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CONSENSUS SEEKING IN MULTI-AGENT SYSTEMS WITH AN ACTIVE LEADER AND COMMUNICATION DELAYS

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In this paper, we consider a multi-agent consensus problem with an active leader and variable interconnection topology. The dynamics of the active leader is given in a general form of linear system. The switching interconnection topology with communication delay among the agents is taken into consideration. A neighbor-based estimator is designed for each agent to obtain the unmeasurable state variables of the dynamic leader, and then a distributed feedback control law is developed to achieve consensus. The feedback parameters are obtained by solving a Riccati equation. By constructing a common Lyapunov function, some sufficient conditions are established to guarantee that each agent can track the active leader by assumption that interconnection topology is undirected and connected. We also point out that some results can be generalized to a class of directed interaction topologies. Moreover, the input-to-state stability (ISS) is obtained for multi-agent system with variable interconnection topology and communication delays in a disturbed environment.

Keywords: multi-agent system, consensus, leader-following, time-delay

Classification: 93E12, 62A10

1. INTRODUCTION

In recent years, the coordination problem of multiple autonomous agents has drawn an increasing attention with rather diverse background such as biology, physics, mathematics, information science, computer science and control science [14, 16]. In conventional or centralized design methods or tasks, couplings or interactions in feedback systems were often viewed as lying somewhere between troubles and undesirable features that should be avoided if at all possible. However, an important aspect of multi-agent systems is to aim at fully exploiting interconnection features in distributed designs. Among the studies of distributed control and collective behavior analysis, the leader-follower problem is important, which have been investigated in different ways recently [1, 2, 7, 15].

As an extension of conventional leader-follower tracking problem, the problem considered in this paper is to study the leader-follower problem where the leader dynamics are different from those of the followers and some states of the leaders may be unknown or unmeasurable. In practice, an active leader may be a moving target in a sensor network, or an evader in a pursuit game, or a reference system that is not completely known. To follow or track such leaders, distributed estimation of the leaders is needed in the leader-following design. On the other hand, distributed design is quite fit to coordinate multi-agent system when the interaction topologies keep switching. In fact, distributed estimation based on distributed observers was considered for multi-agent coordination in [6, 7] to estimate unmeasurable states of the active leader with simple dynamics expression, while a distributed algorithm was proposed for distributed estimation of a general active leader's unmeasurable state variables in [8]. Moreover, the active leader-following problem with first-order agent dynamics was considered even in a stochastic scenario by [11]. Additionally, internal-model approach began in the study of such problems and the problem with a simple topology and agents in the form general linear systems was solved in [19].

There is no doubt that the stability of multi-agent systems is of utmost importance. In real applications, the interacting topology between agents may change dynamically. For example, in the case of interaction via communications, the communication links between vehicles may be unreliable subject to disturbances and/or communication time-delay. However, a well-known fact is that switching of the communication topology and communication time delays may lower the system performance and even cause the network system to diverge or oscillate. Time-delay systems have attracted much attention in recent years [4], even in multi-agent systems for either first-order agents [15] or second-order agents [10, 12, 13].

The motivation of this paper is to extend the results on active leader-following problem to systems involving communication delays among agents under switching topology. Similar distributed observers and control laws proposed by [8] are used to track the active leader. Moreover, to handle the switching topology, an approach based on common Lyapunov function (CLF) is employed to study the consensus and estimation in both noise-free and noise environments. As a special case of our result, we also generalize the result of [8] to a class of directed interaction topologies.

The paper is organized as follows. In section 2, formulation of the problem and some basic results about time-delay systems are presented. Main results are given in section 3. Following that, section 4 provides a simple simulation example, and finally, concluding remarks with discussions of the future work are reported in section 5.

The notation of this paper is standard. Throughout this paper, the following notations are used: R is the real number set. I is an identity matrix with compatible dimension. A^T is denoted as transpose of a matrix A; $\mathbf{1}_n = [1, 1, \ldots, 1]^T$ with proper dimension; For symmetric matrices A and B, $A > (\geq) B$ means $A - B$ is positive (semi-) definite. $\lambda(A)$ represents an eigenvalue of matrix A. For symmetric matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent the minimum and maximum eigenvalue of A respectively. $\|\cdot\|$ denotes Euclidean norm. $\otimes$ denotes the Kronecker product, which satisfies (1) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$; (2) If $A \geq 0$ and $B \geq 0$, then $A \otimes B \geq 0$ [9].

2. PRELIMINARIES

First of all, we introduce some preliminary knowledge that will be used throughout this paper.

2.1. Graph theory

Stability analysis of a group of agents is based on several results of algebraic graph theory. More details are available in [3]. Let $\mathcal{G} = \{\mathcal{V}, \varepsilon, A)$ be a weighted directed graph of order n, where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes, ε is the set of edges and a weighted adjacency matrix $A = [a_{ij}]$ with nonnegative elements. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, \ldots, n\}$. The element a_{ij} associated with the edge of the directed graph is positive, i. e., $a_{ij} > 0 \Leftrightarrow (v_i, v_j) \in \varepsilon$. Throughout the paper, we assume that all the graphs have no edges from a node to itself. Thus, for all $i \in \mathcal{I}$ we have $a_{ij} = 0$. A weighted graph is call undirected if $\forall\,(v_i, v_j) \in \varepsilon \Rightarrow (v_j, v_i) \in \varepsilon$ and $a_{ij} = a_{ji}$. Otherwise, the graph is called a directed graph. If $(v_i, v_j) \in \varepsilon$, then v_j is said to be a neighbor of v_i and we denote the set of all neighbors of node v_i by $\mathcal{N}_i = \{j | (v_i, v_j) \in \varepsilon\}$. A path is a sequence of ordered edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_{j-1}}, v_{i_j})$ where $i_j \in \mathcal{I}$ and $v_{i_j} \in \mathcal{V}$. The degree matrix $D = \{d_1, d_2, \ldots, d_n\} \in \mathcal{R}^{n\times n}$ of graph $\mathcal{G}$ is a diagonal matrix, where diagonal elements $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ for $i = 1, 2, \ldots, n$. Then the Laplacian matrix of $\mathcal{G}$ is defined as $L = D - A \in R^{n\times n}$. Obviously, the Laplacian matrix of any undirected graph is symmetric. Denote the eigenvalues of Laplacian matrix L by $\lambda_1(L), \ldots, \lambda_n(L)$, which satisfy $\lambda_1(L) \le \cdots \le \lambda_n(L)$. Then we know that $\lambda_1(L) = 0$ and $\mathbf{1}_n$ is its eigenvector (see [3]).

In what follows, we mainly consider a graph $\bar{\mathcal{G}}$ associated with the system with n agents (labeled by $v_i, i = 1, 2, \ldots, n$) and one leader (labeled by v_0). A simple and undirected graph $\mathcal{G}$ describes the topology relation of these n followers and $\hat{\mathcal{G}}$ contains $\mathcal{G}$ and v_0 with directed edges from some agents to the leader v_0. The graph $\mathcal{G}$ is allowed to have several components, within every such component all the agents are connected via undirected edges. The graph $\hat{\mathcal{G}}$ is said to be connected if at least one agent in each component of $\mathcal{G}$ is connected to the leader by a directed edge. Let $t_1 = 0, t_2, t_3, \ldots$ be an infinite time sequence at which the interconnection graph of the considered multi-agent system switches. Usually, it is assumed that chattering does not occur, that is, there is a constant $\Delta > 0$, often called dwell time, with $t_{i+1} - t_i \ge \Delta,\ \forall\, i$. Moreover, we assume that there only finite possible interconnection topologies can be switched. Denote $\bar{\mathcal{S}} = \{\hat{\mathcal{G}}_1, \hat{\mathcal{G}}_2, \ldots, \hat{\mathcal{G}}_N\}$ as a set of all possible topology graphs and denote $\mathcal{P} = \{1, 2, \ldots, N\}$ as its index set. To describe the variable interconnection topology, we define a switching signal $\sigma : [0, \infty) \to \mathcal{P}$, which is piecewise constant. Therefore, $\mathcal{N}_i$ and the connection weight $a_{ij} (i, j = 1, \ldots, n)$ are time-varying, and moreover, Laplacian matrix $L_{\sigma(t)} (\sigma(t) \in \mathcal{P})$ associated with the switching interconnection graph is also time-varying, though it is a time-invariant matrix in any interval $[t_i, t_{i+1})$.

2.2. Time-delay systems

In this subsection, we introduce some basic preliminaries related to time-delay systems (see [4, 5]). Consider the following system:

$$\dot{x} = f(t, x_t), \quad t > 0, \tag{1}$$

where $x(t) \in R^n$, $f : R \times \mathcal{C} \to R^n$ with $f(t,0) = 0$ and $\mathcal{C} = \mathcal{C}([-r,0], R^n)$ is the set of continuous functions mapping interval $[-r,0]$ to R^n. r is said to be maximum time delay of the system (1). Equation (1) indicates that the derivative of the state variables x at time t depends on t and $x(\omega)$ for $t - r \le \omega \le t$. As such, to determine the future evolution of the state, it is necessary to specify the initial state variables $x(t)$ in a time interval of length r, say, from $t_0 - r$ to t_0, i. e.,

$$x_{t_0} = \phi,$$

where $\phi \in \mathcal{C}$ is given. In other words $x_{t_0}(\theta) = x(t_0 + \theta), \forall\, \theta \in [-r,0]$ with norm $||\phi||_c = \max_{\theta \in [-r,0]} \|\phi(\theta)\|$.

Then the time-delay system (1) is given by

$$\begin{cases} \dot{x} = f(t, x_t), & t \ge t_0, \\ \dot{x}_{t_0}(\theta) = \phi(\theta), & \theta \in [-r,0]. \end{cases} \tag{2}$$

For an $A > 0$, a function $x(t, t_0, \phi)$ is said to be a solution of (2) on the interval $[t_0 - r, t_0 + A)$ if within this interval $x(t, t_0, \phi)$ is continuous and satisfies the system (2).

The concepts about stability for functional differential equation (2) are given as follows.

Definition 2.1. For the system described by (2), the trivial solution $x(t, t_0, \phi) = 0$ is said to be stable if for any $t_0 \in R$ and any $\epsilon > 0$, there exists a $\mu = \mu(t_0, \epsilon) > 0$ such that $\|\phi\|_c < \mu$ implies $\|x(t, t_0, \phi)\| < \epsilon$ for $t \ge t_0$. It is said to be asymptotically stable if it is stable, and for any $t_0 \in R$ and any $\epsilon > 0$, there exists a $\mu_a = \mu_a(t_0, \epsilon) > 0$ such that $\|\phi\|_c < \mu_a$ implies $\lim_{t\to\infty} x(t, t_0, \phi) = 0$. It is said to be uniformly stable if it is stable and $\mu(t_0, \epsilon)$ can be chosen independently of t_0. It is uniformly asymptotically stable if it is uniformly stable and there exists a $\mu_a > 0$ such that for any $\tilde{\epsilon} > 0$, there exists a $T = T(\mu_a, \tilde{\epsilon})$, such that $\|\phi\|_c < \mu_a$ implies $\|x(t, t_0, \phi)\| < \tilde{\epsilon}$ for $t \ge t_0 + T$ and $t_0 \in R$. It is globally (uniformly) asymptotically stable if it is (uniformly) asymptotically stable and μ_a can be an arbitrarily large, finite number.

The following result is for the stability of system (2) (the details can be found in [4, 5]).

Lemma 2.2. Suppose $f : R \times \mathcal{C} \to R^n$ in (2) takes $R\times$ (bounded sets of $\mathcal{C}$) into bounded sets of R^n, and $\varphi_1, \varphi_2, \varphi_3 : R_{\ge 0} \to R_{\ge 0}$ are continuous nondecreasing functions, $\varphi_1(s) > 0, \varphi_2(s) > 0, \varphi_3(s) > 0$ for $s > 0$ and $\varphi_1(0) = \varphi_2(0) = 0$, φ_2 strictly increasing. If there exists continuously differentiable function $V : R \times R^n \to R$ such that

$$\varphi_1(||x||) \le V(t,x) \le \varphi_2(||x||),\ t \in R,\ x \in R^n.$$

In addition, there exists a continuous nondecreasing function $\varphi(s)$ with $\varphi(s) > s$, $s > 0$ such that the derivative of V along the solution $x(t)$ of (2) satisfies

$$\dot{V}(t,x)|_{(2)} \le -\varphi_3(||x||),$$

if

$$V(t+\theta, x(t+\theta)) \le \varphi(V(t, x(t))),\ \theta \in [-r,0],$$

then system (2) is uniformly asymptotically stable. If, in addition, $\lim_{t\to\infty}\varphi_3(s)=\infty$, then system (2) is globally uniformly asymptotically stable.

In what follows, we will introduce input-to-state stability (ISS) for functional differential equation following [18]. This concept plays an important role in the case of controlling nonlinear time-delay systems.

Consider the following nonlinear time-delay system

$$\begin{cases}\dot{\bar{x}}=g(t,\bar{x}_t,\bar{u}), \quad t\geq t_0,\\ \bar{x}_{t_0}(\theta)=\bar{\phi}(\theta),\ \theta\in[-r,0],\end{cases} \tag{3}$$

where $\bar{x}\in R^n$, $\bar{u}\in R^m$ is bounded and piecewise continuous, $\bar{\phi}\in\mathcal{C}$. Suppose for each initial data, input, and starting time $t_0\geq 0$ there exists $A>0$ and a unique maximal solution $\bar{x}(\cdot)$ defined on $[t_0-r,t_0+A)$.

$\gamma: R_{\geq 0}\to R_{\geq 0}$ is called to a K-function if it is continuous, strictly increasing and $\gamma(0)=0$, and moreover, it is called to be a K_∞-function if it is a K-function and satisfies that $\lim_{t\to\infty}\gamma(t)=\infty$. $\beta: R_{\geq 0}\times R_{\geq 0}\to R_{\geq 0}$ is called to be a KL-function if, for each fixed $t\geq 0$, the function $\beta(\sigma,t)$ is a K-function, and for each fixed $\sigma\geq 0$ it decreases to zero as $t\to\infty$.

Definition 2.3. Let χ be continuous, zero at zero, and nondecreasing function, $\iota\in R_{\geq 0}$, and $\nu_{\bar{x}},\nu_{\bar{u}}\in R_{\geq 0}\bigcup\infty$. The trivial solution of (3) is said to be uniformly ISS with gain χ [and offset ι and restriction $(\nu_{\bar{x}},\nu_{\bar{u}})$] if $\|\bar{\phi}\|_c<\nu_{\bar{x}}$ and $\|\bar{u}\|_\infty<\nu_{\bar{u}}$ imply $A=\infty$ and the following properties holding uniformly in $t_0\geq 0$:

1) for each $\epsilon>0$ there exists $\mu>0$ such that $\|\bar{\phi}\|_c\leq\mu$ implies $\|x(t)\|_\infty\leq\max\{\epsilon,\chi(\|\bar{u}\|_\infty),\iota\}$ and

2) for each $\epsilon>0$, $\upsilon_{\bar{x}}\in(0,\nu_{\bar{x}})$ and $\upsilon_{\bar{u}}\in(0,\nu_{\bar{u}})$ there exists $T>0$ such that $\|\bar{\phi}\|_c\leq\upsilon_{\bar{x}}$ and $\|\bar{u}\|_\infty\leq\upsilon_{\bar{u}}$ imply $\sup_{t\geq t_0+T}\|x(t)\|\leq\max\{\epsilon,\chi(\|\bar{u}\|_\infty),\iota\}$.

When $\iota=0$ and $\bar{u}\equiv 0$, this is the standard definition of uniformly asymptotically stable for trivial solution of a functional differential equation. This definition is consistent with the definition ISS for ordinary differential equations ([17]).

The following lemma is the global ISS version of the Razumikhin-type theorem ([18]) for globally asymptotically stable ([4]).

Lemma 2.4. If there exist K_∞-functions φ_1,φ_2, a continuously differential function $V: R\times R^n\to R_{\geq 0}$, χ_1,χ_2 are continuous, zero at zero, and nondecreasing functions and K-function φ_3 such that

1) $\varphi_1(\|x\|)\leq V(t,x)\leq\varphi_2(\|x\|)$,

2) $V(t,x)\geq\max\{\chi_1(\max_{t-r\leq s\leq t}\|V(s+\theta,x(s+\theta))\|),\chi_2(\max_{t-r\leq s\leq t}\|\bar{u}(s)\|)\}$ implies $\dot{V}(t,x)|_{(3)}\leq-\varphi_3(||x||)$,

3) $\chi_1(s)<s$ for $s>0$, then the trivial solution is globally uniformly ISS with gain $\varphi_1^{-1}\circ\chi_2$.

3. MAIN RESULTS

In this paper, the dynamics of follower-agent i is described by

$$\begin{cases} \dot{x}_i = u_i + \delta_i, & x_i \in R^{n_0} \\ i = 1, \dots, n, \end{cases} \tag{4}$$

where x_i is the state, $\delta_i(t)$ are uncertain and u_i is the input.

The leader of this considered multi-agent system is active, whose underlying dynamics, different from the followers, can be expressed as follows:

$$\begin{cases} \dot{\omega} = \bar{A}\omega, \omega = (\omega_1, \omega_2, \dots, \omega_m)^T \in R^{mn_0}, \\ x_0 = C\omega \in R^{n_0}. \end{cases} \tag{5}$$

Here $\bar{A}=(\bar{a}_{ij})_{mn_0 \times mn_0}$, $\omega=(\omega_1, \omega_2, \dots, \omega_m)^T \in R^{mn_0}$ are respectively the state matrix and state variables. Without loss of generality, we assume n_0=1 in the sequel. The obtained result of this paper is true for any dimension m, because we can revise the expressions via Kronecker product. For simplicity, we take $C^T = (1, 0, \dots, 0)^T \in R^m$, or equivalently, $x_0 = \omega_1$. Clearly, the leader model considered in [7] can be viewed as a special case of (5).

The state matrix $\bar{A}$ is assumed to be known by all following agents, but its initial condition $\omega(0)$ is unknown and x_0 is the only measurable variable.

Our problem is to let all the follower-agents track the active leader. Since we consider tracking problems with time delays, each agent cannot instantly get the information from others and the leader. To obtain distributed design, the relative measurement is employed. Denote the relative error of agent i as

$$\begin{aligned} z_i(t) = &\sum_{j \in N_i(t)} a_{ij}(t)(x_i(t-\tau_i) - x_j(t-\tau_j)) \\ &+ b_i(t)(x_i(t-\tau_i) - x_0(t-\tau_0)). \end{aligned} \tag{6}$$

The time-varying delay $\tau_i(t) > 0$, $i = 0, 1, \dots, n$ are continuously differentiable functions with $0 \le \tau_i \le r_i$.

At time t , $a_{ij}(t)$ and $b_i(t)$ are chosen by

$$a_{ij}(t) = \begin{cases} \alpha_{ij} = \alpha_{ji}, & \text{if agents } i \text{ and } j \text{ are connected at } t, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_i(t) = \begin{cases} \beta_i, & \text{if agent } i \text{ is connected to the leader at } t, \\ 0, & \text{otherwise,} \end{cases}$$

where $\alpha_{ij} > 0$ $(i, j = 1, \dots, n)$ is connection weight constant between agent i and agent j, and $\beta_i > 0$ $(i = 1, \dots, n)$ is connection weight constant between agent i and leader.

To track the leader in the form of (5), follower agents have to estimate the state ω on line. However, the conventional observer design cannot apply here to deal with the state estimation problem because the distributed design is based on local information. Therefore we have to propose a distributed control scheme with online estimation algorithm for each agent to estimate the leader's unmeasurable state variables under switching topologies with time delays.

To solve the tracking problem, similar distributed observers and control laws proposed by [8] are used to track the active leader, which consists of two parts:

1) Neighbor-based feedback law:

$$u_i = \bar{a}_{11} x_i + \sum_{j=2}^{m} \bar{a}_{1j} v_i^{j-1} - \bar{l}_1 z_i, \tag{7}$$

with $\bar{a}_{ij}$ defined in (5) and z_i in (6);

2) Distributed estimation law:

$$v_i^{k-1} = \bar{a}_{k1} x_i + \sum_{j=2}^{m} \bar{a}_{kj} v_i^{j-1} - \bar{l}_k z_i, k = 2, \ldots, m, \tag{8}$$

with $\bar{l}_k$ $(k = 1, 2, \ldots, m)$ to be determine in the following.

Note that u_i in (7) is a local controller of agent i, with neighbor-based estimation rule in form of observer (8), which can be viewed as a distributed (reduced) observer, to estimate the leader's state variables. In other words, each agent cannot "observe" the leader immediately based on the measured information of the leader if it is not connected to the leader. In fact, it has to collect the information of the leader in a distributed way from its neighbor agents.

At first, we consider noise-free case ($\delta_i = 0$, $i = 1, \ldots, n$). Denote

$$\xi_i = \begin{pmatrix} x_i \\ v_i^1 \\ \vdots \\ v_i^{m-1} \end{pmatrix}, \quad \eta_i = \xi_i - \omega \in R^m, \quad i = 1, \ldots, n,$$

and

$$\eta = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{pmatrix} \in R^{mn}, \quad \bar{L} = \begin{pmatrix} \bar{l}_1 \\ \bar{l}_2 \\ \vdots \\ \bar{l}_m \end{pmatrix}, \quad z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix}.$$

Then, after manipulations with combining (4–8), we have

$$\dot{\eta}_i = \bar{A}\eta_i - \bar{L} z_i, i = 1, 2, \ldots, n,$$

or equivalently

$$\dot{\eta} = (I_n \otimes \bar{A})\eta - (I_n \otimes \bar{L}) z.$$

To express conveniently, assume that all time-delays have same form $\tau_j = \tau$, and according the properties of Laplacian matrix we have

$$z(t) = (L_p + B_p) \otimes C\eta(t-\tau)$$

where $p = \sigma(t) \in \mathcal{P}$, L_p is the Laplacian matrix of graph $\mathcal{G}$ associated to n follower-gents, B_p is a $n \times n$ diagonal matrix whose ith diagonal elements is $b_i(t)$.

For convenience, let $H_p = L_p + B_p$. Due to $L_p\mathbf{1} = 0$, the error dynamics of the closed-loop system can be rewritten in a compact form:

$$\dot{\eta}(t) = I_n \otimes \bar{A}\eta(t) - H_p \otimes (\bar{L}C)\eta(t-\tau). \tag{9}$$

Note that, even when the interconnection graph is connected, $b_i(t)$ may be 0 for some i, and therefore, B_p may not be of full rank. It was known that, H_p associated with $\bar{\mathcal{G}}_p$ is positive definite if graph $\bar{\mathcal{G}}_p$ is connected (see [7]).

In the sequel, it is always assume that the interconnection graph $\bar{\mathcal{G}}$ is connected, though the interconnection topology keeps changing.

Define

$$\bar{\lambda} := \max_{p\in\mathcal{P}}\{\lambda_{\max}(H_p)|\forall\, \bar{\mathcal{G}}_p \text{ is connected}\}$$

and

$$\underline{\lambda} := \min_{p\in\mathcal{P}}\{\lambda_{\min}(H_p)|\forall\, \bar{\mathcal{G}}_p \text{ is connected}\}.$$

Based on the fact that the set $\mathcal{P}$ is finite, $\bar{\lambda}$ and $\underline{\lambda}$ are fixed and positive.

Theorem 3.1. Assume that the switching interaction graphs $\bar{\mathcal{G}}_{\sigma(t)}$ are all connected with a given dwell time, the time-delay is sufficiently small, and the system parameters l_i of the consensus protocol has been designed. Controller (7) together with "observer" (8) can guarantee the follower-agents track the active leader, namely,

$$\lim_{t\to\infty} \eta(t) = 0, \tag{10}$$

if there exists a matrix $P = P^T > 0$ such that

$$\begin{cases} P(\bar{A} - \bar{\lambda}\bar{L}C) + (\bar{A} - \bar{\lambda}\bar{L}C)^T P < 0 \\ P(\bar{A} - \underline{\lambda}\bar{L}C) + (\bar{A} - \underline{\lambda}\bar{L}C)^T P < 0. \end{cases} \tag{11}$$

Proof. Choose a common Lyapunov function for system (9):

$$V(\eta) = \eta^T(I_n \otimes P)\eta.$$

To prove the theorem, we consider the dynamics in each interval at first. In any interval $[t_i, t_{i+1})$, the topology graph is fixed and the system matrices are time-invariant. Then we will focus on the discussion in $[t_i, t_{i+1})$, when the system become time-invariant with some fixed $p \in \mathcal{P}$. Let U_p be an orthogonal transformation such that $U_pH_pU_p^T$ is a diagonal matrix $diag\{\lambda_{1p}, \lambda_{2p}, \ldots, \lambda_{np}\}$, where λ_{ip} is ith eigenvalue of matrix H_p.

If (11) is satisfied, there exists positive definite matrix $\bar{Q}$ such that

$$\begin{cases} P(\bar{A}-\bar{\lambda}\bar{L}C)+(\bar{A}-\bar{\lambda}\bar{L}C)^T P \le -\bar{Q} < 0 \\ P(\bar{A}-\underline{\lambda}\bar{L}C)+(\bar{A}-\underline{\lambda}\bar{L}C)^T P \le -\bar{Q} < 0. \end{cases}$$

Due to $\lambda_{ip} \in [\underline{\lambda}, \bar{\lambda}]$, there exist constants $\alpha_{ip} \ge 0$ and $\beta_{ip} \ge 0$ satisfying $\lambda_{ip} = \alpha_{ip}\underline{\lambda} + \beta_{ip}\bar{\lambda}$ and $\alpha_{ip}+\beta_{ip}=1$. From (11), we have

$$\begin{aligned} \Omega_{ip} &:= P(\bar{A}-\lambda_{ip}\bar{L}C)+(\bar{A}-\lambda_{ip}\bar{L}C)^T P = \alpha_{ip}[P(\bar{A}-\underline{\lambda}\bar{L}C) \\ &+ (\bar{A}-\underline{\lambda}\bar{L}C)^T P] + \beta_{ip}[P(\bar{A}-\bar{\lambda}\bar{L}C)+(\bar{A}-\bar{\lambda}\bar{L}C)^T P] \le -\bar{Q} < 0. \end{aligned}$$

Moreover, we have

$$\begin{aligned} &(I\otimes P)(I\otimes \bar{A} - H_p \otimes (\bar{L}C)) + (I\otimes \bar{A} - H_p\otimes(\bar{L}C))^T (I\otimes P) \\ &= (U_p^T \otimes I) diag\{\Omega_{1p},\dots,\Omega_{np}\}(U_p\otimes I) \\ &\le -(U_p^T\otimes I)(I\otimes \bar{Q})(U_p\otimes I) \\ &= -I\otimes\bar{Q} < 0. \end{aligned} \tag{12}$$

Then we consider $\dot{V}(\eta)|_{(9)}$. By Leibniz–Newton formula, we have

$$\begin{aligned} \eta(t-\tau) &= \eta(t) - \int_{t-\tau}^{t} \dot{\eta}(s)\,\mathrm{d}s \\ &= \eta(t) - (I\otimes\bar{A})\int_{-\tau}^{0}\eta(t+s)\,\mathrm{d}s + H_p\otimes(\bar{L}C)\int_{-\tau}^{0}\eta(t+s-\tau)\,\mathrm{d}s. \end{aligned} \tag{13}$$

Thus the delayed differential equation (9) can be rewritten as

$$\begin{aligned} \dot{\eta}(t) &= (I\otimes\bar{A} - H_p\otimes(\bar{L}C))\eta(t) + (H_p\otimes(\bar{L}C\bar{A})\int_{-\tau}^{0}\eta(t+s)\,\mathrm{d}s \\ &\quad - (H_p^2\otimes(\bar{L}C\bar{L}C)\int_{-\tau}^{0}\eta(t+s-\tau)\,\mathrm{d}s. \end{aligned}$$

Then, we have

$$\begin{aligned} &\dot{V}(t)|_{(9)} = \eta^T(t)[(I\otimes\bar{A} - H_p\otimes(\bar{L}C))^T(I\otimes P) \\ &+ (I\otimes P)(I\otimes\bar{A}-H_p\otimes(\bar{L}C))]\eta(t) + 2\eta^T(t)[H_p\otimes(P\bar{L}C\bar{A})]\int_{-\tau}^{0}\eta(t+s)\,\mathrm{d}s \\ &- 2\eta^T(t)[H_p^2\otimes(P\bar{L}C\bar{L}C)]\int_{-\tau}^{0}\eta(t+s-\tau)\,\mathrm{d}s. \end{aligned}$$

Note that $2a^Tb \le a^T\psi a + b^T\psi^{-1}b$ holds for any appropriate positive define matrix ψ, and then with $a^T = -\eta^T[H_p\otimes(P\bar{L}C\bar{A})]$ and $b=\eta(t+s)$, $\psi = (I\otimes P)^{-1}$ we have

$$\begin{aligned} &2\eta^T(t)[H_p\otimes(P\bar{L}C\bar{A})]\int_{-\tau}^{0}\eta(t+s)\,\mathrm{d}s \\ &\le \tau\eta^T(t)[H_p^2\otimes(P\bar{L}C\bar{A}P^{-1}(P\bar{L}C\bar{A})^T)]\eta + \int_{-\tau}^{0}\eta^T(t+s)(I\otimes P)\eta(t+s)\,\mathrm{d}s. \end{aligned}$$

Similarly, with $a = -\eta^T[H_p^2 \otimes (P\bar{L}C\bar{L}C)]$ and $b = \eta(t+s-\tau)$, $\psi = P^{-1}$, we have

$$\begin{aligned}&-2\eta^T(t)[H_p^2 \otimes (P\bar{L}C\bar{L}C)]\int_{-\tau}^{0}\eta(t+s-\tau)\,\mathrm{d}s\\ &\le \tau\eta^T(t)[H_p^4 \otimes (P\bar{L}C\bar{L}CP^{-1}(P\bar{L}C\bar{L}C)^T)]\eta + \int_{-2\tau}^{-\tau}\eta^T(t+s)(I\otimes P)\eta(t+s)\,\mathrm{d}s.\end{aligned}$$

Set $r = \max\{r_1,\ r_2,\ldots,r_n\}$ and $\phi(s) = qs$ for some constant $q > 1$, in case of $V(\eta(t+\theta)) < qV(\eta(t))$, $-r \le \theta \le 0$. Then we have

$$\begin{aligned}\dot{V}(t) &\le -\eta^T(t)\bar{Q}\eta(t) + \tau\eta^T[H_p^2 \otimes (P\bar{L}C\bar{A}P^{-1}(P\bar{L}C\bar{A})^T)]\eta\\ &+ \tau\eta^T[H_p^4 \otimes (P\bar{L}C\bar{L}CP^{-1}(P\bar{L}C\bar{L}C)^T)]\eta + \int_{-\tau}^{0}\eta^T(t+s)(I\otimes P)\eta(t+s)\,\mathrm{d}s\\ &+ \int_{-2\tau}^{-\tau}\eta^T(t+s)(I\otimes P)\eta(t+s)\,\mathrm{d}s\\ &\le -\eta^T(t)(I\otimes\bar{Q})\eta(t) + \tau\eta^T(t)[H_p^2 \otimes P\bar{L}C\bar{A}P^{-1}(P\bar{L}C\bar{A})^T]\eta(t)\\ &+ \tau\eta^T(t)[H_p^4 \otimes P\bar{L}C\bar{L}CP^{-1}(P\bar{L}C\bar{L}C)^T]\eta(t) + 2rq\eta^T(I\otimes P)\eta.\end{aligned}$$

Set $\varrho = \max_{p\in\mathcal{P}}\{\|H_p^2 \otimes P\bar{L}C\bar{A}P^{-1}(P\bar{L}C\bar{A})^T\| + \|H_p^4 \otimes P\bar{L}C\bar{L}CP^{-1}(P\bar{L}C\bar{L}C)^T\|\}$, and then

$$\begin{aligned}\dot{V}(\eta) &\le -\eta^T(I\otimes\bar{Q})\eta + r\varrho\eta^T\eta + 2rq\eta^T(I\otimes P)\eta\\ &\le -[\lambda_{\min}(\bar{Q}) - r\varrho - 2rq\lambda_{\max}(P)]\|\eta\|^2,\end{aligned}$$

where $\lambda_{\min}(\bar{Q})$ denotes the smallest eigenvalue of $\bar{Q}$, $\lambda_{\max}(P)$ denotes the largest eigenvalue of P.

If $r < \frac{\lambda_{\min}(\bar{Q})}{\varrho + 2q\lambda_{\max}(P)}$, then, according to Lemma 2.2, system (9) is globally uniformly asymptotically stable, which implies (10). □

Remark 3.2. As mentioned before, the assumption that all the time delays are the same as τ is not necessary. By Leibniz–Newton formula (13), the equation (9) with different delays τ_j can be expressed as

$$\begin{aligned}\dot{\eta}(t) = (I\otimes\bar{A} - H_p\otimes(\bar{L}C))\eta(t) + \sum_{j=1}^{M}M_j\int_{-\tau_j}^{0}\eta(t+s)\,\mathrm{d}s\\ -\sum_{j=1}^{M}N_j\int_{-\tau_j}^{0}\eta(t+s-\tau_j)\,\mathrm{d}s,\end{aligned} \tag{14}$$

where M is the number of difference time-delays, M_j and N_j are known constant matrix with appropriated dimension. For small enough r, all results of this paper can also be obtained by using (14).

Remark 3.3. According to condition (11), we can obtain feedback parameters l_i, $i = 1, 2, \ldots, m$ by solving the following LMI condition. If there exist positive definite matrix P and matrix K such that

$$\begin{cases} P\bar{A} + \bar{A}^T P - \bar{\lambda}(KC + C^T K^T) < 0 \\ P\bar{A} + \bar{A}^T P - \underline{\lambda}(KC + C^T K^T) < 0, \end{cases}$$

then $\bar{L} = P^{-1}K$.

Moreover, if $(C, \bar{A})$ is detectable, we provide the following result to obtain feedback parameters l_i $(i = 1, 2, \ldots, m)$ from a Riccati equation, which can be generalized to a class of directed interaction topologies.

Theorem 3.4. Assume $(C, \bar{A})$ is detectable and the switching interaction graphs $\bar{\mathcal{G}}_{\sigma(t)}$ are all connected with a given dwell time. If the time-delay bound is sufficiently small, then there exist constants $l_i, i = 1, 2, \ldots, m$ such that controller (7) with "observer" (8) together yields

$$\lim_{t\to\infty} \eta(t) = 0. \tag{15}$$

Namely, the follower-agents can track the active leader.

Proof. Since $(C, \bar{A})$ is detectable and Q is positive definite, it is well known that there is a unique positive definite matrix $\bar{P}$ to satisfy the Riccati equation

$$\bar{A}\bar{P} + \bar{P}\bar{A}^T - \bar{P}C^T C\bar{P} + Q = 0. \tag{16}$$

Furthermore, $\bar{A}^T - C^T C\bar{P}$ is stable (see [8]).

For any positive constant $k > \frac{1}{2}$, taking

$$\bar{L}^T = \frac{k}{\underline{\lambda}} C\bar{P}, \tag{17}$$

we have

$$\begin{aligned} &\bar{P}(\bar{A} - \lambda(H_p)\bar{L}C)^T + (\bar{A} - \lambda(H_p)\bar{L}C)\bar{P} \\ &= -Q + \bar{P}C^T C\bar{P} - 2\lambda(H_p)\frac{k}{\underline{\lambda}}\bar{P}C^T C\bar{P} \le -Q, \end{aligned} \tag{18}$$

where $\lambda(H_p)$ denotes any eigenvalue of matrix H_p. Then we know that matrices $\bar{A} - \lambda(H_p)\bar{L}C$, $p \in \mathcal{P}$ are stable, that is, all eigenvalues of $\bar{A} - \lambda(H_p)\bar{L}C$ are negative, which implies the following inequality

$$P(\bar{A} - \lambda(H_p)\bar{L}C) + (\bar{A} - \lambda(H_p)\bar{L}C)^T P \le -PQP := -\bar{Q} \tag{19}$$

where both $P = \bar{P}^{-1}$ and $\bar{Q}$ are positive definite. From the inequality (19), it is easy to obtain that

$$(I \otimes P)(I \otimes \bar{A} - H_p \otimes (\bar{L}C)) + (I \otimes \bar{A} - H_p \otimes (\bar{L}C))^T (I \otimes P) \le -I \otimes \bar{Q} < 0.$$

The rest proof is omitted, because we can prove by similar line as to prove Theorem 3.1. $\square$

Remark 3.5. If there is no time-delay, the relative error of agent i becomes

$$\begin{aligned} z_i(t) = & \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(x_i(t) - x_j(t)) \\ & + b_i(t)(x_i(t) - x_0(t)), \end{aligned} \tag{20}$$

and our control scheme becomes

$$u_i = \bar{a}_{11}x_i + \sum_{j=2}^{m} \bar{a}_{1j}v_i^{j-1} - \bar{l}_1 z_i, \tag{21}$$

with $\bar{a}_{ij}$ defined in (5) and z_i defined in (6); with the distributed estimation law

$$\dot{v}_i^{k-1} = \bar{a}_{k1}x_i + \sum_{j=2}^{m} \bar{a}_{kj}v_k^{j-1} - \bar{l}_k z_i, \; k = 2, \ldots, m, \tag{22}$$

which is in the same form as given in [8]. Therefore, our result can applied to non-time-delay case directly. The result of Theorem 3.4 is same as the result obtained by [8] in this case. In the next remark, we will point out that Theorem 3.4 can generalize to a class of directed interaction topologies.

Remark 3.6. The condition that $(C, \bar{A})$ is detectable is often used, though it is not necessary. Moreover, we can consider the graph $\bar{\mathcal{G}}_p$ is directed, which means matrix H_p associated with $\bar{\mathcal{G}}_p$ is not symmetric matrix. Assume that all matrices $H_p + H_p^T$ are positive definite. Therefore, define

$$\lambda := \min_{p\in\mathcal{P}}\{\lambda_{\min}(H_p + H_p^T)\}. \tag{23}$$

For any positive constant $k > 1$, taking

$$\bar{L}^T = \frac{k}{\lambda}C\bar{P}, \tag{24}$$

where P is a unique positive definite solution of Riccati equation (16), we have

$$\begin{aligned} & \bar{P}(\bar{A} - \frac{1}{2}\lambda(H_p + H_p^T)\bar{L}C)^T + (\bar{A} - \frac{1}{2}\lambda(H_p + H_p^T)\bar{L}C)\bar{P} \\ & = -Q + \bar{P}C^TC\bar{P} - \lambda(H_p + H_p^T)\frac{k}{\lambda}\bar{P}C^TC\bar{P} \leq -Q, \end{aligned} \tag{25}$$

which implies the following inequality holds by noting that $\bar{L}C\bar{P}$ is symmetric matrix

$$\begin{aligned} & (I\otimes\bar{P})(I\otimes\bar{A} - H_p\otimes(\bar{L}C))^T + (I\otimes\bar{A} - H_p\otimes(\bar{L}C))(I\otimes P) \\ & = (I\otimes\bar{P})[I\otimes\bar{A} - \frac{1}{2}(H_p + H_p^T)\otimes(\bar{L}C)]^T \\ & + [I\otimes\bar{A} - \frac{1}{2}(H_p + H_p^T)\otimes(\bar{L}C)](I\otimes P) \leq -I\otimes\bar{Q} < 0. \end{aligned} \tag{26}$$

Similarly, we can obtain

$$(I \otimes P)(I \otimes \bar{A} - H_p \otimes (\bar{L}C)) + (I \otimes \bar{A} - H_p \otimes (\bar{L}C))^T (I \otimes P) \leq -I \otimes \bar{Q} < 0.$$

From the above analysis, we know that the results of Theorem 3.4 and the non-time-delay case given in [8] is also right in some directed graph cases.

Consider the special case that the graph $\mathcal{G}_p$ associated with all followers is balanced. A weighted graph $\mathcal{G} = (\mathcal{V}, \epsilon, A)$ is said to be balanced if

$$\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji}, \quad i = 1, 2, \ldots, n.$$

Any undirected weighted graph is balanced. Furthermore, a weighted graph is balanced if and only if $\mathbf{1}^T L = 0$ (see [15]). Although the matrix H_p associated with $\bar{\mathcal{G}}_p$ is not symmetric and positive definite, we have: The matrix $H_p^T + H_p$ is positive definite if and only if node 0 is globally reachable in $\bar{\mathcal{G}}_p$(see [12]). Therefore, that all interconnection graph $\mathcal{G}_p$ associated to all followers are balanced and node 0 is globally reachable in $\bar{\mathcal{G}}_p$ can also guarantee solvableness of the tracking problem.

Remark 3.7. Obviously, the consensus problem of multi-agent system is equivalent to stability problem of error system (9). Although the closed-loop multi-agent system and the error system both have mn-order, the consensus condition given in Theorem 3.1 contains two m-order common Lyapunov matrix inequalities, which make the established reduced consensus condition more easier to be checked. Moreover, if $(C, \bar{A})$ is detectable, all control constants l_i can be designed only by solving a Reccati equation. Obviously, the solution $\bar{L}$ obtain by Theorem 3.4 also satisfies the consensus condition given in Theorem 3.1. Thus, the result established by Theorem 3.4 is more conservative than the result of Theorem 3.1.

Next, we study the case when $\delta_i \neq 0$ for some $i = 1, \ldots, n$. Similarly, we can obtain the error dynamics of closed-loop system as follows

$$\dot{\eta}(t) = I_n \otimes \bar{A}\eta(t) - H_p \otimes (\bar{L}C)\eta(t - \tau) + \delta, \tag{27}$$

where $\delta = (\delta^1, \delta^2, \ldots, \delta^n)^T \in R^{mn}$, and $\delta^i = (\delta_i, 0, \ldots, 0) \in R^m$. Now we give the convergent analysis of system (27) in the following theorem.

Theorem 3.8. Suppose that the switching interaction graphs $\bar{\mathcal{G}}_{\sigma(t)}$ are all connected with a given dwell time and the time-delay bound is sufficiently small. Then there are constants $l_i, i = 1, 2, \ldots, n$ such that there is a constant $c_\delta > 0$ with $\lim_{\delta \to 0} c_\delta = 0$ to make

$$\lim_{t \to \infty} [x_i(t) - x_0(t)] \leq c_\delta, \tag{28}$$

hold for the multi-agent system (4)-(5) with local feedback laws (7) and observers (8). Moreover, the considered system (27) is globally uniformly ISS with δ_i $(i = 1, \ldots, n)$ as its inputs.

Proof. Still take $V(\eta) = \eta^T(t)(I \otimes P)\eta(t)$ with P defined in the proof of Theorem 3.4. Take an interval $[t_i, t_{i+1})$ into consideration, during which the graph associated with H_p for some $p \in \mathcal{P}$ is connected and unchanged.

If $V(\eta(t+s)) \leq qV(\eta(t))$, $-2r \leq s \leq 0$, then similar to the discussion in Theorem 3.1, the derivative of $V(t)$ is given by

$$\begin{aligned}
\dot{V}(t)|_{(27)} \;&\leq\; -\eta^T(t)(I \otimes \bar{Q})\eta(t) + \tau\eta^T(t)[H_p^2 \otimes P\bar{L}C\bar{A}P^{-1}(P\bar{L}C\bar{A})^T]\eta(t) \\
&+\; \tau\eta^T(t)[H_p^4 \otimes P\bar{L}C\bar{L}CP^{-1}(P\bar{L}C\bar{L}C)^T]\eta(t) \\
&+\; \eta^T(I \otimes P)\delta + 2rq\lambda_{\max}(P))\eta^T(t)\eta(t) \\
&\leq\; -\lambda_{\min}(\bar{Q}))\eta^T(t)\eta(t) + r\varrho\eta^T(t)\eta(t) + 2rq\lambda_{\max}(P))\eta^T\eta(t) \\
&+\; \frac{2\lambda_{\max}^2(P)}{\lambda_{\min}(\bar{Q})}\delta^T(t)\delta(t) + \frac{\lambda_{\min}(\bar{Q})}{2}\eta^T(t)\eta(t) \\
&\leq\; -(\frac{\lambda_{\min}(\bar{Q})}{2} - r\varrho - 2rq\lambda_{\max}(P))\|\eta(t)\|^2 + \frac{2\lambda_{\max}^2(P)}{\lambda_{\min}(\bar{Q})}\delta^T(t)\delta(t)
\end{aligned}$$

with ϱ defined in Theorem 3.1.

If

$$r < \frac{\lambda_{\min}(\bar{Q})}{2(\varrho + 2q\lambda_{\max}(P))},$$

we have

$$\left(\frac{\lambda_{\min}(\bar{Q})}{2} - r\varrho - 2rq\lambda_{\max}(P)\right)\|\eta\|^2 = \alpha(\|\eta\|),$$

$$\lambda_{\max}^2(P)\|\delta\|^2\frac{2}{\lambda_{\min}(\bar{Q})} = \gamma_\delta(\|\delta\|),$$

which means that

$$\dot{V}(\eta) \leq -\alpha(\|\eta\|) + \gamma_\delta(\|\delta\|).$$

According to Lemma 2.4, system (27) is uniformly globally ISS, which implies (28). □

4. SIMULATION RESULTS

In this section, to illustrate our theoretical results derived in the above section, we will provide a simple simulation example. The multi-agent system include one leader and six followers. The dynamic of leader is considered as: $\ddot{x} = \alpha$, which means that the active leader move with an known acceleration α. Set $\omega_1 = x$, $\omega_2 = \dot{x}$ and $\omega_3 = \ddot{x}$. The dynamic of leader is rewritten as $\dot{\omega} = \bar{A}\omega$, where $\bar{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Interconnection topology is arbitrarily switched with switching period 1 among four graphs $\bar{\mathcal{G}}_i (i = 1, 2, 3, 4)$. The Laplacian matrices $L_i (i = 1, 2, 3, 4)$ for the four subgraphs $\mathcal{G}_i (i = 1, 2, 3, 4)$ are

$$L_1 = \begin{bmatrix} 3.5 & -1.5 & 0 & 0 & 0 & -2 \\ -1 & 5.5 & -2.5 & 0 & -2 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -2 & 5 & -1 & -2 \\ 0 & -2 & 0 & -1 & 5 & -2 \\ -1 & 0 & 0 & -1 & -2 & 4 \end{bmatrix}, L_2 = \begin{bmatrix} 5 & -1 & -2 & 0 & -2 & 0 \\ -1.5 & 4.5 & -1 & 0 & 0 & -2 \\ -2 & -1 & 4 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 & 0 & -2 \\ -1 & 0 & -1 & 0 & 2 & 0 \\ 0 & -2 & 0 & -2 & 0 & 4 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 3 & 0 & 0 & -2 & 0 & -1 \\ 0 & 2 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 \\ -2 & -1 & 0 & 5 & -2 & 0 \\ 0 & -1 & 0 & -2 & 3 & 0 \\ -2 & 0 & -2 & 0 & 0 & 4 \end{bmatrix}, L_4 = \begin{bmatrix} 2 & 0 & 0 & -1 & -1 & 0 \\ 0 & 4 & -2 & 0 & -2 & 0 \\ 0 & -2 & 5 & -2 & 0 & -1 \\ -2 & 0 & -2 & 4 & 0 & 0 \\ -1 & -2 & 0 & 0 & 5 & -2 \\ 0 & 0 & -1 & 0 & -2 & 3 \end{bmatrix}.$$

and the diagonal matrices for the interconnection relationship between the leader and the followers are

$$B_1 = diag(1, 0, 0, 1, 0, 0), \quad B_2 = diag(0, 1, 0, 1, 0, 0),$$

$$B_3 = diag(1, 0, 0, 0, 1, 1), \quad B_4 = diag(0, 0, 1, 1, 0, 0).$$

The initial values of the all agents are randomly produced. The time-delay is taken as 0.07. The control constants are designed using the method proposed by Theorem 3.4. Figure 1 shows that all six follower-agents can track the accelerated motion leader in time-delay case by only using the neighbors position information in the distributed control laws. The Laplacian matrices given in here are not symmetric, but the multi-agent system can also achieve consensus, which verifies our above analysis.

5. CONCLUSIONS

In this paper, we considered an active leader-following problem, where the multi-agent network has communication delays and switching interconnection topologies. The dynamics of the active leader is given in a general form of linear systems. A neighbor-based estimator is designed for each agent to obtain the unmeasurable state variables of the dynamic leader, and then a distributed feedback control rules were designed to track the active leader. The tracking convergence was proved in the noise-free case by constructing a common Lyapunov function, while the input-to-state stability (ISS) was obtained for the time-delay system in the case with disturbances. In this paper, we assume that the dimension of following agent's state is 1 for notational simplicity and would not be lost generality. All result of this paper can be also true for high dimension, and we can revise the expressions via Kronecker product. Due to the conservativeness of the common Lyapunov function method, we also should probe less conservative method in our future work. More generalized and interesting cases on active leader models are still under investigation.

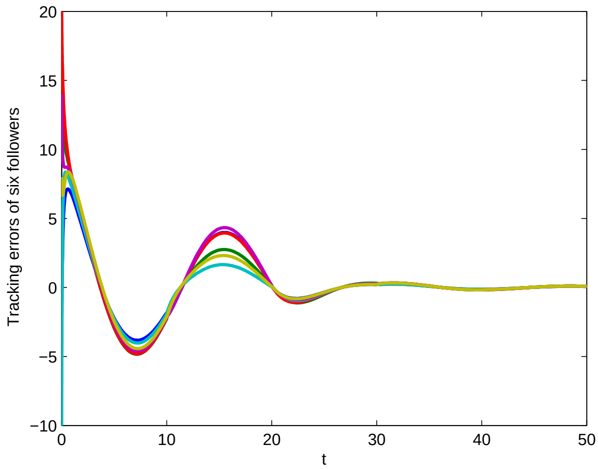

Fig. 1. Tracking errors of six followers with time-delay network.

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