A GENERAL SYNCHRONIZATION METHOD OF CHAOTIC COMMUNICATION SYSTEMS VIA KALMAN FILTERING

Suojun Lu and Liang Chen

A general synchronization method is proposed for a class of nonlinear chaotic systems involving uncertain parameters and nonlinear transmitted signals. Under some mild conditions, it shows that the class of nonlinear chaotic systems can be treated as linear time-varying systems driven by the additive white noise contaminated at the receiver, or the observed output. Synchronization can be achieved by using Kalman filtering technology. We present some sufficient conditions under which the states of the driven system are able to track the states of the drive system asymptotically, and good tracking performance can be obtained in the presence of the additive white noise involved in the observed output.

Keywords: chaotic system, secure communication, synchronization, uncertain parameters, Kalman filtering

AMS Subject Classification: 37D45

1. INTRODUCTION

Synchronization of chaos refers to a process wherein two (or many) chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behavior due to a coupling or to a forcing (periodical or noisy). This ranges from complete agreement of trajectories to locking of phases. In the case of applications to secure communications, a global system is formed by two subsystems, which realize a drive-response (or master-slave) configuration. This implies that one subsystem evolves freely and drives the evolution of the other. As a result, the response system is slaved to follow the dynamics (or a proper function of the dynamics) of the drive system, which, instead, purely acts as an external but chaotic forcing for the response system.

In the context of coupled chaotic elements, many different synchronization states have been studied in the past decade, including complete or identical synchronization (CS) [9], phase (PS) [10] and lag (LS) synchronization [11], generalized synchronization (GS) [7], intermittent lag synchronization (ILS) [4], imperfect phase synchronization (IPS) [15], and almost synchronization (AS) [6]. The natural continuation of these pioneering works was to investigate synchronization phenomena in spatially extended or infinite dimensional systems [3], to test synchronization in experiments or natural systems [5], and to study the mechanisms leading to desynchronization [2]. Recently, chaos synchronization in large-scale complex dynamical networks has received more and more attention and made some great achievements [8, 14].

CS was the first discovered and is the simplest form of synchronization in chaotic systems. It consists in a perfect hooking of the chaotic trajectories of two systems, that is, they remain in step with each other in the course of the time. When the states of the chaotic system are estimated using the Extend Kalman Filtering (EKF), the process within the filter synchronizes to the transmitter (drive system), provided that the estimated states converge to the states of the drive system. The synchronization is robust, and is relatively insensitive to variations in transmitter parameters and to additive noise. However the performance of the EKF is dependent on linearization, and on the magnitude of the signal variations about the equilibrium. That is, if the nonlinear system is close to the linear one, or the signal variations about the equilibrium are relatively small, good performance for EKF can be expected. Otherwise little can be said about its performance.

In this paper we propose a different synchronization method. Motivated by the work of EKF approach [12], we investigate a class of nonlinear chaotic systems where the uncertain parameters are present, and the nonlinearities are due to the output variable. For secure communications, the system output is the transmitted signal, different from the true symbols for communication, which need be estimated based on the received signal. Hence the class of nonlinear chaotic systems is quite general, provided that the bandwidth is large enough for transmission. We will show that under some mild assumptions, the class of nonlinear chaotic systems can be treated as linear time-varying systems, contaminated by the additive white noise at the receiver. The synchronization is then approached via optimal filtering to which the results of Kalman filtering can be applied. Even though linearization is not used, linear techniques are applicable in our proposed synchronization method. We also present some sufficient conditions under which the states of the driven system are able to track the states of the drive system asymptotically, and good tracking performance can be guaranteed in the presence of the additive white noise involved in the observed output. The well-known Lorenz chaotic system is employed to illustrate our proposed synchronization method. This paper is organized as follows. In Section 2, we formulate the problem in detail. Section 3 describes how to design the slave system to achieve synchronization via Kalman Filtering. In Section 4, we take the Lorenz system as an example, and give an illustrative simulation. Finally, we conclude in Section 5.

2. PROBLEM FORMULATION

This paper will focus on synchronization in chaotic communication systems with a new approach under a new problem formulation. The class of nonlinear chaotic systems under consideration has the following form:

$$\dot{x}(t) = F[s(t)]\,x(t) + G[s(t)]\,\theta, \quad s(t) = H[x(t)], \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $s(t) \in \mathbb{R}^n$ is the output signal, $\theta \in \mathbb{R}^p$ is the vector of the uncertain parameters associated with the chaotic nonlinear system, F, G and H are vector-valued functions. We make the following assumption on the state $x(t)$:

Assumption 1. The nonlinear functions $F(\cdot)$, $G(\cdot)$ and $H(\cdot)$ are smooth, $x(t) \in B(\mathbb{R}^n)$ is a bounded closed subset in $\mathbb{R}^n$ including the origin.

The continuity assumption is standard, and has no loss of generality for applications to chaos-based communications. Because state variables are bounded for chaotic nonlinear systems, the boundedness assumption is also valid. For secure communications, the binary message signals are modulated by the uncertain parameter vector θ, which is associated with the states of the chaotic system. The output $s(t)$ is the transmitted signal. At the receiver,

$$y(t) = s(t) + \eta(t) = H[x(t)] + \eta(t), \tag{2}$$

where $\eta(t)$ is the observation noise. It is assumed that $\eta(t)$ is a white process, described by

$$\mathrm{E}[\eta(t)] = 0, \quad \mathrm{E}[\eta(t)\eta'(t)] = D(t)\,D'(t)\,\delta(t-\tau), \tag{3}$$

where $\mathrm{E}[\cdot]$ denotes the expectation, and $\delta(t)$ is the Dirac delta function, which is zero for $t = 0$, and $\int_{-\infty}^{\infty} \delta(t)\,\mathrm{d}t = 1$.

Due to the chaotic behavior of the nonlinear system, the modulated signals embedded in θ are difficult to detect, which are desirable for secure communications. On the other hand, it also increases the difficulty level for the receiver to detect the modulated signal contained in θ, which usually requires estimation of the state vector $x(t)$. Such an approach to signal detection is termed as synchronization. That is, the state vector of the receiver (the slave system) is required to track the state vector of the transmitter (the master system). Because the state trajectories are dependent on the uncertain parameter vector θ, once the synchronization is achieved at the receiver, the parameter vector θ is also available at the receiver. Hence the objective of synchronization is to reconstruct the uncertain parameter vector θ based on the received signal $y(t)$ as in Eq. (2). For this purpose, we rewrite Eq. (1) as

$$\dot{x}(t) = F[y(t)]\,x(t) + G[y(t)]\,\theta + e_s(t), \tag{4}$$

where by Eq. (2), $e_s(t)$ is given by

$$e_s(t) = \{F[s(t)] - F[y(t)]\}\,x(t) + \{G[s(t)] - G[y(t)]\}\,\theta.$$

In general, it is difficult to find an explicit expression of $e_s(t)$ in terms of the observation noise $\eta(t)$. We thus make the following assumption:

Assumption 2. $\mathrm{E}[e_s(t)\,e_s'(\tau)] \le B_s(t)\,B_s'(t)\,\delta(t-\tau) \quad \forall\, x(t) \in B(\mathbb{R}^n)$.

The above tends to yield a conservative estimate for $B_s(t)$ due to the lack of the true information on the state vector. Thus, effort should be made to obtain a more accurate estimate of $B_s(t)$ whenever possible. Our last assumption is the linearity of the transmitted signal as follows.

Assumption 3. $s(t) = H[x(t)] = Hx(t)$.

Assumptions 1, 2, and 3 allow us to represent the nonlinear chaotic system (1) in a linear form. Indeed, since θ is a constant vector, $\dot{\theta} = 0$. Combined with Eq. (4), we obtain an augmented state equation:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} F[y(t)] & G[y(t)] \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix} + e_a(t), e_a(t) = \begin{bmatrix} e_s(t) \\ 0 \end{bmatrix}, \tag{5}$$

and a output equation:

$$y(t) = \begin{bmatrix} H & 0 \end{bmatrix} = \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix} + \eta(t), \tag{6}$$

where $e_s(t)$ satisfies Assumption 2, and $\eta(t)$ is as in Eq. 3). Define $x_a(t) = \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix}$, $A(t) = \begin{bmatrix} F[y(t)] & G[y(t)] \\ 0 & 0 \end{bmatrix}$, $C(t) = \begin{bmatrix} H & 0 \end{bmatrix}$. By the fact that $y(t)$ is the received signal at time t, the nonlinear system described by Eq. (5) and Eq. (6) is equivalent to the following linear time-varying system

$$\dot{x}_a(t) = A(t)\, x_a(t) + e_a(t), \quad y(t) = C(t)\, x_a(t) + \eta(t), \tag{7}$$

by treating $e_a(t)$ as the process noise, and $\eta(t)$ as the observation noise. It follows that synchronization amounts to estimation of $x(t)$ and θ, such that the estimation error converges to zero asymptotically in the noise-free case, and error variance is minimized in the case of noisy measurements. For convenience we modify the state space model (7) into the following linear time-varying system:

$$\begin{aligned} \dot{x}_a(t) &= A(t)\, x_a(t) + B(t)\, v(t), \\ y &= C(t)\, x_a(t) + D(t)\, v(t), \end{aligned} \tag{8}$$

where $v(t)$ is a white noise process with covariance identity. Although the two systems in (7) and (8) are different from each other, their statistical properties are similar due to the assumptions on $\eta(t)$ and $e_s(t)$. More importantly, synchronization problem for the system (7) can be approached via Kalman filtering to the system (8) if the optimal solution exists.

In the following, we provide an example to illustrate our problem formulation.

Example. Consider the well-known Lorenz system described by the following equations:

$$\begin{cases} \dot{x}_1 = c(x_2 - x_1), \\ \dot{x}_2 = -x_1 x_3 + r x_1 - x_2, \\ \dot{x}_3 = x_1 x_2 - b x_3, \end{cases}$$

where r is the parameter, and b, c are constants. It has been proven that the Lorenz system exhibits chaotic behavior when $b = 10,\ c = 8/3$, and r is in the interval of

28 to 32. If $s(t) = x_1(t)$ is the transmitted signal, and $y(t) = x_1(t) + \eta(t)$ is the received signal with $\eta(t)$'s white noise of variance σ^2, then our problem formulation Eq. (4) leads to:

$$\dot{x}(t) = \begin{bmatrix} -c & c & 0 \\ 0 & -1 & -y(t) \\ 0 & y(t) & -b \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ y(t) \\ 0 \end{bmatrix} \theta + \begin{bmatrix} c \\ x_3(t) - r \\ -x_2 \end{bmatrix} \eta(t), \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(t) + \eta(t)$$

with $\theta = r$. Suppose it is known that $|x_3(t)| \leq M_3$, $|x_2(t)| \leq M_2$, and $|r| \leq M$. It follows that the Lorenz system can be written into the same form as the linear time-varying system (8) with

$$A(t) = \begin{bmatrix} -c & c & 0 & 0 \\ 0 & -1 & -y(t) & y(t) \\ 0 & y(t) & -b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B(t) = \begin{bmatrix} 0 \\ -r + x_3(t) \\ x_2(t) \\ 0 \end{bmatrix} \sigma, \tag{9}$$

$$C(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \qquad D(t) = \sigma,$$

where $e_a(t) = B(t)\,v(t)$ and

$$B(t)\,B'(t) \leq \begin{bmatrix} 0 \\ M_r + M_3 \\ M_2 \\ 0 \end{bmatrix} \sigma^2 \begin{bmatrix} 0 & M_r + M_3 & M_2 & 0 \end{bmatrix}.$$

3. SYNCHRONIZATION VIA KALMAN FILTERING

We consider design of the receiver or slave system to achieve synchronization via Kalman filtering. That is, we intend to design an optimal state estimator for the linear time-varying system (7), or (8), such that the error variance is minimum. A critical issue is the convergence of the estimation error in the noise-free case. We begin with the following stability notions.

Definition. Consider the linear time-varying state space equation (8) with the initial condition $x(0) = x_0 \neq 0$. Let $\Phi_A(t, \tau)$ be the associated transition matrix satisfying

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_A(t,\tau) = A(t)\,\Phi_A(t,\tau), \quad \Phi_A(\tau,\tau) = I, \quad \Phi_A = (t,t_0) = \Phi_A(t,\tau)\,\Phi_A(\tau,t_0),$$

where $t \geq \tau \geq t_0$. Then in the noise-free case, i.e, $e_a(t) = 0$, $v(t) = 0$, the solution to the state space equation (8) is given by $x(t) = \Phi_A(t,0)\,x_0$. If $\Phi_A(t,0) \to 0$ as $t \to \infty$, it is called asymptotically stable. If

$$\|\Phi_A(t,0)\| \leq \alpha\gamma^{\beta t}, \quad \alpha > 0, \quad \beta > 0, \quad |\gamma| < 1,$$

Eq. (8) is called exponentially stable.

As discussed above, synchronization amounts to estimation of the state vector as well as the uncertain parameter vector, which are the augmented state vector in Eq. (7). We instead consider the use of the standard state estimator as the receiver for system (8), which has the form:

$$\dot{\hat{x}}_a(t) = A(t)\,\hat{x}_a(t) + L(t)[y(t) - \hat{y}(t)] = [A(t) - L(t)C(t)]\,\hat{x}_a(t) + L(t)\,y(t). \quad (10)$$

The matrix gain $L(t)$ is called state estimation gain, to be designed. The following result is immediate.

Proposition. Consider the state-space system (8) for the noise free case. If there exists a stabilizing state estimation gain $L(t)$ such that $A(t) - L(t)C(t)$ is exponentially stable, then for the noise-free case, the receiver (10) achieves the synchronization asymptotically with exponential convergence.

Proof. Because $L(t)$ exists and is exponentially stabilizing, taking the difference between (8) and (10) gives the error system:

$$\dot{e}(t) = [A(t) - L(t)C(t)]\,e(t) + [B(t) - L(t)\,D(t)]\,v(t) = A_L(t)\,e(t) + B_L(t)\,v(t), \quad (11)$$

where $v(t) = 0$ for the noise-free case, and $e(t) = x_a(t) - \hat{x}_a(t) = \Phi_{A_L}(t,0)\,e(0)$. It follows from the exponential stabilization for the state estimation gain, the error system (11) is exponentially stable for the noise-free case, and $e(t)$ converges to zero exponentially.

Although Proposition provides some guidance to the design of the synchronizer for the noise-free case, it does not have much application because it does not provide a design procedure of the exponential stabilizing state estimation gain. In this regard, Kalman filtering is more suitable for the linear time-varying system (8). Thus, we consider the state estimation gain designed from the DRE (differential Riccati equation) as below:

$$\left\{\begin{array}{rcl} \dot{P}(t) & = & A(t)\,P(t) + P(t)\,A'(t) + B(t)\,B'(t) - L(t)\,D(t)\,D'(t)\,L'(t), \\ L(t) & = & [P(t)C'(t) + B(t)\,D'(t)][D(t)\,D'(t)]^{-1}, \\ P(0) & \geq & 0. \end{array}\right. \quad (12)$$

The above $L(t)$ is called Kalman gain. The next theorem is our main result of this section. □

Theorem. Consider the state estimator in Eq. (10) for the state-space model (8). Suppose that $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, $D(\cdot)$ and $[D(t)\,D'(t)]^{-1}$ are continuous functions, $\delta I \leq B(\cdot)\,B'(\cdot) \leq \Delta I$ for some $\Delta \geq \delta > 0$, and $t \in [0,T]$. Then the DRE in Eq. (12) admits a bounded and continuous solution $P(t) \geq 0, t \in [0,T]$, for each finite $T > 0$, and each initial condition $P(t) \geq 0$. Let $L(t)$ be the state estimation gain designed as in Eq. (12). If the limit $P(T) \geq 0$ exists and is bounded as $T \to \infty$, then the error system (11) is asymptotically stable.

P r o o f. The existence of bounded and continuous solution $P(t) \geq 0$ to the DRE (12) with initial condition $P(0) \geq 0$ is standard. See for example [1]. We notice that the DRE can be rewritten as a differential Lyapunov equation

$$\dot{P}(t) = A_L(t)\,P(t) + P(t)\,A_L'(t) + B_L(t)\,B_L'(t),$$

where $A_L(t) = A(t) - L(t)C(t)$, $B_L(t) = B(t) - L(t)\,D(t)$. Hence, $P(t)$ is also the "controllability gramian" for the error system (11). That is, if $\Phi_{A_L}(t,\tau)$ is the transition matrix associated with the error system (11), the solution $P(t)$ to the DRE has the expression

$$P(t) = \int_0^t \Phi_{A_L}(t,\tau)\,B_L(\tau)\,B_L'(\tau)\,\Phi_{A_L}'(t,\tau)\,\mathrm{d}\tau.$$

The hypothesis $\delta I \leq B(\cdot)\,B'(\cdot) \leq \Delta I$ implies that $\delta \int_0^t \Phi_{A_L}(t,\tau)\,\mathrm{d}\tau \leq P(t) \leq \Delta \int_0^t \Phi_{A_L}(t,\tau)\,\Phi_{A_L}'(t,\tau)\,\mathrm{d}\tau$ for all $t \in [0,T]$, which in turn implies that $\Phi_{A_L}(t,\tau)$ $\Phi_{A_L}'(t,\tau)$ is bounded for all t,τ such that $0 \leq \tau \leq t \leq T$, by the continuity of $A(\cdot), B(\cdot), C(\cdot)$ and $P(\cdot)$. Because the limit $P(T)$ is bounded as $T \to \infty$, $\Phi_{A_L}(T,\tau) \to 0$ for each finite $\tau \geq 0$. Consequently for the noise-free case, the state vector of the error system (11) satisfies

$$\lim_{T\to\infty} e(T) = \lim_{T\to\infty} \Phi(T,0)\,e_0 = 0$$

for any nonzero initial condition $e(0) = e_0 \neq 0$, concluding that the error system (11) is asymptotically stable.

By Kalman filtering theory, the state estimator (10) designed for the state space model (8) is optimal in the presence of white and Gaussian noise, if the state estimation gain $L(t)$ is computed according to Eq. (12). In this case the error variance is given by

$$\mathrm{E}[e'(t)\,e(t)] = \operatorname{trace} \mathrm{E}[e'(t)\,e(t)] = \operatorname{trace} P(t),$$

which is minimum among all linear estimators. Although there is a difference between the state space model (8) from the state space model (7) in the presence of the noise, the state estimator (10) designed for Eq. (8) also works for Eq. (7). Indeed for the noise free case ($v = 0$), the error system (11) has the same expression no matter Eq. (7) or Eq. (8) is used. Hence we have the following result for which the proof is omitted. □

Corollary. Given the state space model (7) and the state estimator (10), the error system can be expressed as

$$\dot{e}(t) = [A(t) - L(t)C(t)]\,e(t) + [e_a(t) - L(t)\,D(t)\,v(t)]. \tag{13}$$

Suppose that the hypotheses in Theorem holds. Then $e(T) \to 0$ as $T \to \infty$ for the noise-free case. If the noise is present, then the error variance $\mathrm{E}[e'(t)\,e(t)] \leq \operatorname{trace} P(t) = \int_0^t \Phi_{A_L}(t,\tau)\,B_L(\tau)\,B_L'(\tau)\,\Phi_{A_L}'(t,\tau)\,\mathrm{d}\tau$, provided that Assumptions 1, 2, and 3 are true.

It should be clear that the asymptotic stability for the error system (13) is equivalent to asymptotic synchronization in the noise-free case, which has been investigated intensively in a number of nonlinear chaotic systems [13]. Our results obtained in this section are quite general, and can be applied to a class of nonlinear chaotic systems satisfying Assumptions 1, 2, and 3. The error variance for state estimation is also a measure for the synchronization error in the average sense. Ideally the smaller the covariance $P(t)$, the better the synchronization. However, due to the requirement that $B(t)\,B'(t) > 0$ for all $t \geq 0$ in order to ensure asymptotic stability as in Theorem, there is a trade-off between the performance, and asymptotic stability.

4. AN ILLUSTRATIVE SIMULATION EXAMPLE

As an illustrative example, we take the Lorenz system as the drive system:

$$\begin{cases} \dot{x}_1 = c(x_2 - x_1), \\ \dot{x}_2 = -x_1 x_3 + r x_1 - x_2, \\ \dot{x}_3 = x_1 x_2 - b x_3, \end{cases}$$
$$y(t) = x_1 + \eta(t).$$

Compared with [13] in which two signals x_1, x_2 are transmitted, we have only $s(t) = x_1(t)$ as the transmitted signal. Let $x_1(t) = y(t) - \eta(t)$, and assume that $\mathrm{E}[\eta(t)\eta'(t)] = \sigma^2\delta(t)$. It was shown in Section 2 that the Lorenz system has the form

$$\begin{aligned} \dot{x}_a(t) &= A(t)\,x_a(t) + B(t)\,v(t), \\ y(t) &= C(t)\,x_a(t) + D(t)\,v(t), \end{aligned}$$

where $A(t)$, $B(t)$, $C(t)$ are given as in Eq. (9), and $D(t) = \sigma$. It has been proved that the Lorenz system is chaotic under $c = 10,\ b = 8/3$, and with r varying between 28 and 32. Following the results in Section 3, we form the driven system (Kalman filter):

$$\dot{\hat{x}}_a(t) = A(t)\hat{x}_a(t) + L[y(t) - \hat{y}(t)], \quad \hat{y}(t) = C(t)\hat{x}_a(t),$$

where $\hat{x}_a(t) = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 & \hat{r} \end{bmatrix}'$, $L(t) = [P(t)C'(t) + B(t)\,D'(t)][D(t)\,D'(t)]^{-1}$, and $\dot{P}(t) = A(t)\,P(t) + P(t)\,A'(t) + B(t)\,B'(t) - L(t)\,D(t)\,D'(t)\,L'(t)$.

We use the bounds described in Section 2, and white noise with $\sigma^2 = 0.01$. The time response of the state $x_1(t)$, the parameter r, their corresponding estimation $\hat{x}_1(t), \hat{r}$ and the estimation error e_{x_1}, e_r are shown in Figure.

5. CONCLUSION

In this paper we propose a new method for synchronizing two chaotic systems based upon Kalman filtering at the receiver. Using the present method, the states of the driven system track the states of the drive system asymptotically under some mild assumptions. In comparison with the existing methods, our method provides a general design procedure for a class of nonlinear chaos systems, guaranteeing the

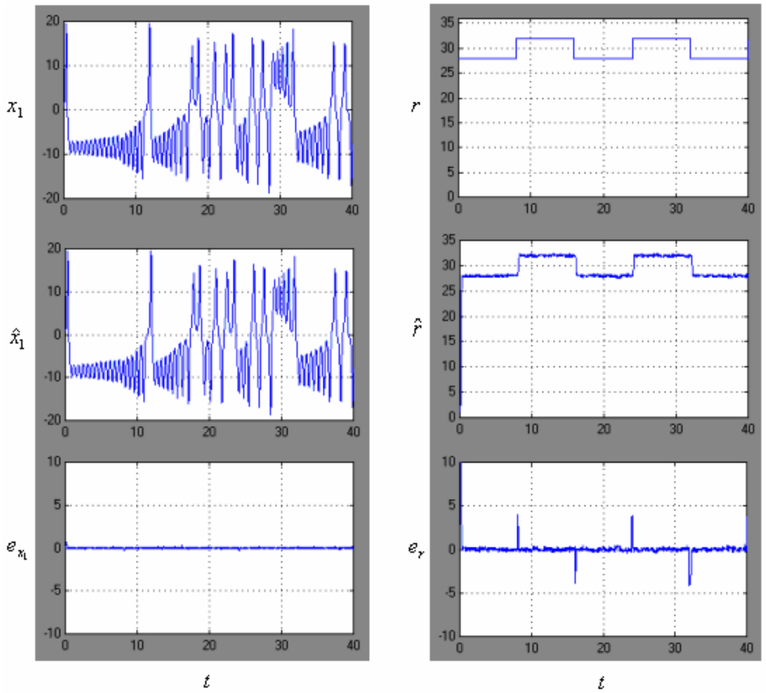

Fig. Time response of the state $x_1(t)$, the parameter r, their corresponding estimation $\hat{x}_1(t)$, $\hat{r}$ and the estimation error e_{x_1}, e_r.

asymptotic convergence of the tracking error to zero in the noise-free case. The Kalman filtering theory is used to estimate all the states and the uncertain parameters. As the estimates converge, the driven system (receiver) synchronizes to the drive system (transmitter). A Lorenz system is given to validate the proposed approach. A drawback for our proposed method is the synchronization performance. Due to the lack of theoretical work on performance of the Kalman filtering for linear time-varying systems in the existing literature, good performance for state estimation, and thus for synchronization is difficult to achieve, which is currently under investigation.

(Received July 19, 2007.)

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