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FINITE-TIME CONSENSUS PROBLEM FOR MULTIPLE NON-HOLONOMIC MOBILE AGENTS

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In this paper, the problem of finite time consensus is discussed for multiple non-holonomic mobile agents. The objective is to design a distributed finite time control law such that the controlled multiple non-holonomic mobile agents can reach consensus within any given finite settling time. We propose a novel switching control strategy with the help of time-rescalling technique and graph theory. The numerical simulations are presented to show the effectiveness of the method.

Keywords: finite time consensus, nonholonomic system, time-rescaling, mobile agents

Classification: 93D15, 93D21

1. INTRODUCTION

Consensus problem for multi-agent systems has received great attention from various research communities recently due to its challenging features and many applications, such as formation control, consensus and flocking [3, 8, 14].

The consensus of the multi-agent systems means that all the states of all agents are required to agree upon certain quantities of interest. In order to achieve the aim, local rules are usually applied to each agent, mainly based on the weighted average of its own information and that of its neighbors [8]. The consensus problem for networks of dynamic agents with fixed and switching topologies was discussed in [14].

Most of the existing consensus control laws for multi-agent are asymptotic consensus laws, that is the states of the agents convergence to the desired value with infinite time. Compared with this, finite time control can provide better disturbances-rejection, fast response and tracking precision [1, 2, 9, 12]. Finite-time stabilizing control has been studied and several finite-time consensus algorithms have been obtained in the references such as [4, 5, 6, 13, 17], just to name a few.

Graph theory results related to consensus control are obtained for linear agents mostly. However, many practical control applications involve agents that are nonlinear and non-holonomic. Therefore, it is necessary to discuss the control of multiple non-holonomic systems. The papers [10, 11] considered cooperative control of only a portion of the state vector of each mobile robot and their proposed methods were specialized to a specific class of robotic system. The paper [3] discussed the cooperative control problem for general nonholonomic agents with limited communication capabilities among neighbors. However, to the best of our knowledge, there are still no any results with respect to finite time consensus for non-holonomic mobile agents. In this paper, based on the results from papers [13] and the time-rescalling technique and switching control technique from paper [7, 15, 16], not only can we solve the finite time consensus problem for non-holonomic mobile agents, but also we can make all the non-holonomic mobile agents reach consensus within any given finite time.

The remainder of this paper is organized as follows. In Section 2, formulation and preliminary results are given. In Section 3 we first present a distributed switching control strategy based on the result from paper [13], and prove the effectiveness of the method, and secondly we employ a time-rescaling technique to reconstruct the distributed finite-time controller to make all the non-holonomic mobile agents reach consensus within any given finite time. In Section 4, we use the numerical simulations to show the effectiveness of our distributed finite time control laws. Finally, some conclusions are drawn in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Algebraic graph theory

In the multi-agent systems, the communication between the agents can be described by the undirected graph $G = (\nu, \varepsilon, A)$, where the set of vertices $\nu = \{1, 2, \cdots, n\}$, set of edges $\varepsilon \subset \nu \times \nu = \{(i, j) : i, j \in \nu\}$, and a adjacency matrix $A = [a_{ij}] \in R^{n\times n}$. If there is an edge from vertex i to vertex j, i.e. $(i, j) \in \varepsilon$, then $a_{ij} = a_{ji} > 0$, the vertex j is called a neighbor of i. The set of neighbors of vertex i is denoted by $N_i = \{j \in \nu : (i, j) \in \varepsilon, j \neq i\}$. In this paper we assume that $a_{ii} = 0, 1 \leq i \leq n$. The degree matrix of G is diagonal matrix $D = diag\{d_1, \ldots, d_n\} \in R^{n\times n}$, where diagonal elements $d_i = \sum_{j\in N_i} a_{ij}$ for $i = 1, \ldots, n$. Then the Laplacian of the weighted graph G is defined as $L = D - A$. A path on G is a non-empty graph $P = (\nu_P, \varepsilon_P, A_P) \subseteq G$ with $\nu_P = \{l_0, l_1, \cdots, l_k\}$ and $\varepsilon \in \{l_0l_1, l_1l_2, \cdots, l_{k-1}l_k\}$, where the $l_i, 1 \leq i \leq k$ are all distinct. A graph G is called connected if and only if any two of its nodes are linked by a path on G.

In this paper, let $\mathbf{1} = [1, 1, \cdots, 1]^T \in R^n$ and $\mathbf{0} = [0, 0, \cdots, 0]^T \in R^n$.

Lemma 2.1. If the undirected graph G is connected, $L(A)$ has the following properties [14]:

1. 0 is a simple eigenvalue of $L(A)$ and $\mathbf{1}$ is the associated eigenvector;

2. $x^T L[A]x = \frac{1}{2}\sum_{i,j=1}^{n} a_{ij}(x_i - x_j)^2$, and the semi-positive definiteness of $L(A)$ implies that all eigenvalue of $L(A)$ are real and not less than zero;

3. The second smallest eigenvalue of $L(A)$, which is denoted by $\lambda_2(L(A))$(the algebraic connectivity of $G(A)$) and satisfies $\lambda_2(L(A)) = \min_{x\neq 0, \mathbf{1}^T x=0} \frac{x^T L[A]x}{x^T x} > 0$, therefore, if $\mathbf{1}^T x = 0$, $x^T L[A]x \geq \lambda_2(L(A))x^T x$.

According to paper [13], we can have the following two lemmas:

Lemma 2.2. Consider the kinematics for n mobile agents, indexed by $i \in \nu$, the kinematics of the ith agent is described by the following form:

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = u_i \end{cases} \tag{1}$$

where $[x_i, v_i]^T$ and u_i are the state and input of agent i respectively. If the graph G is connected, then, the following distributed finite-time control law

$$u_i(t) = k_1 \Big[k_2^p \Big(\sum_{j \in N_i} a_{ij}(x_j - x_i) \Big) - v_i^p \Big]^{\frac{2}{p}-1}, \ i \in \nu \tag{2}$$

can solve the finite-time consensus problem, namely such that state consensus can be achieved within finite time $T_1 \leq \frac{V(0)^{1-d/2}}{b_2(1-d/2)}$, where

$$\begin{aligned}
V(0) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(x_i(0)-x_j(0))^2 + \sum_{i=1}^{n}\int_{v_i^*(0)}^{c_i v_i(0)} \frac{(s^p - v_i^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s, \\
v_i^*(0) &= -k_2\Big[\sum_{j\in N_i} a_{ij}(x_i(0)-x_j(0)\Big]^{1/p}, \\
k_2 &\geq \frac{p2^{1-1/p}+\beta+n\gamma}{1+p} + k_3, \\
k_1 &\geq (2-1/p)2^{1-1/p}k_2^{1+p} \times \Big(\frac{2^{1-1/p}+(\beta+n\gamma)p}{1+p} + \frac{2^{1-1/p}(\beta+n\gamma)}{k_2} + k_3 \Big), \\
b_2 &= k_3/(2b_1^{d/2}), \\
b_1 &= \max\Big\{ \frac{1}{2\lambda_2}, \frac{1}{(2-1/p)k_2^{1+p}} \Big\},
\end{aligned}$$

k_3 is a positive constant, $d = 1+1/p$, $1 < p = p_1/p_2 < 2$, p_1, p_2 are positive odd integers, $\beta = \max_{1\leq i\leq n}\Big\{\sum_{j\in N_i} a_{ij}\Big\}$, $\gamma = \max_{1\leq i,j\leq n}\{a_{ij}\}$.

Lemma 2.3. Consider the kinematics for n mobile agents, indexed by $i \in \nu$, the kinematics of the ith agent is described by the following form:

$$\dot{x}_i = u_i \tag{3}$$

where x_i and u_i are the state and input of agent i respectively. If the graph G is connected, then, the following distributed finite-time control law

$$u_i(t) = k_2 \Big(\sum_{j\in N_i} a_{ij}(x_j - x_i) \Big)^{\frac{1}{p}}, \ i \in \nu \tag{4}$$

can solve the finite-time consensus problem, namely state consensus can be achieved within finite time $T_2 = \frac{V_0(0)^{\frac{p-1}{2p}}}{k_2 \frac{p-1}{2p} (2\lambda_2)^{\frac{1+p}{2p}}}$, where

$$V_0(0) = \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(x_i(0)-x_j(0))^2$$

and the definitions for k_2 and p are the same as in Lemma 2.2.

2.2. Finite time stability and problem formulation

Lemma 2.4. (Hong and Wang [7]) Consider the nonlinear system

$$\dot{x} = f(x), f(0) = 0, x \in R^n,$$

where $f : U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. Suppose there is a C^1 function $V(x)$ defined in a neighborhood $\hat{U} \subset U_0 \in R^n$ of the origin, real numbers $c > 0$ and $0 < \alpha < 1$, such that $V(x)$ is positive definite on $\hat{U}$ and $\dot{V}(x) + cV^\alpha(x) \leq 0$ (along the trajectory) on $\hat{U}$. Then, $V(x)$ approaches 0 in finite time along the trajectory with any initial condition $x(0) \in \hat{U}/\{0\}$, in addition, the finite settling time T satisfies that $T \leq \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}$.

Consider the kinematics for n non-holonomic mobile agents, indexed by $i \in \nu$, the kinematics of the ith agent is described by the following form:

$$\begin{cases} \dot{q}_{1i} = u_{1i} \\ \dot{q}_{2i} = u_{2i} \\ \dot{q}_{3i} = q_{2i}u_{1i} \end{cases} \tag{5}$$

where $q_{*i} = [q_{1i}, q_{2i}, q_{3i}]^T$ and $u_{*i} = [u_{1i}, u_{2i}]^T$ are the state and input of agent i respectively.

This paper aims to find distributed controller

$$\begin{aligned} u_{1i} &= u_{1i}(q_{1k_1}, q_{2k_1}, q_{3k_1}, \cdots, q_{1k_{m_i}} q_{2k_{m_i}}, q_{3k_{m_i}}) \\ u_{2i} &= u_{2i}(q_{1k_1}, q_{2k_1}, q_{3k_1}, \cdots, q_{1k_{m_i}}, q_{2k_{m_i}}, q_{3k_{m_i}}) \end{aligned} \tag{6}$$

with $K_i = \{k_1, \cdots, k_{m_i}\} \subseteq \{i\} \cup N_i$for system (5) with any initial condition such that the system (5) will achieve consensus ($q_{ij} = q_{im} : 1 \leq i \leq 3, 1 \leq j \neq m \leq n$) within finite settling time T.

3. MAIN RESULTS

To solve the finite-time consensus problem, inspired by the idea of paper [7, 15, 16], we divide the system (5) into a first-order subsystem

$$\dot{q}_{1i} = u_{1i} \tag{7}$$

and a second-order subsystem

$$\begin{cases} \dot{q}_{2i} = u_{2i} \\ \dot{q}_{3i} = q_{2i}u_{1i}. \end{cases} \tag{8}$$

3.1. Consensus within finite settling time

Theorem 3.1. Consider the system (5) for all $i \in \nu$, and let $c = [c_1, c_2, \cdots, c_n]^T$ be a suitably selected constant vector with $c_i, 1 \leq i \leq n$ being nonzero constants. If the graph G is connected, then, the following distributed finite-time control law

$$u_{1i} = \begin{cases} c_i & t < T_1 \\ k_2(\sum_{j \in N_i} a_{ij}(q_{1j} - q_{1i}))^{1/p} & t \geq T_1 \end{cases} \tag{9}$$

$$u_{2i} = \frac{k_1}{c_i}[k_2^p(\sum_{j \in N_i} a_{ij}(q_{3j} - q_{3i})) - (c_i q_{2i})^p]^{\frac{2}{p}-1} \tag{10}$$

solves the finite-time consensus problem after time $T = T_1 + T_2$ with $T_1 = \frac{V(0)^{1-d/2}}{b_2(1-d/2)}$ and $T_2 = \frac{V_0(T_1)^{\frac{p-1}{2p}}}{k_2 \frac{p-1}{2p}(2\lambda_2)^{\frac{1+p}{2p}}}$, where

$$\begin{aligned} V_0(T_1) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j \in N_i} a_{ij}(q_{1i}(T_1) - q_{1j}(T_1))^2 \\ V(0) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j \in N_i} a_{ij}(q_{3i}(0) - q_{3j}(0))^2 + \sum_{i=1}^{n}\int_{v_i^*(0)}^{c_i q_{2i}(0)} \frac{(s^p - v_i^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s, \\ v_i^*(0) &= -k_2\Big[\sum_{j \in N_i} a_{ij}(q_{3i}(0) - q_{3j}(0)\Big]^{1/p}, \end{aligned}$$

and the definitions for k_1, k_2, p, d and b_2 is the same as in Lemma 2.2.

Proof.

3.1.1. When $t < T_1$

Because $u_{1i} = c_i$, hence the second-order subsystem is as follows.

$$\begin{cases} c_i \dot{q}_{2i} = c_i u_{2i} \\ \dot{q}_{3i} = c_i q_{2i}. \end{cases} \tag{11}$$

Based on Lemma 2.2, we can get the distributed finite time control law for the second-order subsystem (11) of agent i:

$$c_i u_{2i} = -k_1\Big[(c_i q_{2i})^p + k_2^p\Big(\sum_{j \in N_i} a_{ij}(q_{3i} - q_{3j})\Big)\Big]^{\frac{2}{p}-1},$$

hence, when $1 \leq i \leq n$, the distributed finite time control law

$$u_{2i} = -\frac{k_1}{c_i}\Big[(c_i q_{2i})^p + k_2^p\Big(\sum_{j \in N_i} a_{ij}(q_{3i} - q_{3j})\Big)\Big]^{\frac{2}{p}-1}$$

can make the second-order subsystem (8) reach consensus within finite time settling time $T \le T_1 = \frac{V(0)^{1-d/2}}{b_2(1-d/2)}$, where

$$\begin{aligned} V(0) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_{3i}(0)-q_{3j}(0))^2 + \sum_{i=1}^{n}\int_{v_i^*(0)}^{c_i q_{2i}(0)} \frac{(s^p - v_i^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s, \\ v_i^*(0) &= -k_2\Big[\sum_{j\in N_i} a_{ij}(q_{3i}(0)-q_{3j}(0)\Big]^{1/p}, \end{aligned}$$

and the definitions for k_1, k_2, p, d and b_2 is the same as in Lemma 2.2.

3.1.2. When $t \ge T_1$

All the agents have reached agreement on states q_{2i} and q_{3i}, where $i : 1 \le i \le n$. Thus, we only consider the first order subsystem (7). According to Lemma 2.3, the distributed finite time control laws of agent indexed by i is as following:

$$u_{1i}(t) = k_2\Big\{\sum_{j\in N_i} a_{ij}(q_{1j}-q_{1i})\Big\}^{1/p}, \tag{12}$$

and $\dot{V_0}(t) \le -k_2(2\lambda_2 V_0(t))^{\frac{1+p}{2p}}$ with $V_0(t) = \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_{1i}-q_{1j})^2$. Hence it is not difficult for us to get that all the agents can reach agreement on states q_{1i} after time $T = T_1 + T_2$ with $T_2 = \frac{V_0(T_1)^{\frac{p-1}{2p}}}{k_2\frac{p-1}{2p}(2\lambda_2)^{\frac{1+p}{2p}}}$, where $i : 1 \le i \le n$. Hence we can have: all the agents can reach agreement on states q_{1i}, q_{2i}, q_{3i} after time $T = T_1 + T_2$, where $i : 1 \le i \le n$. This completes the proof. □

3.2. Consensus within any given finite settling time

Theorem 3.2. Consider the system (5) for all $i \in \nu$, and let $c = [c_1, c_2, \cdots, c_n]^T$ be a suitably selected constant vector with $c_i, 1 \le i \le n$ being nonzero constants. If the graph G is connected, then, for any given time T and design parameter $0 < \alpha < 1$, through selecting suitable time-rescaling constants $K_1 \ge 1$ and $K_2 \ge 1$ the following distributed finite-time control law

$$\begin{aligned} u_{1i} &= \begin{cases} c_i & t < \alpha T \\ K_2 k_2\Big(\sum_{j\in N_i} a_{ij}(q_{1j}-q_{1i})\Big)^{1/p} & t \ge \alpha T \end{cases} \\ u_{2i} &= \frac{K_1^2 k_1}{c_i}\Big[k_2^p\Big(\sum_{j\in N_i} a_{ij}(q_{3j}-q_{3i}\Big) - \Big(\frac{c_i q_{2i}}{K_1})^p\Big)\Big]^{\frac{2}{p}-1} \end{aligned}$$

can solve the finite-time consensus problem within $\frac{\overline{T}_1}{K_1} \leq \alpha T, \frac{T_2}{K_2} \leq (1-\alpha)T$ with $\overline{T}_1 = \frac{\overline{V}(0)^{1-d/2}}{b_2(1-d/2)}$ and $T_2 \leq \frac{V_0(\alpha T)^{\frac{p-1}{2p}}}{k_2 \frac{p-1}{2p}(2\lambda_2)^{\frac{1+p}{2p}}}$, where

$$\begin{aligned} V_0(\alpha T) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_{1i}(\alpha T) - q_{1j}(\alpha T))^2 \\ \overline{V}(0) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_{3i}(0) - q_{3j}(0))^2 + \sum_{i=1}^{n}\int_{v_i^*(0)}^{c_i\overline{q}_{2i}(0)} \frac{(s^p - v_i^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s, \\ v_i^*(0) &= -k_2\Big[\sum_{j\in N_i} a_{ij}(q_{3i}(0) - q_{3j}(0)\Big]^{1/p}, \end{aligned}$$

and the definitions for k_1, k_2, p, d and b_2 is the same as in Lemma 2.2.

Proof. For any given finite settling time T, if $T_1 + T_2 \leq T$, the distributed finite time control laws $u_{1i}, u_{2i}, 1 \leq i \leq n$ in the form of Theorem 3.1 solve the finite consensus problem for system (5). If $T_1 + T_2 > T$, we will employ a time-rescaling technique to reconstruct a distributed finite-time controller to make all the agents reach consensus with agent kinematics (5) within a modified settling time $T^* = T_1^* + T_2^* < T$. Take $\overline{t} = K_1 t, \overline{q}_{3i} = q_{3i}, \overline{q}_{2i} = K_1^{-1}q_{2i}, \overline{u}_{2i} = K_1^{-2}u_{2i}, K_1 \geq 1$, From the second order subsystem (8) we can get

$$\begin{cases} \frac{\mathrm{d}\overline{q}_{2i}}{\mathrm{d}\overline{t}} = \overline{u}_{2i} \\ \frac{\mathrm{d}\overline{q}_{3i}}{\mathrm{d}\overline{t}} = u_{1i}\overline{q}_{2i}. \end{cases} \tag{13}$$

Note that when $\overline{t} < \overline{T}_1 = \frac{\overline{V}(0)^{1-d/2}}{b_2(1-d/2)}$ with

$$\overline{V}(0) = V(\overline{q}_{21}(0), \overline{q}_{22}(0), \cdots, \overline{q}_{2n}(0), q_{31}(0), q_{32}(0), \cdots, q_{3n}(0)),$$

$u_{1i} = c_i$ has the same form as in Theorem 3.1, therefore, $\overline{u}_{2j}$ is still in the same form:

$$\overline{u}_{2i} = -\frac{k_1}{c_i}[(c_i\overline{q}_{2i})^p + k_2^p(\sum_{j\in N_i} a_{ij}(q_{3i} - q_{3j}))]^{\frac{2}{p}-1}.$$

Hence

$$u_{2i} = \frac{K_1^2 k_1}{c_i}[(k_2^p(\sum_{j\in N_i} a_{ij}(q_{3j} - q_{3i}) - c_i\overline{q}_{2i})^p)]^{\frac{2}{p}-1}$$

can make all the agents reach consensus with respect to states $q_{2i}, q_{3i}, 1 \leq i \leq n$ within $T_1^* = \frac{\overline{T}_1}{K_1} = \frac{\overline{V}(0)^{1-d/2}}{K_1 b_2(1-d/2)}$. We can select suitable K_1 such that $T_1^* = \frac{\overline{T}_1}{K_1} = \frac{\overline{V}(0)^{1-d/2}}{K_1 b_2(1-d/2)} \leq \alpha T$ with $0 < \alpha < 1$. In the following, we only make all the agents reach agreement with respect to states $q_{1i} : 1 \leq i \leq n$ within finite time $(1-\alpha)T$. If the distributed finite-time control law $u_{1i} = (\sum_{j\in N_i} a_{ij}(q_{1j} - q_{1i}))^{1/p}$ can not make all the agents reach agreement with respect to states $q_{1i} : 1 \leq i \leq n$ within finite time $(1-\alpha)T$, we can take $\hat{t} = K_2 t, \hat{q}_{1i} = q_{1i}, \hat{u}_{1i} = K_2^{-1}u_{1i}, K_2 \geq 1$. Based on the first subsystem (7), we can have

$$\frac{\mathrm{d}\hat{q}_{1i}}{\mathrm{d}\hat{t}} = \hat{u}_{1i}. \tag{14}$$

Note that when $\hat{t} \geq \frac{\overline{T}_1}{K_1} = \frac{\overline{V}(0)^{1-d/2}}{K_1 b_2(1-d/2)}$ with

$$\overline{V}(0) = V(\overline{q}_{21}(0), \overline{q}_{22}(0), \cdots, \overline{q}_{2n}(0), q_{31}(0), q_{32}(0), \cdots, q_{3n}(0)),$$

$\hat{u}_{1i} = k_2(\sum_{j \in N_i} a_{ij}(q_{1j} - q_{1i}))^{1/p}$ has the same form as in Theorem 3.1. Hence $u_{1i} = K_2 k_2(\sum_{j\in N_i} a_{ij}(q_{1j} - q_{1i}))^{1/p}$ can make all the agents reach consensus with respect to states $q_{1i}, 1 \leq i \leq n$ with finite time $T_2^* = \frac{T_2}{K_2} = \frac{V_0(\alpha T)^{\frac{p-1}{2p}}}{K_2 k_2 \frac{p-1}{2p}(2\lambda_2)^{\frac{1+p}{2p}}}$. We can select suitable K_2 such that $T_2^* = \frac{T_2}{K_2} = \frac{V_0(\alpha T)^{\frac{p-1}{2p}}}{K_2 k_2 \frac{p-1}{2p}(2\lambda_2)^{\frac{1+p}{2p}}} \leq (1-\alpha)T$. This completes the proof. □

Remark 3.3. From Theorem 2 we can see that the estimation for the upper bound of the settling time T_1 and T_2 can be made small by making both the values of K_1 and K_1 large.

4. SIMULATIONS

To verify the effectiveness of the proposed distributed finite time control law, we give some simulation results for Section 3.

Here we give a 5-agent system described by an undirected graph $\mathcal{G}$ as shown in Figure 1. Except $a_{13} = a_{31} > 0, a_{34} = a_{43} > 0, a_{24} = a_{42} > 0, a_{25} = a_{52} > 0$, all the

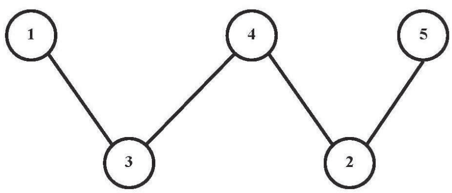

Fig. 1. The communication topology of the system.

other a_{ij} are zeros. In the simulations, we take all the nonzero a_{ij} as $\frac{1}{2}$, and then $L = D - A = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$, $\lambda_2 = 0.1910$, $c_i = \frac{1}{2}, 1 \leq i \leq 5, p = \frac{9}{7}$, $k_3 = 1$.
According to Theorem 3.1, $\beta = \max_{1\leq i\leq n}\{\sum_{j\in N_i} a_{ij}\} = 1, \gamma = \max_{1\leq i,j\leq n}\{a_{ij}\} = 0.5$,

$$\begin{aligned} k_2 &\geq \frac{p2^{1-1/p} + \beta + n\gamma}{1+p} + k_3 \\ &= \frac{\frac{9}{7}2^{\frac{2}{9}} + 1 + 5 \times 0.5}{1 + \frac{9}{7}} + 1 = 3.1874 \end{aligned}$$

hence we can take $k_2 = 3.2$.

$$\begin{aligned} k_1 &\geq (2-1/p)2^{1-1/p}k_2^{1+p} \times \Big(\frac{2^{1-1/p}+(\beta+n\gamma)p}{1+p} + \frac{2^{1-1/p}(\beta+n\gamma)}{k_2} + k_3\Big) \\ &\geq \Big(\frac{11}{9}\Big)2^{\frac{2}{9}}3.2^{\frac{16}{7}} \times \Big(\frac{2^{\frac{2}{9}}+\frac{9}{7}(1+5\times 0.5)}{\frac{16}{7}} + \frac{2^{\frac{2}{9}}(1+5\times 0.5)}{3.2} + 1\Big) \\ &= 96.7887, \end{aligned}$$

k_1 can be taken as 96.8. The initial conditions are randomly selected as follows:

$$\begin{array}{llll} q_{11}(0)=5 & q_{12}(0)=10 & q_{13}(0)=15 & q_{14}(0)=-5 \\ q_{15}(0)=-10 & q_{21}(0)=10 & q_{22}(0)=-5 & q_{23}(0)=10 \\ q_{24}(0)=2 & q_{25}(0)=7 & q_{31}(0)=5 & q_{32}(0)=15 \\ q_{33}(0)=-10 & q_{34}(0)=3 & q_{35}(0)=6 & \end{array}$$

hence

$$\begin{aligned} q_1(0) &= [5 \quad 10 \quad 15 \quad -5 \quad -10]^T \\ q_2(0) &= [10 \quad -5 \quad 10 \quad 2 \quad 7]^T \\ q_3(0) &= [5 \quad 15 \quad -10 \quad 3 \quad 6]^T. \end{aligned}$$

According to Theorem 3.1,

$$\begin{aligned} & \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_{3i}(0)-q_{3j}(0))^2 \\ =\ & \frac{1}{2}q_{30}^T L q_{30} = 154.7500. \end{aligned}$$

$$\begin{aligned} v_1^*(0) &= -k_2\Big[\sum_{j\in N_1} a_{1j}(q_{31}(0)-q_{3j}(0))\Big]^{1/p} \\ &= -3.2\times 7.5^{7/9} = -15.3375 \\ v_2^*(0) &= -k_2\Big[\sum_{j\in N_2} a_{2j}(q_{32}(0)-q_{3j}(0))\Big]^{1/p} \\ &= -3.2\times 10.5^{7/9} = -19.9255 \\ v_3^*(0) &= -k_2\Big[\sum_{j\in N_3} a_{3j}(q_{33}(0)-q_{3j}(0))\Big]^{1/p} \\ &= 3.2\times (14)^{7/9} = 24.9220 \\ v_4^*(0) &= -k_2\Big[\sum_{j\in N_4} a_{4j}(q_{34}(0)-q_{3j}(0))\Big]^{1/p} \\ &= -3.2\times (0.5)^{7/9} = -1.8664 \\ v_5^*(0) &= -k_2\Big[\sum_{j\in N_5} a_{5j}(q_{35}(0)-q_{3j}(0))\Big]^{1/p} \\ &= 3.2\times (4.5)^{7/9} = 10.3087 \end{aligned}$$

$$\int_{v_1^*(0)}^{\frac{q_{21}(0)}{2}} \frac{(s^p - v_1^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s = 48.6332$$
$$\int_{v_2^*(0)}^{\frac{q_{22}(0)}{2}} \frac{(s^p - v_2^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s = 42.6276$$
$$\int_{v_3^*(0)}^{\frac{q_{23}(0)}{2}} \frac{(s^p - v_3^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s = 63.0730$$
$$\int_{v_4^*(0)}^{\frac{q_{24}(0)}{2}} \frac{(s^p - v_4^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s = 0.2895$$
$$\int_{v_5^*(0)}^{\frac{q_{25}(0)}{2}} \frac{(s^p - v_5^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s = 4.3790.$$

Hence we have

$$\begin{aligned}
V(0) &= \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_{3i}(0)-q_{3j}(0))^2 + \sum_{i=1}^{n}\int_{v_i^*(0)}^{c_i q_{2i}(0)} \frac{(s^p - v_i^*(0)^p)^{2-1/p}}{(2-1/p)2^{1-1/p}k_2^{1+p}}\,\mathrm{d}s,\\
&= 154.7500 + 48.6332 + 42.6276 + 63.0730 + 0.2895 + 4.3790\\
&= 313.7524\\
b_1 &= \max\left\{\frac{1}{2\lambda_2}, \frac{1}{(2-1/p)k_2^{1+p}}\right\}\\
&= \max\left\{\frac{1}{2\times 0.1910}, \frac{1}{(11/9)3.2^{16/7}}\right\} = 2.6180\\
b_2 &= k_3/(2b_1^{d/2}) = 1/(2\times 2.6178^{8/9}) = 0.2125\\
T_1 &= \frac{V(0)^{1-d/2}}{b_2(1-d/2)} = \frac{9\times 313.7523^{1/9}}{0.2126} = 80.2056.
\end{aligned}$$

We can get: when $t < T_1$,

$$u_{1i}(t) = 0.5, 1 \le i \le 5, \tag{15}$$

and

$$\begin{aligned}
u_{21} &= 193.6\Big[3.2^{\frac{9}{7}}\big(\tfrac{q_{33}-q_{31}}{2}\big) - \big(\tfrac{q_{21}}{2}\big)^{\frac{9}{7}}\Big]^{\frac{5}{9}}\\
u_{22} &= 193.6\Big[3.2^{\frac{9}{7}}\big(\tfrac{q_{34}+q_{35}}{2} - q_{32}\big) - \big(\tfrac{q_{22}}{2}\big)^{\frac{9}{7}}\Big]^{\frac{5}{9}}\\
u_{23} &= 193.6\Big[3.2^{\frac{9}{7}}\big(\tfrac{q_{31}+q_{34}}{2} - q_{33}\big) - \big(\tfrac{q_{23}}{2}\big)^{\frac{9}{7}}\Big]^{\frac{5}{9}}\\
u_{24} &= 193.6\big[3.2^{\frac{9}{7}}\big(\tfrac{q_{32}+q_{33}}{2} - q_{34}\big) - \big(\tfrac{q_{24}}{2}\big)^{\frac{9}{7}}\Big]^{\frac{5}{9}}\\
u_{25} &= 193.6\Big[3.2^{\frac{9}{7}}\big(\tfrac{q_{32}-q_{35}}{2}\big) - \big(\tfrac{q_{25}}{2}\big)^{\frac{9}{7}}\Big]^{\frac{5}{9}}.
\end{aligned} \tag{16}$$

When $t \geq T_1$,

$$\begin{aligned} q_1(T_1) &= [q_{11}(0) + c_1T_1 \; q_{12}(0) + c_2T_1 \; q_{13}(0) + c_3T_1 \\ & \quad q_{14}(0) + c_4T_1 \; q_{15}(0) + c_5T_1]^T, \end{aligned}$$

based on $c_i = 0.5, 1 \leq i \leq 5$ and Lemma 2.1 we can get

$$V_0(T_1) = \frac{1}{2} q_1(T_1)^T L q_1(T_1) = \frac{1}{2} q_1(0)^T L q_1(0) = 281.25,$$

and moreover

$$T_2 = \frac{V_0(T_1)^{\frac{p-1}{2p}}}{k_2 \frac{p-1}{2p} (2\lambda_2)^{\frac{1+p}{2p}}} = \frac{9 \times 281.25^{1/9}}{3.2(2 \times 0.1910)^{8/9}} = 12.3808,$$

$$u_{2i}(t) = 0, 1 \leq i \leq 5, \tag{17}$$

and

$$\begin{aligned} u_{11} &= (1/2(q_{13} - q_{11}))^{7/9} \\ u_{12} &= (1/2(q_{14} - q_{12}) + 1/2(q_{15} - q_{12}))^{7/9} \\ u_{13} &= (1/2(q_{11} - q_{13}) + 1/2(q_{14} - q_{13}))^{7/9} \\ u_{14} &= (1/2(q_{13} - q_{14}) + 1/2(q_{12} - q_{14}))^{7/9} \\ u_{15} &= (1/2(q_{12} - q_{15}))^{7/9}. \end{aligned} \tag{18}$$

The numerical results in Figures 2, 3 and Figure 4 show that the effectiveness of the proposed controller. From Figure 3 and Figure 4, we can get that the distributed finite time controller (16) can make all the mobile non-holonomic agents reach consensus with respect to states $q_{2i}, q_{3i}, 1 \leq i \leq 5$ within $t < 2.5 < T_1 = 83.8270$. Hence in the simulation when $t \geq T_1 = 83.8270$ we take the distributed finite time controllers as (16)and (17), and the distributed finite time controllers (16)and (17) can make all the mobile non-holonomic agents reach consensus with respect to states $q_{1i}, 1 \leq i \leq 5$ within $t < 90 - 83.8270 = 6.1730 < T_2 = 12.3808$ as demonstrated by Figure 2. For the space limitation, the simulation for Theorem 3.2 is omitted.

5. CONCLUSION

In this paper, the problem of finite time consensus was discussed for multiple non-holonomic mobile agents, and based on the result from paper [13] we proposed a distributed finite-time control law for each agent. And moreover, with help of time-rescaling techniques from papers [7, 15, 16], we have achieved finite-time consensus within any given settling time.

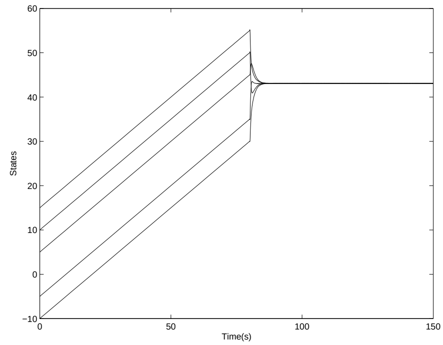

Fig. 2. Trajectories of $q_{1i}, 1 \leq i \leq 5$ with $c = [1, 1, 1, 1, 1]^T$.

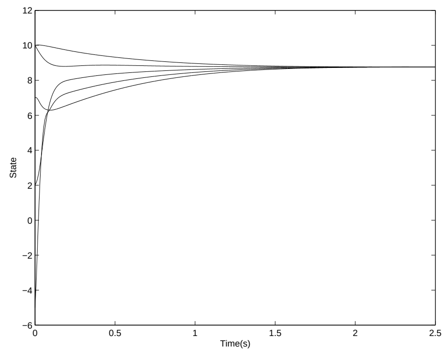

Fig. 3. Trajectories of $q_{2i}, 1 \leq i \leq 5$ with $c = [1, 1, 1, 1, 1]^T$.

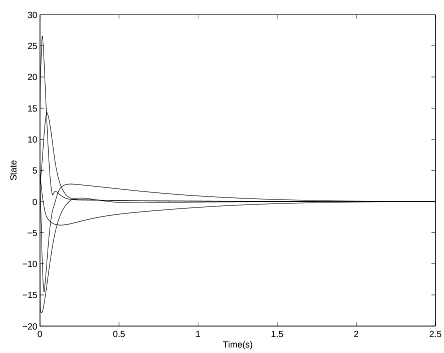

Fig. 4. Trajectories of $q_{3i}, 1 \leq i \leq 5$ with $c = [1, 1, 1, 1, 1]^T$.

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