KYBERNETIKA — VOLUME 47 (2011), NUMBER 4, PAGES 560–571

STOCHASTIC BOTTLENECK TRANSPORTATION PROBLEM WITH FLEXIBLE SUPPLY AND DEMAND QUANTITY

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We consider the following bottleneck transportation problem with both random and fuzzy factors. There exist m supply points with flexible supply quantity and n demand points with flexible demand quantity. For each supply-demand point pair, the transportation time is an independent positive random variable according to a normal distribution. Satisfaction degrees about the supply and demand quantity are attached to each supply and each demand point, respectively. They are denoted by membership functions of corresponding fuzzy sets. Under the above setting, we seek a transportation pattern minimizing the transportation time target subject to a chance constraint and maximizing the minimal satisfaction degree among all supply and demand points. Since usually there exists no transportation pattern optimizing two objectives simultaneously, we propose an algorithm to find some non-dominated transportation patterns after defining non-domination. We then give the validity and time complexity of the algorithm. Finally, a numerical example is presented to demonstrate how our algorithm runs.

Keywords: bottleneck transportation, random transportation time, flexible supply and demand quantity, non-dominated transportation pattern

Classification: 90C35, 90C15, 90C70, 68Q25

1. INTRODUCTION

The purpose of the classical transportation problem is to determine the optimal transportation pattern of a certain good from suppliers to demand customers so that the total transportation cost becomes minimum. It is also called the cost minimizing transportation problem, which has been extensively studied in the literature and several algorithms [2, 4, 5, 9, 14, 15] are available to solve it. Similarly, efficient algorithm have been proposed by Hammer [8], Szwarc [16] and Garfinkel and Rao [6] for solving the time minimizing (bottleneck) transportation problem, assuming that all the transportation are allowed to commence simultaneously. This paper extends the bottleneck transportation problem by considering randomness of transportation time and flexibility of supply and demand quantity. Randomness means that transportation time may change according to many factors. The flexibility reflects on the actual situation that total quantity from suppliers is less than that to demand customers. So two criteria are taken into account in this paper. One is to minimize the transportation time target subject to a chance constraint. The other is to maximize the minimal satisfaction degree with respect to the flexibility of supply and demand quantity. However, there usually exists no transportation pattern optimizing two objectives simultaneously. So we seek some non-dominated transportation patterns after defining non-domination. Our model is an extension of our previous models [3, 11, 12, 17, 18] and a similar but different model due to Lin and Tsai [13]. As for another fuzzy version, we have considered competitive transportation problem [10] also in order to cope with an actual situation. Geetha and Nair [7] have considered a stochastic bottleneck transportation problem and proposed an efficient solution, which is very useful for our subproblem.

The rest of this paper is organized as follows. Our problem is first formulated in Section 2, and then in Section 3 we present an algorithm to find some non-dominated transportation patterns, demonstrate its validity and study its time complexity. Section 4 shows how our algorithm runs using a numerical example. Finally, Section 5 concludes this paper and discusses further research problems.

2. PROBLEM FORMULATION

In this paper, we focus on the following bi-criteria stochastic bottleneck transportation problem with flexible supply and demand quantity.

(1) There exist a set of m supply points $S = \{S_1, S_2, \ldots, S_m\}$ and a set of n demand points $T = \{T_1, T_2, \ldots, T_n\}$.

(2) Edges set A is a set of routes connecting each supply point S_i with each demand point T_j denoted by (i, j), $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

(3) For each route (i, j) from S_i to T_j, the transportation time t_{ij} is an independent positive random variable according to the normal distribution $N(m_{ij}, \sigma_{ij}^2)$. We denote the transportation quantity using each route (i, j) by f_{ij} and assume that these f_{ij} are nonnegative integer decision variables. The following chance constraint is attached:

$$\Pr\{t_{ij} \leq F\} \geq \alpha \quad \text{for every } (i, j) | f_{ij} > 0$$

where $\alpha > 1/2$ and F is also a decision variable to be minimized.

(4) Let s_i, t_j be the total flow value sent from S_i and to T_j, respectively. Different to the usual transportation problem, upper limit of supply quantity for each supply point and lower limit of demand quantity for each demand point are flexible. They are expressed by the following two kinds of membership functions $\mu_{S_i}(s_i)$ and $\mu_{T_j}(t_j)$ for fuzzy supply quantity from S_i and fuzzy demand quantity to T_j, respectively, which characterizing the satisfaction degrees of supply and demand points:

$$\mu_{S_i}(s_i) = \begin{cases} 1 & (s_i \leq a_i) \\ \frac{b_i - s_i}{b_i - a_i} & (a_i < s_i < b_i) \\ 0 & (s_i \geq b_i) \end{cases},$$

$$\mu_{T_j}(t_j) = \begin{cases} 0 & (t_j \le d_j) \\ \frac{t_j - d_j}{e_j - d_j} & (d_j < t_j < e_j) \\ 1 & (t_j \ge e_j) \end{cases},$$

where $a_i < b_i,\ d_j < e_j$ and $a_i,\ b_i,\ d_j,\ e_j$ are positive integers.

We assume that $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} e_j$, otherwise our problem becomes trivial, since all values of membership functions can be set to 1; besides, we assume that $\sum_{i=1}^{m} b_i > \sum_{j=1}^{n} d_j$, otherwise the value of the second objective of our problem is 0. Note that $s_i = \sum_{j=1}^{n} f_{ij}$, $i = 1, 2, \ldots, m$, $t_j = \sum_{i=1}^{m} f_{ij}$, $j = 1, 2, \ldots, n$.

(5) We consider two criteria: one is to minimize F, the other is to maximize the minimal satisfaction degree with respect to the flexibility of supply and demand quantity.

From the above setting, our transportation problem TP can be formulated as follows:

$$\begin{aligned} \text{TP}: \quad & \text{Minimize} && F \\ & \text{Maximize} && \min_{i,j}\{\mu_{S_i}(s_i), \mu_{T_j}(t_j)\} \\ & \text{Subject to} && \Pr\{t_{ij} \le F\} \ge \alpha \ \text{ for every } (i,j)|f_{ij} > 0 \\ &&& \sum_{j=1}^{n} f_{ij} = s_i,\ i = 1, 2, \ldots, m \\ &&& \sum_{i=1}^{m} f_{ij} = t_j,\ j = 1, 2, \ldots, n \\ &&& f_{ij}: \ \text{nonnegative integer},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \end{aligned}$$

Since for every $(i,j)|f_{ij} > 0$,

$$\Pr\{t_{ij} \le F\} \ge \alpha \Leftrightarrow \Pr\left\{\frac{t_{ij} - m_{ij}}{\sigma_{ij}} \le \frac{F - m_{ij}}{\sigma_{ij}}\right\} \ge \alpha$$

and

$$\frac{t_{ij} - m_{ij}}{\sigma_{ij}}$$

is a random variable according to the standard normal distribution $N(0,1)$, the chance constraint is equivalent to the following deterministic constraint:

$$\frac{F - m_{ij}}{\sigma_{ij}} \ge K_\alpha \Leftrightarrow F \ge m_{ij} + K_\alpha \sigma_{ij},$$

where K_α is the α percentile point of the cumulative distribution function of $N(0,1)$ and note that $K_\alpha > 0$ since we assume $\alpha > 1/2$.

Since F should be minimized, then problem TP is transformed into the following equivalent problem P.

$$
\begin{array}{lll}
\mathrm{P}: & \text{Minimize} & \max\limits_{i,j}\{m_{ij}+K_\alpha\sigma_{ij}|f_{ij}>0\} \\
& \text{Maximize} & \min\limits_{i,j}\{\mu_{S_i}(s_i),\mu_{T_j}(t_j)\} \\
& \text{Subject to} & \sum\limits_{j=1}^{n} f_{ij}=s_i,\ i=1,2,\ldots,m \\
& & \sum\limits_{i=1}^{m} f_{ij}=t_j,\ j=1,2,\ldots,n \\
& & f_{ij}:\ \text{nonnegative integer},\ i=1,2,\ldots,m,\ j=1,2,\ldots,n.
\end{array}
$$

Next, we define the bi-objective vector $\mathrm{v}(\mathbf{f})=(v(\mathbf{f})_1,v(\mathbf{f})_2)$ of a transportation pattern $\mathbf{f}=(f_{ij})$ as

$$v(\mathbf{f})_1=\max_{i,j}\{m_{ij}+K_\alpha\sigma_{ij}|f_{ij}>0\},\ v(\mathbf{f})_2=\min_{i,j}\{\mu_{S_i}(s_i),\mu_{T_j}(t_j)\}.$$

Generally, a transportation pattern optimizing two objectives simultaneously does not exist. So we seek some non-dominated transportation patterns, the definition of which is given as follows.

Definition 2.1. Let $\mathbf{f}^a$, $\mathbf{f}^b$ be two transportation patterns, we say that $\mathbf{f}^a$ **dominates** $\mathbf{f}^b$, if $v(\mathbf{f}^a)_1\le v(\mathbf{f}^b)_1$, $v(\mathbf{f}^a)_2\ge v(\mathbf{f}^b)_2$ and at least one inequality holds as a strict inequality. If there exists no transportation pattern dominating $\mathbf{f}$, $\mathbf{f}$ is called a *non-dominated* transportation pattern.

3. SOLUTION PROCEDURE

Note that s_i, t_j are integers, then we can denote ranges of $\mu_{S_i}(s_i)$ and $\mu_{T_j}(t_j)$ with $\{\mu_{S_i,1},\mu_{S_i,2},\ldots,\mu_{S_i,k_i}\}$ and $\{\mu_{T_j,1},\mu_{T_j,2},\ldots,\mu_{T_j,l_j}\}$, respectively, $i=1,2,\ldots,m$, $j=1,2,\ldots,n$. Now sorting them, let the result be $0<\mu^1<\mu^2<\ldots<\mu^g\le 1$, where g is the number of different values of them. In order to solve our bi-criteria problem P, first we solve single criterion subproblems P_u with fixed parameter u which will be given soon.

For fixed $u\in\{1,2,\ldots,g\}$, we only consider $\mu_{S_i}(s_i)\ge\mu^u$, $\mu_{T_j}(t_j)\ge\mu^u$, that is, $s_i\le b_i-\mu^u(b_i-a_i)$, $t_j\ge d_j+\mu^u(e_j-d_j)$, $i=1,2,\ldots,m$, $j=1,2,\ldots,n$.

As it is easily seen, the total supply quantity should be not less than the total demand quantity. Otherwise, the problem becomes infeasible. So we assume

$$\sum_{i=1}^{m}\{b_i-\mu^u(b_i-a_i)\}\ge\sum_{j=1}^{n}\{d_j+\mu^u(e_j-d_j)\},$$

that is,

$$\mu^u \leq \frac{\sum_{i=1}^m b_i - \sum_{j=1}^n d_j}{\sum_{i=1}^m (b_i - a_i) + \sum_{j=1}^n (e_j - d_j)} \triangleq \bar{\mu}.$$

In order to find non-dominated transportation patterns, we consider the following subproblem P_u, $u = 1, 2, \ldots, h$, where $h = \max\{u \in \{1, 2, \ldots, \beta\} | \beta \in \{1, 2, \ldots, g\}, \mu^\beta \leq \bar{\mu} < \mu^{\beta+1}, \sum_{i=1}^m \lfloor b_i - \mu^u(b_i - a_i) \rfloor \geq \sum_{j=1}^n \lceil d_j + \mu^u(e_j - d_j) \rceil\}$, $\lfloor \cdot \rfloor$ is the greatest integer not greater than $\cdot$ and $\lceil \cdot \rceil$ is the smallest integer not smaller than $\cdot$.

$$\begin{aligned}
\mathrm{P}_u: \quad & \text{Minimize} \quad && \max_{i,j}\{m_{ij} + K_\alpha \sigma_{ij} | f_{ij} > 0\} \\
& \text{Subject to} && \sum_{j=1}^n f_{ij} \leq s_i(u), \ i = 1, 2, \ldots, m \\
& && \sum_{i=1}^m f_{ij} \geq t_j(u), \ j = 1, 2, \ldots, n \\
& && f_{ij}: \ \text{nonnegative integer}, \ i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n
\end{aligned}$$

where $s_i(u) = \lfloor b_i - \mu^u(b_i - a_i) \rfloor$, $t_j(u) = \lceil d_j + \mu^u(e_j - d_j) \rceil$.

For fixed $u \in \{1, 2, \ldots, h\}$, a procedure to solve problem P_u is based on a binary search over the values arranged in ascending order of $m_{ij} + K_\alpha \sigma_{ij}$ to find the smallest of these values for which a feasible transportation pattern exists; this value is the optimal value of problem P_u. Next we present the solution procedure in detail.

Compute $m_{ij} + K_\alpha \sigma_{ij}$, $i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n$, and arrange these values in ascending order. Let the result be

$$c^1 < c^2 < \ldots < c^l$$

where l is the number of different values of them.

For $k = 1, 2, \ldots, l$, set

$$c_{ij}^k = \begin{cases} 0 & \text{if } m_{ij} + K_\alpha \sigma_{ij} \leq c^k \\ \infty & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.$$

Denote the matrices $\mathbf{C} = (m_{ij} + K_\alpha \sigma_{ij})_{m \times n}$ and $\mathbf{C}^k = (c_{ij}^k)_{m \times n}$.

For $u = 1, 2, \ldots, h, \ k = 1, 2, \ldots, l$, denote the cost minimizing transportation problem with the above defined cost values as follows:

$$
\begin{aligned}
\mathrm{P}_u^k: \quad & \text{Minimize} && \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k f_{ij} \\
& \text{Subject to} && \sum_{j=1}^{n} f_{ij} \le s_i(u),\ i = 1, 2, \ldots, m \\
& && \sum_{i=1}^{m} f_{ij} \ge t_j(u),\ j = 1, 2, \ldots, n \\
& && f_{ij}:\ \text{nonnegative integer},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n
\end{aligned}
$$

Now we give the following algorithm to solve problem P_1.

Algorithm 1

Step 1 Set $L = 1$ and check whether P_1^L is feasible or not. If feasible, go to Step 4 after setting $q_1 = L$. Otherwise, set $U = l$ and check whether P_1^U is feasible or not. If feasible, go to Step 2. Otherwise, terminate as an infeasibility.

Step 2 When $U - L > 1$, set $K = \lfloor (L + U)/2 \rfloor$ and check whether P_1^K is feasible or not. If feasible, set $U = K$ and repeat Step 2. Otherwise, set $L = K$ and repeat Step 2. When $U - L = 1$, go to Step 3.

Step 3 If P_1^L is feasible, set $q_1 = L$. Otherwise, set $q_1 = U$.

Step 4 A feasible transportation pattern $\mathbf{f}(1)$ for $\mathrm{P}_1^{q_1}$ exists, which is optimal for P_1 and the corresponding c^{q_1} is the optimal value of P_1.

Remark 3.1. $\mathrm{P}_1^{q_1}$ is feasible, it means that there exists a feasible transportation pattern, which is obviously optimal for $\mathrm{P}_1^{q_1}$, and its optimal value is 0.

Next, we show the validity of Algorithm 1.

Proposition 3.2. Algorithm 1 is valid.

P r o o f. Algorithm 1 is based on a binary search to find the smallest $q_1 \in \{1, 2, \ldots, l\}$ for which a feasible transportation pattern $\mathbf{f}(1)$ of $\mathrm{P}_1^{q_1}$ exists and the corresponding c^{q_1} is the optimal value of P_1, it is valid. □

The time complexity of Algorithm 1 is given as follows.

Theorem 3.3. The time complexity of Algorithm 1 is

$$\mathrm{O}((m+n)^3 \log(mn) \log(m+n)).$$

P r o o f. The time complexity of Algorithm 1 follows from the fact that the binary search over $l = \mathrm{O}(mn)$ values has time complexity $\mathrm{O}(\log l)$ and every step in the binary search results in solving a transportation problem whose solution method is of time complexity $\mathrm{O}((m+n)^3\log(m+n))$ [1]. Hence, in the worst case analysis, Algorithm 1 is of time complexity $\mathrm{O}((m+n)^3\log(mn)\log(m+n))$. $\square$

For P_u, $u = 2, 3, \ldots, h$, the algorithm is very similar to P_1; the only difference is we first set $L = q_{u-1}$, where q_{u-1} is the superscript of the optimal value $c^{q_{u-1}}$ of P_{u-1}.

Then, we give the following algorithm for P, where NDT and NDV denote the set of non-dominated transportation patterns and corresponding bi-objective vectors, respectively.

Algorithm 2

Step 1 Set $u = 1$, $NDT = \emptyset$, $NDV = \emptyset$.

Step 2 Solve P_u, we obtain an optimal transportation pattern $\mathbf{f}(u) = (f_{ij}(u))$ and the optimal value is c^{q_u}. If

$$\min_{i,j}\Big\{\mu_{S_i}\Big(\sum_{j=1}^{n} f_{ij}(u)\Big), \mu_{T_j}\Big(\sum_{i=1}^{m} f_{ij}(u)\Big)\Big\} \neq \mu^u,$$

go to Step 3. Otherwise, check whether $\mathbf{f}(u)$ dominates the transportation patterns in NDT or not. If so, update NDT and NDV by deleting the dominated transportation patterns and the corresponding bi-objective vectors, respectively, and then adding $\{\mathbf{f}(u)\}$ and $\{(c^{q_u}, \mu^u)\}$, respectively. Otherwise, update NDT and NDV directly by adding $\{\mathbf{f}(u)\}$ and $\{(c^{q_u}, \mu^u)\}$, respectively. Go to Step 3.

Step 3 If $u = h$, terminate. Otherwise, set $u = u + 1$ and return to Step 2.

The validity of Algorithm 2 may be proved as follows.

Proposition 3.4. Algorithm 2 is valid.

Proof. Algorithm 2 checks all possibilities (i. e., $\mu^1, \mu^2, \ldots, \mu^h$) of the second components of bi-objective vectors, that is, we solve corresponding problems P_1, P_2, …, P_h, which are subproblems of problem P. Then obtain an optimal transportation pattern $\mathbf{f}(u)$ and the optimal value c^{q_u} for each P_u, $u = 1, 2, \ldots, h$. From the definition of non-domination, Algorithm 2 is valid. $\square$

The time complexity of our solution procedure for problem P is shown in the following theorem.

Theorem 3.5. The time complexity of our solution procedure for problem P is

$$\mathrm{O}(M \cdot \max\{\log M, (m+n)^3\log(mn)\log(m+n)\}),$$

where $M = \sum_{i=1}^{m}(b_i - a_i) + \sum_{j=1}^{n}(e_j - d_j)$.

Tab. 1 The values of $a_i,\ b_i,\ d_j,\ e_j$ and the distribution of t_{ij}.

$i\backslash j$	1	2	3	a_i	b_i
1	$N(3, 0.5^2)$	$N(7, 0.4^2)$	$N(4, 1.2^2)$	10	14
2	$N(6, 0.8^2)$	$N(5, 0.3^2)$	$N(1, 0.7^2)$	12	18
3	$N(7, 0.3^2)$	$N(4, 0.6^2)$	$N(8, 1.0^2)$	5	8
d_j	12	6	10		
e_j	15	8	13		

P r o o f. We consider integer problem, then $h \leq M$ holds, so sorting $\mu^1, \mu^2, \ldots, \mu^h$ takes at most $\mathrm{O}(M \log M)$ operations. Solving each P_u, $u = 1, 2, \ldots, h$ takes at most $\mathrm{O}((m{+}n)^3 \log(mn) \log(m{+}n))$ operations. In Step 2, check whether $\mathbf{f}(u)$ dominates the transportation patterns in NDT or not needs at most $\mathrm{O}(h)$ operations. As Step 2 to Step 3 is repeated at most h times. Therefore, the total time complexity is $\mathrm{O}(\max\{M \log M, M(m+n)^3 \log(mn) \log(m+n)\})$. $\square$

4. NUMERICAL EXAMPLE

Consider problem TP with $\alpha = 0.9987$, $t_{ij} \sim N(m_{ij}, \sigma_{ij}^2)$ and the values of a_i, b_i, d_j, e_j are given in Table 1.

Our problem TP reduces to problem P.

$$
\begin{array}{lll}
\mathrm{P}: & \text{Minimize} & \max\limits_{i,j}\{m_{ij} + 3.0\sigma_{ij} | f_{ij} > 0\} \\
& \text{Maximize} & \min\limits_{i,j}\{\mu_{S_i}(s_i), \mu_{T_j}(t_j)\} \\
& \text{Subject to} & \displaystyle\sum_{j=1}^{3} f_{ij} = s_i,\ i = 1, 2, 3 \\
& & \displaystyle\sum_{i=1}^{3} f_{ij} = t_j,\ j = 1, 2, 3 \\
& & f_{ij} :\ \text{nonnegative integer},\ i, j = 1, 2, 3.
\end{array}
$$

Since $s_i,\ t_j$ are integers, sorting the values of $\mu_{S_i}(s_i)$ and $\mu_{T_j}(t_j)$, we obtain

$$0 < \mu^1 = 1/6 < \mu^2 = 1/4 < \mu^3 = 1/3 < \mu^4 = 1/2$$

$$< \mu^5 = 2/3 < \mu^6 = 3/4 < \mu^7 = 5/6 < \mu^8 = 1.$$

Note that, $\bar{\mu} = 12/21$, so $\mu^4 < \bar{\mu} < \mu^5$ and $\beta = 4$. Further, we get $h = 4$.

We only need to solve the following subproblem P_u, $u = 1, 2, 3, 4$.

$$\begin{aligned} \mathrm{P}_u: \quad \text{Minimize} \quad & \max_{i,j}\{m_{ij} + 3.0\sigma_{ij} | f_{ij} > 0\} \\ \text{Subject to} \quad & \sum_{j=1}^{3} f_{ij} \leq s_i(u),\ i = 1, 2, 3 \\ & \sum_{i=1}^{3} f_{ij} \geq t_j(u),\ j = 1, 2, 3 \\ & f_{ij}:\ \text{nonnegative integer},\ i, j = 1, 2, 3 \end{aligned}$$

Compute $m_{ij} + 3.0\sigma_{ij}$, $i, j = 1, 2, 3$, we obtain

$$\mathbf{C} = \begin{pmatrix} 4.5 & 8.2 & 7.6 \\ 8.4 & 5.9 & 3.1 \\ 7.9 & 5.8 & 11.0 \end{pmatrix}.$$

Arrange these values in ascending order, that is,

$$c^1 = 3.1 < c^2 = 4.5 < c^3 = 5.8 < c^4 = 5.9 < c^5 = 7.6$$
$$< c^6 = 7.9 < c^7 = 8.2 < c^8 = 8.4 < c^9 = 11.0.$$

For $k = 1, 2, \ldots, 9$, set

$$c_{ij}^k = \begin{cases} 0 & \text{if } m_{ij} + 3.0\sigma_{ij} \leq c^k \\ \infty & \text{otherwise} \end{cases}, \quad i, j = 1, 2, 3.$$

For $u = 1, 2, 3, 4$, $k = 1, 2, \ldots, 9$, problem P_u^k has the following form:

$$\begin{aligned} \mathrm{P}_u^k: \quad \text{Minimize} \quad & \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}^k f_{ij} \\ \text{Subject to} \quad & \sum_{j=1}^{3} f_{ij} \leq s_i(u),\ i = 1, 2, 3 \\ & \sum_{i=1}^{3} f_{ij} \geq t_j(u),\ j = 1, 2, 3 \\ & f_{ij}:\ \text{nonnegative integer},\ i, j = 1, 2, 3. \end{aligned}$$

Next we solve problem P_1, note that $s_1(1) = 13$, $s_2(1) = 17$, $s_3(1) = 7$, $t_1(1) = 13$, $t_2(1) = 7$, $t_3(1) = 11$.

Algorithm 1 for problem P_1 performs as follows:

Step 1. Set $L = 1$ and P_1^1 is infeasible. Set $U = 9$ and P_1^9 is feasible. Go to Step 2.
Step 2. $U - L = 8 \neq 1$. Set $K = 5$ and P_1^5 is feasible. Set $U = 5$ and repeat Step 2.

Step 2. $U - L = 4 \neq 1$. Set $K = 3$ and P_1^3 is feasible. Set $U = 3$ and repeat Step 2.
Step 2. $U - L = 2 \neq 1$. Set $K = 2$ and P_1^2 is infeasible. Set $L = 2$, repeat Step 2.
Step 2. $U - L = 1$, so go to Step 3.
Step 3. P_1^2 is infeasible, so set $q_1 = 3$.
Step 4. An feasible transportation pattern $\mathbf{f}(1)$ of problem P_1^3 is given as follows: $f_{11} = 13$, $f_{23} = 11$, $f_{32} = 7$. It is optimal for problem P_1 and the optimal value is 5.8.

For P_u, $u = 2, 3, 4$, it is similar to P_1. Solve them, we obtain an optimal transportation pattern and the optimal value for each problem, which are given as follows:

$$\mathrm{P}_2: \ f_{11} = 13, \ f_{23} = 11, \ f_{32} = 7; \text{ the optimal value is 5.8.}$$

$$\mathrm{P}_3: \ f_{11} = 12, \ f_{22} = 1, \ f_{23} = 11, \ f_{31} = 1, \ f_{32} = 6; \text{ the optimal value is 7.9.}$$

$$\mathrm{P}_4: \ f_{11} = 12, \ f_{22} = 3, \ f_{23} = 12, \ f_{31} = 2, \ f_{32} = 4; \text{ the optimal value is 7.9.}$$

Finally, we solve our problem P.

Algorithm 2 for problem P performs as follows:

Step 1. Set $u = 1$, $NDT = \emptyset$, $NDV = \emptyset$.
Step 2. Solve P_1. An optimal transportation pattern $\mathbf{f}(1)$ is given as follows: $f_{11} = 13$, $f_{23} = 11$, $f_{32} = 7$ and the optimal value is 5.8. Since

$$\min\{\mu_{S_1}(13), \mu_{S_2}(11), \mu_{S_3}(7), \mu_{T_1}(13), \mu_{T_2}(7), \mu_{T_3}(11)\} = 1/4 \neq \mu^1 = 1/6,$$

so go to Step 3.
Step 3. $u = 1 \neq 4$, so set $u = 2$ and return to Step 2.
Step 2. Solve P_2. An optimal transportation pattern $\mathbf{f}(2)$ is given as follows: $f_{11} = 13$, $f_{23} = 11$, $f_{32} = 7$ and the optimal value is 5.8. Since

$$\min\{\mu_{S_1}(13), \mu_{S_2}(11), \mu_{S_3}(7), \mu_{T_1}(13), \mu_{T_2}(7), \mu_{T_3}(11)\} = 1/4 = \mu^2,$$

so set $NDT = \{\mathbf{f}(2)\}$, $NDV = \{(5.8, 1/4)\}$, and then go to Step 3.
Step 3. $u = 2 \neq 4$, so set $u = 3$ and return to Step 2.
Step 2. Solve P_3. An optimal transportation pattern $\mathbf{f}(3)$ is given as follows: $f_{11} = 12$, $f_{22} = 1$, $f_{23} = 11$, $f_{31} = 1$, $f_{32} = 6$ and the optimal value is 7.9. Since

$$\min\{\mu_{S_1}(12), \mu_{S_2}(12), \mu_{S_3}(7), \mu_{T_1}(13), \mu_{T_2}(7), \mu_{T_3}(11)\} = 1/3 = \mu^3,$$

and $\mathbf{f}(2)$ is not dominated by $\mathbf{f}(3)$, so set

$$NDT = \{\mathbf{f}(2), \mathbf{f}(3)\}, \ NDV = \{(5.8, 1/4), (7.9, 1/3)\},$$

then go to Step 3.
Step 3. $u = 3 \neq 4$, so set $u = 4$ and return to Step 2.

Step 2. Solve P_4. An optimal transportation pattern $\mathbf{f}(4)$ is given as follows: $f_{11} = 12$, $f_{22} = 3$, $f_{23} = 12$, $f_{31} = 2$, $f_{32} = 4$ and the optimal value is 7.9. Since

$$\min\{\mu_{S_1}(12), \mu_{S_2}(15), \mu_{S_3}(6), \mu_{T_1}(14), \mu_{T_2}(7), \mu_{T_3}(12)\} = 1/3 \neq \mu^4 = 1/2,$$

then go to Step 3.

Step 3. $u = 4$, terminate. A set of some non-dominated transportation patterns and corresponding bi-objective vectors for problem P are given as follows:

$$NDT = \{\mathbf{f}(2), \mathbf{f}(3)\},\ NDV = \{(5.8, 1/4), (7.9, 1/3)\}.$$

5. CONCLUSION

In this paper, we considered a bi-criteria stochastic bottleneck transportation problem with flexible supply and demand quantity and developed an algorithm to find some non-dominated transportation patterns. Further, we proved the validity of the algorithm, studied its time complexity and illustrated it using a numerical example. As a further research problem, we should consider the preference of the route used in a transportation. This case makes the problem three criteria one and we are now attacking this case. Anyway, there remain many other network problems with both random and fuzzy factors to be investigated.

(Received June 8, 2010)

REFERENCES

[1] R. K. Ahuja, J. B. Orlin, and R. E. Tarjan: Improved time bounds for the maximum flow problem. SIAM J. Comput. *18* (1989), 939–954.

[2] A. Charnes and W. W. Cooper: The stepping stone method of explaining linear programming calculations in transportation problems. Management Sci. *1* (1954), 49–69.

[3] M. H. Chen, H. Ishii, and C. X. Wu: Transportation problems on a fuzzy network. Internat. J. Innovative Computing, Information and Control *4* (2008), 1105–1109.

[4] G. B. Dantzig: Application of the simplex method to a transportation problem. In: Activity Analysis of Production and Allocation, Chapter 23, Cowles Commission Monograph *13*. Wiley, New York 1951.

[5] L. R. Ford, Jr. and D. R. Fulkerson: Solving the transportation problem. Management Sci. *3* (1956), 24–32.

[6] R. S. Garfinkel and M. R. Rao: The bottleneck transportation problem. Naval Res. Logist. Quart. *18* (1971), 465–472.

[7] S. Geetha and K. P. K. Nair: A stochastic bottleneck transportation problem. J. Oper. Res. Soc. *45* (1994), 583–588.

[8] P. L. Hammer: Time minimizing transportation problem. Naval Res. Logist. Quart. *16* (1969), 345–357.

[9] F. L. Hitchcock: The distribution of a product from several sources to numerous localities. J. Math. Phys. *20* (1941), 224–230.

[10] H. Ishii: Competitive transportation problem. Central Europ. J. Oper. Res. *12* (2004), 71–78.

[11] H. Ishii and Y. Ge: Fuzzy transportation problem with random transportation costs. Scient. Math. Japon. *70* (2009), 151–157.

[12] H. Ishii, M. Tada, and T. Nishida: Fuzzy transportation problem. J. Japan Soc. Fuzzy Theory and System *2* (1990), 79–84.

[13] F. T. Lin and T. R. Tsai: A two-stage genetic algorithm for solving the transportation problem with fuzzy demands and fuzzy supplies. Internat. J. Innov. Comput. Inform. Control *5* (2009), 4775–4785.

[14] J. Munkres: Algorithms for the assignment and transportation problems. J. Soc. Industr. Appl. Math. *5* (1957), 32–38.

[15] V. Srinivasan and G. L. Thompson: An operator theory of parametric programming for the transportation-I. Naval Res. Logist. Quart. *19* (1972), 205–226.

[16] W. Szwarc: Some remarks on the time transportation problem. Naval Res. Logist. Quart. *18* (1971), 473–485.

[17] M. Tada and H. Ishii: An integer fuzzy transportation Problem. Comput. Math. Appl. *31* (1996), 71–87.

[18] M. Tada, H. Ishii and T. Nishida: Fuzzy transportation problem with integral flow. Math. Japon. *35* (1990), 335–341.

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