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INVARIANT SUBSPACES FOR GRASPING INTERNAL FORCES AND NON-INTERACTING FORCE-MOTION CONTROL IN ROBOTIC MANIPULATION

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This paper presents a parametrization of a feed-forward control based on structures of subspaces for a non-interacting regulation. With advances in technological development, robotics is increasingly being used in many industrial sectors, including medical applications (e. g., micromanipulation of internal tissues or laparoscopy). Typical problems in robotics and general mechanisms may be mathematically formalized and analyzed, resulting in outcomes so general that it is possible to speak of structural properties in robotic manipulation and mechanisms. This work shows an explicit formula for the reachable internal contact forces of a general manipulation system. The main contribution of the paper consists of investigating the design of a feed-forward force-motion control which, together with a feedback structure, realizes a decoupling force-motion control. A generalized linear model is used to perform a careful analysis, resulting in the proposed general geometric structure for the study of mechanisms. In particular, a lemma and a theorem are presented which offer a parametrization of a feed-forward control for a task-oriented choice of input subspaces. The existence of these input subspaces is a necessary condition for the structural non-interaction property. A simulation example in which the subspaces and the control structure are explicitly calculated is shown and widely explicated.

Keywords: subspaces, matrices, manipulators, internal forces

Classification: 93D09, 19L64, 70Q05, 14L24

1. INTRODUCTION

There are many unconventional uses of manipulation mechanisms, such as the coordinated use of a robot's multiple fingers or arms in a cooperative task, the use of the inner links of a robot arm or finger to hold an object, and the exploitation of parallel mechanical structures. These devices can be referred to as "general manipulation systems". A rigorous definition of a "general manipulation system" is provided below. Prattichizzo and Bicchi [17] characterized the structural property of the linearized model of the general manipulation system as being reachable and observable. For a broad overview of the manipulation control problem, refer to [14] and its references. Recent contributions to the topic of manipulation have furthered progress in the geometric approach through the use of linear algebra. Works such as in [10] mark progress in the analysis and synthesis of geometric controller for mechanical systems. Mercorelli and Prattichizzo [12] present a geometric approach which includes procedures that guarantee the robustness of the system against parametric uncertainties in the model. In general, the application of subspace structures to mechanical systems has many advantages [15]. In particular, the geometric approach can be focused on the disturbance decoupling problem [15]; this has attracted many scientists. Furthermore, in [8] and [7] a systematic new analysis is presented based on the computation of condensed forms under orthogonal equivalent transformations. This has both theoretical and practical advantages, including easy and elegant interpretation of the results, and straightforward computer implementation. The earliest use of a matrix oriented to the control of systems was by Basile and Marro ([2] and [1]) and by Wonham and Morse ([21] and [13]). These authors proposed a geometric approach to solve problems such as non-interacting control, observer, and disturbance rejection. In this paper, a linearized model of the general mechanisms of manipulation is used. The linearized analysis is considered to be a fundamental preparatory step towards a full non-linear analysis, which is currently too complex to provide full generality. Finally, it should be noted that there exists a subclass of Cartesian manipulators with which the linearized model provides an exact representation of the complete system dynamics.

The work in [11] investigates the geometric and structural characteristics involved in the control of general mechanisms and manipulation systems. These systems consist of multiple cooperating linkages that interact with a reference member of the mechanism (the "object") by means of contacts on any available part of their links. Grasp and manipulation of an object by the human hand are taken as a paradigmatic example for this class of manipulators.

The main result consists of a general matrix parametrization of a feed-forward control proposed for a task-oriented choice of input subspaces. The existence of these input subspaces is a necessary condition for the structural non-interaction property. The remainder of this paper is organized as follows. Section 2 introduces some notation and provides the linearized dynamics of manipulation systems. In Section 3, the system outputs are specified in terms of object motions and contact forces. Section 4 is aimed at the design of a decoupling controller for a general grasping mechanism. It uses rigid-body object motions and the reachable contact forces together with possible mechanism redundancy. Finally, Section 5 presents the linear structures concerning a parametrization of a feed-forward control.

2. DYNAMIC MODEL

This section derives the linearized model of the dynamics of a general manipulation system. A detailed discussion of this model is presented in [16]. The vector of manipulator joint positions is denoted by $\mathbf{q} \in \Re^q$, $\tau \in \Re^q$ is the vector of joint actuator torques, $\mathbf{u} \in \Re^d$ is the vector locally describing the position and the orientation of a frame attached to the object, and $\mathbf{w} \in \Re^d$ is the vector of forces and torques resulting from external forces acting directly on the object. In the literature, $\mathbf{w}$ is usually referred to as the disturbance vector. The force/torque interaction $\mathbf{t}_i$ (see Figure 1) at the ith contact is accounted for by using a lumped parameter $(\mathbf{K}_i,\ \mathbf{B}_i)$ model of visco-elastic

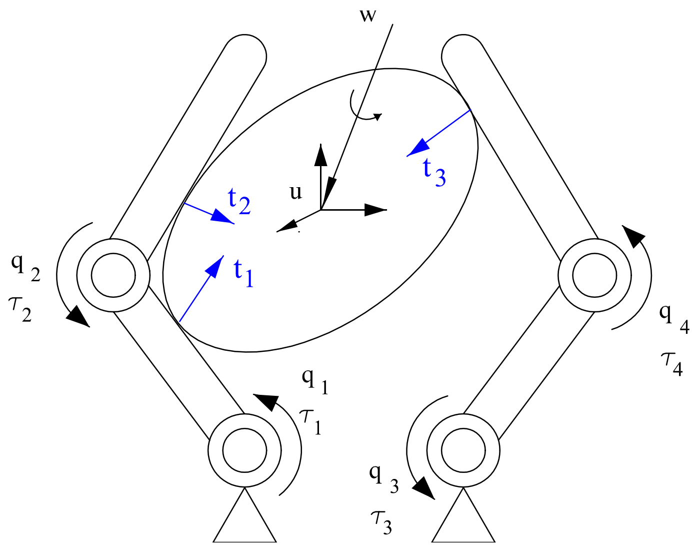

Fig. 1. Vector notation for general manipulation system analysis.

phenomena. According to this model, the contact force vector $\mathbf{t}_i$ is

$$\mathbf{t}_i = \mathbf{K}_i({}^h\mathbf{c}_i - {}^o\mathbf{c}_i) + \mathbf{B}_i({}^h\dot{\mathbf{c}}_i - {}^o\dot{\mathbf{c}}_i), \tag{1}$$

where vectors ${}^h\mathbf{c}_i$ and ${}^o\mathbf{c}_i$ describe the positions of two contact frames, the first on the manipulator and the second on the object. The ith contact spring and damper are anchored. Matrices $\mathbf{K}_i$ and $\mathbf{B}_i$ are symmetric and positive definite (p.d.), and the dimensions of the vectors in (1) depend on the particular model used to describe the contact interaction [19]. The computation and control of the stiffness matrix have been considered in depth by Cutkosky and Kao [9]. To simplify the notation, the contact forces $\mathbf{t}_i$'s, and the contact points ${}^h\mathbf{c}_i$'s and ${}^o\mathbf{c}_i$'s are grouped into vectors $\mathbf{t}$, ${}^h\mathbf{c}$, and ${}^o\mathbf{c}$. Similarly, the $\mathbf{K}_i$'s and $\mathbf{B}_i$'s are grouped to build the block diagonal grasp stiffness and damping symmetric and p.d. matrices $\mathbf{K}$ and $\mathbf{B}$. The Jacobian $\mathbf{J}$ and grasp matrix $\mathbf{G}$ of the manipulation system (see [17]) are defined by linear maps relating the velocities of vectors ${}^h\mathbf{c}$ and ${}^o\mathbf{c}$ with the joint and object velocities $\dot{\mathbf{q}}$ and $\dot{\mathbf{u}}$, respectively:

$${}^h\dot{\mathbf{c}} = \mathbf{J}\dot{\mathbf{q}}, \quad {}^o\dot{\mathbf{c}} = \mathbf{G}^T\dot{\mathbf{u}}. \tag{2}$$

Note that both $\mathbf{J}^T\mathbf{t}$ and $\mathbf{G}\mathbf{t}$ represent the effects of the contact forces $\mathbf{t}$ on the manipulation and object dynamics, whose full non-linear models are:

$$\mathbf{M}_h\ddot{\mathbf{q}} + \mathbf{Q}_h = -\mathbf{J}^T\mathbf{t} + \tau; \quad \mathbf{M}_o\ddot{\mathbf{u}} + \mathbf{Q}_o = \mathbf{G}\mathbf{t} + \mathbf{w}. \tag{3}$$

Here, $\mathbf{M}_h$ and $\mathbf{M}_o$ are inertia symmetric and p.d. matrices, while $\mathbf{Q}_h$ and $\mathbf{Q}_o$ are terms representing the velocity-dependent and gravity forces of the manipulator and the object, respectively. Proceeding with the analysis of the linearized model of the full manipulation system, a reference equilibrium configuration is considered:

$$\mathbf{q} = \mathbf{q}_o, \quad \mathbf{u} = \mathbf{u}_o, \quad \dot{\mathbf{q}} = \dot{\mathbf{u}} = \mathbf{0}, \quad \tau = \tau_o, \quad \mathbf{w} = \mathbf{w}_o \quad \mathbf{t} = \mathbf{t}_o, \quad \text{such that}$$
$$\tau_o = \mathbf{J}^T \mathbf{t}_o \qquad \text{and} \qquad \mathbf{w}_o = -\mathbf{G}\mathbf{t}_o.$$

The linear approximation of the manipulation system in the neighborhood of this equilibrium is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau \delta\tau + \mathbf{B}_w \delta\mathbf{w}, \tag{4}$$

where the state and input vectors are defined as a departure from the reference equilibrium configuration:

$$\begin{array}{rcl} \mathbf{x} & = & \left[\delta\mathbf{q}^T, \delta\mathbf{u}^T, \delta\dot{\mathbf{q}}^T, \delta\dot{\mathbf{u}}^T\right]^T = \left[(\mathbf{q}-\mathbf{q}_o)^T \ (\mathbf{u}-\mathbf{u}_o)^T \ \dot{\mathbf{q}}^T \ \dot{\mathbf{u}}^T\right]^T, \\ \delta\tau & = & \tau - \mathbf{J}^T\mathbf{t}_o, \\ \delta\mathbf{w} & = & \mathbf{w} + \mathbf{G}\mathbf{t}_o, \end{array} \tag{5}$$

and the dynamic, input, and disturbance matrices are

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \quad \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \quad \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix}. \tag{6}$$

To simplify the notation, the symbol δ henceforth will be omitted. According to [16], neglecting gravity, assuming a locally isotropic model of visco-elastic phenomena (stiffness matrix $\mathbf{K}$ is proportional to damping matrix $\mathbf{B}$), and assuming that the local variations of the Jacobian and grasp matrices are small, will all ensure that the blocks $\mathbf{L}_k$ and $\mathbf{L}_b$ in $\mathbf{A}$ can be simply obtained as

$$\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k \qquad\qquad \mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b, \tag{7}$$

where

$$\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o), \quad \mathbf{P}_k = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{K} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix},$$

$$\mathbf{P}_b = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{B} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix}.$$

To be more precise

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I}_q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_u \\ -\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{K}\mathbf{J} & \mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{K}\mathbf{G}^T & -\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{B}\mathbf{J} & \mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{B}\mathbf{G}^T \\ \mathbf{M}_o^{-1}\mathbf{G}\mathbf{K}\mathbf{J} & -\mathbf{M}_o^{-1}\mathbf{G}\mathbf{K}\mathbf{G}^T & \mathbf{M}_o^{-1}\mathbf{G}\mathbf{B}\mathbf{J} & -\mathbf{M}_o^{-1}\mathbf{G}\mathbf{B}\mathbf{G}^T \end{bmatrix}. \tag{8}$$

2.1. A grasp and its geometric property

The following grasp properties, based on the existence of null spaces in the grasp matrices $\mathbf{G}$ and their transposes, influence the dynamic behaviour of the manipulation system ([17] and [16]).

Definition 1. A grasp (or manipulation system) is said "defective" if $\ker(\mathbf{J}^T) \neq \mathbf{0}$.

From (3), notice that $\mathbf{J}^T \in \Re^{(q \times t)}$, where t is the number of components of contact force $\mathbf{t}$. Thus, whenever the manipulation system has fewer than t degrees of freedom (DoF's) q, it exhibits a defective grasp. When the system is defective, directions for $\mathbf{t}$ exist which have no influence on the manipulator dynamics (3). This scenario may be considered to be a common factor in all defective manipulation systems due to their intrinsically low number of DoF's, q. A more detailed discussion of defectivity is provided in [17].

Definition 2. A grasp is said to be "indeterminate" if $\ker(\mathbf{G}^T) \neq \mathbf{0}$.

If the grasp is indeterminate, there exist motions for the objects under which no variations of contact force occur (2). In other words, indeterminacy implies that the object is not firmly grasped.

Definition 3. A manipulation system is said to be "graspable" if $\ker(\mathbf{G}) \neq \mathbf{0}$.

If the system is graspable, it is possible to exert contact forces with zero resultant forces on the object. The forces belonging to the null space of $\mathbf{G}$ are usually referred to in the literature as "internal forces", and they play a fundamental role in controlling the manipulation task. In the absence of internal forces squeezing the object, a manipulator is only accommodating the object, but not grasping it. Should a disturbance to the object occur that is tangential to the manipulator contact, the system cannot reject the disturbance by simply opposing the contact force. It must generate an additional internal force to keep the total contact force in the friction cone, thereby maintaining the contact. The following proposition, reported in [17], concerns the stabilizability of the linear dynamics. Finally, the well-known concept of manipulator redundancy is formalized as follows.

Definition 4. A grasp is said "redundant" if $\ker(\mathbf{J}) \neq \mathbf{0}$.

Proposition 1. If the system is not indeterminate, i. e. $\ker(\mathbf{G}^T) = \mathbf{0}$, then the minimal $\mathbf{A}$–invariant subspace containing $\mathrm{im}(\mathbf{B}_\tau)$, $\min\mathcal{I}(\mathbf{A}, \mathbf{B}_\tau)$, is externally stable.

From here on, non-indeterminacy is assumed, $\ker(\mathbf{G}^T) = \mathbf{0}$. This assumption is a necessary condition for the linearized manipulation system (4) to be stabilizable.

3. INTERNAL FORCES

The main goal of a manipulation task is to control the motion of a manipulated object. An interesting aspect of this work is that the manipulated object is not anchored to the robotic device, but is acted upon through passive (not directly actuated) "joints" with mechanical unilateral contact. Unilateral contacts occur between different parts of the system, and are usually modeled as inequality constraints on the direction of forces and kinematic constraints on rolling and sliding motions. Since contact constraints ensure both object grasp and motion control, it is of a paramount importance to prevent their violation. Assume that a general task is specified in terms of the object motion. Then, the remaining degrees of freedom for controlling the contact phenomena correspond to the "internal forces", which belong to the null space of the grasp matrix $\mathbf{G}$. As previously mentioned, they are considered "internal" since their resultant action on the object dynamics is zero. In the robotic community the importance of such a mathematical characterization of the graspability of an object emerged in the last years. In particular, the importance of a formal mathematical concept of graspability is in the fact that we can define such a measure of the possibility to grasp, hold and manipulate an object. Since the resulting action on the object dynamics is zero, this yields that the object can be squeezed and/or held.

The outputs of the dynamic system (4) must be defined to further investigate force/motion control, by considering the "rigid-body coordinate object motions", the "reachable internal contact forces", and the "manipulator dynamic redundancy" (following [17]).

3.1. Rigid–body coordinate object motions

Rigid-body kinematics are of particular interest in the control of manipulation systems. Rigid-body kinematics have been studied in a quasi-static setting [5], and in terms of unobservable subspaces [16]. In both cases, they were described by a matrix $\mathbf{\Gamma}$ whose columns form a basis for

$$\mathsf{ker}\begin{bmatrix}\mathbf{J} & -\mathbf{G}^T\end{bmatrix} = \mathsf{im}(\mathbf{\Gamma}), \qquad \text{where} \qquad \mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_r & \mathbf{\Gamma}_{qc} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_{uc} & \mathbf{\Gamma}_i \end{bmatrix} \qquad \text{and} \qquad \mathbf{J}\mathbf{\Gamma}_{qc} = \mathbf{G}^T\mathbf{\Gamma}_{uc}, \tag{9}$$

$\mathbf{\Gamma}_r$ is a basis matrix (b.m.)[1] of the subspace of redundant manipulator motions $\mathsf{ker}(\mathbf{J})$, $\mathbf{\Gamma}_i$ is a b.m. of the subspace of indeterminate object motions $\mathsf{ker}(\mathbf{G}^T)$, and $\mathbf{\Gamma}_{qc}$ and $\mathbf{\Gamma}_{uc}$ are conformal partitions of a complementary basis matrix (c.b.m.).[2] Coordinated rigid-body motions of the mechanisms are defined in [5] as motions of the manipulator $\delta\mathbf{q}$ and of the object $\delta\mathbf{u}$, such that

$$\mathsf{im}\begin{bmatrix}\delta\mathbf{q} \\ \delta\mathbf{u}\end{bmatrix} \in \mathsf{im}\begin{bmatrix}\mathbf{\Gamma}_{qc} \\ \mathbf{\Gamma}_{uc}\end{bmatrix}$$

Physically, rigid-body displacements are those that do not involve a variation of contact forces, thus the name "rigid". The object-motion regulated output $\mathbf{e}_{uc}$ is chosen as the projection of object motions $\mathbf{u}$ onto the subspace of rigid-body object motions $\mathsf{im}(\mathbf{\Gamma}_{uc})$:

$$\mathbf{e}_{uc} = \mathbf{E}_{uc}\mathbf{x}; \qquad \text{with} \qquad \mathbf{E}_{uc} = \mathbf{\Gamma}_{uc}^P \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{10}$$

where $\mathbf{\Gamma}_{uc}^P$ is the projection matrix

$$\mathbf{\Gamma}_{uc}^P = \mathbf{\Gamma}_{uc}(\mathbf{\Gamma}_{uc}^T\mathbf{\Gamma}_{uc})^{-1}\mathbf{\Gamma}_{uc}^T. \tag{11}$$

[1] $\mathbf{V}$ is called a basis matrix of a subspace $\mathcal{V}$, if it is full column rank (f.c.r.) and $\mathsf{im}(\mathbf{V}) = \mathcal{V}$.

[2] $\mathbf{W}$ is called a complementary basis matrix of $\mathcal{V}$ to $\mathcal{X}$, if it is f.c.r. and $\mathsf{im}(\mathbf{W}) \oplus \mathcal{V} = \mathcal{X}$.

Notice that matrix $\mathbf{M}_0$ does not play a role due to the considered subspace of the displacements, as they do not involve any variation of contact forces.

3.2. Reachable internal contact forces

Contact forces $\mathbf{t}$ are exerted by the manipulating system on the object in order to maintain a grasp, to reject disturbance wrenches $\mathbf{w}$, and to control the object motion. The control of contact forces is fundamental to manipulation control, and improved control leads to finer manipulation. In [16], the reachable subspace of contact forces as outputs of the dynamic system (4) was studied, with results reported in the next proposition. Define $\delta\mathbf{t}$ as the departure of contact force vector $\mathbf{t}$ from the reference equilibrium $\mathbf{t}_o$ (5). Its first order approximation can be easily evaluated by substituting differential kinematics (2) in $\mathbf{t}$, the grouped vector of $\mathbf{t}_i$'s (1). Hence

$$\mathbf{t} = \mathbf{C}_t\mathbf{x}, \qquad \text{where} \quad \mathbf{C}_t = \begin{bmatrix} \mathbf{KJ} & -\mathbf{KG}^T & \mathbf{BJ} & -\mathbf{BG}^T \end{bmatrix}. \tag{12}$$

The regulated force output $\mathbf{e}_{ti}$ is defined as the projection of the contact force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$. Then, the output matrix is defined as

$$\mathbf{e}_{ti} = \mathbf{E}_{ti}\mathbf{x}, \quad \text{with} \quad \mathbf{E}_{ti} = (\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{C}_t = \begin{bmatrix} \mathbf{Q}_k & \mathbf{0} & \mathbf{Q}_\beta & \mathbf{0} \end{bmatrix}, \tag{13}$$

$$\text{where } \mathbf{Q}_k = (\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ} \text{ and } \mathbf{Q}_\beta = (\mathbf{I} - \mathbf{BG}^T(\mathbf{GBG}^T)^{-1}\mathbf{G})\mathbf{BJ}.$$

Notice that $\mathsf{im}(\mathbf{Q_k}) = \mathsf{im}(\mathbf{Q}_\beta)$, under the hypothesis $\mathsf{im}(\mathbf{K}) = \mathsf{im}(\mathbf{B})$. The third output $\mathbf{e}_{qr}$ is now introduced, taking into account the possible redundancy of the mechanism. Whenever the analysis is not static, the inertia matrix $\mathbf{M}_h$ must be considered in characterizing the redundancy. Therefore, the redundancy output matrix $\mathbf{E}_{qr}$ is defined as

$$\mathbf{e}_{qr} = \mathbf{E}_{qr}\mathbf{x}, \qquad \text{with} \quad \mathbf{E}_{qr} = \begin{bmatrix} \mathbf{\Gamma}_{qr}^P\mathbf{M}_h & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{14}$$

where $\mathbf{\Gamma}_{qr}^P$ is the projection matrix onto $\mathsf{ker}(\mathbf{J})$ whose b.m. is $\mathbf{\Gamma}_{qr}$

$$\mathbf{\Gamma}_{qr}^P = \mathbf{\Gamma}_{qr}(\mathbf{\Gamma}_{qr}^T\mathbf{\Gamma}_{qr})^{-1}\mathbf{\Gamma}_{qr}^T. \tag{15}$$

Proposition 1. According to Definition 3, the reachable subspace of contact forces $\mathbf{t}$, under the hypothesis $\mathbf{K}$ is proportional to $\mathbf{B}$,is

$$\mathcal{R}_{t,\tau} = \mathbf{C}_t\mathsf{min}\mathcal{I}(\mathbf{A}, \mathbf{B}_\tau) = \mathsf{min}\mathcal{I}(\mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{KJ}).$$

Control of the contact forces belonging to the null space of the grasp matrix $\mathbf{G}$ is normally an area of great interest of the research in this field. Obviously, in general, the null space of $\mathbf{G}$ is not completely reachable. The importance of the reachability of internal forces in grasping was clarified in [6], where the principle of virtual work was used to describe the subspace of *active* internal forces, and in [17] where the *asymptotically reachable* internal forces were studied as a steady state behaviour of a suitable transfer function. In this work the *reachable internal forces subspace* as an intersection of subspaces is characterized.

Definition 5. The reachable internal forces subspace $\mathcal{R}_{ti,\tau}$ is

$$\mathcal{R}_{ti,\tau} = \mathcal{R}_{t,\tau} \cap \mathsf{ker}(\mathbf{G}).$$

The following theorem provides an explicit formula for the reachable internal forces subspace:

Theorem 1. Under the hypothesis $\mathbf{K}$ is proportional to $\mathbf{B}$ then

$$\mathcal{R}_{ti,\tau} = \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\ \mathbf{C}_t) = \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\ \mathbf{KJ}).$$

P r o o f. The theorem statement is equivalent to

$$\mathcal{R}_{ti,\tau} \supseteq \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}) \tag{16}$$

$$\mathcal{R}_{ti,\tau} \subseteq \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}). \tag{17}$$

From Definition 5 and Proposition 1, the inclusion (16) turns into

$$\big(\mathsf{min}\mathcal{I}(\mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{KJ}) \cap \mathsf{ker}(\mathbf{G})\big) \supseteq \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}). \tag{18}$$

It is known that

$$\mathsf{ker}(\mathbf{G}) \supseteq \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}), \tag{19}$$

because the matrix $(\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}$ is a projection onto the null space of $\mathbf{G}$. Moreover,

$$\begin{aligned}\mathsf{min}\mathcal{I}(\mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{KJ}) &\supseteq \mathsf{im}\begin{bmatrix} \mathbf{KJ} & \mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{GKJ} \end{bmatrix} \\ &\supseteq \mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}),\end{aligned} \tag{20}$$

because $\mathbf{M}_o^{-1}$ and $(\mathbf{GKG}^T)^{-1}$ are nonsingular matrices. Hence, (18) follows from (19) and (20). Now, instead of proving the inclusion (17), its orthogonal complement is considered

$$\mathcal{R}_{ti,\tau}^{\perp} \supseteq (\mathsf{im}((\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}))^{\perp}. \tag{21}$$

Again from Definition 5, the previous relationship is equivalent to

$$\mathcal{R}_{ti,\tau}^{\perp} = \mathsf{im}(\mathbf{G}^T) + \mathcal{R}_{t,\tau}^{\perp} \supseteq \mathsf{ker}(\mathbf{J}^T\mathbf{K}(\mathbf{I} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GK}))$$

and being $\mathsf{im}(\mathbf{G}^T)$ the null space of the projection matrix $(\mathbf{I} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GK})$ the following relationship is obtained

$$\mathsf{im}(\mathbf{G}^T) + \mathcal{R}_{t,\tau}^{\perp} \supseteq \mathsf{im}(\mathbf{G}^T) + \mathsf{im}(\mathbf{I} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GK}) \cap \mathsf{ker}(\mathbf{J}^T\mathbf{K}).$$

Now, to prove (21) and end the theorem's proof, it will suffice to show that

$$\mathcal{R}_{t,\tau}^{\perp} \supseteq \mathsf{im}(\mathbf{I} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GK}) \cap \mathsf{ker}(\mathbf{J}^T\mathbf{K})$$

and this is trivial by considering the orthogonal

$$\mathcal{R}_{t,\tau} = \mathsf{min}\mathcal{I}(\mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{KJ}) \subseteq \mathsf{ker}(\mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G} - \mathbf{I}) + \mathsf{im}(\mathbf{KJ}). \qquad \square$$

According to this result, the subspace of reachable internal forces is obtained by projector $\mathbf{I}-\mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G}$ acting on the column space of $\mathbf{C}_t$. Notice that Theorem 1 states the equality of $\mathcal{R}_{ti,\tau}$ with the active internal forces in [6] and with the asymptotically reachable internal forces in [17]. In order to specify consistent control outputs, the suggestion of Theorem 1 is followed. In fact, it is possible to choose as regulated force output $\mathbf{e}_{ti}$ the projection of the contact force vector $\mathbf{t}$ on the null space of $\mathbf{G}$. Then the output matrix is defined as follows

$$\mathbf{e}_{ti}=\mathbf{E}_{ti}\mathbf{x}; \quad \text{with} \quad \mathbf{E}_{ti}=(\mathbf{I}-\mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{C}_t=\begin{bmatrix} \mathbf{Q}_k & \mathbf{0} & \mathbf{Q}_\beta & \mathbf{0}\end{bmatrix}, \tag{22}$$

where

$$\mathbf{Q}_k=(\mathbf{I}-\mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J} \tag{23}$$

and

$$\mathbf{Q}_\beta=(\mathbf{I}-\mathbf{B}\mathbf{G}^T(\mathbf{G}\mathbf{B}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{B}\mathbf{J}. \tag{24}$$

It should be noted that $\mathsf{im}(\mathbf{Q}_\mathbf{k})=\mathsf{im}(\mathbf{Q}_\beta)$ under the hypothesis $\mathsf{im}(\mathbf{K})=\mathsf{im}(\mathbf{B})$.

4. DESIGN OF A NON-INTERACTING CONTROLLER

This section describes the design of a decoupling controller for a general grasping mechanism with respect to the rigid-body object motions and the reachable contact forces, together with the possible mechanism redundancy. A geometric approach is used in this analysis. The earliest geometric approaches to non-interacting control were due to Basile and Marro ([1, 2]) and to Wonham and Morse ([13, 21], and [20]).

Definition 6. A control law for the dynamic system (4) is *non-interacting* with respect to the regulated outputs $\mathbf{e}_{uc},\mathbf{e}_{ti}$, and $\mathbf{e}_{qr}$, if there exists partitions τ_{uc},τ_{ti}, and τ_{qr} of the input vector τ such that for zero initial conditions, each input $\tau_{(\cdot)}$ (with all other inputs, identically zero) only affects the corresponding output $e_{(\cdot)}$.

In [18] and [4], it was shown that for the aforementioned outputs $\mathbf{t}_i$ and $\mathbf{u}_c$, there exists a decoupling and stabilizing state feedback matrix $\mathbf{F}$, along with three input partition matrices $\mathbf{U}_{ti}$, $\mathbf{U}_{uc}$, and $\mathbf{U}_{qr}$ such that, for the dynamic triples

$$\begin{gathered}(\mathbf{E}_{ti},\, \mathbf{A}+\mathbf{B}_\tau\mathbf{F},\, \mathbf{B}_\tau\mathbf{U}_{ti})\,,\\ (\mathbf{E}_{uc},\, \mathbf{A}+\mathbf{B}_\tau\mathbf{F},\, \mathbf{B}_\tau\mathbf{U}_{uc})\,,\\ (\mathbf{E}_{qr},\, \mathbf{A}+\mathbf{B}_\tau\mathbf{F},\, \mathbf{B}_\tau\mathbf{U}_{qr})\,,\end{gathered} \tag{25}$$

it holds:

$$\mathcal{R}_{ti}=\mathsf{min}\mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F},\,\mathbf{B}_\tau\mathbf{U}_{ti})\subseteq\mathsf{ker}(\mathbf{E}_{uc})\cap\mathsf{ker}(\mathbf{E}_{qr}),\quad \mathbf{E}_{ti}\mathcal{R}_{ti}=\mathsf{im}(\mathbf{E}_{ti}), \tag{26}$$

$$\mathcal{R}_{uc}=\mathsf{min}\mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F},\,\mathbf{B}_\tau\mathbf{U}_{uc})\subseteq\mathsf{ker}(\mathbf{E}_{ti})\cap\mathsf{ker}(\mathbf{E}_{qr}),\quad \mathbf{E}_{uc}\mathcal{R}_{uc}=\mathsf{im}(\mathbf{E}_{uc}), \tag{27}$$

$$\mathcal{R}_{qr}=\mathsf{min}\mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F},\,\mathbf{B}_\tau\mathbf{U}_{qr})\subseteq\mathsf{ker}(\mathbf{E}_{ti})\cap\mathsf{ker}(\mathbf{E}_{uc}),\quad \mathbf{E}_{qr}\mathcal{R}_{qr}=\mathsf{im}(\mathbf{E}_{qr}). \tag{28}$$

Here,

$$\mathsf{min}\mathcal{I}(\mathbf{A},\mathsf{im}(\mathbf{B}))=\sum_{i=0}^{n-1}\mathbf{A}^i\mathsf{im}(\mathbf{B})$$

is a minimum $\mathbf{A}$–invariant subspace containing $\mathsf{im}(\mathbf{B})$. Moreover, the partition matrices $\mathbf{U}_{uc}$ and $\mathbf{U}_{ti}$ satisfy the following relationships

$$\begin{aligned}\mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{uc}) &= \mathsf{im}(\mathbf{B}_\tau)\cap\mathcal{R}_{uc},\\ \mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{ti}) &= \mathsf{im}(\mathbf{B}_\tau)\cap\mathcal{R}_{ti},\\ \mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{qr}) &= \mathsf{im}(\mathbf{B}_\tau)\cap\mathcal{R}_{qr}.\end{aligned} \tag{29}$$

The stabilizing matrix $\mathbf{F}$ is such that

$$(\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{uc}\subseteq\mathcal{R}_{uc},\quad (\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{ti}\subseteq\mathcal{R}_{ti},\quad (\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{qr}\subseteq\mathcal{R}_{qr}. \tag{30}$$

Considering

$$\mathbf{U}=[\mathbf{U}_{ti},\mathbf{U}_{uc},\mathbf{U}_{qr},\mathbf{U}_c],$$

where $\mathbf{U}_c$ is defined in a complementary fashion. It is assumed that the intersections in (26) are invariant subspaces. If this is not the case, there are several other ways to realize the non-interacting controller, for instance, as described in [3].

5. MAIN RESULTS: PARAMETRIZATION OF THE FEED-FORWARD CONTROL AND GEOMETRIC STRUCTURES

Subspaces $\mathsf{im}(\mathbf{T}_{qr})$ and $\mathsf{im}(\mathbf{T}_h)$ are defined such that

$$\mathbf{T}_{qr}=\begin{bmatrix}\mathbf{\Gamma}_{qr} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Gamma}_{qr}\\ \mathbf{0} & \mathbf{0}\end{bmatrix},\quad \mathbf{T}_h=\begin{bmatrix}\mathbf{\Gamma}_h & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Gamma}_h\\ \mathbf{0} & \mathbf{0}\end{bmatrix}. \tag{31}$$

Subspace $\mathsf{im}(\mathbf{\Gamma}_{qr})$ is a basis matrix for $\mathsf{ker}(\mathbf{J})$, and represents a basis matrix of the redundant movements subspace, according to Definition (4). Subspace $\mathsf{im}(\mathbf{\Gamma}_h)$ is defined as follows:

$$\mathbf{\Gamma}_h=\text{b.m. of }\mathsf{im}(\mathbf{M}_h^{-1}\mathbf{J}^T)\cap\mathsf{max}\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ},\mathsf{ker}(\mathbf{GKJ})). \tag{32}$$

$\mathsf{max}\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ},\mathsf{ker}(\mathbf{GKJ}))$ represents a basis matrix of the subspace, and characterizes the controlled manipulator movements that do not produce object movements ($\mathsf{ker}(\mathbf{GKJ})$). Recall from Section 1 that $\mathsf{im}(\mathbf{M}_h^{-1}\mathbf{J}^T)$ is the term responsible for the effect of the contact forces on the manipulator; thus it can be seen how $\mathbf{\Gamma}_h$ is a basis matrix of the "identically internal forces". Regarding the above notation, recall that the maximal $\mathbf{S}$–invariant subspace contained in $\mathcal{V}$ is indicated by $\mathsf{max}\mathcal{I}(\mathbf{S},\mathcal{V})$. The following discussion outlines the calculation of $\mathcal{R}_{\mathcal{K}_{(\cdot)}}$, as defined in (26), (27) and (28). For practical purposes the subspaces included in $\mathcal{R}_{\mathcal{K}_{(\cdot)}}$ are calculated. It is very easy to show that

$$\mathsf{ker}(\mathbf{E}_{ti})=\mathsf{ker}\begin{bmatrix}\mathbf{Q}_k & \mathbf{0} & \mathbf{Q}_\beta & \mathbf{0}\end{bmatrix}^3\supseteq\mathsf{im}(\mathbf{L}_{ti}),$$

where

$$\mathbf{L}_{ti}=\begin{bmatrix}\mathbf{\Gamma}_{qr} & \mathbf{0} & \mathbf{\Gamma}_{qc} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{\Gamma}_{uc} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Gamma}_{qr} & \mathbf{0} & \mathbf{\Gamma}_{qc}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Gamma}_{uc}\end{bmatrix}. \tag{33}$$

[3] It is very easy to show that $\mathsf{ker}(\mathbf{Q}_k)=\mathsf{ker}(\mathbf{Q}_\beta)$.

The inclusion is easily shown using the definition of $\mathbf{Q}_k$ and $\mathbf{Q}_\beta$. The following remark shows this aspect explicitly.

Remark 1. The null subspace of $\mathbf{Q}_{(\cdot)}$ can be calculated very easily; in fact, $\mathsf{ker}(\mathbf{Q}_{(\cdot)}) = \mathsf{ker}(\mathbf{J}) + \mathcal{V}$ where $\mathcal{V} = \{\mathbf{v}|\mathbf{KJv} \in \mathsf{ker}(\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G}) = \mathsf{im}(\mathbf{KG}^T), \mathbf{v} \notin \mathsf{ker}(\mathbf{J})\}$. From (9) it is easy to show that: $\mathcal{V} = \mathsf{im}(\mathbf{\Gamma}_{qc})$ and thus:

$$\mathsf{ker}(\mathbf{Q}_{(\cdot)}) = \mathsf{im}(\mathbf{\Gamma}_{qr}) + \mathsf{im}(\mathbf{\Gamma}_{qc}). \tag{34}$$

In the same way $\mathsf{ker}(\mathbf{E}_{uc}) = \mathsf{ker}\begin{bmatrix} \mathbf{0} & \mathbf{\Gamma}_{uc}^T & \mathbf{0} & \mathbf{0} \end{bmatrix} \supseteq \mathsf{im}(\mathbf{L}_{uc})$ where

$$\mathbf{L}_{uc} = \begin{bmatrix} \mathbf{\Gamma}_{qr} & \mathbf{0} & \mathbf{\Gamma}_h & \mathbf{0} & \mathbf{S}_q\mathbf{Z} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_u & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_{qr} & \mathbf{0} & \mathbf{\Gamma}_h & \mathbf{0} & \mathbf{S}_q\mathbf{Z} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_u \end{bmatrix}, \tag{35}$$

with $\mathbf{X}_u =$ b.m. of $\mathsf{ker}(\mathbf{\Gamma}_{uc}^T) \cap \mathsf{im}(\mathbf{S}_u)$,

$$\begin{aligned} \mathsf{im}(\mathbf{S}_q) &= \min\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KG}^T), \\ \mathsf{im}(\mathbf{S}_u) &= \min\mathcal{I}(\mathbf{M}_o^{-1}\mathbf{GKG}^T, \mathbf{M}_o^{-1}\mathbf{GKJ}), \end{aligned} \tag{36}$$

and $\mathbf{Z}$ such that

$$\mathsf{im}(\mathbf{M}_o^{-1}\mathbf{GKJS}_q\mathbf{Z}) = \mathsf{im}(\mathbf{M}_o^{-1}\mathbf{GKJS}_q) \cap \mathsf{ker}(\mathbf{\Gamma}_{uc}^T). \tag{37}$$

Recall that $\mathbf{\Gamma}_h$ is a basis matrix of

$$\mathrm{im}(\mathbf{M}_h^{-1}\mathbf{J}^T) \cap \max\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathsf{ker}(\mathbf{GKJ})). \tag{38}$$

Subspace $\mathsf{im}(\mathbf{S}_q)$ can be interpreted as the subspace of the forces on the manipulator which are generated by the object movements. Similarly, subspace $\mathsf{im}(\mathbf{S}_u)$ can be interpreted as the subspace of the forces on the object which are generated by the manipulator. Relationship (37) is not physically interpretable.

About subspace $\mathsf{ker}(\mathbf{E}_{qr}) = \mathsf{ker}\begin{bmatrix} \mathbf{\Gamma}_{qr}^P\mathbf{M}_h & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$, recall that $\mathbf{\Gamma}_r^P$ is the projection matrix onto $\mathsf{ker}(\mathbf{J})$ whose b.m. is $\mathbf{\Gamma}_{qr}$

$$\mathbf{\Gamma}_{qr}^P = \mathbf{\Gamma}_{qr}(\mathbf{\Gamma}_{qr}^T\mathbf{\Gamma}_{qr})^{-1}\mathbf{\Gamma}_{qr}^T \tag{39}$$

and, for a given $\mathbf{S}$-basic matrix it holds $\mathsf{ker}(\mathbf{S}^T) = (\mathsf{im}(\mathbf{S}))^\perp$, and then it follows that $\mathsf{ker}(\mathbf{E}_{qr}) \supseteq \mathsf{im}(\mathbf{L}_{qr})$ where

$$\mathbf{L}_{qr} = \begin{bmatrix} \mathbf{X}_{qr} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{X}_{qr} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_u \end{bmatrix}, \tag{40}$$

and $\mathbf{X}_{qr} =$ b.m. of $\mathsf{ker}(\mathbf{\Gamma}_{qr}^T)$. If

$$\mathbf{S}_{ti} = \begin{bmatrix} \mathbf{\Gamma}_h & \mathbf{0} & \mathbf{S}_q\mathbf{Z} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_u & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_h & \mathbf{0} & \mathbf{S}_q\mathbf{Z} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{X}_u \end{bmatrix}, \tag{41}$$

then from (40), the subspace $\mathrm{im}(\mathbf{L}_{qr})$ includes all of the state space except for the redundant movements subspace, $\mathsf{im}(\mathbf{S}_{ti}) \subseteq \mathsf{im}(\mathbf{L}_{uc}) \cap \mathsf{im}(\mathbf{L}_{qr})$.

If $\mathbf{S}_{uc} = \begin{bmatrix} \boldsymbol{\Gamma}_{qc} & \mathbf{0} \\ \boldsymbol{\Gamma}_{uc} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_{qc} \\ \mathbf{0} & \boldsymbol{\Gamma}_{uc} \end{bmatrix}$, then, from (33) and (40), $\mathrm{im}(\mathbf{S}_{uc}) \subseteq \mathsf{im}(\mathbf{L}_{ti}) \cap \mathrm{im}(\mathbf{L}_{qr})$.

If $\mathbf{S}_{qr} = \begin{bmatrix} \boldsymbol{\Gamma}_{qr} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_{qr} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, then, from (33) and (35), $\mathsf{im}(\mathbf{L}_{ti}) \cap \mathsf{im}(\mathbf{L}_{uc}) = \mathsf{im}(\mathbf{S}_{qr})$.

Remark 2. Subspaces $\mathsf{im}(\mathbf{S}_{ti})$, $\mathsf{im}(\mathbf{S}_{uc})$ and $\mathsf{im}(\mathbf{S}_{qr})$ are controlled invariant subspaces. In fact, considering the matrix defined in (8), it is straightforward to show that $\mathbf{A}\mathsf{im}(\mathbf{S}_{(\cdot)}) \subseteq \mathsf{im}(\mathbf{S}_{(\cdot)}) + \mathsf{im}(\mathbf{B}_{\tau})$.

Lemma 1.

$$\mathsf{rank}\begin{bmatrix} \boldsymbol{\Gamma}_h & \mathbf{S}_q\mathbf{Z} \end{bmatrix} = \mathsf{rank}(\boldsymbol{\Gamma}_h) + \mathsf{rank}(\mathbf{S}_q\mathbf{Z}) = q - r - c.$$

Proof. The first equality comes from the null intersection between $\mathsf{im}(\boldsymbol{\Gamma}_h)$ and $\mathsf{im}(\mathbf{S}_q\mathbf{Z})$. In fact from (38) $\mathsf{im}(\boldsymbol{\Gamma}_h)$ is a subspace of $\max\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathsf{ker}(\mathbf{GKJ}))$ which, from (36), is orthogonal to $\mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q)$. The proof of the second equality of the lemma begins with the following relation.

$$\max\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathsf{ker}(\mathbf{GKJ})) = \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q)^{\perp},$$

and it follows that

$$\mathsf{im}(\mathbf{M}_h^{-1}\mathbf{J}^T) \subseteq \max\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathsf{ker}(\mathbf{GKJ})) \oplus \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q).$$

Now, from (36) $\mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q) \subseteq \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{J}^T)$. From the above mentioned inclusion and from definition (38) it follows that

$$\begin{aligned} \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{J}^T) &= \mathbf{M}_h^{-1}\mathbf{J}^T \cap \left(\max\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathsf{ker}(\mathbf{GKJ})) \oplus \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q)\right) \\ &= \left(\mathbf{M}_h^{-1}\mathbf{J}^T \cap \max\mathcal{I}(\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{KJ}, \mathsf{ker}(\mathbf{GKJ}))\right) \oplus \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q) \\ &= \mathsf{im}(\boldsymbol{\Gamma}_h) \oplus \mathsf{im}(\mathbf{M}_h^{-1}\mathbf{S}_q). \end{aligned}$$

It follows that

$$\mathsf{rank}(\boldsymbol{\Gamma}_h) + \mathsf{rank}(\mathbf{S}_q) = \mathsf{rank}(\mathbf{M}_h^{-1}\mathbf{J}^T) = \mathsf{rank}(\mathbf{J}) = q - r$$

and

$$\mathsf{rank}(\boldsymbol{\Gamma}_h) = q - r - \mathsf{rank}(\mathbf{S}_q). \tag{42}$$

It remains to calculate $\mathsf{rank}(\mathbf{S}_q\mathbf{Z})$. Recalling that $\mathbf{S}_q$ and $\mathbf{Z}$ are basis matrices and from (37) $\mathsf{rank}(\mathbf{Z}) \leq \mathsf{rank}(\mathbf{S}_q)$, then

$$\mathsf{rank}(\mathbf{S}_q\mathbf{Z}) = \mathsf{rank}(\mathbf{Z}). \tag{43}$$

From the definition of $\mathbf{Z}$ in (37) it follows that

$$\mathsf{rank}(\mathbf{Z}) = \mathsf{rank}(\mathbf{S}_q) - \mathsf{rank}(\mathbf{Z}^{\perp}), \tag{44}$$

where $\mathsf{rank}(\mathbf{S}_q)$ is the number of components $\mathbf{z} \in \mathbf{Z}$. The last part of this demonstration consists of estimating $\mathsf{rank}(\mathbf{Z}^{\perp})$, which from (37) is

$$\mathsf{rank}(\mathbf{Z}^{\perp}) = \mathsf{rank}(\mathbf{S}_q^T \mathbf{J}^T \mathbf{K} \mathbf{G}^T \mathbf{M}_o^{-1} \mathbf{\Gamma}_{uc}).$$

From (36), it is easy to show that $\mathsf{ker}(\mathbf{S}_q^T) \subseteq \mathsf{ker}(\mathbf{GKJ})$, and thus

$$\mathsf{ker}(\mathbf{S}_q^T) \cap \mathsf{im}(\mathbf{J}^T \mathbf{K} \mathbf{G}^T) = \mathbf{0}$$

and

$$\mathsf{rank}(\mathbf{Z}^{\perp}) = \mathsf{rank}(\mathbf{J}^T \mathbf{K} \mathbf{G}^T \mathbf{M}_o^{-1} \mathbf{\Gamma}_{uc}). \tag{45}$$

Now

$$\mathsf{rank}(\mathbf{Z}^{\perp}) = \mathsf{rank}(\mathbf{J}^T \mathbf{K} \mathbf{G}^T \mathbf{M}_o^{-1} \mathbf{\Gamma}_{uc}) = \mathsf{rank}(\mathbf{\Gamma}_{uc}) = c. \tag{46}$$

If (45) is transposed, then

$$\mathsf{rank}(\mathbf{Z}^{\perp}) = \mathsf{rank}(\mathbf{\Gamma}_{uc}^T \mathbf{M}_o^{-1} \mathbf{GKJ}),$$

and from (9)

$$\mathsf{rank}(\mathbf{Z}^{\perp}) = \mathsf{rank}(\mathbf{\Gamma}_{uc}^T \mathbf{M}_o^{-1} \mathbf{G} \mathbf{K} \mathbf{G}^T \mathbf{\Gamma}_{uc}) = \mathsf{rank}(\mathbf{\Gamma}_{uc}),$$

where the last equality follows because matrix $\mathbf{\Gamma}_{uc}^T \mathbf{M}_o^{-1} \mathbf{G} \mathbf{K} \mathbf{G}^T \mathbf{\Gamma}_{uc}$ has full rank. Finally, from (43), (44) and (46), it can be concluded:

$$\mathsf{rank}(\mathbf{S}_q \mathbf{Z}) = \mathsf{rank}(\mathbf{S}_q) - c.$$

Now, if this last result with (42) is compared

$$\mathsf{rank} \begin{bmatrix} \mathbf{\Gamma}_h & \mathbf{S}_q \mathbf{Z} \end{bmatrix} = q - r - c.$$

$\square$

Theorem 2. Given the system in (4), then the following relationship holds:

$$\mathsf{im}(\mathbf{B}_\tau \mathbf{U}_c) = \Big(\mathsf{im}(\mathbf{B}_\tau \mathbf{U}_{ti}) \oplus \mathsf{im}(\mathbf{B}_\tau \mathbf{U}_{uc}) \oplus \mathsf{im}(\mathbf{B}_\tau \mathbf{U}_{qr})\Big)^{\perp} \cap \mathsf{im}(\mathbf{B}_\tau) = \mathbf{0},$$

where

$$\mathsf{im}(\mathbf{B}_\tau \mathbf{U}_{uc}) = \mathcal{R}_{uc} \cap \mathsf{im}(\mathbf{B}_\tau) \supseteq \mathsf{im}(\mathbf{S}_{uc}) \cap \mathsf{im}(\mathbf{B}_\tau),$$
$$\mathsf{im}(\mathbf{B}_\tau \mathbf{U}_{ti}) = \mathcal{R}_{ti} \cap \mathsf{im}(\mathbf{B}_\tau) \supseteq \mathsf{im}(\mathbf{S}_{ti}) \cap \mathsf{im}(\mathbf{B}_\tau),$$
$$\mathsf{im}(\mathbf{B}_\tau \mathbf{U}_{qr}) = \mathcal{R}_{qr} \cap \mathsf{im}(\mathbf{B}_\tau) \supseteq \mathsf{im}(\mathbf{S}_{qr}) \cap \mathsf{im}(\mathbf{B}_\tau),$$

$\mathbf{B}_\tau$ is the inputs map defined in (4) and subspaces $\mathsf{im}(\mathbf{S}_{ti})$, $\mathsf{im}(\mathbf{S}_{uc})$, and $\mathsf{im}(\mathbf{S}_{qr})$ are the controlled invariant subspaces defined above.

P r o o f. The previous intersections are calculated:

$$\mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{uc}) \supseteq \mathsf{im}(\mathbf{S}_{uc}) \cap \mathsf{im}(\mathbf{B}_\tau), \text{where } \mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{uc}) \supseteq \mathsf{im}(\mathbf{U}_2) = \mathsf{im}\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{\Gamma}_{qc} \\ \mathbf{0} \end{bmatrix}, \tag{47}$$

the second intersection is

$$\mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{ti}) \supseteq \mathsf{im}(\mathbf{S}_{ti}) \cap \mathsf{im}(\mathbf{B}_\tau), \text{where } \mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{ti}) \supseteq \mathsf{im}(\mathbf{U}_1) = \mathsf{im}\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{X}_{qh} \\ \mathbf{0} \end{bmatrix}, \tag{48}$$

with $\mathsf{im}(\mathbf{X}_{qh}) = \mathsf{im}(\mathbf{\Gamma}_h) \oplus \mathsf{im}(\mathbf{S}_q\mathbf{Z})$.

At the end, the third intersection

$$\mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{qr}) \supseteq \mathsf{im}(\mathbf{S}_{qr}) \cap \mathsf{im}(\mathbf{B}_\tau), \text{where } \mathsf{im}(\mathbf{B}_\tau\mathbf{U}_{qr}) \supseteq \mathsf{im}(\mathbf{U}_3) = \mathsf{im}\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{\Gamma}_{qr} \\ \mathbf{0} \end{bmatrix}. \tag{49}$$

$$\Big(\mathsf{im}(\mathbf{U}_1) \oplus \mathsf{im}(\mathbf{U}_2) \oplus \mathsf{im}(\mathbf{U}_3)\Big)^\perp = \Big(\big(\mathsf{im}(\mathbf{U}_1)\big)^\perp \cap \big(\mathsf{im}(\mathbf{U}_2)\big)^\perp \cap \big(\mathsf{im}(\mathbf{U}_3)\big)^\perp\Big)^\perp,$$

it follows

$$\ker\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{X}_{qh}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Gamma}_{qc}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Gamma}_{qr}^T & \mathbf{0} \end{bmatrix} = \mathsf{im}\begin{bmatrix} \mathbf{I}_q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_u \end{bmatrix}^4,$$

where

$$\mathsf{im}(\mathbf{P}) = \ker(\mathbf{X}_{qh}^T) \cap \ker(\mathbf{\Gamma}_{qc}^T) \cap \ker(\mathbf{\Gamma}_{qr}^T).$$

To prove the theorem, it is enough to show that

$$\mathsf{im}(\mathbf{P}) = \ker(\mathbf{X}_{qh}^T) \cap \ker(\mathbf{\Gamma}_{qc}^T) \cap \ker(\mathbf{\Gamma}_{qr}^T) = \mathbf{0}.$$

Subsequently, it is sufficient to prove that

$$\big(\mathsf{im}(\mathbf{P})\big)^\perp = \mathsf{im}(\mathbf{X}_{qh}) \oplus \mathsf{im}(\mathbf{\Gamma}_{qc}) \oplus \mathsf{im}(\mathbf{\Gamma}_{qr}) = \Re^q, \tag{50}$$

or equivalently that

$$\big(\mathsf{im}(\mathbf{P})\big)^\perp \text{has dimention equal to q.} \tag{51}$$

Finally, the following relationship must be proven to show (51):

$$\mathsf{rank}(\mathbf{X}_{qh}) = q - r - c. \tag{52}$$

Relationship (52) was shown in lemma 1. □

[4] Note that for a given $\mathbf{S}$-basic matrix it holds:

$$\ker(\mathbf{S}^T) = \big(\mathsf{im}(\mathbf{S})\big)^\perp.$$

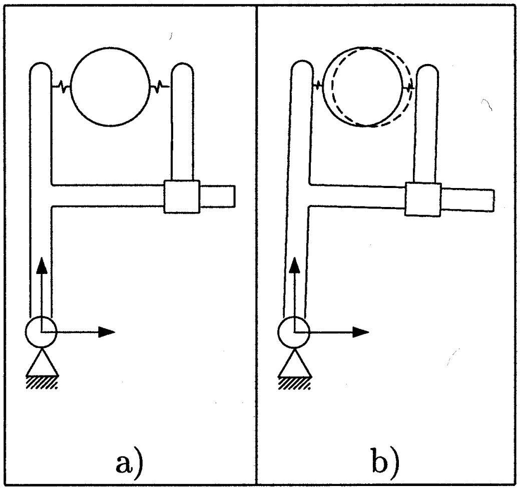

Fig. 2. Cross-section of manipulator.

6. AN APPLICATION EXAMPLE

As already mentioned, a decoupling control law consists of a feedback and a feedforward control law. In this section these two control structures are explicitly calculated. Numerical results are reported for the gripper described in Figure 2. This system is a planar device without redundant movements and two degrees of freedom, a prismatic and a rotoidal joint. Joint variables are positive when links move left. In the reference frame, the contacts are $\mathbf{c}_1 = (0,2)$, $\mathbf{c}_2 = (1,2)$, and the object center of mass is $\mathbf{c}_b = (0.5,2)$. As previously explained, $\mathbf{J} = \mathbf{H}\frac{\delta \mathbf{c}^m}{\delta \mathbf{q}}$ and $\mathbf{G}^T = \mathbf{H}\frac{\delta \mathbf{c}^o}{\delta \mathbf{u}}$, the identity matrix is assumed in the presented case matrix $\mathbf{H}$. The inertia matrices of the object and manipulator are assumed to be normalized to the identity matrix. The contact behavior is assumed isotropic at the contacts. Given that $\mathbf{q} = [q_1, q_2]^T$. In general $\mathbf{c}_1^m = (2\cos q_1, 2 - 2sinq_1)$, $\mathbf{c}_2^m = (2\cos q_1 - q_2, 2)$, the Jacobian matrix, and its linearisation around the point $q_1 = \frac{\pi}{2}$ assume the following values:

$$\mathbf{J} = \begin{bmatrix} -2\sin q_1 & 0 \\ -2\cos q_1 & 0 \\ -2\sin q_1 & -1 \\ -2\sin q_1 & 0 \end{bmatrix}; \mathbf{J}_l = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}.$$

The grasp matrix was once assumed $\mathbf{u} = [x, y, \theta]^T$ to be the vector of the generalised coordinates for the object. Then, the contact points could be represented as follows $\mathbf{c}_1^o = (x + \cos\theta, 1 + y + \sin\theta)$, $\mathbf{c}_2^o = (1 + x - \cos\theta, 1 + y - \sin\theta)$. The grasp matrix and its linearisation around $\theta = 0$ have the following form:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\sin\theta & \cos\theta & \sin\theta & -\cos\theta \end{bmatrix}; \mathbf{G}_l = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}. \tag{53}$$

According to Theorem 1 it is to check the dimension of the subspace of the reachable internal contact forces. The test to be done is the following:

$$\mathsf{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}) \subseteq \mathsf{ker}(\mathbf{G}). \tag{54}$$

Condition (54) is easy to be checked. In fact,

$$\mathsf{ker}(\mathbf{G}) = \mathsf{im}\begin{bmatrix} 0 \\ -0.7071 \\ 0 \\ 0.7071 \end{bmatrix}, \tag{55}$$

and

$$\mathsf{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\ \mathbf{K}\mathbf{J}) = \mathsf{im}\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}. \tag{56}$$

The tests in (55) and (56) guarantee that the internal forces are reachable. In this straightforward example it is, by observing the structure of the manipulator, intuitively understandable that the two internal contact forces are reachable. If $\mathsf{ker}\begin{bmatrix}\mathbf{J} & -\mathbf{G}_l^T\end{bmatrix} = \mathsf{im}\,(\mathbf{\Gamma})$ is calculated, then it follows:

$$\mathbf{\Gamma} = \begin{bmatrix} 0.0000 \\ 0.8165 \\ -0.4082 \\ 0.0000 \\ -0.4082 \end{bmatrix}; \tag{57}$$

where

$$\mathbf{\Gamma}_{qc} = \begin{bmatrix} 0.0000 \\ 0.8165 \end{bmatrix}; \mathbf{\Gamma}_{uc} = \begin{bmatrix} -0.4082 \\ 0.0000 \\ -0.4082 \end{bmatrix}; \mathbf{\Gamma}_{qr} = \begin{bmatrix} \mathbf{0} \end{bmatrix}. \tag{58}$$

The manipulator described in Figure 2 does not present redundance movements and this yields $\mathbf{\Gamma}_{qr} = 0$. It is possible to calculate

$$\mathbf{S}_q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{59}$$

and also to calculate $\mathsf{ker}(\mathbf{G}\mathbf{K}\mathbf{J}) = 0$ which yields $\mathbf{\Gamma}_h = 0$. In the analysed case, $\mathsf{im}(\mathbf{M}_o^{-1}\mathbf{G}\mathbf{K}\mathbf{J}\mathbf{S}_q) \subseteq \mathsf{ker}\,(\mathbf{\Gamma}_{uc})^T$, then $\mathbf{Z} = \mathbf{I}$. According to (47), (48) and (49) the feed-forward structure is already calculated. In fact, Figure 3 shows the proposed control scheme structure in which the concept of the feed-forward control law is visible through matrix $\mathbf{U}$. Matrix $\mathbf{U}$ is defined as follows:

$$\mathbf{U} = \begin{bmatrix} \mathbf{B}_\tau\mathbf{U}_{uc} & \mathbf{B}_\tau\mathbf{U}_{ti} \end{bmatrix}. \tag{60}$$

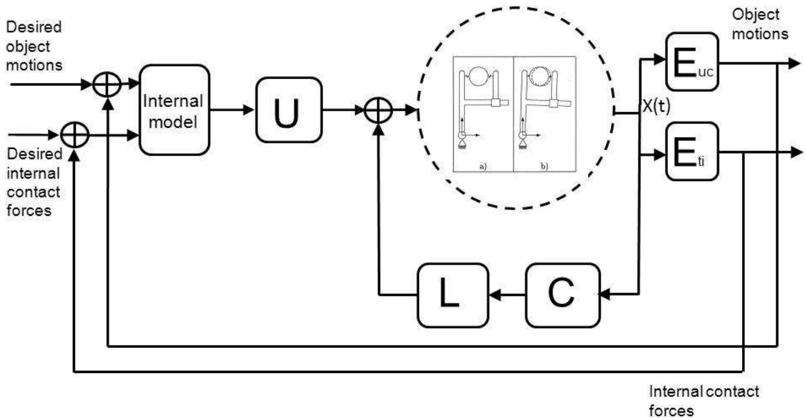

Fig. 3. Control scheme.

From this scheme the concept of the conditioned and controlled-invariant subspace as an algebraic feedback is visible. Considering the main results shown in [11], which are reported here below, it is possible to build the decoupling feedback law which, together with the feed-forward control law, can obtain a robust decoupling. In fact, it is possible to show that a robust decoupling controller is obtained through matrix $\mathbf{L}$. This matrix represents a static-output feedback control law, together with the above defined matrix $\mathbf{U}$ which represents the static feed-forward control law. According to the structures and the results in [11], then

$$\begin{aligned} &\text{a)} \quad \min\ \mathcal{I}\ ((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau \mathbf{LC}), \mathbf{B}_\tau \mathbf{U}_{uc}) \subseteq \ker \mathbf{E}_{ti}; \\ &\text{b)} \quad \operatorname{im} \mathbf{E}_{ti}(\Delta k, \Delta b) = \mathbf{E}_{ti}(\Delta k, \Delta b) \min\ \mathcal{I}\ ((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau \mathbf{LC}), \mathbf{B}_\tau \mathbf{U}_{uc})\,; \end{aligned} \tag{61}$$

$$\begin{aligned} &\text{c)} \quad \min\ \mathcal{I}\ ((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau \mathbf{LC}), \mathbf{B}_\tau \mathbf{U}_{ti}) \subseteq \ker \mathbf{E}_{uc}; \\ &\text{d)} \quad \operatorname{im} \mathbf{E}_{uc}(\Delta k, \Delta b) = \mathbf{E}_{uc}(\Delta k, \Delta b) \min\ \mathcal{I}\ ((\mathbf{A}(\Delta k, \Delta b) + \mathbf{B}_\tau \mathbf{L}_{ti} \mathbf{C}), \mathbf{B}_\tau \mathbf{U}_{ti})\,. \end{aligned} \tag{62}$$

The sensed outputs are weighted by the coefficients of the matrix $\mathbf{L}$. It will be shown that the decoupling control of the internal forces can be obtained by means of an algebraic output feedback control from the sensed output consisting of contact forces $\mathbf{t}$ and of manipulator joint positions $\mathbf{q}$. These have an output relationship for the linearised model denoted by the following:

$$\begin{aligned} \mathbf{y}_m &= \mathbf{Cx} \\ \mathbf{C} &= \begin{bmatrix} \mathbf{I}_{q\times q} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{KJ} & -\mathbf{KG}^T & \mathbf{BJ} & -\mathbf{BG}^T \end{bmatrix}. \end{aligned} \tag{63}$$

According to equations (61) all the object movements $\mathbf{E}_{uc}$ remain inside the subspace $\mathsf{im}(\mathbf{E}_{uc})\ \forall\,\mathbf{K}$ and $\mathbf{B}$. This means that, thanks to the decoupling control law, the object motions do not influence the subspace of the internal contact forces. According to equations (62) all the internal contact forces $\mathbf{E}_{ti}$ remain inside the subspace $\mathsf{im}(\mathbf{E}_{ti})\ \forall\,\mathbf{K}$ and $\mathbf{B}$. To conclude, thanks to the decoupling control law which is characterised by matrix $\mathbf{U}$ and matrix $\mathbf{L}$, the internal contact forces do not influence the subspace of the object motions. In the presented case the outputs which describe the object motions and the internal contact forces are the following:

$$\mathbf{E}_{uc} = \begin{bmatrix} 0.71 & -0.71 & 0 & 0 & 0 & 1.42 & -1.42 & 0 & 0 & 0 \end{bmatrix}, \tag{64}$$

$$\mathbf{E}_{ti} = \begin{bmatrix} 0 & 0 & -0.58 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{65}$$

From equation (61) and (62) it is also possible to calculate matrix $\mathbf{L}$. In the case presented matrix $\mathbf{L}$ assumes the following values:

$$\mathbf{L} = \begin{bmatrix} -9.3 & -9.3 & 0 & 4.7 & 0 & 3.7 \\ -9.3 & -9.3 & 0 & 3.7 & 0 & 4.7 \end{bmatrix}. \tag{66}$$

Matrix $\mathbf{L}$ guarantees also the stability of the system. From Figures 4, it is visible how the contact forces "compensate". Essentially, no movements are allowed, and the desired force on the object is obtained. The dynamics of the forces, represented on the lower (left and right) part of Figure 4 are due to the particular choice of eigenvalues that characterise the force answer of the system. Figures 5 show the case in which the center of mass is moving but the force acting on it remains constant. The robustness, with respect to the variations of $\mathbf{K}$ and $\mathbf{B}$, is widely explained in [11]. The robust decoupling controller exits if some structural conditions are satisfied. These structural conditions are satisfied by those mechanisms that present some symmetry in their geometric structure. In fact, a symmetric structure always uses symmetric contact forces to guarantee the existence of the compensation mentioned above.

7. CONCLUSIONS

A generalized linear model is used and a careful analysis is performed for the design of a force/motion controller. This work shows an explicit formula for the reachable internal contact forces of a general manipulation system. The main contribution of the paper consists of investigating the design of a feed-forward force-motion control which, together with a feedback structure, realizes a decoupling force-motion control. Structural geometric properties are proposed, and a general matrix parametrization of a feed-forward control for a task-oriented choice of input subspaces is provided. The existence of these input subspaces is a necessary condition for the structural non-interaction property. In particular, a theorem is shown which offers a general parametrization of the pre-compensator. Work on the synthesis of the force/motion non-interacting control law of manipulation systems continues. Specifically, the synthesis of non-interacting force/motion controllers for defective devices appears to be straightforward to be implemented in several application examples.

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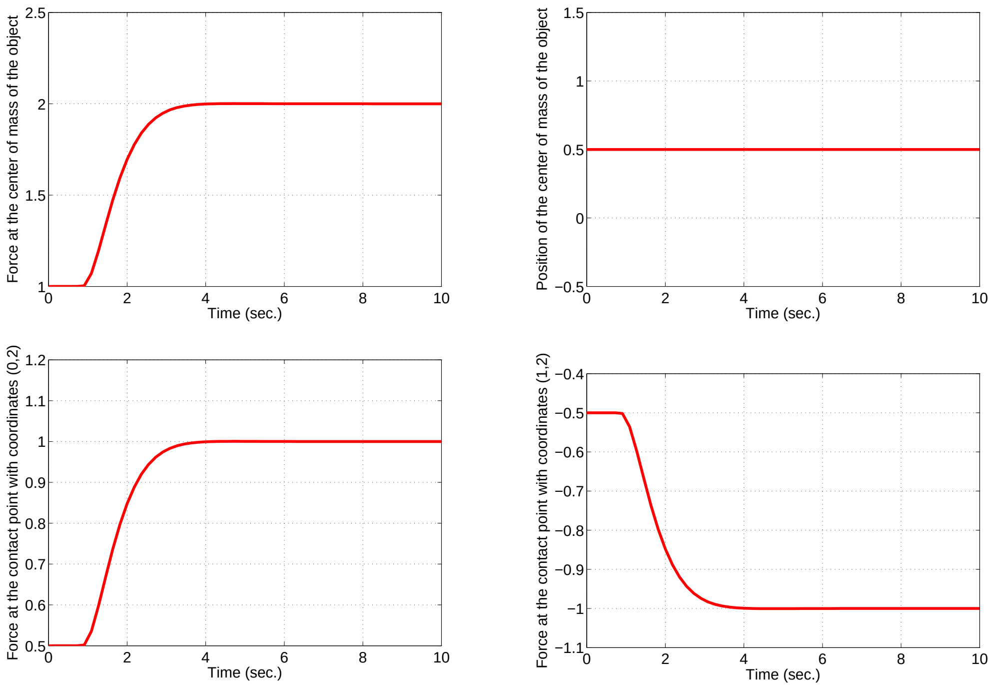

Fig. 4. Upper-Left: Resulting horizontal squeezing force on the center of mass of the object. Upper-Right: Resulting horizontal position of the center of mass of the object. Lower-Left: Force acting at the contact point with coordinates (0, 2). Lower-Right: Force acting at the contact point with coordinates (1, 2).

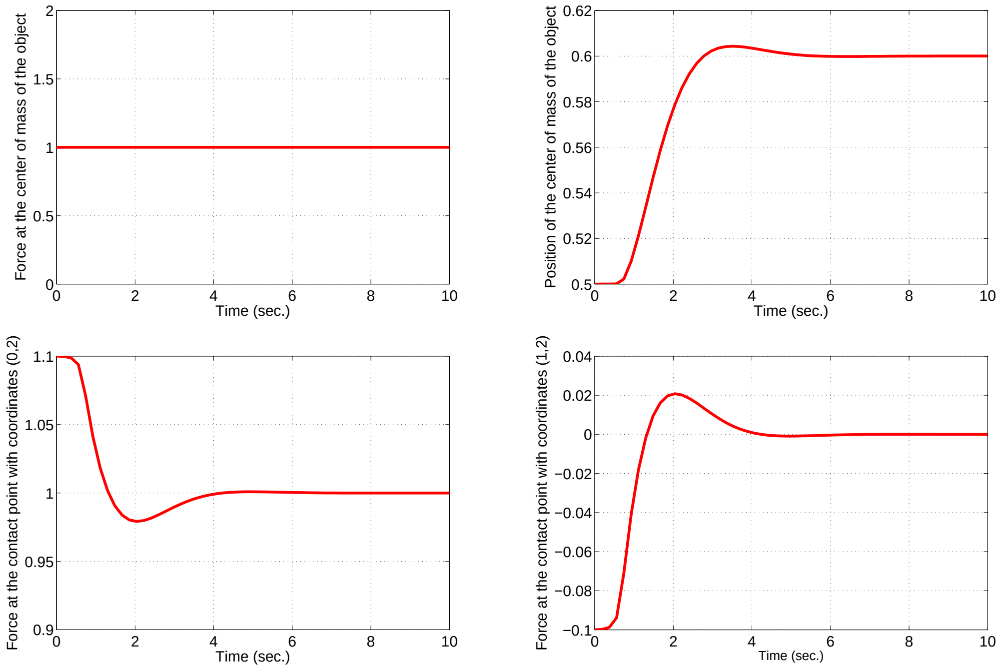

Fig. 5. Upper-Left: Resulting horizontal squeezing force on the center of mass of the object. Upper-Right: Resulting horizontal position of the center of mass of the object. Lower-Left: Force acting at the contact point with coordinates (0, 2). Lower-Right: Force acting at the contact point with coordinates (1, 2).

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