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**MOMENTS AND MOMENT INVARIANTS  
IN PATTERN RECOGNITION**

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Thanks to thorough psychological research, we already know that about 80 % of information entering the human mind is accepted by optical channels, i. e., by our sight. From the point of view of this fact, the analysis of visual information, like pattern recognition, remote sensing, computer vision, virtual environment design, their specialized sub-branches and applied methodology in two or three dimensions appear extremely important for the progress of information sciences and information technology.

Nevertheless, the above fields of research are very complicated. They deal with complex phenomena and multilaterally structured processes, as well as with massive multidimensional data of various types whose processing is connected with enormous computational complexity. It means that the ability to cope with visual information by information technology tools depends, namely, on two branches of its methodological support - on the existence of lucid and manageable formal theoretical concepts adequate to the essence of the modeled phenomena, and on the power of actual information technology, used for their practical processing. The progress in these two directions was not simple, and it was usually realized in small, however numerous, steps described in a great number of specialized research papers. The publication of a comprehensive summary of the present state of art was waiting for its authors till now. It regards, namely, the significant tools and methods of visual information processing with special stress on the theory and application of moments and moment invariants.

It is possible to say that the referred monograph bridges the gap mentioned above and offers, in its eight chapters, a representative summary of the approach to the moment-based processing of visual information and its special parts like pattern recognition or computer vision.

The seven main chapters of the volume, numbered as Chapter 2 – Chapter 8, follow after brief *Preface* and compendious Chapter 1, *Introduction to Moments*, offering a brief overview of basic concepts and their motivation. They can be very roughly grouped in two clusters. The first one, Chapters 2–5, deals rather with fundamental concepts of the theory, and their properties, meanwhile the second cluster, consisting of the remaining three chapters, is oriented to their advanced or more specialized derivatives, partly motivated by applications. The volume is finished by condensed *Conclusion*, and the *Index*.

Regarding the orientation of particular chapters, then Chapter 2, *Moment Invariants to Translation, Rotation and Scaling*, characterizes mainly the moment invariants connected with the simplest spatial transforms, named it its heading, and other important classical types of invariants. Their presentation is followed by the description of general method based on complex moments. It is also shown, that the basis of invariants being independent and complete for their description exists and it is relatively small.

The previous simple spatial transformations are, in Chapter 3, *Affine Moment Invariants*, extended by affine transforms of spatial coordinates. The attention is focused on three main methods leading to the extraction of their moment invariants, the method based on Cayley–Aronhold extraction, graph method, and the method of normalized moments. Effective methods for the elimination of reducible and dependent invariants are described, and their properties are derived.

The next Chapter 4, *Implicit Invariants to Elastic Transformations*, is devoted to a novel concept of invariants, called “implicit invariants” to deformations. This concept is formally specified and its properties are analyzed. It is shown that the implicit moment invariants significantly enlarge the class of pattern recognition problems pertinent to the image deformations which can be processed by the methods based on moments and moment invariants.

Essentially different kind of moment invariant is shown and studied in Chapter 5. Its heading, *Invariants to Convolution*, well characterizes its topic. It is focused on the invariants to convolution/blurring, and the main results presented in this chapter characterize the invariants with respect to image blur and the method of their derivation regardless of the convolution kernel, provided that they display certain degree of symmetry. Moreover, the concept of so called combined invariants, which is derived in this chapter, as well, enables to recognize objects in the degraded scene without any restoration.

The remaining chapters form the second of the clusters mentioned above. The first one of them, Chapter 6 headed *Orthogonal Moments*, includes, analyses and comments various types of moments named in its heading, as well. In general, two main types of them are distinguished – those being orthogonal on rectangle, and those orthogonal on unit disc. In the second part of this chapter, the problem of image reconstruction from its moments is demonstrated and the better adequacy of the orthogonal moments in comparison with the geometric ones is explained.

The computing complexity of the moment invariants and, naturally, of the moments themselves, is discussed and analyzed in Chapter 7, *Algorithms for Moment Computation*. Both problems of complexity, the one of moment invariants and the complexity of moment computation, are closely interconnected, and both of them are essential for the efficiency and applicability of the moment-based pattern recognition. It is shown that various algorithms for moment calculation form two major groups of methods. The methods attempting to decompose the object into non-overlapping regions of simple shape, and those evaluating the double integral over the object by means of Green’s Theorem. Both of these groups and their properties are characterized and discussed. The chapter is completed by effective algorithms for calculation of geometric and selected orthogonal moments for binary and gray-level objects.

The last chapter, numbered as Chapter 8 and named *Applications*, is fully characterized by its heading. It is devoted to interesting and inspirational examples of the applications of moments and moment invariants in image analysis and recognition. They illustrate the theoretical concepts and results by means of their use in a wide scale of practical problems covering such fields like image registration, object recognition, medical imaging, content-based image retrieval, focus/defocus measurement, forensic investigation, robot navigation or digital watermarking.

As indicated above, already, the referred volume is the first comprehensive and widely drawn monograph devoted to the moment invariants and moment-based approach to the image processing and pattern recognition methods. This fact predestinates the book to respect several principal target groups of potential readers. First, it is to address the post-graduate students (and, strictly speaking, to give a hand to their teachers) and, in the case of the first chapters, also to the pre-graduate ones. Second, it is expected to serve as a comprehensive handbook for specialists (including the advanced ones) in the visual information processing and pattern recognition, especially when the recognition is connected with moment-based methods. Finally, it would be disposable even for specialists in others, but related, fields of research, wishing to find specific but clearly formulated facts in the field of interest of this volume. The monograph deserves to say that it excellently copes with all of these aims.

This result was achieved thanks to several principles, respected by the authors. The topics included in particular chapters cover recent state of art of the relevant themes, and the chapters include all essential knowledge regarding the purpose of the book. The presentation of particular topics is clear and it consequently respects the mathematical style in the theoretical parts. Nevertheless, illustrative examples allow good understanding of the explained concepts, and well demonstrate their heuristic meaning.

Moreover, the organization of the chapters and sections respects the needs of the target groups of readers. It is structured in usually short and naturally ordered units, so that the field of interest can be easily found. Moreover, even the lists of relevant references are inserted at the end of every chapter which method simplifies the reading of the book, namely for those who need to get acquainted with selected segment of the monograph, only. The references are well chosen and representative for the topics treated in every chapter. Also the items of the Index concluding the volume are selected with a good idea about the needs of the target group of readers.

Summarizing the previous paragraphs, it can be concluded that the referred monograph represents a significant and, exactly speaking, hardly fungible contribution to the literature on visual information processing. It will be useful not only for specialist and students being deeply interested in this field of information sciences but also for anybody who comes into occasional contact with it and needs a handbook for easy orientation in the problems and concepts mentioned in the heading of this volume.

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