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CUT PROPERTIES OF RESEMBLANCE

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The resemblance relation is used to reflect some real life situations for which a fuzzy equivalence is not suitable. We study the properties of cuts for such relations. In the case of a resemblance on a real line E we show that it determines a special family of crisp functions closely connected to its cut relations. Conversely, we present conditions which should be satisfied by a collection of real functions in E in order that this collection determines a resemblance relation.

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1. INTRODUCTION

Some real life situations can be successfully described by means of a suitable fuzzy relation. One of such situations is for example a relationship of "being similar". Clearly any fuzzy relation to describe this relationship should be reflexive and symmetric, but usually not transitive. Hence a fuzzy equivalence is not a good candidate here. More suitable relations are fuzzy nearnesses (introduced in [4]) and resemblances (introduced in [1, 2]) which we investigate in this paper.

The aim of the paper is to investigate fuzzy resemblance in the framework of cut relations. A general properties of cuts in addition to theorems of decomposition and synthesis by cuts are evaluated in Sections 3 and 4. In the latter we also investigate aftersets of resemblance. Section 5 is devoted to resemblances on a real line as a domain. In order to represent such a resemblance by crisp relations (cuts) or real functions, an additional element $(-\infty)$ is added to its domain. Under these conditions we describe special real functions determined by a resemblance; we also express cut relations in terms of the mentioned functions. Conversely, we show that a family of real functions with special (mentioned) properties can be used for a synthesis of a resemblance on a real line.

2. PRELIMINARIES

Throughout the paper we use some basic notions from the theory of binary relations. On finite sets, these are represented by the corresponding characteristic functions,

i. e., by the table which for a relation ρ on a nonempty set X has entries from $\{0, 1\}$. In addition, for $x \in X$ the **ρ-afterset** (or simply **afterset**) of x is the set $\rho(x) := \{y \in X \mid x\rho y\}$.

For a **fuzzy set** $A : X \to [0, 1]$ on a nonempty set X and $\alpha \in [0, 1]$, we denote by A_α the **α-cut** of A:

$$A_\alpha := \{x \in X \mid A(x) \geq \alpha\},$$

and by A_α^+ the **strong α-cut** of A:

$$A_\alpha^+ := \{x \in X \mid A(x) > \alpha\}.$$

The analogue notation is used here for an α-cut and a strong α-cut (R_α and R_α^+, respectively) of a **fuzzy** (binary) **relation** $R : X^2 \to [0, 1]$ on X:

$$\begin{array}{rcl} R_\alpha & := & \{(x, y) \in X^2 \mid R(x, y)) \geq \alpha\}, \\ R_\alpha^+ & := & \{(x, y) \in X^2 \mid R(x, y)) > \alpha\}. \end{array}$$

Obviously, 0-cut is equal to X^2 and strong 1-cut is equal to $\emptyset$.

The relation of a **fuzzy nearness** has been introduced in [4] in the following way:

Definition 2.1. A binary fuzzy relation N on a real line with values in the interval $[0, 1]$ is called a fuzzy nearness if for all $x, y, t \in \mathbb{R}$ the following hold:

- $N(x, x) = 1$;
- $N(x, y) = N(y, x)$;
- if t is between x and y then $N(x, t) \geq N(x, y)$.

This definition can be generalized for an arbitrary metric space (X, ρ). The obtained notion, called a **fuzzy resemblance relation**, can be found in [1] and [2]:

Definition 2.2. Let (X, ρ) be a metric space. Then $R : X^2 \to [0, 1]$ is a resemblance relation, if

- $R(x, x) = 1$ for all $x \in X$,
- $R(x, y) = R(y, x)$ for all $x, y \in X$,
- if $\rho(x, y) \leq \rho(x, z)$ then $R(x, y) \geq R(x, z)$.

Remark 2.3. It is easy to see that in case of a usual metric on the real line, the third condition is equivalent with

$$\text{if } x \leq y \leq z \text{ then } R(x, y) \geq R(x, z) \text{ and } R(y, z) \geq R(x, z). \tag{1}$$

We deal with such fuzzy relations and their cuts. For the brevity we speak about the resemblance relation and omit the word fuzzy.

For some more properties of cut relations, and generally for the connection of crisp cut structures and the corresponding fuzzy ones see [5, 6, 7].

3. CUTS OF RESEMBLANCE RELATION

If $\alpha \in [0,1]$ then by R_α we understand the α-cut of a fuzzy relation R. The connection between a resemblance relation R and its cuts is described in the following proposition.

Proposition 3.1. Let (X, ρ) be a metric space. A fuzzy relation R is a resemblance on (X, ρ), if and only if its cuts fulfill the following conditions:

(i) $(x, x) \in R_\alpha$ for all $x \in X, \alpha \in [0,1]$,

(ii) if $(x, y) \in R_\alpha$, then $(y, x) \in R_\alpha$ for all $x, y \in X, \alpha \in [0,1]$,

(iii) if $\rho(x, y) \leq \rho(x, z)$ and $(x, z) \in R_\alpha$ then $(x, y) \in R_\alpha$ for all $x, y, z \in X, \alpha \in [0,1]$.

Proof. (i) Let R be a resemblance relation on the metric space (X, ρ). If $x \in X, \alpha \in [0,1]$, then $R(x, x) = 1 \geq \alpha$ and so $(x, x) \in R_\alpha$.

(ii) If $(x, y) \in R_\alpha$, then $R(y, x) = R(x, y) \geq \alpha$ and then also $(y, x) \in R_\alpha$.

(iii) Let $\rho(x, y) \leq \rho(x, z)$, let $(x, z) \in R_\alpha$. Then from the definition of R we have $R(x, y) \geq R(x, z) \geq \alpha$ and thus $(x, y) \in R_\alpha$.

To show the converse let us remind that if the α-cuts R_α of a fuzzy relation R are given, then the values of R can be expressed in the following way: $R(x, y) = \sup\{\alpha \mid (x, y) \in R_\alpha \}$.

Suppose that the conditions from our statement for the cuts of R are fulfilled. Then from the condition (i) we obtain

$$R(x, x) = \sup\{\alpha \mid (x, x) \in R_\alpha\} = \sup\{\alpha \mid \alpha \in [0,1]\} = 1.$$

The fulfillment of the second condition for R is a consequence of the condition (ii) in the statement.

To show the third condition for the resemblance let us take $x, y, z, \in X, \rho(x, y) \leq \rho(x, z)$. Then from the condition (iii) we get $(x, z) \in R_\alpha$ implies $(x, y) \in R_\alpha$ for any α. Thus for any fixed α we have $\{\alpha \mid (x, y) \in R_\alpha\} \supseteq \{\alpha \mid (x, z) \in R_\alpha\}$ and so $R(x, y) \geq R(x, z)$. Therefore R is a resemblance. □

In the following proposition we show how a resemblance can be constructed from a suitable collection of crisp relations.

Proposition 3.2. Let $\Phi = \{S_\alpha \mid \alpha \in [0,1]\}$ be a family of relations fulfilling the conditions (i) – (iii) from the previous statement and satisfying the following:

$$\text{for every family } \Psi \subseteq \Phi, \quad \bigcap_{S_\alpha \in \Psi} S_\alpha = S_{\sup\{\alpha \mid S_\alpha \in \Psi\}}. \tag{2}$$

Moreover, let $S_0 = X^2$.

Then there is a resemblance such that Φ is the set of all its cuts.

P r o o f. First we observe that from condition (2) it follows that $\alpha \le \beta$ implies $S_\beta \subseteq S_\alpha$, for all $\alpha, \beta \in [0,1]$.

Put $R(x,y) = \sup\{\alpha \mid (x,y) \in S_\alpha\}$. We show that R is a resemblance relation.

As $(x,x) \in S_\alpha$ for all $\alpha \in [0,1]$, then by the definition of R we have $R(x,x) = 1$. The symmetry of R also follows immediately from the condition (ii) for all S_α. To show the last condition of a resemblance take $x,y,z \in X$ such that $\rho(x,z) \le \rho(x,y)$. Then from the condition (iii) for the elements of Φ we have $\{\alpha \mid (x,z) \in S_\alpha\} \supseteq \{\alpha \mid (x,y) \in S_\alpha\}$. Considering suprema of these sets we obtain $R(x,z) \ge R(x,y)$. Thus R is a resemblance relation.

Finally we have to show that the elements of Φ are the cuts of R. We have to prove that $R_\beta = S_\beta$, for every $\beta \in [0,1]$.

Take an arbitrary $\beta \in [0,1]$. Then $(x,y) \in R_\beta$ if and only if $R(x,y) \ge \beta$. This is equivalent to the inequality $\sup\{\alpha \mid (x,y) \in S_\alpha\} \ge \beta$.

Denote $\sup\{\alpha \mid (x,y) \in S_\alpha\}$ by γ. Since family $\{S_\alpha \mid (x,y) \in S_\alpha\}$ is closed under intersection, then

$$\bigcap_{(x,y)\in S_\alpha} S_\alpha = S_\gamma,$$

and $(x,y) \in S_\gamma$. By $\gamma \ge \beta$, we have that $S_\gamma \subseteq S_\beta$ and $(x,y) \in S_\beta$.

To prove the opposite, if $(x,y) \in S_\beta$, then S_β is a member of the family $\{S_\alpha \mid (x,y) \in S_\alpha\}$, and $\beta \le \sup\{\alpha \mid (x,y) \in S_\alpha\}$, that is $R(x,y)$ by the definition. Hence, $(x,y) \in R_\beta$. □

4. AFTERSETS OF RESEMBLANCE RELATION

In this section we show how a resemblance R can be constructed from a system of sets in the universe, so that this system is the family of cuts for fuzzy sets which are particular R-aftersets, as defined in the sequel.

Let (X,ρ) be a metric space and $R : X^2 \to [0,1]$ a resemblance relation.

For $x \in X$, the **fuzzy R-afterset** of x is the fuzzy set R_x on X, defined by $R_x(y) := R(x,y)$, for every $y \in X$.

For $\alpha \in [0,1]$ and $x \in X$, the α-cut of R_x, denoted by $(R_x)_\alpha$, is obviously a crisp R-afterset of x of the α-cut relation R_α of R. We call it the **α, R-afterset** of x:

$$(R_x)_\alpha = \{y \mid R(x,y) \ge \alpha\}.$$

In the following, α, R-aftersets of x, $x \in X, \alpha \in [0,1]$, will be called **R-aftersets**.

Next we present some properties of aftersets of a resemblance relation.

The first two are obvious, they are general properties of cuts:

(I) Let $x \in X$ is fixed, and let Φ be a family of all α, R-aftersets of x. Then for every family $\Psi \subseteq \Phi$,

$$\bigcap_{(R_x)_\alpha\in\Psi} (R_x)_\alpha = (R_x)_{\sup\{\alpha \mid (R_x)_\alpha \in \Psi\}}. \tag{3}$$

(II) $(R_x)_0 = X$, for every $x \in X$.

It is easy to see that a consequence of property (3) is following:
if $\alpha \le \beta$ then $(R_x)_\alpha \supseteq (R_x)_\beta$.

Proposition 4.1. Let (X, ρ) be a metric space and $R : X^2 \to [0,1]$ a resemblance relation. Then the following hold.

(III) For every $x \in X$ and every $\alpha \in [0,1]$, $x \in (R_x)_\alpha$.

(IV) If $x \in (R_y)_\alpha$ then $y \in (R_x)_\alpha$.

(V) For all $x, y, z \in X$ and every $\alpha \in [0,1]$ from $\rho(x,y) \leq \rho(x,z)$ it follows that $z \in (R_x)_\alpha$ implies $y \in (R_x)_\alpha$.

Proof. Straightforward: (III), (IV) and (V) follow from the respective properties of R, given in Definition 2.2. □

Theorem 4.2. Let (X, ρ) be a metric space and

$$\mathcal{S} = \{(S_x)_\alpha \mid x \in X, \alpha \in [0,1]\}$$

a family of subsets of X indexed by elements of X and α, satisfying properties (I) – (V). Then the mapping $R : X^2 \to [0,1]$ defined by

$$R(x,y) := \sup\{\alpha \mid y \in (S_x)_\alpha\}$$

is a resemblance relation and the family of α, R-aftersets of x is exactly the family $\mathcal{S}$.

Proof. R is a mapping from X^2 to $[0,1]$. By (III), $R(x,x) = 1$. By (IV), R is a fuzzy symmetric relation. By (V), from $\rho(x,y) \leq \rho(x,z)$ it follows that $R(x,y) \geq R(x,z)$.

Further, we prove that for every $x \in X$, $\beta \in [0,1]$, we have $(S_x)_\beta = (R_x)_\beta$.

Let $y \in (R_x)_\beta$, then $R(x,y) \geq \beta$. Hence, $\sup\{\alpha \mid y \in (S_x)_\alpha\} \geq \beta$. Denote $\{(S_x)_\alpha \mid y \in (S_x)_\alpha\}$ by Ψ and $\sup\{\alpha \mid y \in (S_x)_\alpha\}$ by γ. Then, by property (I),

$$\bigcap_{(S_x)_\alpha \in \Psi} (S_x)_\alpha = (S_x)_\gamma.$$

Since y belongs to any member of family Ψ, we have that $y \in (S_x)_\gamma$. By $\gamma \geq \beta$, we have that $(S_x)_\gamma \subseteq (S_x)_\beta$ and thus $y \in (S_x)_\beta$.

To prove that opposite inclusion, suppose that $y \in (S_x)_\beta$. Then $(S_x)_\beta \in \Psi$. Hence, $\beta \leq \gamma = R(x,y)$ and and hence $y \in (R_x)_\beta$. □

In the following we tackle similar, but a more difficult problem: starting with a non indexed collection of subsets of X associated to every $x \in X$, under which conditions there is a resemblance relation R, such that the family of α, R- aftersets is the starting collection.

A starting point is the following theorem, which is formulated here for the first time, but the proof is very similar to the proof of Theorem 1 from [3], so we skip it.

Theorem 4.3. Let X be a nonempty set and for each $x \in X$ let $\mathcal{R}_x$ be a collection of nonempty subsets of X being a chain under $\subseteq$. Then there is a fuzzy relation R on X, such that for every $x \in X$, $\mathcal{R}_x$ is the collection of nonempty cuts of R-afterset of x.

The open problem is whether the obtained relation is resemblance relation provided that the starting collection satisfies properties (I) – (V).

5. RESEMBLANCE ON THE REAL LINE

In this part we investigate a resemblance whose domain is the real line with an addition of a bottom element ($-\infty$) under the natural order. With such a domain we are able to obtain a representation of a resemblance by two families of special crisp relations. In addition, these families of relations are used for a description of cut-relations of a resemblance. The metric used in this part is shift-invariant.

Let E be the real line completed with $-\infty$ from the lower side. If $R : E^2 \to [0,1]$ is a resemblance, then $R(x, -\infty) = 0$, for every $x \in E \setminus \{-\infty\}$.

Proposition 5.1. Let R be a resemblance on E, let $x \in E, \delta > 0$. Then $R(x, x - \delta) = R(x, x + \delta)$.

Proof. As $\rho(x, x-\delta) \le \rho(x, x+\delta)$, we have $R(x, x-\delta) \ge R(x, x+\delta)$. Assuming the opposite inequality $\rho(x, x-\delta) \ge \rho(x, x+\delta)$ we obtain the opposite inequality for R and hence the required equality. □

The consequence of the third property of resemblance and the previous proposition is, that for an arbitrary $x \in E$ the function $f(t) = R(x,t)$ is nondecreasing for $t \le x$ and nonincreasing for $t \ge x$.

In the following, we show that each resemblance in E gives rise to a set of functions with special properties.

Theorem 5.2. Let R be a resemblance on E. For any $\alpha \in [0,1], x \in E$ put

$$f_\alpha(x) = \sup\{y \in E \mid y \le x, R(x,y) \le \alpha\}.$$

Then $\{f_\alpha\}_{\alpha\in[0,1]}$ is a system of nondecreasing real functions fulfilling the following:

(A) if $\alpha, \beta \in [0,1], \alpha < \beta$, then $f_\alpha \le f_\beta$,

(B) $f_\alpha(x) \le x$ for all $\alpha \in [0,1], x \in E$,

(C) If for any $x \in X$, $f_\alpha(x) < f_\beta(x)$, then $\alpha < \beta$.

(D) $f_1(x) = x$ for every x.

Proof. Using the convention $\sup \emptyset = -\infty$, f_α are well defined functions on E, for every $\alpha \in [0,1]$.

Further, take an arbitrary $\alpha \in [0,1]$. We show that f_α is nondecreasing. Take $x \le z$. Then,

$$f_\alpha(x) = \sup\{y \in E \mid y \le x, R(x,y) \le \alpha\}.$$

We consider y such that $y \le x \le z$. Since R is a resemblance, $R(z,y) \le R(x,y) \le \alpha$. Therefore, $\{y \in E \mid y \le x, R(x,y) \le \alpha\} \subseteq \{y \in E \mid y \le z, R(x,z) \le \alpha\}$ and thus $f_\alpha(x) \le f_\alpha(z)$.

Conditions (A), (B), (C) and (D) are straightforward from the definition of f_α. □

Remark 5.3. Starting from a resemblance R on E, in the analogous way (due to the symmetry of R), for $\alpha \in [0,1]$ we can obtain another system of nondecreasing real functions:

$$h_\alpha(y) = \sup\{x \in E \mid x \le y, R(x,y) \le \alpha\}.$$

This system also fulfills the following :

- if $\alpha, \beta \in [0,1], \alpha < \beta$, then $h_\alpha \leq h_\beta$,
- $h_\alpha(y) \leq y$ for all $\alpha \in [0,1], y \in E$.
- If for any $x \in X$, $h_\alpha(x) < h_\beta(x)$, then $\alpha < \beta$.
- $h_1(x) = x$, for every x.

Using the foregoing families of functions connected to a resemblance R on E, we characterize its cut-relations $R_\alpha, \alpha \in [0,1]$, as follows.

Theorem 5.4. Let R be a resemblance on E such that it is continuous in the second coordinate. Let $\{f_\alpha \mid \alpha \in [0,1]\}$ and $\{h_\alpha \mid \alpha \in [0,1]\}$ be the families of functions defined in the previous proposition and in Remark. Then, for every $\alpha \in [0,1]$,

$$R_\alpha^+ = \{(x,y) \mid f_\alpha(x) < y \leq x\} \cup \{(x,y) \mid h_\alpha(y) < x \leq y\}.$$

P r o o f. Since R is symmetric, it is also continuous in the first coordinate.

Let $(x,y) \in R_\alpha^+$. Then $R(x,y) > \alpha$. Suppose that $y \leq x$. We prove that $f_\alpha(x) < y$. Indeed, by the definition of $f_\alpha(x)$, we have that $f_\alpha(x)$ is the supremum of elements y for which $R(x,y) \leq \alpha$. By the continuity, $R(x, f_\alpha(x)) \leq \alpha$. If $y \leq f_\alpha(x) \leq x$ would be true, then we would have $R(x, f_\alpha(x)) \geq R(x,y) > \alpha$, a contradiction. Therefore, $f_\alpha(x) < y$.

In case $x \leq y$ we would obtain analogously $(x,y) \in \{(x,y) \mid h_\alpha(y) < x \leq y\}$.

To prove the opposite inclusion, suppose that $(x,y) \in \{(x,y) \mid f_\alpha(x) < y \leq x\}$ (another case is proved analogously).

Suppose that $R(x,y) \leq \alpha$. Then, $y \leq f_\alpha(x)$ by the definition of f_α. Therefore, there must be $R(x,y) > \alpha$, and $(x,y) \in R_\alpha^+$. □

Next we show how to construct a resemblance on E, using a particular family of real functions. It turns out that such a family is already described by some of the properties listed in Theorem 5.2.

Theorem 5.5. Let $\{f_\alpha\}_{\alpha\in[0,1]}$ be a system of nondecreasing real functions such that $f_\alpha(x) \leq x$ for all $\alpha \in [0,1], x \in E$ and $f_\alpha \leq f_\beta$ for $\alpha < \beta$. Then

$$R(x,y) = \sup\{\alpha \in [0,1] \mid f_\alpha(\max\{x,y\}) < \min\{x,y\}\}$$

is a resemblance relation on E.

P r o o f. We show that R is a resemblance relation.

Assuming $\sup\emptyset = 0$ we see that R attains values in $[0,1]$, hence it is a fuzzy relation.

As the inequality $f_\alpha(x) \leq x$ holds for all α, we get

$$R(x,x) = \sup\{\alpha \in [0,1] \mid f_\alpha(x) < x\} = 1.$$

The symmetry $R(x,y) = R(y,x)$ follows from the symmetry of the functions max and min.

Now let us take $x, y, z \in E,\ x < y < z$. We show that $R(x,y) \geq R(x,z)$.

For our choice of x, y, z we have

$$\begin{aligned} R(x,y) &= \sup\{\alpha \in [0,1] \mid f_\alpha(y) < x\}, \\ R(x,z) &= \sup\{\alpha \in [0,1] \mid f_\alpha(z) < x\}. \end{aligned}$$

As each f_α is nondecreasing, $f_\alpha(z) < x$ implies $f_\alpha(y) < x$ and therefore $R(x,y) \geq R(x,z)$. This shows that R is a resemblance relation. □

The previous propositions show how it is possible to create a resemblance from a family of real functions and vice versa. A natural question is, whether starting from a system of functions, creating a resemblance (Theorem 5.5) and using it to generate a function system (Theorem 5.2) we return to the original one. In general the answer is negative, unless we expect an additional property of $\{f_\alpha\}_{\alpha\in[0,1]}$.

To simplify the formulation in the following proposition, let us call the resemblance created in Theorem 5.5 as a generated resemblance and the function system from Theorem 5.2 as a generated function system.

Theorem 5.6. Let $\{f_\alpha\}_{\alpha\in[0,1]}$ be a system of real functions with properties from Theorem 5.5.

Moreover, let the mapping $\alpha \mapsto f_\alpha(x)$ be continuous for every $x \in E$. Let R be a generated resemblance and let $\{g_\alpha\}_{\alpha\in[0,1]}$ be a function system generated by R. Then $f_\alpha = g_\alpha$ for all $\alpha \in [0,1]$.

Proof. Let us choose an arbitrary $x \in E, \alpha \in [0,1]$. We show that $f_\alpha(x) = g_\alpha(x)$.

From the Proposition 5.2 we obtain

$$g_\alpha(x) = \sup\{y \in E \mid y \leq x, R(x,y) \leq \alpha\}.$$

Take an arbitrary $\alpha \in [0,1]$. Using the formula for generated resemblance from Theorem 5.5 and restricting to $y \leq x$ we have

$$g_\alpha(x) = \sup\{y \in E \mid y \leq x, \sup\{\beta \in [0,1] \mid f_\beta(x) < y\} \leq \alpha\}.$$

We can notice that $f_\alpha(x)$ belongs to the family $\{y \in E \mid y \leq x, \sup\{\beta \in [0,1] \mid f_\beta(x) < y\} \leq \alpha\}$. Indeed, $f_\alpha(x) \leq x$ and $\sup\{\beta \in [0,1] \mid f_\beta(x) < f_\alpha(x)\} \leq \alpha$, since from $f_\beta(x) < f_\alpha(x)$ it follows that $\beta < \alpha$.

Now, we prove that $f_\alpha(x)$ is the supremum of this family. Suppose that there is y such that $f_\alpha(x) < y$ and $y \leq x$ and $\sup\{\beta \in [0,1] \mid f_\beta(x) < y\} \leq \alpha$.

Since $f_1(x) = x$ and $f_\alpha(x) < y < f_1(x)$, by the continuity, we have that there is γ, such that $f_\alpha(x) < f_\gamma(x) < y$. Now we have that $\alpha < \gamma$, which contradicts the fact that $\sup\{\beta \in [0,1] \mid f_\beta(x) < y\} \leq \alpha$. Hence, $f_\alpha(x) = g_\alpha(x)$. □

6. CONCLUSION

We have investigated some basic properties of resemblance relations, as a suitable tool for modeling some real-life problems. We have concentrated on cut and generally crisp properties of such relations, particularly in the case of resemblances on the real line. The framework in which these relations have been investigated are classical fuzzy relations with the co-domain being a unit interval $[0, 1]$. To generalize these settings we intend to deal with the lattice-valued case, which is from the point of view of cuts even more convenient for applications.

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