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DECENTRALIZED CONTROL FOR LARGE–SCALE SYSTEMS WITH TIME–VARYING DELAY AND UNMATCHED UNCERTAINTIES

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Many real-world systems contain uncertainties and with time-varying delays, also, they have become larger and more complicated. Hence, a new decentralized variable structure control law is proposed for a class of uncertain large-scale system with time varying delay in the interconnection and time varying unmatched uncertainties in the state matrix. The proposed decentralized control law for the large-scale time-varying delay system is realized independently through the delayed terms and it can drive the trajectories of the investigated systems onto the sliding mode. Further, the proposed control law can be successfully applied to stabilize a class of uncertain large-scale time-varying delay system with matched and unmatched uncertainties. The so-called sliding coefficient matching condition can be extended for the decentralized variable structure control of the uncertain large-scale time-varying delay systems. Furthermore, in the sliding mode, the investigated system with matched and unmatched uncertainties still bears the insensitivity to the uncertainties and disturbances, which is the same as the systems with just matched uncertainties do. Finally, an illustrative example is given to verify the validity of the proposed decentralized variable structure control law. It has been shown that the proposed decentralized control law is effective for all subsystems of the investigated system. However, the traditional decentralized variable structure control law is not applicable to the investigated system with unmatched uncertainties. It is worth noting that the traditional large-scale system is only a special case in this work.

Keywords: decentralized variable structure control, uncertain large-scale systems, time-delayed systems, unmatched uncertainty

Classification: 93A15, 93B12, 93C10, 93D05

1. INTRODUCTION

In recent year, the control of system becomes more difficult because of most systems have become larger and more complicated [2, 3, 4, 6, 8, 12, 13, 16, 17, 18, 23]. Frequently, several control substations existing in a class of real-world systems, each of them is responsible for the operation of a portion of the overall system. If such a situation arising in a control system design then it can be called as decentralized control. Also, such an overall system may be called as a large-scale system (LSS) and it always accompanies with difficulty and complexity of system control. To control large-scale systems, decentralized control is much preferred because it attempts to avoid difficulties in information gathering, computing and system structuring. In other words, the advantage of the decentralized control design can decompose the interconnected system into several lower-order subsystems, then the burden of computation can be shared by all the subsystem control and the design complexity will be reduced [2, 6, 8, 16, 17, 23].

In addition to the situation that system becomes larger and more complicated, the uncertainties and time delays existing in the system also become interesting issues in recent years. The time delay phenomenon is quite frequently found in various engineering systems, for examples, chemical processes, biological systems and hydraulic/pneumatic systems. In fact, the existence of time delay usually leads to the degradation, or even instability of system performance, so the existence of time delay renders the control problem much more complex and difficult. Through the different approaches, the problem of the stabilization for uncertain time-delay systems has been studied in [1, 5, 7, 9, 11, 15, 19, 21, 22, 25]. On the other hand, the presence of uncertainties is commonly due to parameter variations, input disturbances or external perturbations. For LSS, the lumped uncertainties may arise from the individual subsystems or in the links within each subsystem. To control the uncertain systems, it has been shown that variable structure control (VSC) is a very effective method [10, 24]. By introducing sliding modes in the control system, one can achieve fast response, stabilization, disturbance rejection, and insensitive to plant parameter variations, and so on. As a consequence, based on VSC theory, the stabilization problem of LSS systems with uncertainties and the systems with time delay have received considerable attention in recent control system design [2, 8, 17, 23]. However, among the above papers, either the matching condition is required for some of them [1, 2, 5, 6, 7, 8, 11, 12, 16, 17, 25] or some of them needed time delays are constant or the derivative of the time-varying delay has to be less than one [1, 4, 5, 9, 11, 12, 17, 19]. In practice, the systems' uncertainties may be unmatched, the delay existing in the developed systems may be time varying and the derivative of the time varying delay may not be less than one. We are going to make an attempt to release these constraints in this work.

In this paper, a new decentralized VSC (DVSC) law is proposed for a class of uncertain LSS with time varying delay in the interconnection and time varying unmatched uncertainties in the state matrix. The proposed DVSC law for the interconnected system is realized independently through the delay terms. In the sliding mode, the investigated large-scale time-varying delay (LSTVD) system with matched and unmatched uncertainties still bears the insensitivity to the uncertainties and disturbances, which is the same as the systems with just matched uncertainties do. Furthermore, the proposed control law can be applied to the uncertain LSTVD systems with matched and unmatched uncertainties. However, the above desirable properties cannot be guaranteed by the traditional DVSC design for the LSTVD systems just with matched uncertainties. It is worth noting that the traditional large-scale system is only a special case in this work. The rest of the paper is organized as follows. System description and assumptions are introduced in Section 2. The switching surface design is presented in Section 3. Section 4 elaborates the design of the proposed DVSC law. Illustrative example and concluding remarks are given in Sections 5 and 6, respectively.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

A class of uncertain large-scale system, composed of N linked subsystems, with matched and unmatched uncertainties and with time delay in the interconnection, is given as follows.

$$\dot{x}_i(t) = \big(A_i+\Delta A_i(t)\big)x_i(t)+B_iu_i(t)+\sum_{j\neq i}^{N} A_{ij}x_j\big(t-d_{ij}(t)\big)+B_if_i(x_i,x_j,t), \quad (1)$$

$$\text{for } i,\, j = 1,2,\dots,N,$$

where $x_i(t) \in R^{n_i}$ is the state variable, $u_i(t) \in R$ is the control input, $f_i(x_i, x_j, t) \in R$ is the external disturbance of the ith subsystem. $d_{ij}(t)$ denotes the nonnegative time varying delay present in the interconnections and are bounded by d_m for all $d_{ij}(t)$. $x_j(t - d_{ij}(t))$ is the delayed state which can be measured at the time $t - d_{ij}(t)$. It is assumed that $x_i(t)$ has corresponding continuous vector-valued initial functions for all $-d_m \leq t < 0$. State matrix A_i, uncertain time-varying matrix $\Delta A_i(t)$, input matrix B_i, and interconnection matrix A_{ij} are of appropriate dimensions. Throughout the remainder of this paper, $W^\top$ denotes the transpose of W, $\|W\|$ represents the Euclidean norm when W is a vector, or the Frobenius norm when W is a matrix. Argument $*$ of $d(*)$ is always omitted unless otherwise stated. For system (1), the following assumptions are made for each subsystem in this paper.

Assumption 1. The matrix pair (A_i, B_i) for each subsystem of (1) is controllable, so that there exists a matrix $K_i \in R^{1\times n_i}$, such that the eigenvalues of the matrix

$$\underline{A}_i = A_i + B_iK_i \quad (2)$$

lie in the open left half plane.

Assumption 2. There exists matrix G_{ij} for each of the interconnection matrix A_{ij}, such that the following matching conditions hold

$$A_{ij} = B_iG_{ij}, \quad \|G_{ij}\| \leq m_{ij}, \quad i,\, j = 1,2,\dots,N \quad (3)$$

where m_{ij} is a nonnegative constant.

Assumption 3. For each of the external disturbance $f_i(x_i, x_j, t)$, there exist known nonnegative constants k_{i1}, p_{ij} and k_{i2}, such that

$$\|f_i(x_i, x_j, t)\| \leq k_{i1}\|x_i\| + \sum_{j\neq i}^{N} p_{ij}\|x_j\| + k_{i2}. \quad (4)$$

Assumption 4. For each of the uncertain time-varying system matrix $\Delta A_i(t)$ is unmatched, and it is of the form [14]

$$\Delta A_i(t) = D_i F_i(t)\, E_i, \tag{5}$$

where D_i and E_i are known constant matrices with appropriate dimension and $F_i(t)$ is unknown but satisfies $F_i^\top(t)\, F_i(t) \le I$, here I is an identity matrix with appropriate dimensions.

Note that the matched uncertainties describing in (3) and (4) are bounded, and they are a technical assumption for mathematical completeness. Also, the unmatched uncertainty describing in (5) is attributable to Petersen [14], and all bounded uncertainties can be decomposed as such a form. Although the conditions posed upon the controlled system seem to be restrictive, the main contribution of this paper as compared to some existing papers on VSC is that the developed large-scale system allows for unmatched uncertainties, time-varying delays and the derivative of the time-varying delay may not be less than one.

For such a class of the uncertain LSTVD system with matched and unmatched uncertainties, there are two phases in the DVSC law design. First, it has to choose an appropriate sub-manifold for each subsystem, so that the sliding motion on the manifold has the desired performance. Second, one needs to select a discontinuous control law to enforce the dynamics of the composite system to be asymptotically stable and insensitive to the LSTVD system with matched and unmatched uncertainties.

3. SWITCHING SURFACE DESIGN

In this section, we will choose an appropriate switching surface for each subsystem [10], so that the sliding motion on the manifold has the desired performance. Let the associated switching surface for each subsystem be defined as

$$s_i(t) = C_i x_i(t), \tag{6}$$

where $s_i \in R$ and $C_i \in R^{1\times n_i}$. Assuming that the matrix product $C_i B_i$ is chosen to be nonsingular, without loss of generality, let $C_i = B_i^g$, the generalized inverse of B_i, to result in $C_i B_i = 1$.

Hence, the switching surface of the composite system is given as

$$S(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_N(t) \end{bmatrix}^\top \tag{7}$$

In the sliding mode, the associate sliding manifolds will satisfy the following equations.

$$s_i(t) = 0 \quad \text{and} \quad \dot{s}_i(t) = 0. \tag{8}$$

Substituting Eq. (1) into (8), one has

$$\dot{s}_i(t) = C_i\Big[\big(A_i + \Delta A_i(t)\big)x_i + B_i u_i + \sum_{j\neq i}^{N} A_{ij} x_j(t-d_{ij}) + B_i f_i\Big] = 0. \tag{9}$$

Using $C_iB_i = 1$, the equivalent control in the sliding manifold is given by

$$u_{ieq} = -C_i\Big[\big(A_i + \Delta A_i(t)\big)x_i + \sum_{j\neq i}^{N} A_{ij}x_j + B_i f_i\Big]. \tag{10}$$

The equivalent dynamic equation for each subsystem is given by

$$\dot{x}_i(t) = \big(I - B_iC_i\big)\big[A_i + \Delta A_i(t)\big]x_i, \quad i = 1, 2, \ldots, N, \tag{11}$$

where I is an $n_i \times n_i$ identity matrix.

Note that the dynamics of the LSTVD system with matched and unmatched uncertainties cannot be dominated by the sliding mode, and it can be said that the well-known invariance property of VSC is not held for the LSTVD systems with unmatched time-varying uncertainties. Furthermore, one can see the fact that the invariance property of VSC required the uncertainties satisfy the matching condition will limit the application of the VSC approach. To overcome the above fact, the so-called sliding coefficient matching condition [20] could be extended to the LSTVD systems to make the invariance property also hold for the large-scale time-delayed systems with unmatched time-varying uncertainties.

Here, we assume that the matrix E_i of the uncertain time-varying system matrix $\Delta A_i(t)$ satisfies the sliding coefficient matching condition, i. e., there exists a matrix H_i with appropriate dimension, such that

$$E_i = H_iC_i, \tag{12}$$

where C_i is the coefficients of the sliding surface function. Substituting Eq. (5) and (12) into (11), then using Eq. (6) and (8), one gets

$$\begin{aligned} \dot{x}_i(t) &= (I - B_iC_i)\big[(A_i + D_iF_i(t)\,H_iC_i)\,x_i(t)\big] \\ &= (I - B_iC_i)\,A_ix_i(t). \end{aligned} \tag{13}$$

Remark 1. Note that the dynamics of a LSTVD system is dominated by the sliding mode, and the invariance condition also holds for such a LSTVD system. From the above analysis, it can be concluded that the uncertain LSS with time delay possesses the same invariance property in the sliding mode no matter whether they are with matched or unmatched uncertainties.

4. DECENTRALIZED VARIABLE STRUCTURE CONTROL DESIGN

In this section, we will design a new DVSC law, which can make the state trajectories of each subsystem toward the sliding surface. The following lemmas will be used to construct an applicable DVSC law.

Lemma 1. If the following condition holds

$$\sum_{i=1}^{N} \frac{s_i^{\top}(t)\,\dot{s}_i(t)}{\|s_i(t)\|} < 0, \tag{14}$$

then the existence of the sliding mode of the composite system (7) is guaranteed.

Lemma 2. (Tao et al. [20]) If $F_i^\top(t)\,F_i(t) \le I$, then

$$2a_i^\top F_i(t)\,b_i \le a_i^\top a_i + b_i^\top b_i, \quad \forall\, a_i,\, b_i \in R^{n_i}. \tag{15}$$

The desired DVSC law is suggested as follow to meet the above condition (14).

$$u_i(t) = -\frac{s_i}{\|s_i\|}\,Q_i(t), \quad i = 1, 2, \ldots, N, \tag{16}$$

where

$$\begin{aligned} Q_i(t) \;&=\; h_i\Big[\big(\|C_iA_i\| + k_{i1}\big)\|x_i\| + \sum_{j\neq i}^{N}(m_{ji} + p_{ji})\max\big(\|x_i\|\big) + k_{i2} \\ &\quad + \frac{1}{2}\big(C_iD_iD_i^\top C_i^\top + x_i^\top E_i^\top E_i x_i\big)\Big], \quad \text{with } h_i > 1. \end{aligned} \tag{17}$$

Here, the term $\max(\|x_i\|)$ given in the above equation denotes the maximum value $(\|x_i(s)\|)$ for $i = 1, 2, \ldots, N$ and for $(t - d_m) \le s < t$.

Now, the following theorem is established.

Theorem 1. Consider the uncertain LSTVD system (1) subjected to the matched and unmatched uncertainties. If the applied decentralized SMC is given by Eq. (16) and (17), then the global reaching condition (14) of the sliding mode of the uncertain large-scale time-delayed system (1) is satisfied.

P r o o f. Let the Lyapunov function candidate of the system (1) be as follow

$$V(t) = \sum_{i=1}^{N}\|s_i\|, \tag{18}$$

then

$$\dot V(t) = \sum_{i=1}^{N}\frac{s_i^\top \dot s_i}{\|s_i\|}. \tag{19}$$

Substituting Eq. (1) and (6) into (19), and using $C_iB_i = 1$, one obtains

$$\begin{aligned} \dot V(t) \;&=\; \sum_{i=1}^{N}\frac{s_i^\top}{\|s_i\|}C_i\Big\{\big[A_i + \Delta A_i\big]x_i + \sum_{j\neq i}^{N}A_{ij}x_j(t - d_{ij}(t)) + B_iu_i + B_if_i\Big\} \\ &=\; \sum_{j=1}^{N}\frac{s_i^\top}{\|s_i\|}\Big\{C_iA_ix_i + \sum_{j\neq i}^{N}C_iA_{ij}x_j(t - d_{ij}(t)) + f_i + C_i\Delta A_ix_i + u_i\Big\}. \end{aligned} \tag{20}$$

By using Eqs. (3), (4), (5) and property $\|AB\| \le \|A|\,\|B\|$, one has

$$\begin{aligned} \dot V(t) \;&\le\; \sum_{i=1}^{N}\Big[\big(\|C_iA_i\| + k_{i1}\big)\|x_i\| + \sum_{j\neq i}^{N}(m_{ij} + p_{ij})\max\Big(\|x_j\|\Big) \\ &\quad + k_{i2} + C_iD_iF_iE_ix_i + \frac{s_i^\top}{\|s_i\|}\,u_i\Big]. \end{aligned} \tag{21}$$

Applying the result (15) and the proposed DVSC law (16), one obtains

$$\begin{aligned}\dot V(t) \;\le\;& \sum_{i=1}^{N}\Big\{(1-h_i)\Big[\big(\|C_iA_i\|+k_{i1}\big)\|x_i\|+k_{i2}+\frac{1}{2}\big(C_iD_iD_i^\top C_i^\top+x_i^\top E_i^\top E_ix_i\big)\Big]\\ &+\sum_{j\neq i}^{N}\big(m_{ij}-p_{ij}\big)\max\big(\|x_j\|\big)-h_j\sum_{j\neq i}^{N}\big(m_{ji}+p_{ji}\big)\max\big(\|x_i\|\big)\Big\}\\ =\;& \sum_{i=1}^{N}\Big\{(1-h_i)\Big[\big(\|C_iA_i\|+k_{i1}\big)\|x_i\|+k_{i2}+\frac{1}{2}\big(C_iD_iD_i^\top C_i^\top+x_i^\top E_i^\top E_ix_i\big)\\ &+\sum_{j\neq i}^{N}\big(m_{ji}+p_{ji}\big)\max\big(\|x_i\|\big)\Big]\Big\}. \end{aligned} \tag{22}$$

Because $h_i > 1$, it is clear that $\dot V(t) < 0$. Hence, the motion of the sliding mode is asymptotically stable, and the proof is completed. □

Remark 2. To satisfy the reaching condition of the uncertain LSTVD system, the proposed DVSC law is implemented independent of delay interconnection terms through the term $\max(\|x_i\|)$.

5. ILLUSTRATIVE EXAMPLE

In this section, to verify the effectiveness of the proposed DVSC law, we apply our new variable structure controller to a class of uncertain LSTVD systems with matched and unmatched uncertainties through a numerical example with a series of computer simulation. Consider a large-scale system, composed of 2 linked subsystems, with matched and unmatched uncertainties and with time delay in the interconnection as follows.

$$\dot x_i(t) = \big(A_i+\Delta A_i(t)\big)\,x_i(t)+B_iu_i(t)+\sum_{j\neq i}^{N}A_{ij}x_j(t-d_{ij})+B_if_i(x_i,x_j,t),\;\; i=1,2. \tag{23}$$

For both subsystems, the corresponding parameters used in numerical simulations are taken as follows.

$$\begin{aligned} A_1 &= \begin{bmatrix}-1.1 & 1\\ 2 & -2.1\end{bmatrix},\quad \Delta A_1(t)=\begin{bmatrix}4\cos(t) & 2\cos(t)\\ 2\cos(t) & \cos(t)\end{bmatrix},\quad A_{12}=\begin{bmatrix}0&0\\1&1\end{bmatrix},\quad B_1=\begin{bmatrix}0\\1\end{bmatrix},\\ f_1 &= 0.3\big(0.2-0.5\sin(x_{12})\big)\|x_1\|+0.1\big(0.5-0.3\cos(t)\big)\|x_2(t-d_{12})\|+0.1\cos(x_{11}),\\ d_{12} &= 1.5\big(1+\cos(t)\big),\\ A_2 &= \begin{bmatrix}-1 & 1\\ 3.2 & -3.1\end{bmatrix},\quad \Delta A_2(t)=\begin{bmatrix}3\sin(t) & \sin(t)\\ 6\sin(t) & 2\sin(t)\end{bmatrix},\quad A_{21}=\begin{bmatrix}0&0\\1&2\end{bmatrix},\quad B_1=\begin{bmatrix}0\\1\end{bmatrix},\\ f_2 &= 0.4\big(0.7-0.5\cos(x_{21})\big)\|x_2\|-0.1\big(0.5+0.2\sin(t)\big)\|x_1(t-d_{21})\|+0.1\sin(x_{22})\\ d_{21} &= 1.5\big(1+\sin(t)\big). \end{aligned}$$

Based on Eq. (5), one can see that

$$D_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad F_1(t) = \cos(t), \quad E_1 = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

and

$$D_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad F_2(t) = \sin(t), \quad E_2 = \begin{bmatrix} 3 & 1 \end{bmatrix}.$$

It is clear that $\Delta A_1(t)$ and $\Delta A_2(t)$ both are unmatched.

According to Eq. (3), (4) and (17), the following parameters can be used for the simulation

$$\begin{aligned} &m_{12} = 1.5, \quad k_{11} = 0.25, \quad p_{12} = 0.1, \quad k_{12} = 0.15, \quad h_1 = 1.1, \\ &m_{21} = 2.25, \quad k_{21} = 0.5, \quad p_{21} = 0.1, \quad k_{22} = 0.15, \quad h_2 = 1.1. \end{aligned}$$

In addition, for both subsystems, the initial values of $x_1(0)$ and $x_2(0)$ are arbitrarily taken as $[\,x_{110}\ x_{120}\,] = [\,1\ \ -1\,]^\top$ and $[\,x_{210}\ x_{220}\,]^\top = [\,1\ \ -1\,]^\top$, respectively.

By using the method in [10], the switching surfaces for each subsystem are selected as

$$\begin{aligned} s_1(t) &= 2x_{11}(t) + x_{12}(t), \\ s_2(t) &= 3x_{21}(t) + x_{22}(t). \end{aligned}$$

In order to eliminate the control chattering, the boundary layer condition is commonly used, and the proposed DVSC law (16) is modified as follow:

$$u_i(t) = -\frac{s_i}{\|s_i\| + e} Q_i(t), \quad i = 1, 2, \ldots, N, \tag{24}$$

where $e = 0.01$.

Based on Eq. (24) and (17), we obtain

$$u_1(t) = -\frac{s_1}{\|s_1\| + e} Q_1(t) \quad \text{and} \quad u_2(t) = -\frac{s_2}{\|s_2\| + e} Q_2(t), \tag{25}$$

where

$$\begin{aligned} Q_1(t) &= 1.1\Big\{\Big(\sqrt{0.05} + 0.25\Big)\|x_1\| + 2.35 \max\|x_i\| + 0.15 + \frac{1}{2}\big[25 + (2x_{11} + x_{12})^2\big]\Big\}, \\ Q_2(t) &= 1.1\Big\{\Big(\sqrt{0.05} + 0.5\Big)\|x_2\| + 1.6 \max\|x_i\| + 0.15 + \frac{1}{2}\big[25 + (3x_{21} + x_{22})^2\big]\Big\}. \end{aligned}$$

Figures 1–2 show the time responses of subsystem i: (a) states $x_{i1}(t)$ and $x_{i2}(t)$, (b) control input $u_i(t)$, (c) phase plane $x_{i2}(x_{i1})$ and (d) switching surface $s_i(t)$ for the uncertain LSTVD systems with matched and unmatched uncertainties under the proposed DVSC law (25). From these simulation results, we can see that the proposed DVSC (25) does work effectively for the large-scale time-delayed systems with matched and unmatched uncertainties.

As the fact that the well-known invariance property of VSC just required the uncertainties satisfy the matching condition, many VSC schemes for large-scale systems just satisfied the matching condition have been proposed [2, 6, 8, 12, 16, 17, 23].

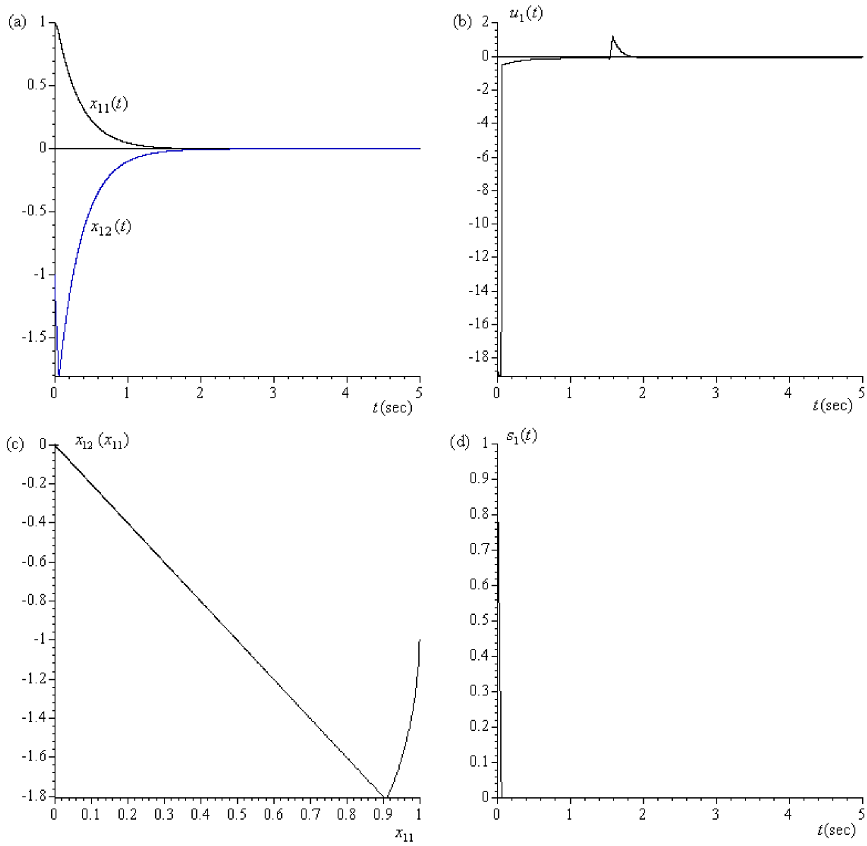

Fig. 1. Time responses of subsystem 1 under the proposed DVSC law (25): (a) States $x_{11}(t)$ and $x_{12}(t)$. (b) Control input $u_1(t)$. (c) Phase plane $x_{12}(x_{11})$ and (d) Switching surface $s_1(t)$.

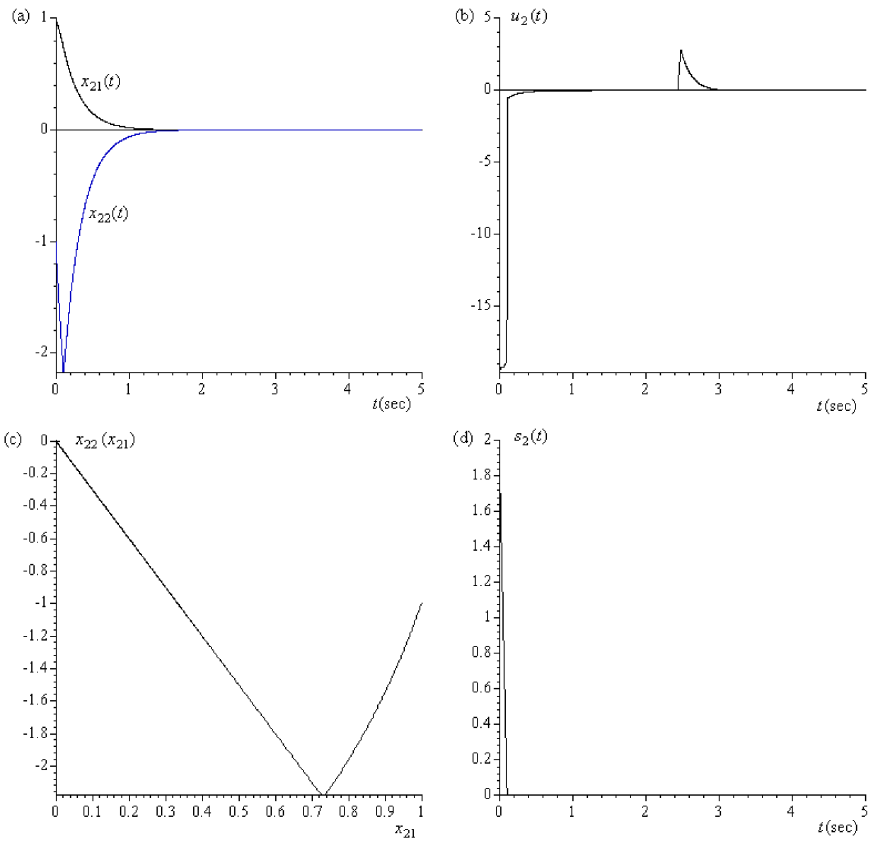

Fig. 2. Time responses of subsystem 2 under the proposed DVSC law (25): (a) States $x_{21}(t)$ and $x_{22}(t)$. (b) Control input $u_2(t)$. (c) Phase plane $x_{22}(x_{21})$ and (d) Switching surface $s_2(t)$.

However, if one disregarded the existence of unmatched uncertainties, then the performances of the systems may be poor or even the systems will be unstable, so that the existence of unmatched uncertainty renders the control problem much more complex and difficult.

For comparisons, we consider the case that $\Delta A_i(t) = 0$, so the investigated LSTVD system (23) just with the matched uncertainties. Then following the traditional DVSC design procedure for this case, we have the following traditional DVSC law.

$$u_{i,tra}(t) = -\frac{s_i}{\|s_i\|} h_i \Big[\big(\|C_i A_i\| + k_{i1}\big) \|x_i\| + \sum_{j \neq i}^{N} (m_{ji} + p_{ji}) \max\big(\|x_i\|\big) + k_{i2} \Big], \quad \text{with } h_i > 1. \tag{26}$$

Figures 3–4 show the time responses of subsystem i: (a) states $x_{i1}(t)$ and $x_{i2}(t)$, (b) control input $u_i(t)$, (c) phase plane $x_{i2}(x_{i1})$ and (d) switching surface $s_i(t)$ for the uncertain LSTVD systems with matched and unmatched uncertainties under the traditional DVSC law (26). From simulation results, one can see that the traditional DVSC controller (26) cannot control an uncertain LSTVD system with unmatched uncertainties to obtain the desired dynamics. It has been shown that the proposed DVSC law (25) is effective for all subsystems with matched and unmatched uncertainties. However, the traditional DVSC law (26) is not applicable to an uncertain large-scale time-delayed system with unmatched uncertainties.

6. CONCLUSIONS

To deal with the existence of time-varying delay in the interconnection terms and the presence of unmatched uncertainties in the state matrix, a new DVSC law is proposed to stabilize a class of uncertain LSTVD systems in this paper. The so-called sliding coefficient matching condition has been extended for the DVSC of the uncertain LSTVD systems. Also, we have shown that the proposed DVSC design guarantees the global reaching condition of the sliding mode for the LSTVD systems with matched and unmatched uncertainties, and the well-known invariance condition also holds for this study. Further, it is worth to note that the traditional LSTVD systems with matched uncertainties are only special cases in this work. Moreover, it has been shown that the proposed DVSC law can be applied to the uncertain LSTVD systems with matched and unmatched uncertainties, which is not always achieved in the traditional DVSC design for the LSTVD systems are just with matched uncertainties.

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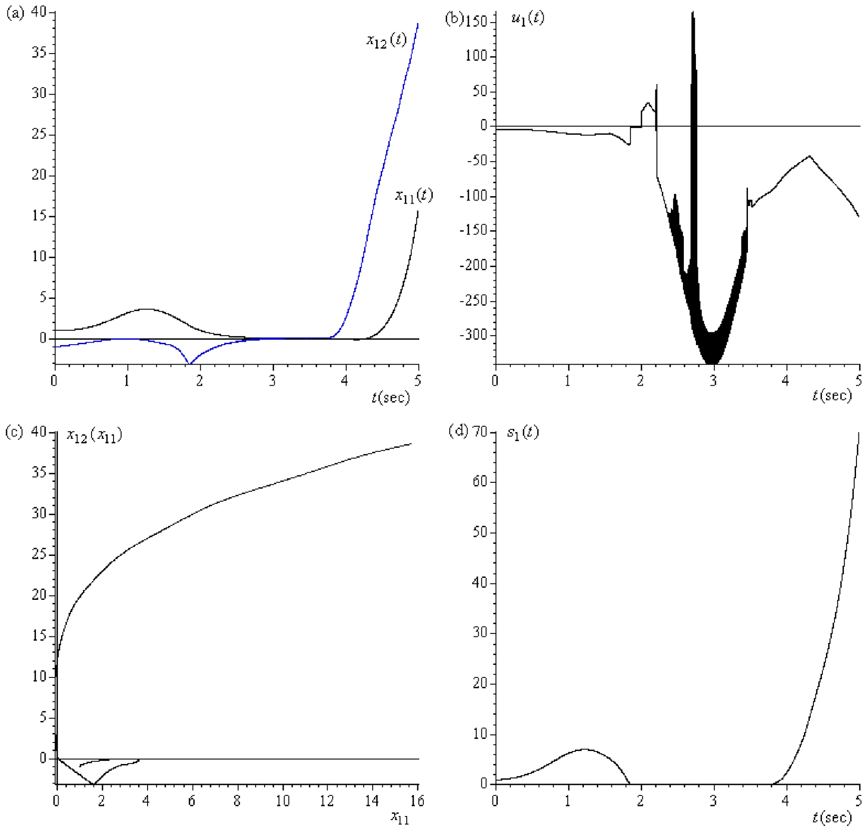

Fig. 3. Time responses of subsystem 1 under the traditional DVSC law (26): (a) States $x_{11}(t)$ and $x_{12}(t)$. (b) Control input $u_1(t)$. (c) Phase plane $x_{12}(x_{11})$ and (d) Switching surface $s_1(t)$.

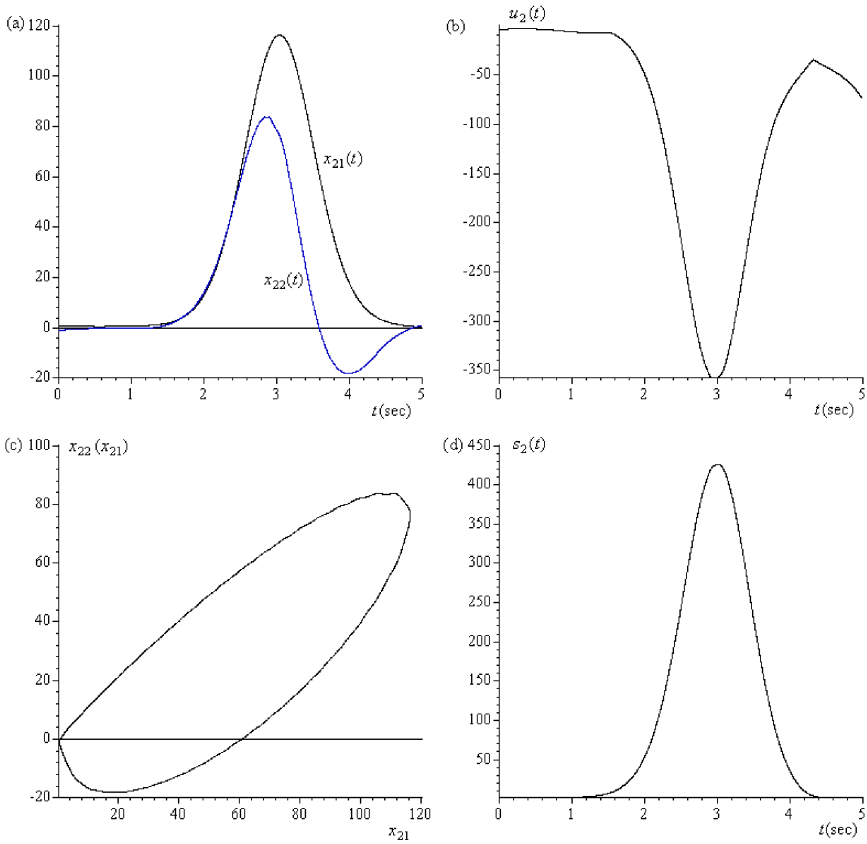

Fig. 4. Time responses of subsystem 2 under the traditional DVSC law (26): (a) States $x_{21}(t)$ and $x_{22}(t)$. (b) Control input u_2(t). (c) Phase plane $x_{22}(x_{21})$ and (d) Switching surface $s_2(t)$.

REFERENCES

[1] C. H. Chou and C. C. Cheng: Design of adaptive variable structure controllers for perturbed time-varying state delay systems. J. Franklin Inst. *338* (2001), 35–46.

[2] C. H. Chou and C. C. Cheng: A decentralized model reference adaptive variable structure controller for large-scale time-varying delay systems. IEEE Trans. Automat. Control *48* (2003), 1213–1217.

[3] W. Chen and J. Li: Decentralized output-feedback neural control for systems with unknown interconnections. IEEE Trans. Systems Man Cybernet. *38* (2008), 258–266.

[4] Z. Ding: Decentralized output regulation of large scale nonlinear systems with delay. Kybernetika *45* (2009), 33–48.

[5] E. Fridman and U. Shaked: An improved stabilization method for linear time-delay systems. IEEE Trans. Automat. Control *47* (2002), 1931–1937.

[6] K. C. Hsu: Decentralized variable structure model-following adaptive control for interconnected systems with series nonlinearities. Internat. J. Systems Sci. *29* (1998), 365–372.

[7] K. C. Hsu: Adaptive variable structure control design for uncertain time-delay systems with nonlinear input. Dynamics and Control *8* (1998), 341–354.

[8] K. C. Hsu: Decentralized sliding mode control for large-scale time-delayed systems with series nonlinearities. ASME J. Dynamic Systems, Measurement, and Control *121* (1999), 708–713.

[9] C. Hua, Q. Wang, and X. Guan: Memoryless state feedback controller design for time delay systems with matched uncertain nonlinearities. IEEE Trans. Automat. Control *53* (2008), 801–807.

[10] J. Y. Hung, W. B. Gao and J. C. Hung: Variable structure control: A survey. IEEE Trans. Ind. Electronics *40* (1993), 2–22.

[11] N. Luo and M. Dela Sen: State feedback sliding mode control of a class of uncertain time delay systems. IEE Proc. Part D – Control Theory Appl. *140* (1993), 261–274.

[12] M. S. Mahmoud: Stabilizing control for a class of uncertain interconnected systems. IEEE Trans. Automat. Control *39* (1994), 2484–2488.

[13] I. R. Petersen: Robust H_∞ control of an uncertain system via a stable decentralized output feedback controller. Kybernetika *45* (2009), 101–120.

[14] I. R. Petersen: A stabilization algorithm for a class of uncertain linear systems. Systems Control Lett. *8* (1987), 351–357.

[15] K. K. Shyu and Y. C. Chen: Robust tracking and model following for uncertain time-delay systems. Internat. J. Control *62* (1995), 589–600.

[16] K. K. Shyu, W. J. Liu, and K. C. Hsu: Decentralized variable structure control of zncertain large-scale systems containing a dead-zone. IEE Proc. Part D – Control Theory Appl. *150* (2003), 467–475.

[17] K. K. Shyu, W. J. Liu, and K. C. Hsu: Design of large-scale time-delayed systems with dead-zone input via variable structure control. Automatica *41* (2005), 1239–1246.

[18] D. D. Siljak: Decentralized Control of Complex Systems. Academic, San Diego 1991.

[19] X. M. Sun, W. Wang, G. P. Liu, and J. Zhao: Stability analysis for linear switched systems with time-varying delay. IEEE Trans. Systems Man Cybernet. *38* (2008), 528–533.

[20] C. W. Tao, W. Y. Wang, and M. L. Chan: Design of sliding mode controllers for bilinear systems with time varying uncertainties. IEEE Trans. Systems Man Cybernet. *34* (2004), 639–645.

[21] J. Wu, Z. Weng, Z. Tian, and S. Shi: Fault tolerant control for uncertain time-delay systems based on sliding mode control. Kybernetika *44* (2008), 617–632.

[22] S. Xu and J. Lam: On equivalence and efficiency of certain stability criteria for time-delay systems. IEEE Trans. Automat. Control *52* (2007), 95–101.

[23] J. Yan, J. S. Tsai, and F. Kung: Robust decentralized stabilization of large-scale delay systems via sliding mode control. ASME J. Dynamic Systems, Measurement, and Control *119* (1997), 307–312.

[24] K. D. Young, V. I. Utkin, and Ü. Özgüner: A control engineer's guide to sliding mode control. IEEE Trans. Control Systems Technology *7* (1999), 328–342.

[25] F. Zheng, Q.-G. Wang, and T. H. Lee: Adaptive robust control of uncertain time delay systems. Automatica *41* (2005), 1375–1383.

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