in light of the result given in [14], we provide a distributed control law based on an adaptive internal model to solve the DOR problem of multi-agent systems with an uncertain leader. The adaptive internal model asymptotically provides an estimation of the uncertain parameters in the leader. By utilizing the concept of the input-to-state stability of the cascaded nonlinear systems, we prove that, under the designed distributed control, the overall system admits a bounded solution over $[0,\infty)$ for any initial conditions, and the regulated outputs converge to zero asymptotically.

The rest of the paper is organized as follows. Section 2 introduces some preliminaries on the multi-agent systems and presents the DOR problem. Section 3 presents a distributed control with the help of an adaptive internal model. Section 4 gives a numerical example. Section 5 contains some conclusions.

2. PROBLEM FORMULATION

We first give a brief introduction of graph theory (see [2] for details). A digraph $\mathcal{G} = (\mathcal{V},\mathcal{E})$ consists of a set of nodes $\mathcal{V} = \{0,1,2,\dots,N\}$ and an edge set $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$. If $(i,j)\in\mathcal{E}$, then node i is said to be the father of node j and node j is the child of node i. All the fathers of node i constitute an in-neighboring set of node i and will be denoted by $\mathcal{N}_i$. If the digraph $\mathcal{G}$ contains a sequence of edges of the form $(i_1,i_2),(i_2,i_3),\dots,(i_k,i_{k+1})$, then the set $\{(i_1,i_2),\dots,(i_k,i_{k+1})\}$ is called a path of $\mathcal{G}$ from i_1 to i_{k+1}, and node i_1 is said to be reachable from node i_{k+1}. If $i_{k+1} = i_1$, the the path is called a loop. If a node is reachable from every other node of the digraph, then the node is called globally reachable.

The adjacency matrix of $\mathcal{G}$ is denoted as $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{(N+1)\times(N+1)}$, where for $i,j = 0,1,\dots,N$, $a_{ij} = 1$ if $(i,j)\in\mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian of a digraph $\mathcal{G}$ is denoted by $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{(N+1)\times(N+1)}$, where for $i,j = 0,1,\dots,N$, $l_{ii} = \sum_{j=0}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$.

Consider a multi-agent system composed of N interconnected agents as follows:

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + Ev \\ y_i &= Cx_i,\ i = 1,\dots,N \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state of ith agent, $u_i, y_i \in \mathbb{R}$ are control input and measurement output, respectively. $v \in \mathbb{R}^{n_v}$ is the exogenous signal representing both the reference input and the disturbance, and is generated by the leader system expressed as

$$\dot{v} = S(\omega)v \tag{2}$$

where $\omega \in \mathbb{R}^{n_\omega}$ represents the uncertain parameter in (2). Let $y_0 = -Fv \in \mathbb{R}$ be the output of leader (2), which represents the desirable reference trajectory. For agent i, the regulated output is defined as

$$e_i(t) = y_i - y_0. \tag{3}$$

A digraph $\mathcal{G}$ can be used to describe the information exchange of a multi-agent system with regarding the N agents as nodes. Considering the multi-agent system (1) with the leader (2), a digraph $\mathcal{G}$ with $N+1$ nodes can be defined, in which node 0 is associated with the leader and the other N nodes are associated with the N agents of system (1). The edge set $\mathcal{E}$ contains an edge (i,j) if agent j can get the measurement output y_i of agent i.

To construct the distributed control law, we first define the neighbor based regulated error e_{iv} for agent i as follows:

$$e_{iv} = \begin{cases} e_i, & (0,i) \in \mathcal{E} \\ \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}(y_i - y_j) = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}(e_i - e_j), & (0,i) \notin \mathcal{E} \end{cases} \tag{4}$$

where $|\mathcal{N}_i|$ is the cardinality of the set $\mathcal{N}_i$.

In this paper, the distributed output regulation (DOR) problem is formulated as: for system (1) with leader (2), find the following distributed control law

$$u_i = f_i(z_i),\ \ \dot{z}_i = g_i(z_i, e_{iv}) \tag{5}$$

where $z_i \in \mathbb{R}^{n_{z_i}}$, with n_{z_i} to be defined later, f_i and g_i are possible nonlinear functions of their arguments vanishing at the origin, such that, for all initial conditions of the closed-loop system and any $\omega \in \mathbb{R}^{n_\omega}$, the solution of the closed-loop system exists and is bounded over $[0,\infty)$, and moreover,

$$\lim_{t\to\infty} e_i(t) = 0.$$

To solve the problem, some standard assumptions are listed.

Assumption 2.1. All eigenvalues of $S(\omega)$ are distinct with zero real parts for all ω.

Assumption 2.2. (A,B) is stabilizable, and (C,A) is detectable.

Assumption 2.3.

$$\operatorname{rank}\begin{pmatrix} A-\lambda I_n & B \\ C & 0 \end{pmatrix} = n+1, \quad \forall\lambda\in\sigma(S(\omega)) \tag{6}$$

where $\sigma(S(\omega))$ denotes the spectrum of $S(\omega)$.

Assumption 2.4. The digraph $\mathcal{G}$ contains no loop and the node 0 is globally reachable.

Remark 2.5. Assumptions 2.1 to 2.3 are standard requirements for guaranteeing the solvability of the output regulation problem with unknown exosystems by the centralized control method (see [13, 14]). Assumption 2.1 means that the leader can produce sinusoidal signals with arbitrary unknown frequencies. Assumption 2.4 is used in [17] to describe the interconnection graph for the multi-agent system.

Under Assumption 2.3, for $\omega \in \mathbb{R}^{n_\omega}$, there exists a solution X, U to the regulator equations

$$\begin{aligned} X(\omega)S(\omega) &= AX(\omega) + BU(\omega) + E \\ 0 &= CX(\omega) + F. \end{aligned} \tag{7}$$

It can be verified that there exists an integer s such that

$$\frac{d^s Uv}{dt^s} - \alpha_{s-2}(\omega)\frac{d^{s-2}Uv}{dt^{s-2}} - \cdots - \alpha_{s-2\lceil \frac{s}{2}\rceil}(\omega)\frac{d^{s-2\lceil \frac{s}{2}\rceil}Uv}{dt^{s-2\lceil \frac{s}{2}\rceil}} = 0 \tag{8}$$

with

$$\lceil \frac{s}{2} \rceil = \begin{cases} s/2, & s \text{ is even} \\ (s-1)/2, & s \text{ is odd} \end{cases}$$

where, according to Caley-Hamilton theorem, $\alpha_j(\omega)$'s are real numbers such that all roots of polynomial

$$\mathcal{P}(\lambda) = \lambda^s - \alpha_{s-2}(\omega)\lambda^{s-2} - \cdots - \alpha_{s-2\lceil \frac{s}{2}\rceil}(\omega)\lambda^{s-2\lceil \frac{s}{2}\rceil} \tag{9}$$

belong to the set containing all eigenvalues of matrix $S(\omega)$ and are distinct with zero real parts for $\omega \in \mathbb{R}^{\omega}$. Define $\alpha = [\alpha_{s-2\lceil \frac{s}{2}\rceil} \ \cdots \ \alpha_{s-2}]^T$.

Performing the coordinate transformation

$$\bar{x}_i = x_i - Xv, \quad \tau_j = US^{j-1}v, \ j = 1, \ldots, s$$

converts the systems (1) and (2) into

$$\begin{aligned} \dot{\bar{x}}_i &= A\bar{x}_i + Bu_i - B\Gamma\tau \\ \dot{\tau} &= \Phi(\alpha)\tau \end{aligned} \tag{10}$$

with the regulated output $e_i = C\bar{x}_i$ and $\tau = [\tau_1 \ \cdots \ \tau_s]^T$, where

$$\Phi(\alpha) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \alpha_1(\omega) & 0 & \cdots & \alpha_{s-2}(\omega) & 0 \end{bmatrix}, \ \Gamma = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, \ \text{when } s \text{ is odd}$$

$$\Phi(\alpha) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_0(\omega) & 0 & \alpha_2(\omega) & \cdots & \alpha_{s-2}(\omega) & 0 \end{bmatrix}, \ \Gamma = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, \ \text{when } s \text{ is even.}$$

Another assumption is needed to guarantee the parameter convergence in the following analysis.

Assumption 2.6. The initial condition v_0 excites all oscillatory modes of the τ subsystem in (10).

Remark 2.7. Assumption 2.6 is a requirement of minimality for $\Phi(\alpha)$. If this condition does not hold, the dimension of $\Phi(\alpha)$ can be reduced leaving out the part which is not excited by the initial condition. Following a similar argument as Theorem 4.1 of [11], $\tau(t)$ is PE (Persistence of Excitation).

3. MAIN RESULTS

In this section, we present a solution for the DOR problem of (1) with an unknown leader (2). To this end, we construct an observable pair first. Let $\mu = [\mu_1, \ldots, \mu_s]$ be such that the polynomial

$$\bar{\mathcal{P}}(\lambda) = \lambda^{s-1} + \frac{\mu_{s-1}}{\mu_s}\lambda^{s-2} + \cdots + \frac{\mu_2}{\mu_s}\lambda + \frac{\mu_1}{\mu_s}$$

is stable, $G \in \mathbb{R}^n$ be such that the matrix $(A - GC)$ is Hurwitz, and $\Delta = G\mu$. Then, from Assumption 2.3, there exists solution $(\Pi(\alpha), M(\alpha))$ to the equations:

$$\begin{aligned} \Pi(\alpha)\Phi(\alpha) &= A\Pi(\alpha) - BM(\alpha) - \Delta \\ 0 &= C\Pi(\alpha) - \mu. \end{aligned} \tag{11}$$

The above obtained pair $(M(\alpha), \Phi(\alpha))$ has the following observability property.

Lemma 3.1. The pair $(M(\alpha), \Phi(\alpha))$ is observable.

Proof. Consider the following linear equations

$$(\lambda_0 I - \Phi(\alpha))z = 0, \ \forall \lambda_0 \in \sigma(\Phi(\alpha)) \tag{12}$$

$$\mu z = 0. \tag{13}$$

From (12), we see that $z_j = \lambda_0^{j-1} z_1, j = 2, \ldots, s$. Substituting this expression of z_j into (13) gives $\mu_s \bar{\mathcal{P}}(\lambda_0) z_1 = 0$. Because, for each $\lambda_0 \in \sigma(\Phi(\alpha))$, $\bar{\mathcal{P}}(\lambda_0) \neq 0$, $z_1 = 0$, which implies

$$\text{Rank}\left(\begin{bmatrix} \lambda_0 I - \Phi(\alpha) \\ \mu \end{bmatrix}\right) = s, \quad \forall \lambda \in \sigma(\Phi(\alpha)).$$

By using Popov–Belevitch–Hautus (PBH) test, the pair $(\mu, \Phi(\alpha))$ is observable. On the other hand, consider another group of linear equations

$$(\lambda_0 I - \Phi(\alpha))z = 0, \ \forall \lambda_0 \in \sigma(\Phi(\alpha)) \tag{14}$$

$$M(\alpha) z = 0. \tag{15}$$

Post-multiplying both sides of (11) by z, from (14) and (15), we have

$$\begin{aligned} (A - GC - \lambda_0 I)\Pi(\alpha)z &= 0 \\ C\Pi(\alpha)z - \mu z &= 0. \end{aligned} \tag{16}$$

Because $A - GC$ is Hurwitz and λ_0 has a zero real part, the matrix $(A - GC - \lambda_0 I)$ is nonsingular. Therefore, $\Pi(\alpha)z = 0$, which means $\mu z = 0$. From the observability of $(\mu, \Phi(\alpha))$, $z = 0$. Then,

$$\text{Rank}\left(\begin{bmatrix} \lambda_0 I - \Phi(\alpha) \\ M(\alpha) \end{bmatrix}\right) = s.$$

By using PBH test again, the conclusion follows. □

With the above observable pair, we can perform the transformation

$$\tilde{x}_i = \bar{x}_i - \Pi(\alpha)\mathcal{O}^{-1}\tau, \quad \theta = \mathcal{O}^{-1}\tau$$

on (10) with

$$\mathcal{O} = \begin{bmatrix} M(\alpha) \\ M(\alpha)\Phi(\alpha) \\ \cdots \\ M(\alpha)\Phi^{s-1}(\alpha) \end{bmatrix}$$

and obtain the following form:

$$\begin{aligned} \dot{\tilde{x}}_i &= A\tilde{x}_i + Bu_i + \Delta\theta \\ \dot{\theta} &= \Phi(\alpha)\theta \\ e_i &= C\tilde{x}_i + \mu\theta. \end{aligned} \tag{17}$$

Based on (17), we consider a distributed dynamic feedback control

$$\begin{aligned} \dot{\eta}_i &= A\eta_i + Bu_i + \Delta\zeta_i + G(e_{iv} - C\eta_i - \mu\zeta_i) \\ \dot{\zeta}_i &= \Phi(\hat{\alpha}_i)\zeta_i + \bar{G}(e_{iv} - C\eta_i - \mu\zeta_i) \\ \dot{\hat{\alpha}}_i &= \Lambda\Omega_i(e_{iv} - C\eta_i - \mu\zeta_i) \\ u_i &= K\eta_i + \varphi(\hat{\alpha}_i)\zeta_i \end{aligned} \tag{18}$$

where K is selected such that $(A + BK)$ is Hurwitz, and

$$\begin{aligned} \varphi(\hat{\alpha}_i) &= M(\hat{\alpha}_i) + K\Pi(\hat{\alpha}_i) \\ \bar{G} &= [0 \ \ \cdots \ \ 0 \ \ \bar{g}_1 \ \ \bar{g}_2]^T \in \mathbb{R}^s \\ \Omega_i &= [\zeta_{i,s-2\lceil s/2\rceil+1} \ \ \zeta_{i,s-2\lceil s/2\rceil+3} \ \ \cdots \ \ \zeta_{i,s-3} \ \ \zeta_{i,s-1}]^T \\ \Lambda &= \mathrm{diag}(\Lambda_1, \ldots, \Lambda_{\lceil s/2\rceil}) \end{aligned}$$

with $\bar{g}_1 = 1/\mu_s$, $\bar{g}_2 > 0$, $\Lambda_j > 0$, for $j = 1, \ldots, \lceil s/2\rceil$.

Remark 3.2. The dynamics in (18) constitutes an adaptive internal model. $\hat{\alpha}_i$'s are used to estimate unknown vector α which depends on the uncertain parameter ω.

Under the distributed control (18), we conclude our main result.

Theorem 3.3. Under Assumptions 2.1−2.6, for any initial conditions and $\omega \in \mathbb{R}^{n_\omega}$, the solution of the closed-loop system composed of (17) and (18) exists and is bounded over $[0, +\infty)$, and moreover, for each $i = 1, \ldots, N$, e_i converges to zero asymptotically. Namely, the distributed control (18) solves the DOR problem of multi-agent system (1) with the unknown leader (2).

Proof. Let $\delta_i = \tilde{x}_i - \eta_i$, $\epsilon_i = \theta - \zeta_i$, and $\varepsilon_i = \hat{\alpha}_i - \alpha$. Then, the error dynamics can be represented in the following form: for $(0, i) \in \mathcal{E}$,

$$\begin{aligned} \dot{\delta}_i &= (A - GC)\delta_i \\ \dot{\epsilon}_i &= (\Phi(\alpha) - \bar{G}\mu)\epsilon_i + (\Phi(\alpha) - \Phi(\hat{\alpha}_i))\zeta_i - \bar{G}C\delta_i \\ \dot{\varepsilon}_i &= \Lambda\Omega_i(C\delta_i + \mu\epsilon_i) \end{aligned} \tag{19}$$

and for $(0,i) \notin \mathcal{E}$,

$$\begin{aligned}
\dot{\delta}_i &= (A-GC)\delta_i + \frac{1}{|\mathcal{N}_i|} G \sum_{j\in\mathcal{N}_i} e_j \\
\dot{\epsilon}_i &= (\Phi(\alpha) - \bar{G}\mu)\epsilon_i + (\Phi(\alpha) - \Phi(\hat{\alpha}_i))\zeta_i - \bar{G}C\delta_i + \frac{1}{|\mathcal{N}_i|}\bar{G}\sum_{j\in\mathcal{N}_i} e_j \\
\dot{\varepsilon}_i &= \Lambda\Omega_i(C\delta_i + \mu\epsilon_i) - \frac{1}{|\mathcal{N}_i|}\Lambda\Omega_i \sum_{j\in\mathcal{N}_i} e_j.
\end{aligned} \tag{20}$$

In what follows, four steps are given for the proof.

Step 1: Due to the special form of matrices $\Phi(\alpha), \bar{G}$ and μ, the ϵ_i subsystem in (19) can be decomposed into the following form

$$\begin{aligned}
\dot{\epsilon}_{i,j} &= \epsilon_{i,j+1}, \ j = 1, \dots, s-2 \\
\dot{\epsilon}_{i,s-1} &= -\sum_{k=1}^{s-1} \frac{\mu_k}{\mu_s}\epsilon_{i,k} - \bar{g}_1 C\delta_i \\
\dot{\epsilon}_{i,s} &= -\bar{g}_2 \sum_{k=1}^{s} \mu_k \epsilon_{i,k} - \bar{g}_2 C\delta_i \\
&\quad - \sum_{k=1}^{\lceil s/2\rceil} \left(\alpha_k \epsilon_{i,s-2(\lceil s/2\rceil - k)-1} + \varepsilon_{i,k}\zeta_{i,s-2(\lceil s/2\rceil-k)-1}\right).
\end{aligned} \tag{21}$$

Taking $\bar{\delta}_i = [\delta_i^T \ \ \epsilon_{i,1} \ \ \cdots \ \ \epsilon_{i,s-1}]^T$, we obtain

$$\begin{aligned}
\dot{\bar{\delta}}_i &= \bar{A}\bar{\delta}_i \\
\dot{\epsilon}_{i,s} &= -\beta\bar{\delta}_i - \gamma\epsilon_{i,s} - \Omega_i^T \varepsilon_i \\
\dot{\varepsilon}_i &= \Lambda\Omega_i(\bar{C}\bar{\delta}_i + \bar{\mu}\epsilon_{i,s})
\end{aligned} \tag{22}$$

where $\bar{A}$, β, γ, $\bar{C}$ and $\bar{\mu}$ are defined as

$$\begin{aligned}
\bar{A} &= \begin{bmatrix} (A-GC) & 0 \\ [0 \ \cdots \ 0 \ \bar{g}_1]^T C & \Xi \end{bmatrix} \\
\beta &= \begin{cases} [-\bar{g}_2\mu_1 \quad -\bar{g}_2\mu_2 - \alpha_1 \quad \cdots \quad -\bar{g}_2\mu_{s-1} - \alpha_{s-2}], & s \text{ is odd} \\ [-\bar{g}_2\mu_1 - \alpha_0 \quad -\bar{g}_2\mu_2 \quad \cdots \quad -\bar{g}_2\mu_{s-1} - \alpha_{s-2}], & s \text{ is even} \end{cases} \\
\gamma &= -\bar{g}_2\mu_s, \ \bar{C} = [C \ \ \mu_1 \ \ \cdots \ \ \mu_{s-1}], \ \ \bar{\mu} = \mu_s
\end{aligned} \tag{23}$$

with

$$\Xi = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \dots & \dots & \ddots & \dots \\ -\mu_1/\mu_s & -\mu_2/\mu_s & \cdots & -\mu_{s-1}/\mu_s \end{bmatrix}.$$

Since G and μ are chosen to satisfy that $\bar{A}$ is Hurwitz, $\bar{\delta}_i(t)$ subsystem is exponentially stable, namely, there exists a quadratic Lyapunov function

$$V_i = \frac{1}{2}\bar{\delta}_i^T P \bar{\delta}_i$$

satisfying

$$
\begin{aligned}
c_{i,1}\|\bar{\delta}_i\|^2 \leq V_i \leq c_{i,2}\|\bar{\delta}_i\|^2 \\
\dot{V}_i \leq -c_{i,3}\|\bar{\delta}_i\|^2
\end{aligned} \tag{24}
$$

for some positive constants $c_{i,1}, c_{i,2}$ and $c_{i,3}$.

Under Assumption 2.6, $\tau(t)$ is PE, so is $\theta(t)$. Applying Lemma 13.5 in [10], for $j = 1, \dots, \lceil s/2 \rceil$, $\zeta_{i,s-2(\lceil s/2\rceil-j)-1} = \theta_{s-2(\lceil s/2\rceil-j)-1} - \epsilon_{i,s-2(\lceil s/2\rceil-j)-1}$ is also PE. Hence, by using Lemma B.2.3 in [12], we have that, when $\bar{\delta}_i = 0$, the trajectories $(\epsilon_{i,s}(t), \varepsilon_i(t))$ of $(\epsilon_{i,s}, \varepsilon_i)$ subsystem in (22) satisfy

$$
\|(\epsilon_{i,s}(t), \varepsilon_i(t))\| \leq \bar{c}_i \mathrm{e}^{-\varpi_i t}\|(\epsilon_{i,s}(0), \varepsilon_i(0))\|, \ \forall t \geq 0
$$

for some $\bar{c}_i > 0$ and $\varpi_i > 0$. By using converse Lyapunov function theorem, there is a continuously differentiable function $\bar{V}_i(t, \epsilon_{i,s}, \varepsilon_i)$ satisfying

$$
\begin{aligned}
d_{i,1}\|(\epsilon_{i,s}, \varepsilon_i)\|^2 \leq \bar{V}_i \leq d_{i,2}\|(\epsilon_{i,s}, \varepsilon_i)\|^2 \\
\dot{\bar{V}}_i|_{\bar{\delta}_i=0} \leq -d_{i,3}\|(\epsilon_{i,s}, \varepsilon_i)\|^2 \\
\left\| \left(\frac{\partial \bar{V}_i}{\partial \epsilon_{i,s}}, \ \frac{\partial \bar{V}_i}{\partial \varepsilon_i} \right) \right\| \leq d_{i,4}\|(\epsilon_{i,s}, \varepsilon_i)\|
\end{aligned} \tag{25}
$$

for some positive constants $d_{i,1}, d_{i,2}, d_{i,3}, d_{i,4}$. Thus, along the trajectories of $(\epsilon_{i,s}, \varepsilon_i)$ subsystem in (22) without the restriction $\bar{\delta}_i = 0$, we have

$$
\begin{aligned}
\dot{\bar{V}}_i &\leq -d_{i,3}\|(\epsilon_{i,s}, \varepsilon_i)\|^2 + \frac{\partial \bar{V}_i}{\partial \epsilon_{i,s}}(-\beta\bar{\delta}_i) + \frac{\partial \bar{V}_i}{\partial \varepsilon_i}(\Lambda\Omega_i\bar{C}\bar{\delta}_i) \\
&\leq -\frac{1}{2}d_{i,3}\|(\epsilon_{i,s}, \varepsilon_i)\|^2 + \frac{d_{i,4}^2}{2d_{i,3}}\left(\|\beta\|^2\|\bar{\delta}_i\|^2 + \|\Lambda\|^2\|\Omega_i\|^2\|\bar{C}\|^2\|\bar{\delta}_i\|^2\right).
\end{aligned} \tag{26}
$$

Because

$$
\Omega_i = \begin{bmatrix} \theta_{s-2\lceil s/2\rceil+1} \\ \theta_{s-2\lceil s/2\rceil+3} \\ \cdots \\ \theta_{s-1} \end{bmatrix} - \begin{bmatrix} \epsilon_{i,s-2\lceil s/2\rceil+1} \\ \epsilon_{i,s-2\lceil s/2\rceil+3} \\ \cdots \\ \epsilon_{i,s-1} \end{bmatrix}
$$

we have $\|\Omega_i\|^2 \leq 2\|\theta\|^2 + 2\|\bar{\delta}_i\|^2$. Since all eigenvalues of $\Phi(\alpha)$ are distinct with zero real parts, $\theta(t)$ is bounded for all $t \geq 0$. Without loss of generality, we assume that $\|\theta(t)\| \leq \breve{B}$, $\breve{B} > 0$. Thus,

$$
\dot{\bar{V}}_i \leq -\frac{1}{2}d_{i,3}\|(\epsilon_{i,s}, \varepsilon_i)\|^2 + \frac{d_{i,4}^2}{2d_{i,3}}\left(\left(\|\beta\|^2 + 2\breve{B}^2\|\Lambda\|^2\|\bar{C}\|^2\right)\|\bar{\delta}_i\|^2 + 2\|\Lambda\|^2\|\bar{C}\|^2\|\bar{\delta}_i\|^4\right). \tag{27}
$$

Let $W_i = k_{i,1}V_i + k_{i,2}V_i^2 + \bar{V}_i$ with

$$
k_{i,1} \geq \frac{1}{c_{i,3}}\left(1 + \frac{d_{i,4}^2}{2d_{i,3}}\left(\|\beta\|^2 + 2\breve{B}^2\|\Lambda\|^2\right)\right), \quad k_{i,2} \geq \frac{1}{2c_{i,1}c_{i,3}}\left(1 + \frac{d_{i,4}^2}{d_{i,3}}\breve{B}^2\|\Lambda\|^2\right).
$$

Then, the time derivative of W_i along the trajectories of the system (22) can be computed as

$$\begin{aligned} \dot{W}_i|_{(22)} &\leq -k_{i,1}c_{i,3}\|\bar{\delta}_i\|^2 - 2k_{i,2}c_{i,1}c_{i,3}\|\bar{\delta}_i\|^4 - \frac{1}{2}d_{i,3}\|(\epsilon_{i,s},\varepsilon_i)\|^2 \\ &\quad + \frac{d_{i,4}^2}{2d_{i,3}}\left(\left(\|\beta\|^2 + 2\breve{B}^2\|\Lambda\|^2\|\bar{C}\|^2\right)\|\bar{\delta}_i\|^2 + 2\|\Lambda\|^2\|\bar{C}\|^2\|\bar{\delta}_i\|^4\right) \\ &\leq -\left(k_{i,1}c_{i,3} - \frac{d_{i,4}^2}{2d_{i,3}}\left(\|\beta\|^2 + 2\breve{B}^2\|\Lambda\|^2\|\bar{C}\|^2\right)\right)\|\bar{\delta}_i\|^2 \\ &\quad -\left(k_{i,2}c_{i,1}c_{i,3} - \frac{d_{i,4}^2}{d_{i,3}}\|\Lambda\|^2\|\bar{C}\|^2\right)\|\bar{\delta}_i\|^4 - \frac{1}{2}d_{i,3}\|(\epsilon_{i,s},\varepsilon_i)\|^2 \\ &\leq -\|\bar{\delta}_i\|^2 - \|\bar{\delta}_i\|^4 - \frac{1}{2}d_{i,3}\|(\epsilon_{i,s},\varepsilon_i)\|^2 \end{aligned} \tag{28}$$

from which, by using Lyapunov stability result, we conclude that the system (19) is globally asymptotically stable.

Step 2 : The time derivative of function W_i along the trajectories of system (20) is given as

$$\begin{aligned} \dot{W}_i|_{(20)} &\leq -\|\bar{\delta}_i\|^2 - \|\bar{\delta}_i\|^4 - \frac{1}{2}d_{i,3}\|(\epsilon_{i,s},\varepsilon_i)\|^2 \\ &\quad + \frac{k_1 + 2k_2V_i}{|\mathcal{N}_i|}\bar{\delta}_i^T P \begin{bmatrix} G^T & 0 & \cdots & 0 & -\bar{g}_1 \end{bmatrix}^T \sum_{j\in\mathcal{N}_i} e_j \\ &\quad + \frac{\partial \bar{V}_i}{\partial \epsilon_{i,s}}\left(\frac{\bar{g}_2}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} e_j\right) - \frac{\partial \bar{V}_i}{\partial \varepsilon_i}\left(\frac{1}{|\mathcal{N}_i|}\Lambda\Omega_i\sum_{j\in\mathcal{N}_i} e_j\right). \end{aligned} \tag{29}$$

Because

$$\begin{aligned} &\frac{k_1 + 2k_2V_i}{|\mathcal{N}_i|}\bar{\delta}_i^T P \begin{bmatrix} G^T & 0 & \cdots & 0 & -\bar{g}_1 \end{bmatrix}^T \sum_{j\in\mathcal{N}_i} e_j \\ &\leq \frac{1}{2}\|\bar{\delta}_i\|^2 + \frac{1}{4}\|\bar{\delta}_i\|^4 + \frac{k_{i,1}^2\|P\|^2(\|G\|^2 + \bar{g}_1^2)}{2|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} e_j^2 + \frac{216k_{i,2}^4\|P\|^4(\|G\|^4 + \bar{g}_1^4)}{|\mathcal{N}_i|^2}\sum_{j\in\mathcal{N}_i} e_j^4 \end{aligned}$$

and

$$\begin{aligned} &\frac{\partial \bar{V}_i}{\partial \epsilon_{i,s}}\left(\frac{\bar{g}_2}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} e_j\right) - \frac{\partial \bar{V}_i}{\partial \varepsilon_i}\left(\frac{1}{|\mathcal{N}_i|}\Lambda\Omega_i\sum_{j\in\mathcal{N}_i} e_j\right) \\ &\leq \frac{1}{4}d_{i,3}\|(\epsilon_{i,s},\varepsilon_i)\|^2 + \frac{1}{4}\|\bar{\delta}_i\|^4 + \frac{d_{i,4}^2\left(\bar{g}_2^2 + 4\breve{B}^2\|\Lambda\|^2\right)}{d_{i,3}|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} e_j^2 + \frac{16d_{i,4}^4\|\Lambda\|^4}{|\mathcal{N}_i|^2 d_{i,3}^2}\sum_{j\in\mathcal{N}_i} e_j^4 \end{aligned}$$

we conclude

$$\dot{W}_i|_{(20)} \leq -\frac{1}{2}\|\bar{\delta}_i\|^2 - \frac{1}{2}\|\bar{\delta}_i\|^4 - \frac{1}{4}d_{i,3}\|(\epsilon_{i,s},\varepsilon_i)\|^2 + \varrho_{i,1}\sum_{j\in\mathcal{N}_i} e_j^2 + \varrho_{i,2}\sum_{j\in\mathcal{N}_i} e_j^4 \tag{30}$$

where

$$\varrho_{i,1} = \frac{k_{i,1}^2\|P\|^2(\|G\|^2+\bar{g}_1^2)}{2|\mathcal{N}_i|} + \frac{d_{i,4}^2\left(\bar{g}_2^2+4\breve{B}^2\|\Lambda\|^2\right)}{d_{i,3}|\mathcal{N}_i|}$$
$$\varrho_{i,2} = \frac{216k_{i,2}^4\|P\|^4(\|G\|^4+\bar{g}_1^4)}{|\mathcal{N}_i|^2} + \frac{16d_{i,4}^4\|\Lambda\|^4}{|\mathcal{N}_i|^2 d_{i,3}^2}. \tag{31}$$

Noting that $e_j = C\bar{x}_j$, the system (20) is ISS with respect to input $\bar{x}_j,\ j\in\mathcal{N}_i$.

Step 3: The $\bar{x}_i$ subsystem in (10) can be rewritten as

$$\dot{\bar{x}}_i = (A+BK)\bar{x}_i - BK\delta_i - B\varphi(\alpha)\epsilon_i + B\tilde{\varphi}(\varepsilon_i)\zeta_i \tag{32}$$

where $\tilde{\varphi}(\varepsilon_i) = \varphi(\hat{\alpha}_i) - \varphi(\alpha)$. Since $\tilde{\varphi}(0) = 0$, according to Lemma 3.5 of [10], there exists a class $\mathcal{K}$ function $\rho(\cdot)$ such that

$$\|\tilde{\varphi}(\epsilon_i)\|^2 \le \rho(\|\epsilon_i\|). \tag{33}$$

Let Q be such that $Q(A+BK)+(A+BK)^TQ = -I$, and consider the Lyapunov function $\bar{W}_i = \bar{x}_i^TQ\bar{x}_i$. Then, the time derivative of $\bar{W}_i$ can be obtained as

$$\begin{aligned}\dot{\bar{W}}_i &= -\|\bar{x}_i\|^2 + 2\bar{x}_i^TQ\left(-BK\delta_i - B\varphi(\alpha)\epsilon_i + B\tilde{\varphi}(\varepsilon_i)\zeta_i\right)\\ &\le -1/2\|\bar{x}_i\|^2 + 4\|QBK\|^2\|\delta_i\|^2 + 4\|QB\|^2\|\varphi(\alpha)\|^2\|\epsilon_i\|^2\\ &\quad +4\|QB\|^2(\breve{B}^2\rho(\|\epsilon_i\|) + \rho^2(\|\epsilon_i\|) + \|\varepsilon_i\|^4).\end{aligned} \tag{34}$$

From which, the $\bar{x}_i$ subsystem (32) is ISS with respect to inputs $\delta_i, \epsilon_i, \varepsilon_i$. Since $e_i = C\bar{x}_i$, with regarding e_i as output of $\bar{x}_i$ subsystem, the $\bar{x}_i$ subsystem is also input-to-output stable.

Step 4: According to [2], under Assumption 2.4, we can relabel the nodes such that, if $(i,j)\in\mathcal{E}$, then $i<j$. In what follows, we using the relabeled index of nodes. Obviously, agent 1 is such that $(0,1)\in\mathcal{E}$. Thus, according to Theorem 10.5.2 of [9], the system composed of $(\delta_1,\epsilon_1,\varepsilon_1,\bar{x}_1)$ is globally asymptotically stable.

For agent 2, if $(0,2)\in\mathcal{E}$, then, similar to agent 1, the system composed of $(\delta_2,\epsilon_2,\varepsilon_2,\bar{x}_2)$ is globally asymptotically stable. Otherwise, the system of $(\delta_2,\epsilon_2,\varepsilon_2)$ is the dynamics (20) with $i=2$, which is ISS with respect to $\bar{x}_1$. Since $\bar{x}_1$ subsystem is globally asymptotically stable, by utilizing Theorem 10.5.2 of [9], the system of $(\delta_2,\epsilon_2,\varepsilon_2)$ is also globally asymptotically stable. Also, because $\bar{x}_2$ is ISS with respect to $(\delta_2,\epsilon_2,\varepsilon_2)$, $\bar{x}_2$ subsystem is globally asymptotically stable. In both cases, the system $(\delta_1,\epsilon_1,\varepsilon_1,\bar{x}_1,\delta_2,\epsilon_2,\varepsilon_2,\bar{x}_2)$ is globally asymptotically stable.

In a recursive manner, we can prove that the system of $(\delta_1,\epsilon_1,\varepsilon_1,\bar{x}_1,\ldots,\delta_N,\epsilon_N,\varepsilon_N,\bar{x}_N)$ is globally asymptotically stable.

From the boundedness of $\theta(t)$ and the relationship between $(\delta_i,\epsilon_i,\varepsilon_i,\bar{x}_i)$ and $(\eta_i,\zeta_i,\hat{\alpha}_i,\tilde{x}_i)$, the solution of the closed-loop system composed of (17) and (18) exists and is bounded over $[0,+\infty)$. Moreover, because $e_i = C\bar{x}_i$, $\lim_{t\to\infty} e_i = 0, i=1,\ldots,N$. The conclusion follows. □

Remark 3.4. In this paper, we only discuss the case when the agent system (1) has a single input and a single output. However, our results can be extended to the system possessing the multi-input and multi-output.

Remark 3.5. Note that, the equations (11) can be rewritten in the standard form of a linear equation: $\mathcal{Q}(\alpha)\chi = \nu$, where

$$\mathcal{Q}(\alpha) = \Phi(\alpha)^T \otimes \begin{bmatrix} I_n & 0_{n\times 1} \\ 0_{1\times n} & 0_1 \end{bmatrix} - I_s \otimes \begin{bmatrix} A & -B \\ C & 0 \end{bmatrix}$$
$$\chi = \mathrm{vec}\left(\begin{bmatrix} \Pi(\alpha) \\ M(\alpha) \end{bmatrix}\right),\ \nu = \mathrm{vec}\left(\begin{bmatrix} -\Delta \\ -\mu \end{bmatrix}\right)$$

with $\otimes$ denoting the Kronecker product of matrices and $\mathrm{vec}(\cdot)$ being a vector-valued function of a matrix such that, for any $X = [X_1 \ \cdots \ X_m] \in \mathbb{R}^{n\times m}$, $\mathrm{vec}(X) = [X_1^T \ \cdots \ X_m^T]^T$. Thus, the solution $(\Pi(\alpha), M(\alpha))$ of the equation (11) depends on the determinant of matrix $\mathcal{Q}(\alpha)$. Under Assumption 2.3, $\det(\mathcal{Q}(\alpha)) \neq 0$ for each α. However, it may happen that $\det(\mathcal{Q}(\hat{\alpha}_i))$ crosses the zero value for some estimation $\hat{\alpha}$. In order to avoid this situation, as in [14], we make a slight change on function $\varphi(\hat{\alpha}_i)$ by selecting $\varphi(\hat{\alpha}_i) = \det^2(\mathcal{Q}(\hat{\alpha}_i))\,\psi(\hat{\alpha}_i)/\max(\varpi, \det^2(\mathcal{Q}(\hat{\alpha}_i)))$, where $\psi(\hat{\alpha}_i) = (M(\hat{\alpha}_i) + K\Pi(\hat{\alpha}_i))$, and ϖ is such that $\varpi < \min_{i=1,\ldots,N} \det^2(\mathcal{Q}(\hat{\alpha}_{iss}))$, with $\hat{\alpha}_{iss}$ being the steady state of $\hat{\alpha}_i$. The existence of class $\mathcal{K}$ functions $\rho(\cdot)$ in (33) can still be guaranteed when the slight change is made on $\varphi(\cdot)$. Because the convergence of $\hat{\alpha}_i$ to α is independent of $\bar{x}_i$, under Assumption 2.1−2.6, we still have $\hat{\alpha}_i \to \alpha = \hat{\alpha}_{iss}$ and $\det(\mathcal{Q}(\hat{\alpha}_{iss})) \neq 0$ for each $i = 1, \ldots, N$.

4. A NUMERICAL EXAMPLE

Consider the distributed output regulation problem of a group of four double-integrator systems with sinusoidal disturbances:

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= u_i + 0.5v_2, \quad i = 1, 2, 3, 4 \\ e_i &= x_{i,1} - v_1 \end{aligned} \tag{35}$$

with the unknown leader

$$\begin{aligned} \dot{v}_1 &= wv_2 \\ \dot{v}_2 &= -wv_1,\ v(0) = [v_{10} \ \ v_{20}]^T \end{aligned} \tag{36}$$

which is an unforced harmonic oscillator with an uncertain frequency $w \in \mathbb{R}$. We assume that w can be an arbitrarily positive number. The interconnection of the multi-agent system is given in Figure 1.

In our example, we have

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ E = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix} \\ C &= [1 \ \ 0],\ F = [-1 \ 0] \\ S(w) &= \begin{bmatrix} 0 & w \\ -w & 0 \end{bmatrix}. \end{aligned} \tag{37}$$

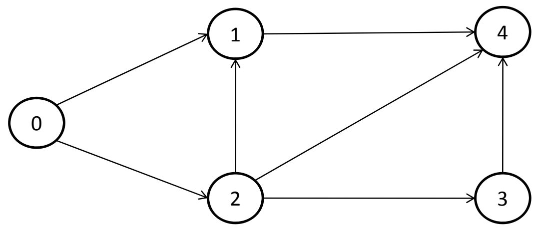

Fig. 1. Communication digraph for the example.

It can be verified that Assumptions 2.1 − 2.4 are satisfied. Moreover, the solution to the regulator equations associated to (35) is

$$X = I_2, \quad U = [-1 \quad -0.5]$$

and $U(w)v = -v_1 - 0.5v_2 = (v_{10}w - \frac{v_{20}}{w})\sin(wt) - (v_{10} + 0.5v_{20})\cos(wt)$. Obviously, $U(w)v$ satisfies

$$\frac{d^2U(w)v}{dt^2} = -w^2U(w)v.$$

Therefore, the τ subsystem in (10) is given with

$$\tau = [U(w)v \quad \tfrac{dU(w)v}{dt}]^T$$
$$\Phi(\alpha) = \begin{bmatrix} 0 & 1 \\ \alpha_0 & 0 \end{bmatrix}, \ \Gamma = [1 \ 0], \ \alpha = \alpha_0 = -w^2$$

and the non-zero initial condition of (36) with $w \neq 0$ excites all modes of τ subsystem.

Therefore, a distributed control taking the form of (18) is designed with the following parameters:

$$\begin{aligned} K &= [-1 \quad -1], \ G = [1 \quad 1]^T, \ \mu = [1 \quad 1] \\ \bar{g}_1 &= \bar{g}_2 = 1, \ \Lambda = 50 \\ \Delta &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \ \hat{\varphi}(\hat{\alpha}_i) = [-3 - 3\hat{\alpha}_i \quad -5 - \hat{\alpha}_i], \ i = 1, 2, 3, 4. \end{aligned} \tag{38}$$

The simulation result is shown in Figures 2 and 3. In the simulation, the initial conditions are selected as $x_{1,1}(0) = 1, x_{1,2}(0) = -4, x_{2,1}(0) = 2, x_{2,2}(0) = -3, x_{3,1}(0) = 3, x_{3,2}(0) = -2, x_{4,1}(0) = 4, x_{4,2}(0) = -1, v_{10} = 3, v_{20} = 3$ and the initial conditions for the dynamic regulator are all zero. The uncertain parameter is chosen as $w = 2.2$. The numerical results demonstrate the effectiveness of the distributed control by showing the regulated output e_i in Figure 2. Moreover, estimated parameter $\hat{\alpha}_i$ converges to $\alpha = -4.84$ as shown in Figure 3.

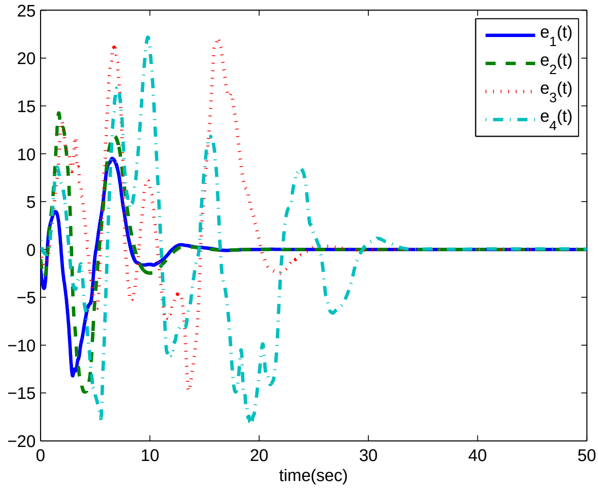

Fig. 2. Response of the regulated outputs e_i of the followers.

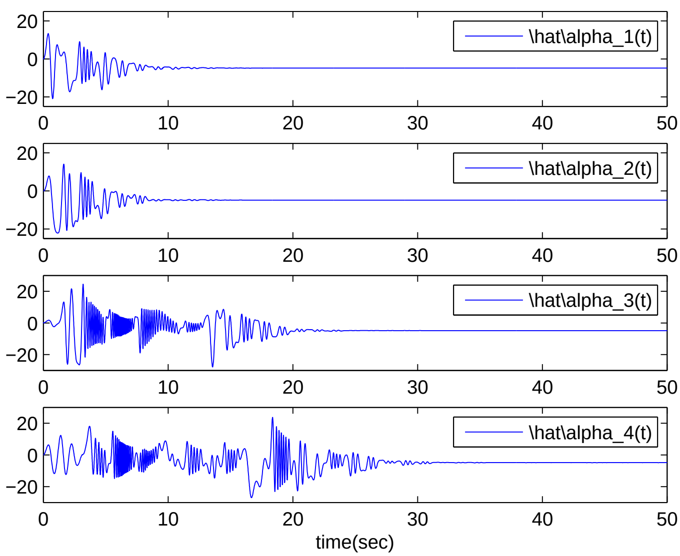

Fig. 3. Response of the estimated parameters $\hat{\alpha}_i$.

5. CONCLUSIONS

In this paper, we studied the distributed output regulation problem of the leader-following multi-agent systems. We focused on the case where the leader contains some uncertain parameters and proposed a distributed control based on an adaptive internal model. By using the results on the input-to-state stability of the cascaded systems, we proved that the designed distributed control guarantees that all agents can track the uncertain leader.

ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China under Grant 61273090.

(Received November 5, 2012)

REFERENCES

[1] B. A. Francis and W. M. Wonham: The internal model priciple of control theory. Automatica *12* (1976), 457–465.

[2] C. Godsil and G. F. Royle: Algebraic Graph Theory. Springer–Verlag, New York 2001.

[3] Y. Hong, J. Hu, and L. Gao: Tracking control for multi-agent consensus with an active leader and variable topology. Automatica *42* (2006), 1177–1182.

[4] Y. Hong, L. Gao, D. Cheng, and J. Hu: Lyapunov-based approach to multi-agent systems with switching jointly connected interaction. IEEE Trans. Automat. Control *52* (2007), 943–948.

[5] Y. Hong, X. Wang, and Z. Jiang: Distribued output regualtion of leader-follower multi-agent systems. Internat. J. Robust Nonlinear Control *23* (2003), 48–66.

[6] J. Hu and G. Feng: Distributed tracking control of leader-follower multi-agent systems under noisy measurement. Automatica *46* (2010), 1382–1387.

[7] J. Huang and Z. Chen: A general framework for tackling the output regulation problem. IEEE Trans. Automat. Control *49* (2004), 2203–2218.

[8] A. Isidori and C. I. Byrnes: Output regulation of nonlinear systems. IEEE Trans. Automat. Control *35* (1990), 131–140.

[9] A. Isidori: Nonlinear Control Systems II. Springer–Verlag, New York 1999.

[10] H. K. Khalil: Nonlinear Systems. Second Edition. Macmillan, New York 1992.

[11] L. Liu, Z. Chen, and J. Huang: Parameter convergence and minimal internal model with an adaptive output regulation problem. Automatica *45* (2009), 1206–1311.

[12] R. Marino and P. Tomei: Nonlinear Control Design: Geometric, Adaptive and Robust. Prentice-Hall, London 1995.

[13] R. Marino and P. Tomei: Output regulation for linear systems via adaptive internal model. IEEE Trans. Automat. Control *48* (2003), 2199–2202.

[14] G. Obregón-Pulido, B. Castillo-Toledo, and A. G. Loukianov: A structurally stable globally adaptive internal model regulator for MIMO linear systems. IEEE Trans. Automat. Control *56* (2011), 160–165.

[15] R. Olfati, J. A. Fax, and R. M. Murray: Consensus and cooperation in networked multi-agent systems. Proc. IEEE *95* (2007), 215–223.

[16] Y. Su and J. Huang: Cooperative output regulation of linear multi-agent systems IEEE Trans. Automat. Control *57* (2012), 1062–1066.

[17] X. Wang, Y. Hong, J. Huang, and Z. Jiang: A distributed control approach to a robust output regulation problem for multi-agent linear systems. IEEE Trans. Automat. Control *55* (2010), 2891–2895.

[18] X. Wang and F. Han: Robust coordination control of switching multi-agent systems via output regulation approach. Kybernetika *47* (2011), 755–772.

[19] D. Xu and Y. Hong: Distributed output regulation of nonlinear multi-agent systems based on networked internal model. In: Proc. Chinese Contol Conference, Hefei 2012, pp. 6483–6488.

Xinghu Wang, Department of Automation, University of Science and Technology of China, Hefei, Anhui 230027. P. R. China.
e-mail: xinghuw@mail.ustc.edu.cn

Haibo Ji, Department of Automation, University of Science and Technology of China, Hefei, Anhui 230027. P. R. China.
e-mail: jihb@ustc.edu.cn

Chuanrui Wang, Department of Automation, University of Science and Technology of China, Hefei, Anhui 230027. P. R. China.
e-mail: hugh@mail.ustc.edu.cn