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OPTIMUM BEAM DESIGN VIA STOCHASTIC PROGRAMMING

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The purpose of the paper is to discuss the applicability of stochastic programming models and methods to civil engineering design problems. In cooperation with experts in civil engineering, the problem concerning an optimal design of beam dimensions has been chosen. The corresponding mathematical model involves an ODE-type constraint, uncertain parameter related to the material characteristics and multiple criteria. As a result, a multi-criteria stochastic nonlinear optimization model is obtained. It has been shown that two-stage stochastic programming offers a promising approach to solving similar problems. A computational scheme for this type of problems is proposed, including discretization methods for random elements and ODE constraint. An approximation is derived to implement the mathematical model and solve it in GAMS. The solution quality is determined by an interval estimate of the optimality gap computed by a Monte Carlo bounding technique. The parametric analysis of a multi-criteria model results in efficient frontier computation. Furthermore, a progressive hedging algorithm is implemented and tested for the selected problem in view of the future possibilities of parallel computing of large engineering problems. Finally, two discretization methods are compared by using GAMS and ANSYS.

Keywords: optimum engineering design, stochastic programming, multi-objective programming, Monte Carlo methods, progressive hedging algorithm

Classification: 90C15, 90C29, 65C05, 49M27

1. INTRODUCTION

Many optimum design problems in civil and mechanical engineering lead to optimization models constrained by differential equations. Specifically, shape-based optimization is recently under focus, e. g. [15]. There are advanced techniques how to deal with it, see [4]. In the case of real-world design problems, uncertain parameters that can be modelled by random elements are also involved, see e. g. [10] for civil engineering cases and [8] for mechanical engineering problems. In general, the problems can be modelled by stochastic optimal control formulations. However, there are several bottlenecks: (1) there are not enough input data to obtain realistic instances of models, (2) solution techniques often significantly vary even for small model changes, (3) theoretical features are often studied for a generalization rather than real-world problems. In addition, the decision process in the optimum design problems discussed is more stage-based than the continuously dynamic one. So, we have previously shown for artificial textbook cases ([17, 18]) that two-stage stochastic programming is also a promising approach for such problems as is for shape optimization, see [3] for the first original models. We learn from engineers that they are often satisfied enough with a significant improvement of the existing design, i. e., with suboptimal solutions. They also prefer widely applicable robust algorithmic schemes to efficient algorithms for specialised cases. Therefore, the model-based approximation with its quality verified by comparing the existing and suboptimal solutions is further chosen and the development of the computational scheme is illustrated by a fundamental "building-stone-like" engineering example. Before we proceed, we should emphasize that further steps improving the proposed scheme must follow. They can focus, e. g., on dealing with reliability terms [11] either directly or by new approximating penalty based methods, see [2].

2. PROBLEM STATEMENT AND THE UNDERLYING PROGRAM

An optimization problem in civil engineering describing the deflection of a beam has been recommended by specialists dealing with similar problems, see e. g. [10]. The optimization aims to obtain an optimal design of beam cross section dimensions while minimizing weight (1), maximizing rigidity (2) and minimizing deflection (3), as shown by the below model:

$$
\begin{aligned}
&\min \ \rho a b l && (1)\\
&\max \ \frac{E(\xi) a b^3}{12} && (2)\\
&\min \ v(\xi, x) && (3)\\
&\text{s.t.} \ E(\xi)\frac{ab^3}{12}\frac{\mathrm{d}^4 v}{\mathrm{d}x^4}(\xi, x) = h(x),\ x \in \langle 0, l\rangle && (4)\\
&\qquad v(\xi, 0) = 0,\ \frac{\mathrm{d}v}{\mathrm{d}x}(\xi, 0) = 0 && (5)\\
&\qquad v(\xi, l) = 0,\ \frac{\mathrm{d}v}{\mathrm{d}x}(\xi, l) = 0 && (6)\\
&\qquad \left| E(\xi)\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}(\xi, x)\frac{b}{2}\right| \le \sigma_{\mathrm{limit}} && (7)\\
&\qquad a_{\min} \le a \le a_{\max} && (8)\\
&\qquad b_{\min} \le b \le b_{\max}, && (9)
\end{aligned}
$$

where ρ is the beam density, l is the beam length, x is the related space coordinate, ξ is a random outcome, Ξ is a sample space, $E(\xi)$ is random Young's modulus (because of the varying uncertain material characteristics [11]), $h(x)$ is a load, a, b are decision variables (dimensions of the cross section) and $v(\xi, x)$ is a deflection. The ODE (4) describes the transverse deflection of the beam, the boundary conditions for clamped end points given by (5) and (6) mean that there are zero transverse deflections and their slopes. Furthermore, maximum stress $\sigma_{\max}$ given as $\sigma_{\max}(x) = \pm E\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}(x)\frac{b}{2}$ must be bounded for safety reasons. The limiting value σ_{limit} relates to the proportional limit which marks the end of the area of elastic behaviour described by

Hooke's law where the stress is proportional to the relative deformation [6], see constraint (7). Finally, the dimensions of the beam cross section must be bounded, see (8) and (9). The underlying program (1) – (9) is syntactically correct but its semantics is not discussed, as is usual in stochastic programming (see [13]). Therefore, the beam problem is not solved from the stochastic-optimal-control point of view. A deterministic reformulation is further defined in Section 3 so that the mean value of objective functions (1) – (3) is taken and constraints (4) – (7) are almost surely satisfied. This model-based approximation satisfies the requirements of the model robustness and solution suboptimality, see Section 1.

3. TWO–STAGE STOCHASTIC PROGRAM

The model-based approximation of model (1) – (9) is carried out in two steps. First, a scenario-based approach for a random variable approximation is used, see [13]. We assume that random variables $E(\xi)$ and $v(\xi, x)$ have discrete probability distributions with a finite number R of equiprobable scenarios $E(\xi_s)$ and $v(\xi_s, x)$ with probabilities $p_s = P(\{\xi_s\}) = \frac{1}{R}$, respectively. The second step consists in discretizing of the space coordinate x in objective functions and constraints. Following the recommendation of [1], we use a simple finite difference method [7] with a uniform grid spacing for $N+1$ points: $x_i = id$, $i = 0, \ldots, N$, $d = \frac{l}{N}$. Derivatives are replaced by central difference formulas and, after some simplification, difference equations are obtained.

For multiple objectives, we employ a weighted-sum approach typically used in multi-objective optimization [14].

Hence, the underlying program (1) – (9) is approximated by a large deterministic nonlinear program:

$$\min_{a,b,\mathbf{V}_s} \left(-\alpha \sum_{s=1}^{R} p_s \frac{E_s a b^3}{12 c_{\text{rigid}}} + \beta \frac{\rho a b l}{c_{\text{weight}}} + \gamma \sum_{s=1}^{R} \sum_{i=0}^{N} p_s \frac{V_{s,i}}{c_{\text{defl}}} \right) \tag{10}$$

$$\text{s.t.} \quad a b^3 \mathbb{K} E_s \mathbf{V}_s = \mathbf{f},\ s = 1, \ldots, R \tag{11}$$

$$V_{s,0} = 0,\ V_{s,N} = 0,\ s = 1, \ldots, R \tag{12}$$

$$b \mathbb{C} E_s \mathbf{V}_s \le d^2 \sigma_{\text{limit}} \mathbf{g},\ s = 1, \ldots, R \tag{13}$$

$$a_{\min} \le a \le a_{\max} \tag{14}$$

$$b_{\min} \le b \le b_{\max}, \tag{15}$$

$$\text{where } \mathbb{K} = \begin{pmatrix} 7 & -4 & 1 & 0 & 0 & \ldots & 0 \\ -4 & 6 & -4 & 1 & 0 & \ldots & 0 \\ 1 & -4 & 6 & -4 & 1 & \ldots & 0 \\ & & & \vdots & & & \\ 0 & \ldots & 1 & -4 & 6 & -4 & 1 \\ 0 & \ldots & 0 & 1 & -4 & 6 & -4 \\ 0 & \ldots & 0 & 0 & 1 & -4 & 7 \end{pmatrix},\ \mathbf{f} = \begin{pmatrix} 12 d^4 h_1 \\ \vdots \\ 12 d^4 h_{N-1} \end{pmatrix},$$

$$\mathbb{C}=\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -2 & 1 & 0 & \dots & 0 \\ 1 & -2 & 1 & \dots & 0 \\ & & \vdots & & \\ 0 & \dots & 1 & -2 & 1 \\ 0 & \dots & 0 & 1 & -2 \\ 0 & \dots & 0 & 0 & 1 \end{pmatrix}, \ \mathbf{g} = \begin{pmatrix} 1 \\ 2 \\ 2 \\ \vdots \\ 2 \\ 2 \\ 1 \end{pmatrix}.$$

α, β, $\gamma > 0$ are the weighting coefficients, $\alpha + \beta + \gamma = 1$ and $c_{\text{rigid}}, c_{\text{weight}}, c_{\text{defl}}$ are the typical values of rigidity, weight and deflection of the beam (i. e. normalizing constants). These values are obtained as the optimal values of the objective functions of three single-objective optimization problems. $\mathbf{V}_s = (V_{s,1}, \dots, V_{s,N-1})^T$ is the approximation of $v(\xi, x)$ and $E_s = E(\xi_s)$, $s = 1, \dots, R$.

4. RESULTS AND SOLUTION QUALITY

The results are presented for input data and the related formulas carefully discussed with specialists. For better scaling, we do not compute with SI units but use units common in engineering computations instead, i. e. length is given in mm (milimeters), weight is given in t (tons) and stress is given in MPa (megapascals). The load is quadratic: $h(x) = -80\frac{x^2}{l^2} + 80\frac{x}{l}$, the length of steel beam is $l = 1000$ mm with density $\rho = 7.85 \cdot 10^{-9}$ tmm^{-3}. The stress limit is $\sigma_{\text{limit}} = 100$ MPa. Number of grid points is $N = 50$, we assume $R = 100$ scenarios and the bounding values of beam dimensions are $a_{\min} = b_{\min} = 10$ mm, $a_{\max} = b_{\max} = 100$ mm. The weighting coefficients are chosen as: $\alpha = 0.3$; $\beta = 0.45$; $\gamma = 0.25$. We assume a random Young's modulus: $E_s = 2 \cdot 10^5$ MPa $+ E_{\text{random},s}$ where $E_{\text{random},s} \sim U(-1 \cdot 10^4, 5 \cdot 10^4)$ MPa. The randomness of Young's modulus E can be caused by different types of heat–treating processing of steel such as normalization, soft annealing, annealing etc. Program (10) – (15) is implemented in GAMS with the CONOPT solver and ran on a laptop with Intel Core 2Duo 2GHz and 2GB RAM. The optimal objective function value is $z = 2.13$. The optimal dimensions are $a = 22.4$ mm, $b = 100$ mm and we use it as a candidate solution $\hat{\mathbf{a}} = (a, b)^T$ for a Monte Carlo bounding technique.

It is important to assess the quality of the solution $\hat{\mathbf{a}}$. We use a Monte-Carlo-bounding-technique concept to determine the solution quality proposed by Morton et al. [9]. Therefore, we estimate the optimality gap as a measure of the solution quality:

$$G(\hat{\mathbf{a}}) = \min_{\mathbf{v}(\xi)} \mathbb{E}\{F(\xi, \hat{\mathbf{a}}, \mathbf{v}(\xi))\} - \min_{\mathbf{a},\, \mathbf{v}(\xi)} \mathbb{E}\{F(\xi, \mathbf{a}, \mathbf{v}(\xi))\} \tag{16}$$

where $\mathbb{E}\{F(\xi, \mathbf{a}, v(\xi))\}$ denotes the objective function value of (10) – (15) and $\mathbf{v}(\xi)$ denotes a random vector with realizations $\mathbf{V}_s$, $s = 1, \dots, R$ and probabilities p_s, $s = 1, \dots, R$ as discussed in the previous section. The gap is estimated by averaging, i. e., n_g samples from $E(\xi)$ each having size n ($E(\xi^{ij})$, $i = 1, \dots, n$, $j = 1, \dots, n_g$) are generated. Then the point estimate $\bar{G}_{n,n_g}(\hat{\mathbf{a}})$ of $G(\hat{\mathbf{a}})$ is:

$$\bar{G}_{n,n_g}(\hat{\mathbf{a}}) = \frac{1}{n_g} \sum_{j=1}^{n_g} \left[\min_{\mathbf{v}(\xi)} \frac{1}{n} \sum_{i=1}^{n} F(\xi^{ij}, \hat{\mathbf{a}}, \mathbf{v}(\xi^{ij})) - \min_{\mathbf{a}, \mathbf{v}(\xi)} \frac{1}{n} \sum_{i=1}^{n} F(\xi^{ij}, \mathbf{a}, \mathbf{v}(\xi^{ij})) \right] \tag{17}$$

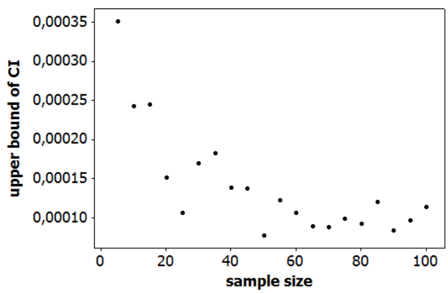
with the $(1-\alpha)$-level confidence interval for the optimality gap being given as follows:

$$G(\hat{\mathbf{a}}) \in \left[0, \bar{G}_{n,n_g}(\hat{\mathbf{a}}) + \frac{t_{1-\alpha}(n_g-1)s_{n_g}(\hat{\mathbf{a}})}{\sqrt{n_g}}\right], \tag{18}$$

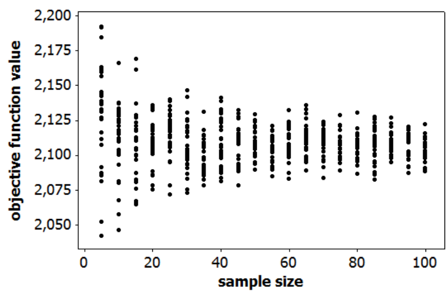
where $t_{1-\alpha}(n_g-1)$ is the $(1-\alpha)$-quantile of t-distribution with n_g-1 degrees of freedom and $s_{n_g}(\hat{\mathbf{a}})$ is the sample standard deviation.

The number of batches is $n_g = 30$ and we repeatedly increase the sample size ($n = 5, 10, \ldots, 100$) to see the behaviour of the optimality gap. The CPU time was about 66 min for PC with AMD Sempron 1.5 GHz and 496 MB RAM.

An engineer using the technique in question can see how the width of the confidence interval of the optimality gap roughly decreases with an increasing sample size and our candidate solution approaches the solution of the true optimization problem (see Figure 1 a)). Furthermore, it can be seen in Figure 1 b) how the variation of the objective function values for a fixed sample size also decreases with an increasing sample size.



a)



b)

Fig. 1. a) Upper bound of confidence interval of optimality gap,
b) Dispersion of objective function values versus sample size.

5. EFFICIENT FRONTIER AND PARAMETRIC ANALYSIS

As the next step, we discuss model (10) – (15) from the multi-objective viewpoint with an optimal solution being replaced by the concepts of efficient points and efficient frontier [14]. A feasible solution to a multi-objective optimization model is an efficient point if no other feasible solution scores at least as well in all objective functions and strictly better in one. The entire set of efficient points for the model is the efficient frontier. The set of points on the efficient frontier can be computed by repeated optimization of (10) – (15) for various values of α, β, γ. New constraints enforcing the achievement levels for all but one criterion (see (2), (3)) are added and the remaining criterion related to (1) is treated as a single-objective function. In our case, we add the following two constraints: $\mathbb{E}\left(\frac{E(\xi)ab^3}{12}\right) \geq \vartheta$, $\mathbb{E}\left(\mathbf{v}(\xi)\right) \leq \omega$ where the parameters ϑ and ω are varied within the range of the relevant rigidity and deflection values ($\vartheta \in (1.3 \cdot 10^{11}; 1.8 \cdot 10^{12})$ Nmm^2, $\omega \in (0.7; 9.4)$ mm).

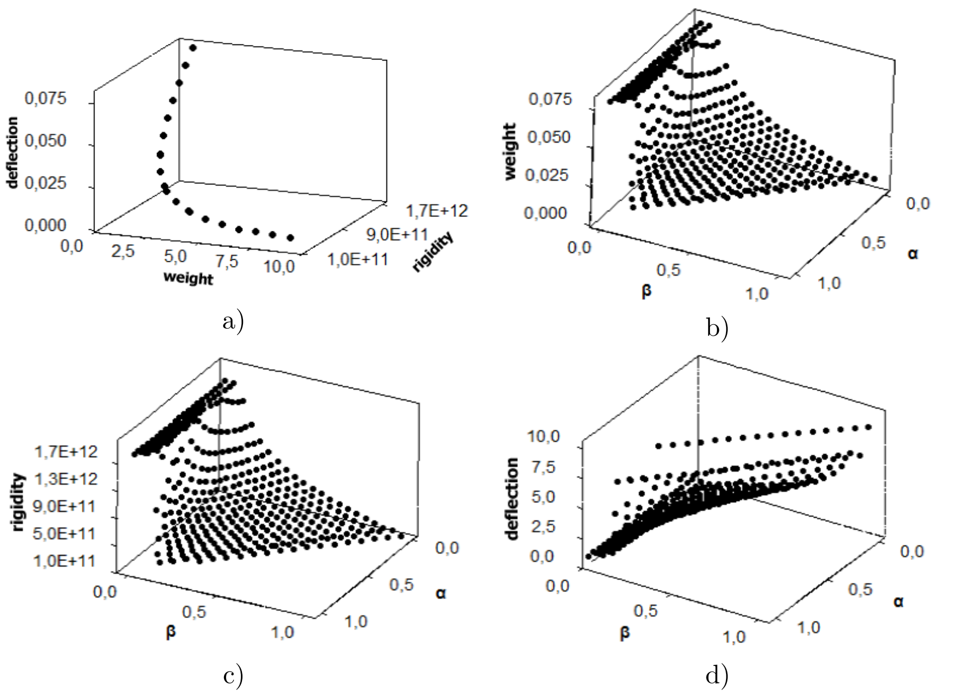


Fig. 2. a) Efficient frontier, b) – d) Beam weight, rigidity and deflection versus weighting coefficients α, β.

The efficient frontier for our three-objective problem degenerates into a curve (see Figure 2 a)) because of the impact on the feasible region of the physical characteristics involved. This means that different efficient points produce the same point in the objective function value space.

Since we employ the weighted-sums approach in our problem (10) – (15), we are interested in a parametric analysis with respect to the weighting coefficients typically required by engineers. The weighting coefficient α is varied from 0 to 1 by increments of 0.05. The weighting coefficient β is varied from 0 to $1-\alpha$ and γ is computed as $\gamma = 1-\alpha-\beta$. For every value of α excluding the last one ($\alpha = 1$), we have 20 values of β and γ.

Figure 2 b) – d) shows the effect of the weighting coefficients α and β on the optimal values of rigidity, weight and deflection. Figure 3 shows the same for optimal values of dimensions a and b. The effect of the weighting coefficients is qualitatively the same for rigidity, weight and dimension a.

We have obtained two extreme solutions and many intermediate solutions by varying the weighting coefficients:

- $a = 10\,\text{mm}$, $b = 89.4\,\text{mm}$ for $\beta = 1-\alpha$, $\alpha \in \langle 0; 0.75\rangle$ (see Figure 4 a))
  This solution corresponds to minimum weight, maximum deflection and minimum rigidity.

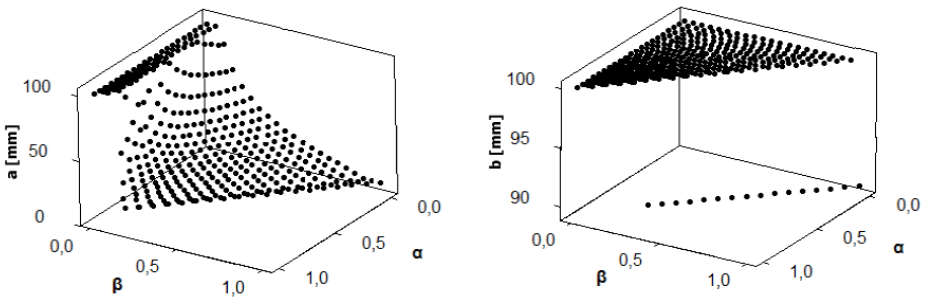
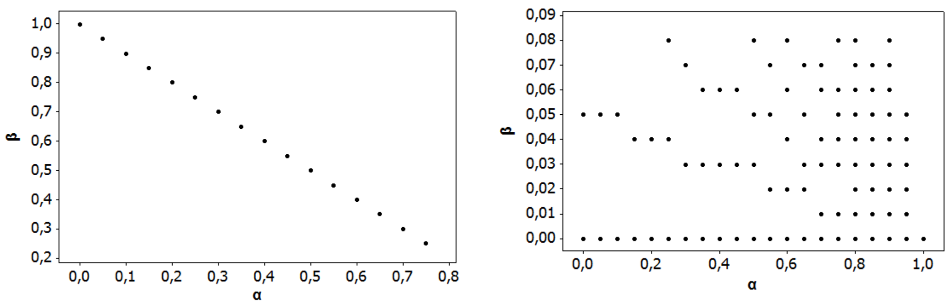


Fig. 3. Beam dimensions a and b versus weighting coefficients α, β.



a) b)

Fig. 4. Relationship between α and β for dimensions
a) $a = 10\,\mathrm{mm}$, $b = 89.4\,\mathrm{mm}$, b) $a = 100\,\mathrm{mm}$, $b = 100\,\mathrm{mm}$.

- $a = 10\,\mathrm{mm}$, $b = 100\,\mathrm{mm}$ for $\beta = 1 - \alpha$, $\alpha \in \langle 0.8; 0.9\rangle$ or $\beta = -0.95\alpha + 0.95$, $\alpha \in \langle 0; 0.85\rangle$ or $\beta = -0.91\alpha + 0.9$, $\alpha \in \langle 0; 0.35\rangle$ (see Figure 5 a))
  This is an intermediate solution. The second and third equations for β have been obtained by regression.
- $a \in (10; 100)\,\mathrm{mm}$, $b = 100\,\mathrm{mm}$ otherwise
  These are also intermediate solutions (see Figure 3).
- $a = 100\,\mathrm{mm}$, $b = 100\,\mathrm{mm}$ for $\beta = 0$, $\alpha \in \langle 0; 1\rangle$ or $\beta \in (0; 0.08\rangle$, α varying (see Figure 4 b))
  This solution corresponds to maximum weight, minimum deflection and maximum rigidity.

6. PROGRESSIVE HEDGING ALGORITHM

As complex engineering problems lead to large optimization models, we have tested using parallel computational techniques for the test beam problem. The Progressive Hedging Algorithm (PHA) proposed by Rockafellar and Wets [12, 16] has been

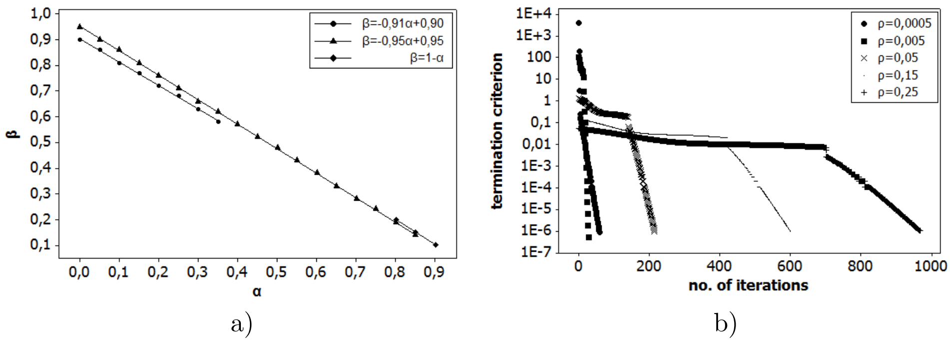


Fig. 5. a) Relationship between α and β for dimensions $a = 10$ mm, $b = 100$ mm, b) Convergence of the termination criterion value.

chosen. It is a decomposition method and includes nonanticipativity constraints in the objective function as a penalty term. The advantage of this algorithm is that we obtain a separable program whose independent scenario subprograms can be solved in parallel. Let us denote $\mathbf{a} = (a, b)^T = (a_1, a_2)^T$. The structure of the PHA for our selected example is the following:

Step 0: Set $\mathbf{w}_s^{(0)} = \mathbf{0}$, choose $\hat{\mathbf{a}}^{(0)}$, penalty parameter $\rho > 0$, tolerance ε, set $k = 1$.

Step 1: For all $s = 1, \ldots, R$ solve the approximation program:

$$\min_{\mathbf{a}, \mathbf{V}_s} F_s(\xi_s, \mathbf{a}, \mathbf{V}_s) + (\mathbf{w}_s^{(k-1)})^T \mathbf{a} + \frac{\rho}{2} \|\mathbf{a} - \hat{\mathbf{a}}^{(k-1)}\|^2$$

where $F_s(\xi_s, \mathbf{a}, \mathbf{V}_s)$ is the objective function value of s^{th} scenario subprogram of (10) – (15). We denote the optimal solution as $\mathbf{a}_s^{(k)}$.

Step 2: Compute the estimate:

$$\hat{\mathbf{a}}^{(k)} = \sum_{s=1}^{R} p_s \mathbf{a}_s^{(k)}$$

and update the weight vector:

$$\mathbf{w}_s^{(k)} = \mathbf{w}_s^{(k-1)} + \rho(\mathbf{a}_s^{(k)} - \hat{\mathbf{a}}^{(k)}).$$

Step 3: If the termination inequality $\|\hat{\mathbf{a}}^{(k)} - \hat{\mathbf{a}}^{(k-1)}\|^2 + \sum_{s=1}^{R} p_s \|\mathbf{a}_s^{(k)} - \hat{\mathbf{a}}^{(k)}\|^2 \leq \varepsilon$ defined by [5] is satisfied then the solution $\hat{\mathbf{a}}^{(k)}$ is optimal with the given tolerance ε, otherwise set $k = k + 1$ and return to step 1.

The values of the parameters are the same as in Section 4 excepting the number of scenarios. We have tested PHA with $R = 10$ scenarios instead of 100 scenarios to avoid excessive computational complexity for the testing non-parallel implementation. The initial estimate for the dimensions is $\hat{\mathbf{a}}^{(0)} = (100; 100)$ mm, which corresponds to maximum rigidity. The tolerance is set to $\varepsilon = 10^{-6}$ because it roughly conforms to the accuracy of one decimal place in length, which is fully sufficient in engineering practice.

The optimal dimensions are $a = 22.5\,\text{mm}$, $b = 100\,\text{mm}$. It can be seen from the Figures 5 b), 6 and Table that the penalty parameter ρ plays the key role for the computational process convergence properties of the algorithm. Unfortunately, there is no exact rule to determine the best value of this parameter ρ. We have estimated that, for our example, the best value lies in the interval (0.001; 0.01). For larger values of ρ the convergence process will take much more time.

Table. Convergence properties of PHA.

Parameter ρ	**No. of iterations**	**CPU time** [min]
0.0005	59	1
0.001	32	0.7
0.005	28	0.5
0.01	50	1
0.05	215	4
0.1	410	9
0.15	599	15
0.2	784	22
0.25	966	38

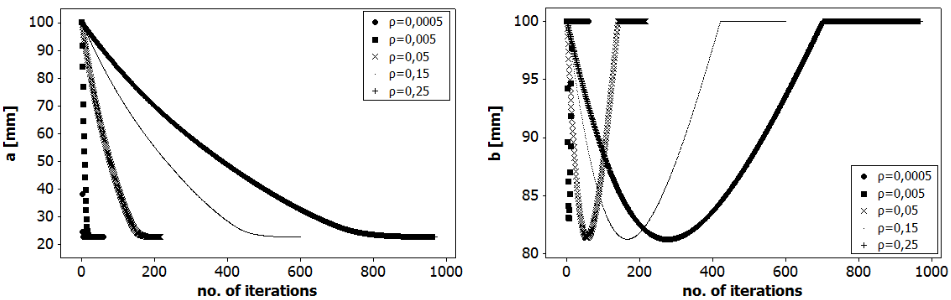


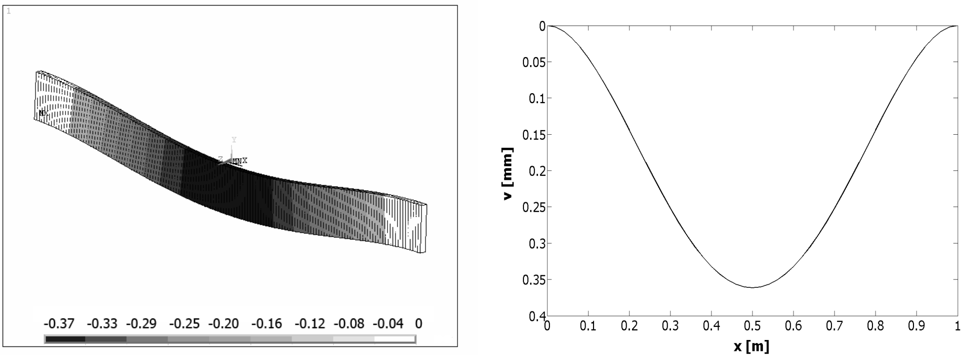
Fig. 6. Convergence of the beam dimensions a and b.

7. COMPARISON OF FDM AND FEM – CASE STUDY

We have been asked by the potential users of the proposed computational scheme whether our approach with a simple discretization method and the algebraic modelling system GAMS provides results comparable with the results from black-box-like systems widely used by engineers. We have compared the GAMS implementation involving a finite difference method (FDM) and the ANSYS 11.0 (Ansys Inc., Canonsburg, PA, USA) model based on a finite element method (FEM). This comparison is made for a deterministic version of our civil engineering optimization problem (1) – (9) with $\alpha = \beta = 0.5$ and $E = 2.1 \cdot 10^5$ MPa.

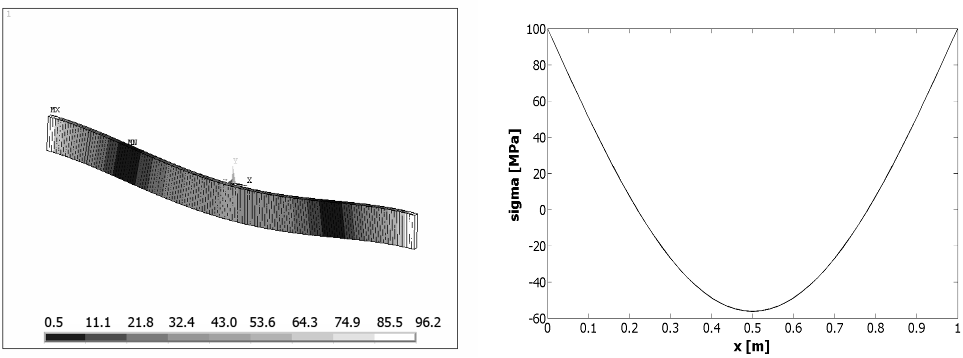
The optimal solution obtained by GAMS with FDM is: $a = 10$ mm, $b = 89.4$ mm, $z = 0.47$, the optimal solution from ANSYS with FEM is: $a = 11.2$ mm, $b = 84.5$ mm, $z = 0.49$. The GAMS results are slightly better but there is only a small difference between them and the results computed by ANSYS.

The deflection in optimized cases is quantitatively and qualitatively the same for both computing systems and discretization methods (see Figure 7). The maximum deflection of 0.37 mm occurs in the middle of the beam while it decreases towards the beam ends. Also the maximum stress in optimized cases is quantitatively and qualitatively nearly the same for both computing systems and discretization methods (see Figure 8). The difference is only in signs – the absolute value of stress is plotted in ANSYS while both positive and negative values are plotted in GAMS. A maximum tensile stress of about 100 MPa occurs at the ends of the beam while a maximum compression stress of about -54 MPa occurs in the middle of the beam.



a) b)

Fig. 7. Deflection computed by a) ANSYS and b) GAMS.



a) b)

Fig. 8. Maximum stress computed by a) ANSYS and b) GAMS.

8. CONCLUSIONS

The applicability of a two-stage stochastic programming approach to civil-engineering-optimum-design problems with random parameters involved has been discussed. A recommended beam design problem has been used as a test case. Thanks to a modelling-based approximation approach focusing on suboptimal solution search we can avoid difficulties with the huge amounts of input data required and problems with the implementation of various algorithms that often appear in real-world applications of stochastic optimal control-related models. The choice of the model is suitable for the prototype case implementations in such modelling languages as GAMS and for further parallel computations by using scenario decomposition as the PHA. The solution quality can be tested by the Monte-Carlo technique presented. An FDM gives acceptable results for the test example by presented results, however, it can be replaced by an FEM or FVM (finite volume method) in future discretization schemes for advanced cases. In general, the computational scheme proposed (scenario-based two-stage stochastic program, modelling language implementation, parallelization, solution quality evaluation, verification of results by the FEM solver) seems sufficiently robust for future applications to similar and advanced optimum design problems. There is also a future challenge to motivate engineers to use this approach because they may still prefer the black-box-like computing system with an appropriate preprogrammed mathematical model chosen (i. e., with no equations describing the physical behaviour necessary).

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