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REJECTION OF NONHARMONIC DISTURBANCES IN NONLINEAR SYSTEMS

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This paper proposes an asymptotic rejection algorithm on the rejection of nonharmonic periodic disturbances for general nonlinear systems. The disturbances, which are produced by nonlinear exosystems, are nonharmonic and periodic. A new nonlinear internal model is proposed to deal with the disturbances. Further, a state feedback controller is designed to ensure that the system's state variables can asymptotically converge to zero, and the disturbances can be completely rejected. The proposed algorithm can be used in many applications, e. g. active vibration control, and the avoidance of nonharmonic distortion in nonlinear circuits. An example is shown that the proposed algorithm can completely reject the nonharmonic periodic disturbances generated from a Van der Pol circuit.

Keywords: disturbance rejection, nonharmonic periodic disturbances, global stability, Van der Pol circuit, vibration control

Classification: 93E12, 62A10, 62F15

1. INTRODUCTION

Oscillatory disturbances produced by nonlinear exosystems can easily affect the performance of many engineering systems, such as turbo-machines, motors/generators, flexible structures, and communication circuits. The system's output response of such oscillatory input signals is referred to as nonharmoniously forced vibration [5, 15], which may cause undesirable effects, e. g., noise, fatigue, precision and durability reduction, unreliability, and unscheduled shutdowns. Reduction/Cancellation of the unwanted vibration is very important for the system's stabilization, and has been a hotspot in various research fields for many years.

Extensive research has been focused on the problem of complete rejection of external inaccessible disturbances, especially since the raise of the internal model principle [7, 14, 24]. The internal model principle was initially developed for linear systems, but its applications have also been extended to some nonlinear control problems [2, 8, 16, 19]. Huang et al. [16, 19] are the pioneering papers that give the solvability condition for robust nonlinear output regulation problem in terms of internal model principle. According to the principle, the outputs of a linear dynamic system, namely exosystem, are treated as deterministic external disturbances. The influence on the systems' response, which is caused by exosystem's disturbances, can be suitably reduplicated on the feedback path of a closed-loop system.

Asymptotic rejection of sinusoidal disturbance, which is one kind of the most common deterministic disturbances, has been widely studied. The autonomous system that generates the disturbance is often referred to as the exosystem. Huang et al. [17, 20] not only handle disturbance rejection but also handle tracking. The signals do not have to be sinusoidal, and can be anything generated by a known exosystem. References [32, 35] also handle both disturbance rejection and tracking with the exosystem unknown. Reference [32] gives semi-global result, while reference [35] gives global result. References [3, 11, 27] deal with disturbance rejection with the exosystem unknown. Ding [11] proposes a disturbance rejection algorithm, which can globally reject unknown sinusoidal disturbances in single-input nonlinear systems. Some recent results on stabilization and output regulation of nonlinear systems are reported in [12, 26, 30]. A related problem is formulated as output regulation, which concerns with the disturbance rejection as well as the stabilization of dynamic systems [9, 12, 13, 17, 18, 20, 22, 26, 30, 31, 32, 35]. A common assumption of these studies that the exosystem under consideration must be linear. Local results for output regulation for nonlinear systems are reported in [20, 22]. Recently, global output regulation has been addressed in the literature for nonlinear systems in the output feedback form [9, 17, 31], and the results have been extended to deal with unknown linear exosystems. Moreover, in both [17] and [18], global output regulation for lower triangular systems are given. More recently, some progresses are reported on output regulation with nonlinear exosystems [4, 6, 10, 28, 29, 34]. Ramos et al. [29] presents a result in terms of sufficient conditions of the state feedback generalized output regulation problem for nonlinear systems with nonautonomous exosystem. Byrnes et al. [4] proposes an algorithm, which uses high gain internal models, to ensure the semi-global output regulation of nonlinear exosystems. Huang et al. [6] is the first paper to provide a framework to study the robust output regulation problem with nonlinear exosystem and is the only paper which actually addresses the existence of the steady state generator and internal model when the exosystem is nonlinear. Ding [10] proposes an output regulation algorithm for a class of nonlinear systems in the output feedback form. A new nonlinear internal model is constructed based on high gain design and the Hermite-Birkhoff interpolation. The approach is then extended to the application of circle criterion [1] in [34]. Similarly, Chen et al. [5] proposes an asymptotic rejection algorithm, which uses the same internal model as in [34], to achieve the asymptotic rejection of nonharmonic disturbances and ensure semi-global stability of the whole systems. Some recent results on output regulation and disturbance rejection of nonlinear systems with nonlinear exosystems are also reported in [23, 33]. Sun et al. [33] gives the global robust output regulation result for output feedback systems which include the disturbance rejection problem as a special case. Jiang et al. [23] proposes an asymptotic rejection algorithm to achieve the asymptotic rejection of nonharmonic disturbances for a class of uncertain nonlinear systems and ensure global stability of the whole systems.

In this paper, we propose an asymptotic rejection algorithm on rejection of nonharmonic periodic disturbances for a class of nonlinear single-input systems. The disturbances, which are produced by nonlinear exosystems, are nonharmonic and periodic. A new internal model is proposed to deal with the disturbances. Further, a state feedback controller is designed to ensure that the system's state variables can asymptotically converge to zero, and the disturbances can be completely rejected. An example is performed to demonstrate that the proposed algorithm can completely reject the nonharmonic periodic disturbances produced by nonlinear exosystems. Our result is different from that in [5] in the following senses. First, in comparison with the result in [5], our algorithm extends rejection of nonlinear disturbances for nonlinear dynamic systems from systems in the strict feedback form and output feedback form to general nonlinear systems. Second, in comparison with the result in [5], we will guarantee that the overall system is global stability while, in [5], the overall system can only be guaranteed semi-global stability. The main contributions of the paper are the following: (i) nonlinear internal model for a class of nonlinear disturbances; (ii) nonlinear regulator with complete servocompensation of the class of nonlinear disturbances; (iii) the proposed control design enlarges the class of nonlinear systems of which nonharmonic periodic disturbances can be asymptotically rejected.

The outline of the paper is as follows. Section 2 describes a class of uncertain nonlinear systems with a nonlinear exosystem as disturbance source. Some assumptions are also given in Section 2. Section 3 is concerned with the nonlinear internal model design. In section 4, the global robust stabilization analysis is given to determine the controller. Section 5 illustrates an example to demonstrate the whole design procedure of the proposed method. Finally, a conclusion is given in Section 6.

2. PROBLEM FORMULATION

Consider a single-input nonlinear system

$$\dot{x} = f(x) + g(x)(u - v(w)) \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control, $v(w)$ is a function of w, and $w \in R^s$ is nonharmonic periodic disturbance vector which generated from a nonlinear exosystem

$$\dot{w} = s(w). \tag{2}$$

Assumption 1. The flows of vector field $s(w)$ are bounded and converge to periodic solutions.

Assumption 2. There exists a function $r(x) : R^n \to R^s$ such that $(\partial r(x)/\partial x)g(x) = K$, a nonzero constant vector in R^s.

Assumption 3. Consider the disturbance free system

$$\dot{x} = f(x) + g(x)u \tag{3}$$

there exists a control law

$$u = \alpha(x) \tag{4}$$

such that the closed-loop system (3)−(4) is asymptotically stable. Moreover, there exists a Lyapunov function $V(x)$ such that

$$\alpha_1(x) \leq V(x) \leq \alpha_2(x) \tag{5}$$

$$\frac{\partial V(x)}{\partial x}(f(x) + g(x)\alpha(x)) \leq -\alpha_3(x) \tag{6}$$

$$\left|\frac{\partial V(x)}{\partial x}g(x)\right|^2 \leq \alpha_3(x) \tag{7}$$

where $\alpha_i,\ i = 1, 2, 3$ are class K_∞ functions.

The problem considered in this brief is described by the following definition.

Definition 1. (Asymptotic Disturbance Rejection With Global Stability): for any prescribed compact subsets $D_w \subset R^s$, find a feedback control u such that, for all $w(0) \in D_w$, and any initial conditions of the plant and the controller, the solution of the closed-loop system exists and is bounded for all $t \geqslant 0$, and $\lim_{t\to\infty} x(t) = 0$.

Remark 1. It is noted that the disturbance $v(w)$ considered in this paper is matched, i. e., the disturbance $v(w)$ is injected in the input channel. However, we can easily use of iterative design called adaptive backstepping to extend the proposed method to more general disturbance-strict-feedback case. So the matching condition is not crucial and is excepted here to avoid tedious mathematics and to focus on the main idea.

Remark 2. Based on Assumption 1, we can know the periodic solutions of the exosystems can include many functions, such as harmonic functions and the limit cycles of nonlinear dynamic systems. For example, a Van der Pol circuit can be modeled as

$$\begin{aligned} \dot{w}_1 &= w_2 - \varsigma(\tfrac{1}{3}w_1^3 - w_1) \\ \dot{w}_2 &= -w_1 \end{aligned} \tag{8}$$

where $\varsigma > 0$ can be treated as a tuning parameter for adjusting the period of current/voltage cycle. The eigenvalues of the Jacobian matrix at the origin of (8) are $\frac{1}{2}(\varsigma \pm \sqrt{\varsigma^2 - 4})$. When $\varsigma \geq 2$, the eigenvalues are positive; when $0 < \varsigma \leq 2$, the eigenvalues are complex conjugates with positive real parts. So origin is an unstable equilibrium point and there exists a limit cycle.

Remark 3. Assumption 2 is a condition for observablity of the disturbances from the system state. If the vector field g is a nonzero constant, there always exists a solution of $r(x) = Kx$ for a nonzero K. For a general non-constant vector field $g(x)$, solutions can still be found [21].

Remark 4. According to the inverse Lyapunov Theorem [25], (5) and (6) are automatically satisfied if the closed-loop system is asymptotically stable. (7) is always satisfied if the closed-loop system is exponentially stable. However, there exists systems that the conditions in Assumption 3 are all satisfied, but the systems are not exponentially stable [25].

3. INTERNAL MODEL DESIGN

The asymptotic rejection algorithm proposed in this paper adopts an indirect approach, i. e. the disturbance is estimated first and then the estimated disturbance is used for control design for disturbance rejection. Thus, the crucial step for solving the disturbance rejection problem is to design an internal model which can be used to estimate the disturbance. For the internal model design of nonlinear exosystem, the following assumption is needed.

Assumption 4. For the exosystem (2), there exists an immersion system

$$\begin{aligned} \dot{\eta} &= F\eta + G\gamma(J\eta) \\ v(w) &= H\eta \end{aligned} \tag{9}$$

where $\eta \in R^r$, the known matrices F, G, H and J have appropriate dimensions, and the pair (F, H) is observable, and there exists a positive definite matrix $P_{\hat{\eta}}$ satisfying $P_{\hat{\eta}}G + J^T = 0$, and the nonlinear function

$$\gamma(J\eta) = \begin{bmatrix} \gamma_1(\sum_{i=1}^{r} J_{1i}\eta_i) \\ \vdots \\ \gamma_m(\sum_{i=1}^{r} J_{mi}\eta_i) \end{bmatrix} \quad \text{satisfies that} \quad (v_1 - v_2)^T(\gamma(v_1) - \gamma(v_2)) \geq 0.$$

Remark 5. Assumption 4 bases on the assumption made on the exosystems in [5], and we can know this is a condition for which the circle criterion [1] can be applied. For the Van der Pol circuit, let $\eta = w$ and choose the matrix parameters as follows:

$$F = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}, \quad J = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & -1 \end{bmatrix},$$
$$\gamma_1(s) = \tfrac{1}{3}s^3, \quad \gamma_2(s) = 0, \quad P_{\hat{\eta}} = \mathrm{diag}(1/2, 2).$$

It can be seen that Assumption 4 is satisfied.

We design the following internal model as

$$\dot{\hat{\eta}} = (F - KH)(\hat{\eta} - r(x)) + G\gamma(J(\hat{\eta} - r(x))) + Ku + (\partial r(x)/\partial x)f(x) \tag{10}$$

where $K \in R^s$ satisfies Assumption 2, such that $F_0 = F - KH$ is Hurwitz, so there exists positive definite matrices $P_{\hat{\eta}}$ and $Q_{\hat{\eta}}$ satisfying

$$P_{\hat{\eta}}F_0 + F_0^T P_{\hat{\eta}} = -Q_{\hat{\eta}}. \tag{11}$$

Remark 6. It is noted that there exist positive definite matrices $P_{\hat{\eta}}$ and $Q_{\hat{\eta}}$ satisfying

$$\begin{aligned} P_{\hat{\eta}}F_0 + F_0^T P_{\hat{\eta}} &= -Q_{\hat{\eta}} \\ P_{\hat{\eta}}G + J^T &= 0. \end{aligned} \tag{12}$$

In particular, if G and J^T are two column vectors, and the pair (F_0, G) is controllable, and the pair (J, F_0) is observable, and the triple (F_0, G, J) satisfies the strictly positive real condition $Re[-J(jwI - F_0)^{-1}G] > 0, \forall w \in R$, then there exists a solution of (12) from the well known Meyer–Kalman–Yacubovic Theorem.

Define an auxiliary error

$$\tilde{\eta} = \eta - \hat{\eta} + r(x). \tag{13}$$

Thus, we have

$$\begin{aligned}
\dot{\tilde{\eta}} &= \dot{\eta} - \dot{\hat{\eta}} + (\partial r(x)/\partial x)(f(x) + g(x)(u - v(w))) \\
&= F\eta + G\gamma(J\eta) - (F - KH)(\hat{\eta} - r(x)) \\
&\quad -G\gamma(J(\hat{\eta} - r(x))) - Ku - (\partial r(x)/\partial x)f(x) \\
&\quad +(\partial r(x)/\partial x)(f(x) + g(x)(u - H\eta)) \\
&= F_0\tilde{\eta} + G\gamma(J\eta) - G\gamma(J(\eta - \tilde{\eta})). \qquad (14)
\end{aligned}$$

4. CONTROL DESIGN

Based on the internal model (10) and Assumption 3, the control input can be designed as

$$u = \alpha(x) + H(\hat{\eta} - r(x)). \tag{15}$$

Define a Lyapunov function candidate

$$W = V(x) + \tilde{\eta}^T P_{\hat{\eta}} \tilde{\eta}. \tag{16}$$

Its derivative along the system dynamics (1) and the auxiliary error dynamic (14) is given by

$$\begin{aligned}
\dot{W} &= \frac{\partial V(x)}{\partial x}(f(x) + g(x)(u - v(w))) + \tilde{\eta}^T(P_{\hat{\eta}}F_0 + F_0^T P_{\hat{\eta}})\tilde{\eta} \\
&\quad +2\tilde{\eta}^T P_{\hat{\eta}} G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \\
&= \frac{\partial V(x)}{\partial x}(f(x) + g(x)\alpha(x)) + \frac{\partial V(x)}{\partial x} g(x) H(\hat{\eta} - r(x)) \\
&\quad -\frac{\partial V(x)}{\partial x} g(x) H(\tilde{\eta} + \hat{\eta} - r(x)) - \tilde{\eta}^T Q_{\hat{\eta}} \tilde{\eta} \\
&\quad +2\tilde{\eta}^T P_{\hat{\eta}} G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \\
&\leq \frac{\partial V(x)}{\partial x}(f(x) + g(x)\alpha(x)) - \frac{\partial V(x)}{\partial x} g(x) H\tilde{\eta}
\end{aligned}$$

$$\begin{aligned} & -\lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 + 2\tilde{\eta}^T P_{\hat{\eta}} G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \\ \leq \quad & -\alpha_3(x) + \left|\frac{\partial V(x)}{\partial x} g(x)\right| \|H\tilde{\eta}\| - \lambda_{\min}(Q_{\hat{\eta}})\|\tilde{\eta}\|^2 \\ & +2\tilde{\eta}^T P_{\hat{\eta}} G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \end{aligned} \tag{17}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalues of a matrix. It follows from Assumption 4 that $P_{\hat{\eta}}G = -J^T$, and thus we have

$$\begin{aligned} & 2\tilde{\eta}^T P_{\hat{\eta}} G(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \\ & = -2(J\eta - J(\eta - \tilde{\eta})))^T(\gamma(J\eta) - \gamma(J(\eta - \tilde{\eta}))) \leq 0. \end{aligned} \tag{18}$$

Then using $2ab \leq ca^2 + c^{-1}b^2$ to the second term on the right hand side of (17) gives

$$\left|\tfrac{\partial V(x)}{\partial x} g(x)\right| \|H\tilde{\eta}\| \leq \tfrac{1}{2}\left|\tfrac{\partial V(x)}{\partial x} g(x)\right|^2 + \tfrac{1}{2}\|H\|^2\|\tilde{\eta}\|^2. \tag{19}$$

Substituting (18) and (19) into (17) gives

$$\dot{W} \leq -\tfrac{1}{2}\alpha_3(x) - (\lambda_{\min}(Q_{\hat{\eta}}) - \tfrac{1}{2}\|H\|^2)\|\tilde{\eta}\|^2. \tag{20}$$

Choose $Q_{\hat{\eta}}$ and H, such that

$$d = \lambda_{\min}(Q_{\hat{\eta}}) - \tfrac{1}{2}\|H\|^2 > 0 \tag{21}$$

we have

$$\dot{W} \leq -\tfrac{1}{2}\alpha_3(x) - d\|\tilde{\eta}\|^2. \tag{22}$$

Therefore we can conclude that all the variables are bounded. Moreover, from the invariant set theorem, we have $\lim_{t\to\infty} x(t) = 0$ and $\lim_{t\to\infty} \tilde{\eta} = 0$.

The result of this section is summarized in the following.

Theorem 1. Suppose that there exist positive definite matrices $P_{\hat{\eta}}$ and $Q_{\hat{\eta}}$, and a nonzero constant vector $K \in R^s$ such that $F_0 = F - KH$ is Hurwitz and (12) and (21) hold, respectively. With Assumptions 1–4, then the internal model (10) and the control input (15) solve the asymptotic rejection problem for the system (1) with the nonharmonic periodic disturbance generated from the nonlinear exosystem (2).

5. ILLUSTRATIVE EXAMPLE

To illustrate the proposed approach, concentrating on the design of the nonlinear internal model, we consider a nonlinear system described by

$$\begin{aligned} \dot{x}_1 &= 2x_2 + x_1^2 + (u - v(w)) \\ \dot{x}_2 &= -x_2 + \tfrac{1}{1+x_2^2}(u - v(w)) \end{aligned} \tag{23}$$

where the nonharmonic periodic disturbance $v(w) = w_1 - w_2$ is the measurable output of the following Van der Pol circuit

$$\begin{aligned} \dot{w}_1 &= w_2 - \varsigma(\tfrac{1}{3}w_1^3 - w_1) \\ \dot{w}_2 &= -w_1 \end{aligned} \tag{24}$$

where $1.6 \leq \varsigma \leq 2.4$. The set D_w in Definition 1 is set as $\{w||w_1| \leq 2, |w_2| \leq 3\}$.

For the Van der Pol circuit, let $\eta = w$ and choose the matrix parameters as in Remark 5, then Assumption 4 is satisfied.

The disturbance-free system is stabilized by setting $u = \alpha(x)$ as

$$\alpha(x) = -6x_1 + x_2 + x_1x_2 - 2x_1^2 + x_2^3 + \tfrac{1}{3}x_1x_2^3. \tag{25}$$

In fact, if we choose the Lyapunov function for the disturbance-free system as

$$V(x) = \tfrac{1}{2}(x_2 + \tfrac{1}{3}x_2^3 - x_1)^2 + \tfrac{1}{2}x_1^2 \tag{26}$$

we have

$$\begin{aligned} &\tfrac{\partial V(x)}{\partial x}(f(x) + g(x)\alpha(x)) = -3(x_2 + \tfrac{1}{3}x_2^3 - x_1)^2 - 3x_1^2 \\ &\tfrac{\partial V(x)}{\partial x}g(x) = x_1. \end{aligned} \tag{27}$$

With reference to Assumption 3, we have

$$\tfrac{3+\sqrt{5}}{4}\|x\|^2 \leq V(x) \leq (2 + (1 + \tfrac{1}{3}\|x\|^2))\|x\|^2 \tag{28}$$

$$\tfrac{\partial V(x)}{\partial x}(f(x) + g(x)\alpha(x)) \leq -\|x\|^2 \tag{29}$$

$$\left|\tfrac{\partial V(x)}{\partial x}g(x)\right|^2 \leq \|x\|^2. \tag{30}$$

Therefore the system with the disturbance-free control design satisfies Assumption 3 with $\alpha_1(x) = \frac{3+\sqrt{5}}{4}\|x\|^2$, $\alpha_2(x) = (2 + (1 + \frac{1}{3}\|x\|^2))\|x\|^2$, $\alpha_3(x) = \|x\|^2$.

With

$$r(x) = \begin{bmatrix} 4x_1 & -x_2 - \tfrac{1}{3}x_2^3 \end{bmatrix}^T \tag{31}$$

we have

$$K = \tfrac{\partial r(x)}{\partial x}g(x) = \begin{bmatrix} 4 & -1 \end{bmatrix}^T. \tag{32}$$

Hence, Assumption 2 is also satisfied. Note that Assumption 1 is automatically satisfied from the statement that disturbance is a nonharmonic periodic disturbance.

In addition, let $P_{\hat{\eta}} = \mathrm{diag}(1/2, 4)$, and then

$$F_0 = F - KH = \begin{bmatrix} -2 & 5 \\ 0 & -1 \end{bmatrix} \tag{33}$$

$$Q_{\hat{\eta}} = \begin{bmatrix} 2 & -5/2 \\ -5/2 & 8 \end{bmatrix} \tag{34}$$

which satisfies (12) and (21).

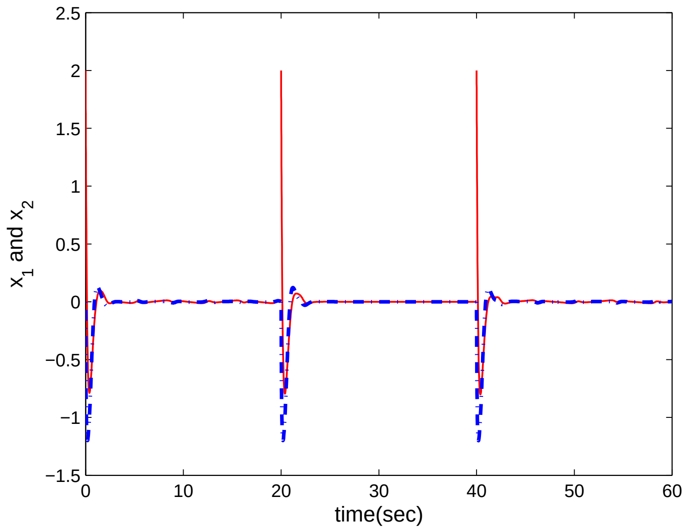

Fig. 1. Time response of states x_1 and x_2(solid line: x_1; dash-dotted line: x_2).

Based on the proposed control method, the internal model and the control input are designed as follows

$$\begin{aligned} \dot{\hat{\eta}}_1 &= -2\hat{\eta}_1 + 5\hat{\eta}_2 + 8x_1 + 13x_2 + 4x_1^2 \\ &\quad -\tfrac{2}{3}(\hat{\eta}_1 - 4x_1)^3 + \tfrac{5}{3}x_2^3 + 4u \\ \dot{\hat{\eta}}_2 &= -\hat{\eta}_2 + \tfrac{2}{3}x_2^3 - u \end{aligned} \tag{35}$$

$$u = \alpha(x) + \hat{\eta}_1 - 4x_1 - \tfrac{1}{3}x_2^3 - \hat{\eta}_2 - x_2. \tag{36}$$

In the simulation, let the initial condition be $x(0) = [2, 0]^T$, $\hat{\eta}(0) = [0, 0]^T$ and $w(0) = [1, -1]^T$. To make the problem more interesting, we allow the parameter ς to be uncertain. To characterize the uncertainty, let

$$\varsigma = \begin{cases} 1.6, & \text{if } 0 \le t \le 20; \\ 2.0, & \text{if } 20 < t \le 40; \\ 2.4, & \text{if } 40 < t \le 60. \end{cases} \tag{37}$$

The time response of system states are shown in Figure 1. It can be observed that the disturbances are rejected completely. Figure 2 shows the phase portrait of the Van der Pol circuit. The corresponding control input is given in Figure 3. As shown in Figure 4 and Figure 5 the disturbances are successfully reproduced by the designed internal model even if the parameter ς changes. It can be seen that the controller can tolerate certain uncertain parameter and has a satisfactory performance.

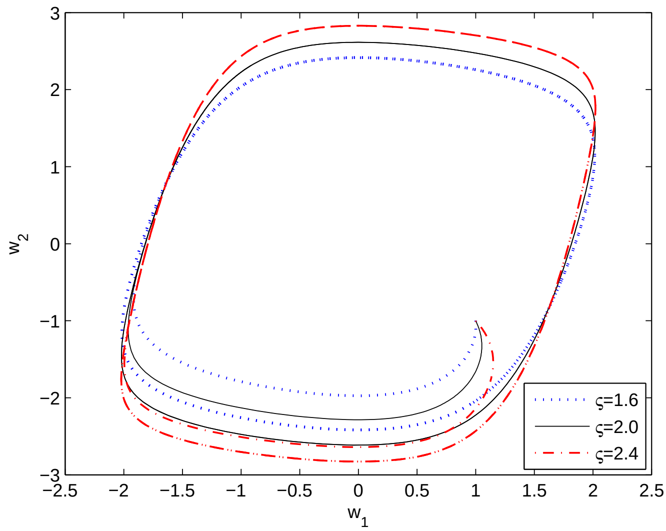

Fig. 2. Phase portrait of the exosystem.

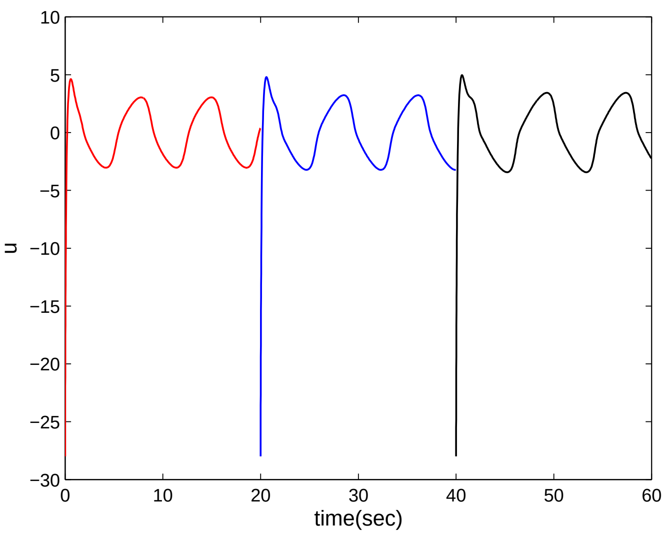

Fig. 3. Control input u.

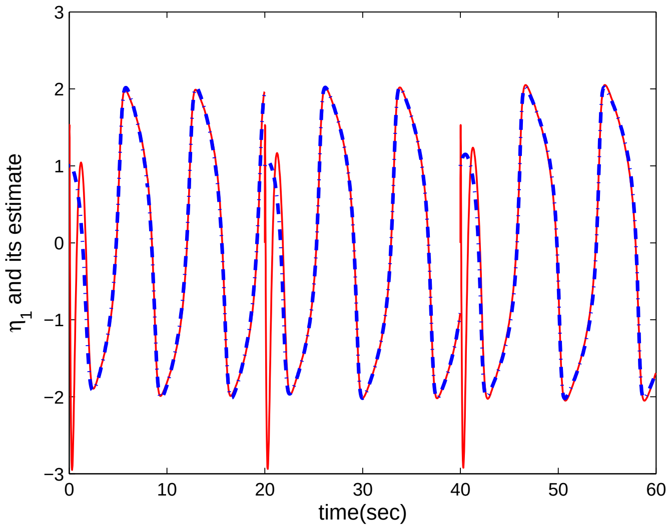

Fig. 4. The state of internal model $\hat{\eta}_1$ (dash-dotted line: η_1; solid line: $\hat{\eta}_1$).

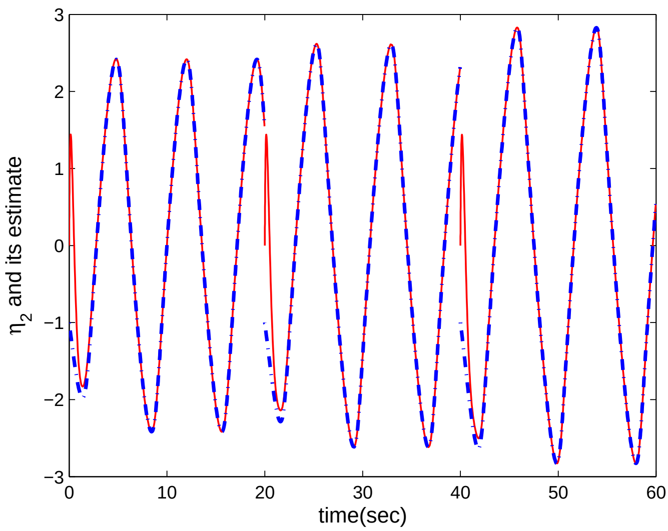

Fig. 5. The state of internal model $\hat{\eta}_2$ (dash-dotted line: η_2; solid line: $\hat{\eta}_2$).

6. CONCLUSIONS

In this paper, a global asymptotic rejection algorithm is proposed to design a state feedback controller for single-input systems in the presence of nonharmonic periodic disturbances. In order to reject disturbances generated from a nonlinear exosystem, a new internal model is constructed. It is shown that the proposed controller can ensure the system's state variables asymptotically converge to zero, and the disturbances can be completely rejected. Simulation results illustrate the effectiveness of our algorithm.

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