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DESIGN OF REACHING PHASE FOR VARIABLE STRUCTURE CONTROLLER BASED ON HOUSEHOLDER TRANSFORMATION

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In this paper a design of reaching phase for variable structure controller with sliding mode of an uncertain dynamic system based on Householder transformation method is considered. The proposed method reduces the number of switching gain vector components and performs satisfactorily while the external disturbance component does not satisfy the matching conditions. Subsequently the stability of the global system is studied and furthermore, the design of switched gain matrix elements based on fuzzy logic approach provides useful result for smooth control actions. The efficacy of the proposed method is demonstrated by considering a load-frequency control problem of interconnected power system.

Keywords: switching function, reachability, Householder transformation, variable structure control, fuzzy logic, interconnected power systems, Lyapunov function

AMS Subject Classification: 93B12, 93C42, 93D05, 93D15

1. INTRODUCTION

Variable-Structure Control (VSC) with sliding modes is widely recognized in control research community due to its insensitiveness to parametric uncertainties and disturbances [10]. Sliding-mode control design generally involves two main steps: firstly, the selection of a sliding surface which induces a stable reduced-order dynamics assigned by the designer, and secondly the synthesis of a switching control law to force the closed-loop system trajectories onto and subsequently remain on the sliding surface. It is powerful in controlling the system with bounded unknown disturbance and can provide very robust performance and transient performance [2, 3]. The system state trajectory in the period before reaching the sliding surface is known as the reaching phase in the control literature. A comprehensive guide on sliding-mode control for control engineers is given in [7] and most salient feature of variable structure sliding mode control is that it is completely robust to matched uncertainties. However, there are even more systems which unfortunately are affected by mismatched uncertainties and do not enjoy nice matching condition. Thus the system behavior in the sliding mode is not invariant to the mismatched uncertainty; the system performance can not be assured insensitive due to parametric uncertainties. Other remarkable advantages of sliding mode control approach are the simplicity of its implementation and the order-reduction of the closed loop system [4]. Pole assignment or Linear Quadratic (LQ) techniques are often used as a component of sliding mode control. However, to be fair, one should also point out two foremost assumptions in the application of VSC with sliding mode control. First, most of the design techniques for sliding-mode control assume that all the system states are accessible to the control law and second, assume the possible occurrence of real sliding mode (chattering phenomena) instead of the ideal one. However, since a discontinuous control action is involved, chattering will take place and the steady-state performance of the system will be degraded. To overcome this problem, numerous schemes have been reported in the literature and one of the most common techniques to alleviate this drawback is to introduce a boundary layer about the sliding plane [9]. The basic idea consists of introducing a boundary layer of the switching surface in which the control law is chosen to a continuous approximation of the discontinuous function when the trajectory of system is inside the boundary layer. A robust sliding mode controller design based on the derivative of control instead of the control itself reduces the effect of chattering. However, the boundary approach provides no guarantee of convergence to the sliding mode and involves a trade off between chattering and robustness. Reduced chattering may be achieved without sacrificing robust performances by combining the attractive features of fuzzy control with sliding mode control [1].

In this paper, we shall discuss how to design a reaching phase based on Householder method with a state-feedback control law. The control law consists of linear feedback term plus a discontinuous term, which guarantee that the sliding mode exits and is globally reachable under a very mild restriction. This paper extends the work of White et al.[11] in order to design a simple sliding mode with variable structure controller based on Householder method. This in turn reduces the number of switching gain vector components as compared to White et al.[11] method and moreover, for the non-switched gain vector components no additional inequality constrains are required to drive the state trajectory into the sliding surface. The significant advantage of the proposed method is addressed for full/reduced switching control gains. A fuzzy logic approach is also adopted in order to avoid hard switching control gains and subsequently the corresponding smooth control signals ensure the reaching conditions and decrease the reaching phase time.

This paper is organized as follows. In Section 2, a mathematical description of the problem is given. Reaching phase design technique, based on Householder method is developed in Section 3. Subsequently, the stability of the sliding mode state trajectories is studied in the same section. In Section 4, design of an equivalent switch gain matrix based fuzzy logic approach is considered. In Section 5, the effectiveness of the proposed VSC control scheme based on Householder method is demonstrated by considering the load-frequency control problem of interconnected power systems. Section 6 provides a brief conclusion.

2. PROBLEM FORMULATION

Consider a linear time-invariant system described by

$$\dot{X}(t) = AX(t) + BU(t) \tag{1}$$
$$Y(t) = CX(t) \tag{2}$$

where $X(t)$ = state vector $\in \Re^{n\times 1}$, $U(t)$ = input vector $\in \Re^{m\times 1}$ and $Y(t)$ = output vector $\in \Re^{p\times 1}$. It is assumed that the system is completely controllable. All the states are directly measurable and the linear system is assumed to be in regular form and the state equation (1) explicitly is described by following pair of equations:

$$\dot{X}_1(t) = A_{11}X_1(t) + A_{12}X_2(t) \tag{3}$$
$$\dot{X}_2(t) = A_{21}X_1(t) + A_{22}X_2(t) + B_2U(t) \tag{4}$$

where $X_1(t) \in \Re^{(n-m)\times 1}$, $X_2(t) \in \Re^{m\times 1}$, $B = \begin{bmatrix} 0 & B_2 \end{bmatrix}^T$ and $B_2 \in \Re^{m\times m}$. If the original system is not in a form of equations (3) and (4), it is required to transform the system (1) into a regular form by using a linear transformation matrix [4].

Before we propose the new VSC based on Householder method, a brief discussion on sliding surface is given below.

$$\sigma = SX(t). \tag{5}$$

With no loss of generality, we can rewrite the equation (5) in more explicit form

$$\begin{aligned}\sigma_{m\times 1}(t) &= S_1X_1(t) + S_2X_2(t)\\ &= S_1X_1(t) + X_2(t)\end{aligned} \tag{6}$$

where $S_1(t) \in \Re^{m\times(n-m)}$, $S_2 \in \Re^{m\times m}$ with $S_2 = I_{m\times m}$. If the system state trajectory is on the sliding surface,

$$\sigma(t) = S_1X_1(t) + X_2(t) = 0$$

and, thus

$$X_2(t) = -S_1X_1(t). \tag{7}$$

Substituting equation (7) into equation (3), we get

$$\dot{X}_1(t) = (A_{11} - A_{12}S_1)X_1(t). \tag{8}$$

It can be noted that the reduced order dynamics of equation (8) on the sliding surface is independent of control input $U(t)$ and exhibits a state feedback structure where S_1 and A_{12} represent a 'state feedback' matrix and an 'input' matrix, respectively. If the system (A_{11}, A_{12}) is stabilizable, it is possible to find the optimal control law, a 'feedback' control gain S_1, such that the control law minimizes performance index

$$J = \int_0^\infty \left[X_1^TQX_1 + X_2^TRX_2\right] \mathrm{d}t \tag{9}$$

where the lower limit of the integration refers to the initiation of sliding, $Q \geq 0$ and $R > 0$. This optimal gain S_1 minimizes index J and asymptotically stabilizes $X_1(t)$. It is needless to state that the system exhibits desirable dynamical behaviour when its trajectory is confined to the sliding surface ($\sigma = SX = 0$). A necessary condition for the system state trajectory to remain on the sliding surface $\sigma = 0$ is $\dot{\sigma}(X,t) = 0$ and the equivalent control for the nominal system has the form

$$U_{\text{eq}} = -(SB)^{-1}SAX(t) = -K_{\text{eq}}X(t). \tag{10}$$

Then equivalent control gain 'K_{eq}' can then be obtained from the above equation and the closed-loop system ($A - BK_{\text{eq}}$) having same ($n-m$) eigenvalues as that of reduced order system (8) and remaining 'm' eigenvalues are at equilibrium point. For the system (3), it is assumed that the control law

$$U(t) = U_f(t) + U_s(t) = -K_fX(t) - \Delta K_sX(t) \tag{11}$$

is employed with the choice of fixed control gain K_f (with $\Delta K_s = 0_{m\times n}$) such that the closed-loop system has $(n-m)$ eigenvectors lying with in the null space of S and the remaining eigenvectors will exhibit the range-space dynamics of S. On the other hand, the role of switched dynamically gain vector ΔK_s is to maintain a switching function σ as close to zero as possible and also to drive the state vector into the null space of S. Consider a linear uncertain dynamic system described by the following state space form

$$\dot{X}(t) = (A+\Delta A)X(t) + (B+\Delta B)U(t) + \Gamma d(t) \tag{12}$$

$$Y(t) = CX(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} X(t) \tag{13}$$

where $X(t) \in \Re^{n\times 1}$ is the measured current value of the state, $U(t) \in \Re^{m\times 1}$ is the control function, $Y(t) \in \Re^{p\times 1}$ is the output of the system, $d \in \Re^{r\times 1}$ is the external unknown constant disturbance vector bounded by $\|d\| \le d_{\max}$, A, B, Γ, C, are constant matrices with appropriate dimensions, with B of full rank, and the matrices ΔA, ΔB represent uncertainty of the system matrix and input matrix, respectively.

Assumption I.

(i) (Matching Conditions) There exist matrices of appropriate dimensions F and E such that [6, 12]

$$\Delta A = BF, \quad \Delta B = BE, \quad \|E\| \le \mu < 1 \tag{14}$$

are satisfied, then the sliding mode is invariant due to parameter perturbation. The physical meaning of matching condition (14) is that all parameter uncertainties enter the system through the control input matrix or channel. The constraint imposed on E is to ensure that the level of the uncertainty ΔB is not so large. It is assumed that the external disturbance component does not satisfy the matching condition.

(ii) the pair (A, B) is completely controllable.

Assume that a sliding mode control is employed for controlling the system under structural assumption, all uncertain elements can be lumped and the system (12) can be written as

$$\dot{X}(t) = AX(t) + BU(t) + B\eta_p(t) + f_d(t) \tag{15}$$

where $f_d(t) = \Gamma d(t)$ and $\eta_p \in \Re^{m\times 1}$ represents the system total uncertainty or total perturbation [11] and it is given by

$$\eta_p(t,X) = FX(t) + EU(t). \tag{16}$$

Solely based on the knowledge of the bound on the uncertainty, we consider the assumption given below.

Assumption II.

There are positive constants c_0 and c_1 such that [12]

$$\|\eta_p(t,X)\|_2 \le c_0 + c_1\|X\|_2 = \rho(t,X) \quad \text{for all } (t,X) \tag{17}$$

where $\rho(t,X)$ is the upper bound of the norm $\|\eta_p(t,X)\|_2$, c_0 and c_1 are estimated by solving a pair of differential equations and it is discussed in Section 3.

We now consider the system (15) that satisfying the assumption II and assume $X(t)$ be the solution of (15) at 't' forced by the input $(U(t), \eta_p(t)\cdot f_d(t))$. The basic stability condition question is: find a control strategy $U(t, X(t))$ such that the system has a sliding mode and the origin is uniformly asymptotically stable in the large.

SMC design is broken down into two phases. The first phase involves constructing a switching surface so that the system restricted to the switching surface produces a desired behavior. For convenience, it is assumed that the system (15) is in regular form

$$\begin{aligned}\dot{X}_1(t) &= A_{11}X_1(t) + A_{12}X_2(t) + f_{d1} \\ \dot{X}_2(t) &= A_{21}X_1(t) + A_{22}X_2(t) + B_2(U+\eta_p) + f_{d2} \qquad (18)\\ Y(t) &= \begin{bmatrix} C_1 & C_2 \end{bmatrix}\left[\frac{X_1(t)}{X_2(t)}\right] \qquad (19)\end{aligned}$$

where $f_d = \begin{bmatrix} f_{d1}^T & f_{d2}^T \end{bmatrix}^T$. It is to be noted that the first part of the external disturbance vector f_{d1} directly affects the state $X_1(t)$ even after the system states are on the sliding mode. This in turn drives the system states away from the sliding surface and finally system response deviates from the desired behavior.

Associated Control Law. In this section, we present the new sliding surface as

$$\begin{aligned}\sigma_{m\times 1}(t) &= SX(t) + W\int_0^t (Y(t) - Y_{\text{ref}}(t))\mathrm{d}t \\ &= SX(t) + WZ(t) \\ &= \begin{bmatrix} S_1 & S_2 \end{bmatrix}\left[\frac{X_1(t)}{X_2(t)}\right] + WZ(t) \\ &= \begin{bmatrix} S_1 & W & S_2 \end{bmatrix}\begin{bmatrix} X_1(t) \\ Z(t) \\ X_2(t) \end{bmatrix} \\ &= S_aX_a(t) \end{aligned} \tag{20}$$

and the corresponding control law that drives the states on the sliding surface is given by

$$U(t) = -(K_{af} + \Delta K_{as}) X_a(t) + V_p^* + V_f^* \tag{21}$$

where the choice of $K_{af}$is in such a way so that $(n+p-m)$ eigenvectors of augmented closed-loop system (using equation (21)) are in the null space of S_a. The switched gain matrix ΔK_{as} maintains switching vector σ as close to zero as possible. The terms V_p^* and V_f^* represent the additional control terms to suppress the effect of uncertainty and external disturbance. In addition to the switching gain matrix ΔK_{as}, the terms V_p^* and V_f^* drive the system trajectories toward the switching surface until the trajectory hits the sliding surface.

Consider the augmented system and it is described by using the equations (18)–(20)

$$\begin{aligned}\dot{X}_a(t) &= \begin{bmatrix} A_{11} & 0 & A_{12} \\ C_1 & 0 & C_2 \\ A_{21} & 0 & A_{22} \end{bmatrix} X_a(t) + \begin{bmatrix} 0 \\ 0 \\ B_2 \end{bmatrix} U(t) + \begin{bmatrix} 0 \\ 0 \\ B_2 \end{bmatrix} \eta_p(t) + \begin{bmatrix} f_{d1} \\ 0 \\ f_{d2} \end{bmatrix} \\ &= A_a X_a(t) + B_a U(t) + B_a \eta_p(t) + f_{ad} \end{aligned} \tag{22}$$

where $X_a(t) = \begin{bmatrix} X_1^T(t) & Z^T(t) & X_2^T(t) \end{bmatrix}_{n+p}^T$ and $Y_{\text{ref}} = 0$. Using the expression (21) in equation (22) the dynamic model of the closed-loop system is

$$\dot{X}_a(t) = A_{ac} X_a(t) - B_a \Delta K_{as} X_a(t) + B_a V_p^* + B_a V_f^* + B_a \eta_p(t,\ X_a(t)) + f_{ad} \tag{23}$$

where $A_{ac} = (A_a - B_a K_{af})$, and $f_{ad} = \begin{bmatrix} f_{d1}^T & 0 & f_{d2}^T \end{bmatrix}^T$. As we have mentioned earlier that the selection of K_{af} is made in such a way so that $(n+p-m)$ eigenvectors of A_{ac} are placed in the null space of S_a with $S_2 = I_{m \times m}$ and the matrix K_{af} can be calculated using the following expression

$$S_{ai} A_{ac} = \lambda_{ri} S_{ai}, \quad i = 1, 2, \ldots, m$$

where $S_a = \begin{bmatrix} S_{a1}^T & S_{a2}^T & \cdots & S_{am}^T \end{bmatrix}_{m \times m}^T$ is assumed to be the left eigenvectors of the matrix A_{ac} corresponding to the eigenvalues $\lambda_{r1},\ \lambda_{r2},\ \ldots,\ \lambda_{rm}$ respectively. Switching surface S_a is designed by following the steps as discussed in Section 2 (from equations (5)–(9)). It can be noted that the matrix K_{af} can be determined in such away so that the range space $(n+p-m)$ eigenvalues of the A_{ac} are placed at desired locations and the corresponding distinct left eigenvectors of A_{ac} are within the null space of S_a. So, the state $X(t)$ lying in the null space of S_a implies that $\dot{X}(t)$ will also lie in the null space.

3. REACHING PHASE DESIGN BASED ON HOUSEHOLDER TRANSFORMATION TECHNIQUE

In this design technique, a suitable Householder transformation is selected in such a way so that the sliding surface equation (20) can be rewritten as

$$H\sigma(t) = HS_aX_a(t) \Rightarrow \bar{\sigma}(t) = \begin{bmatrix} \bar{\sigma}_1(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix} = HS_aX_a(t) \tag{24}$$

where $H_{m\times m}$ is the Householder transformation matrix which is symmetric and orthogonal. Differentiating equation (22) and using equation (23) we will obtain the following expressions

$$\begin{aligned}
\dot{\bar{\sigma}}(t) &= HS_a\dot{X}_a(t) \\
&= HS_a\left[A_{ac}X_a(t) - B_a\Delta K_{as}X_a(t) + B_aV_p^* + B_aV_f^* + B_a\eta_p\left(t,\ X_a(t)\right) + f_{ad}\right] \\
&= H\,\mathrm{diag}\left[\lambda_{r1}, \lambda_{r2}, \ldots, \lambda_{rm}\right] \cdot \left[S_aX_a\right] \\
&\quad - \left[HS_aB_a\Delta K_{as}X_a(t) + HS_aB_a(V_p^* + V_f^*) + HS_a(B_a\eta_p(t, X_a(t) + f_{ad})\right] \\
&\quad (\mathrm{Note}, S_aA_{ac} = \mathrm{diag}\left[\lambda_{r1}, \lambda_{r2}, \ldots, \lambda_{rm}\right] S_a).
\end{aligned}$$

Now, a Singular–Value–Decomposition is employed on S_a which will convert the above equation in the following form

$$\begin{aligned}
\dot{\bar{\sigma}}(t) \;\; = \;\; & H\,\mathrm{diag}\left[\lambda_{r1}, \lambda_{r2}, \ldots, \lambda_{rm}\right]\left[WDV^TX_a(t)\right] - \left[HB_2\Delta K_{as}VV^TX_a(t)\right. \\
& \left. + HB_2V_p^* + HB_2V_f^* + HB_2\eta_p\left(t,\ X_a(t)\right) + HS_af_{ad}(t)\right]
\end{aligned}$$

where $W_{m\times m}$ and $V_{(n+p)\times(n+p)}$ are the orthogonal matrices and $D_{m\times(n+p)}$ is the rectangular matrix with diagonal elements are the singular values ($\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m \geq 0$) of S_a [8]. The main idea of using the Householder transformation technique is to obtain reaching phase conditions in a simpler form by exploiting the structure and properties of both D and H matrices and moreover, with a view to reduce the number of switching gain vector components. Using the properties of D and H, the above equation is rewritten as

$$\dot{\bar{\sigma}}(t) = \overline{H}D\overline{X}_a(t) - \overline{\Delta K}_{as}^*\overline{X}_a(t) + \overline{V}_p^* + \overline{V}_f^* + \overline{\eta}_p^* + \overline{f}_{ad}^* \tag{25}$$

where

$$\overline{H} = H\,\mathrm{diag}\left[\begin{matrix}\lambda_{r1} & \lambda_{r2} & \cdots & \lambda_{rm}\end{matrix}\right]W, \quad \overline{\Delta K}_{as}^* = HB_2\Delta K_{as}V, \quad \overline{V}_p^* = HB_2V_p^*,$$

$$\overline{V}_f^* = HB_2V_f^*, \quad \overline{\eta}_p^* = HB_2\eta_p\left(t,\ X_a(t)\right), \quad \overline{f}_{ad}^* = HS_af_{ad}$$

and the transformed state is given by $\overline{X}_a(t) = V^TX_a(t)$.

The equation (25) can be rewritten in different form as

$$\dot{\bar{\sigma}}_1(t) = \sum_{k=1}^{m}\left(\alpha_k \overline{H}_{1k} - \overline{\Delta K}^*_{as,1k}\right)\bar{x}_{a,k}(t) - \sum_{j=m+1}^{n+p} \overline{\Delta K}^*_{as,ij}\bar{x}_{a,j}(t) + \overline{V}^*_{p,1} + \overline{V}^*_{f,1} + \bar{\eta}^*_{p,1} + \overline{f}^*_{ad,1}$$

$$0 = \sum_{k=1}^{m}\left(\alpha_k \overline{H}_{ik} - \overline{\Delta K}^*_{as,ik}\right)\bar{x}_{a,k}(t) - \sum_{j=m+1}^{n+p} \overline{\Delta K}^*_{as,ij}\bar{x}_{a,j}(t) + \overline{V}^*_{p,i} + \overline{V}^*_{f,i} + \bar{\eta}^*_{p,i} + \overline{f}^*_{ad,i} \quad i = 2,\dots,m. \tag{26}$$

A sufficient condition for the existence of sliding motion in the vicinity of the sliding surface $\sigma(t) = 0$ is

$$\sigma^T(t)\dot{\sigma}(t) < 0 \Rightarrow \sigma^T(t)H^T H\dot{\sigma}(t) < 0 \tag{27}$$

(since H is a Householder matrix)

$$\Rightarrow \quad \bar{\sigma}^T(t)\dot{\bar{\sigma}}(t) < 0 \Rightarrow \bar{\sigma}_1^T(t)\dot{\bar{\sigma}}_1(t) < 0. \tag{28}$$

It should be pointed out that the Householder transformation matrix H is a function of time and thus the transformed sliding surface equation is also time varying. To formulate a switching control law that assures condition (28), we need to satisfy the following inequality conditions

(i) $\overline{\Delta K}^*_{as,1k} = \begin{cases} mf \cdot |\overline{H}_{1k}\alpha_k| & \text{for } \bar{\sigma}_1\bar{x}_{a,k}(t) > 0 \\ -mf \cdot |\overline{H}_{1k}\alpha_k| & \text{for } \bar{\sigma}_1\bar{x}_{a,k}(t) < 0 \end{cases}$ for $k = 1, 2, \dots, m$,
where $mf > 1$ is the multiplying factor.

(ii) $\overline{\Delta K}^*_{as,1j} = \begin{cases} \geq 0 & \text{for } \bar{\sigma}_1\bar{x}_{a,j}(t) > 0 \\ \leq 0 & \text{for } \bar{\sigma}_1\bar{x}_{a,j}(t) < 0 \end{cases}$ $j = (m+1), (m+2), \dots, (n+p)$

(iii) $\overline{V}^*_{p,1} = \begin{cases} -\|HB_2\|\,\|\eta_p(t, X_a)\|_2 & \text{for } \bar{\sigma}_1 > 0 \\ \|HB_2\|\,\|\eta_p(t, X_a)\|_2 & \text{for } \bar{\sigma}_1 < 0 \end{cases}$

(iv) $\overline{V}^*_{f,1} = \begin{cases} -\|\overline{f}^*_{ad}\|_\infty & \text{for } \bar{\sigma}_1 > 0 \\ \|\overline{f}^*_{ad}\|_\infty & \text{for } \bar{\sigma}_1 < 0 \end{cases}$

(v) $\overline{\Delta K}^*_{as,ik} = \alpha_k \overline{H}_{ik}$ for $i = 2, 3, \dots, m$ and for $k = 1, 2, \dots, m$.

(vi) $\overline{\Delta K}^*_{as,ij} = 0$ for $i = 2, 3, \dots, m$ and for $j = (m+1), (m+2), \dots, (n+p)$
since $\bar{\sigma}_i = 0$, $\bar{\sigma}_i\dot{\bar{\sigma}}_i = 0$ for $i = 2, 3, \dots, m$.

$$\text{(vii)} \quad \left.\begin{array}{l} \overline{V}^*_{p,i} = 0 \\ \overline{V}^*_{f,i} = 0 \end{array}\right\}$$

where $|g|_\infty = \max_i |g_i|$, $\|\eta_p\,(t,\; X_a)\|_2$ are the infinity norm and Euclidean norm of a vector, respectively and $\|H\,B_2\|$ is the spectral norm of the matrix HB_2.

Consider the equations (16) with assumption II and the upper bound of the norm $\|\eta_p\,(t,\; X_a)\|$ is synthesized as

$$\|\eta_p\,(t,\; X_a)\| \leq \rho(t,\; X_a) = c_0(t,\; X_a) + c_1(t,\; X_a)\|X_a\|_2 \tag{29}$$

where, $c_0(t, X_a)$ and $c_1(t, X_a)$ are parameters. These parameters are computed using the following dynamic equations [12].

$$\dot{c}_0(t, X_a) = q_0\|B_a^T S_a^T \sigma\|_2 \tag{30}$$

$$\dot{c}_1(t,\; X_a) = q_1\|B_a^T S_a^T \sigma\|\;\|X_a\|_2. \tag{31}$$

It may be noted that the matrix ΔK_{as} can be expressed in terms of transformed switching gain matrix $\Delta K_{as} = (HB_2)^{-1}\overline{\Delta K^*_{as}}V^T$. It is important to note that the reachability condition for multi-input system with a state feedback switch gain using Householder technique can be obtained by adopting only 2^j reduced switching control actions and moreover it does not require any stringent condition need to be satisfied to ensure reachability. A detail comparative study on number of switching and additional stringent conditions between the proposed method and White et al. method [11] is given in the table.

Table 1. Comparative study on number of switching and stringent condition.

Description	Proposed Method (Householder)	White et al. Method
Number of states $= n$ Number of inputs $= m$ Number of switching structures: (a) Full switching for gain $\overline{\Delta K^*_{as}}$ (b) Reduce switching for gain $\overline{\Delta K^*_{as}}$ (up to jth component, $j \geq m$)	 2^n 2^j	 2^{mn} 2^{mj} Inequality conditions should be satisfied.

3.1. Equivalent control law in presence of disturbances and system uncertainties

Equivalent control determines the behavior of the system restricted to the switching surface and a necessary condition for the state trajectory to remain on the sliding surface σ is $\dot{\sigma}\,(t, X_a(t)) = 0$. The motion in the sliding mode may be determined by differentiating (20) with respect to time and inserting the value of $\dot{X}_a$ given in (22) gives

$$\dot{\sigma} = S_a\left[A_a X_a(t) + B_a U_{\text{eq}}(t) + B_a \eta_p(t,\ X_a) + f_{ad}\right] = 0 \tag{32}$$

and equivalent control law in the sliding mode is obtained from (32) as

$$\begin{aligned} U^*_{\text{eq}}(t) &= -(S_aB_a)^{-1}\left[S_aA_aX_a(t) + S_aB_a\eta_p(t,\ X_a) + S_af_{ad}\right] \\ &\Rightarrow (I_m + E)U^*_{\text{eq}}(t) = -\left[(S_aB_a)^{-1}(S_aA_aX_a + S_af_{ad}) + F_aX_a(t)\right] \\ &\Rightarrow U^*_{\text{eq}}(t) = -(I_m + E)^{-1}\left[(S_aB_a)^{-1}(S_aA_aX_a + S_af_{ad}) + F_aX_a(t)\right] \end{aligned} \tag{33}$$

where $F_a = \left[\ F_1^T \quad 0_{m\times p} \quad F_2^T\ \right]^T_{m\times(n+p)}$.

Using the following relation [8] $\frac{1}{1+\|E\|} \le \|(I_m + E)^{-1}\| \le \frac{1}{1-\|E\|}$ in equation (33), we obtain

$$U^*_{\text{eq}}(t) = -\frac{\gamma}{1-\mu}\times \left[(S_aB_a)^{-1}(S_aA_aX_a + S_a\begin{bmatrix}\operatorname{sign}(f_{ad,1})|f_{ad,1}|_{\max} \\ \operatorname{sign}(f_{ad,2})|f_{ad,2}|_{\max} \\ \vdots \\ \operatorname{sign}(f_{ad,n+p})|f_{ad,n+p}|_{\max}\end{bmatrix} + F_aX_a(t))\right] \tag{34}$$

where $\|E\|$ is the spectral norm of E, $|f_{ad,i}|_{\max}$ is the upper bound of $|f_{ad,i}|$ and $0 < \gamma < 1$. Here, we need to adjust γ in such away so that the control law $U_{\text{eq}}(t)$ will drive the states on the sliding surface and the corresponding control law (34) can then be expressed in terms of states $X(t)$ and rewritten in the following form

$$\begin{aligned} U^*_{\text{eq}}(t) = -\alpha\Bigg\{&[L_1\, L_2]\begin{bmatrix} X_1(t) \\ X_2(t)\end{bmatrix} \\ &+ \sum_{i=1}^{n-m} S_{1,i}\operatorname{sign}(f_{ad,i})\,|f_{ad,i}|_{\max} + \sum_{j=n-m+1}^{n} e_j\operatorname{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\Bigg\} \\ = -\alpha&\left\{LX(t) + \sum_{i=1}^{n-m} S_{1,i}\operatorname{sign}(f_{ad,i})\,|f_{ad,i}|_{\max} + \sum_{j=n-m+1}^{n} e_j\operatorname{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\right\} \\ = -\alpha&LX(t) - \alpha\left\{\sum_{i=1}^{n-m} S_{1,i}\operatorname{sign}(f_{ad,i})\,|f_{ad,i}|_{\max} + \sum_{j=n-m+1}^{n} e_j\operatorname{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\right\} \\ = -\alpha&U_{\text{eq}}(t) - \alpha\left\{\sum_{i=1}^{n-m} S_{1,i}\operatorname{sign}(f_{ad,i})\,|f_{ad,i}|_{\max} + \sum_{j=n-m+1}^{n} e_j\operatorname{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\right\} \end{aligned} \tag{35}$$

where $L_i = (B_2)^{-1}(S_1A_{1,i} + WC_i + A_{2,i}) + F_i,\ i = 1, 2,\ e_j$ is the unit vector whose jth element is 1 and $\alpha = \frac{\gamma}{1-\mu}$.

3.2. Stability study on the sliding surface based on LMI technique

Theorem. The uncertain system in equation (15) is boundedly stable on the sliding surface σ if the following inequality conditions

$$\begin{array}{c} \left[\ (A-\alpha BL)^T P + P(A-\alpha BL) + \gamma Q + 2\beta PPB \ B^T P \gamma I \ \right] < 0 \\ 2\beta\lambda_{\min}(P) + \gamma\lambda_{\min}(Q) > \frac{1}{\gamma}\lambda_{\max}(PBB^TP) + \frac{\|f_d^T\|_2 \|P\|_2}{\|X\|_2} \end{array} \tag{36}$$

are satisfied, where β and γ are the positive constants and α is the tuning parameter of the control law (35).

Proof. Consider a Lyapunov function candidate $V(X(t)) = X^T(t)PX(t)$ of the system (15). Taking derivative of $V(X(t))$ along the sliding trajectories, using assumptions I, II and combining with (35), we obtain

$$\begin{aligned} \dot{V}(X(t)) &= X^T(t)(A^TP + PA)X(t) + 2U_{\text{eq}}^{*T} B^T PX(t) \\ &\quad + 2\eta_p^T B^T PX(t) + 2f_d^T PX(t) \\ &= X^T(t)(A^TP + PA)X(t) - 2\alpha(LX(t) + M)^T B^T PX(t) \\ &\quad + 2\eta_p^T B^T PX(t) + 2f_d^T PX(t) \end{aligned}$$

where 'M' is assumed as equal to the 2nd part of the right hand side of equation (35).

$$\begin{aligned} \dot{V}(X(t)) &= X^T(t)(A^TP + PA)X(t) - 2\alpha X^T(t)L^TB^TPX(t) \\ &\quad + 2(\eta_p^T - \alpha M^T)B^TPX(t) + 2f_d^TPX(t) \\ &= X^T(t)(A^TP + PA)X(t) - 2\alpha X^T(t)L^TB^TPX(t) \\ &\quad + 2\bar{\eta}_p^T B^TPX(t) + 2f_d^TPX(t) \end{aligned}$$

where $\bar{\eta}_p^T = (\eta_p - \alpha M)^T B^T$. From the above equation, we obtain

$$\begin{aligned} &\dot{V}(X(t)) \le X^T(t)\left(A^TP + PA - 2\alpha X^T(t)L^TB^TPX(t) + 2\beta P\right. \\ &\quad \left. + \gamma Q - \gamma^{-1}PBB^TP) - \gamma^{-1}PBB^TP\right) X(t) \\ &\quad + X^T(t)(\gamma^{-1}PBB^TP - 2\beta P - \gamma Q)\, X(t) + \bar{f}_d^T PX(t), \end{aligned} \tag{37}$$

where $Q > 0$ and $\bar{f}_d^T = 2\bar{\eta}_p^T + 2f_d^T$

$$\begin{aligned} &\dot{V}(X(t)) \le X^T(t)\left(A^TP + PA - 2\alpha X^T(t)L^TB^TPX(t)\right. \\ &\quad \left. + 2\beta P + \gamma Q - \gamma^{-1}PBB^TP\right) X(t) \\ &\quad + \left[\lambda_{\max}(\gamma^{-1}PBB^TP) - (2\beta\lambda_{\min}(P) + \gamma\lambda_{\min}(Q)) + \frac{\|\bar{f}_d^T\|_2 \|P\|_2}{\|X\|_2}\right] \|X\|_2^2. \end{aligned} \tag{38}$$

Examination of equation (39) revels that sufficient conditions for $\dot{V} < 0$ are

$$\begin{bmatrix} (A-\alpha BL)^TP + P(A-\alpha BL) + \gamma Q + 2\beta P & PB \\ B^TP & \gamma I \end{bmatrix} < 0 \tag{39}$$

$$2\beta\lambda_{\min}(P) + \gamma\lambda_{\min}(Q) > \frac{1}{\gamma}\lambda_{\max}(PBB^TP) + \frac{\|\bar{f}_d^T\|_2 \|P\|_2}{\|X\|_2}. \tag{40}$$

It can be noted that the solution of LMI equation (39) along with the above additional condition ensures $\dot{V} < 0$. Thus, we conclude that the sliding mode state trajectories of the uncertain system (15) and (16) under the equivalent control action (35) are robustly asymptotically stable. Thus, we have successfully developed a new constructive reaching phase design based on Householder method and subsequently the stability condition for completely uncertain system is established. Hence, the system is boundedly stable. □

4. REACHING MODE GAIN COMPONENTS ($\overline{\Delta K}^*_{as,ij}$) BASED ON FUZZY LOGIC APPROACH

Fuzzy logic control (FLC) has recently proved to be a successful control approach for complex system. It is well known that each control method always has its advantages and drawbacks, or we can say that all control techniques have their individual characteristic features. Combining several control theories to design a new controller may have possibly better system performance than one based on single control theory only. In this section, the design of switch gain control components based on fuzzy logic approach with sliding modes is adopted with a view to achieve good dynamic system response and smooth control action. Here we recall the reaching phase control law (21) for our convenience and ready reference.

$$U(t) = -(K_{af} + \Delta K_{as}) X_a(t) + V_p^* + V_f^* \tag{41}$$

where the feedback gain K_{af} is kept constant, but the proper choice of fuzzy switching gain ΔK_{as} can accelerate the state trajectories to reach the sliding hyper plane, and thus the dynamic performances will may be improved. The function of each part of the control (21) is already discussed in detail in Section 2. Now, we consider the design procedure of the fuzzy switching gain matrix $\overline{\Delta K}^*_{as}$ as a part of the control signal that will drive the state trajectories from any initial state condition to the sliding surface.

4.1. Design of switched gain matrix elements $\overline{\Delta K}^*_{as,ij}$ based on fuzzy logic approach

We have considered $\bar{\sigma}_1\bar{x}_{a,x}$ (see equation (26)) as the linguistic input variable and linguistic variable $\overline{\Delta K}^*_{as,1k}$ is quantized into six linguistic variables. The universe of discourse for each membership function is selected based on some trails and these are shown in Figure 1.

Based on the expressions (25) – (26) (derived in the previous section), we calculate fuzzy logic based switch gain matrix elements $\overline{\Delta K}^*_{as,1k}$ using the following decision rules. For $k = 1, 2, \ldots, m$:

R_1: If $\bar{\sigma}_1\bar{x}_{a,k}$ is PL then $\overline{\Delta K}^*_{as,1k}$ is PL
R_2: If $\bar{\sigma}_1\bar{x}_{a,k}$ is PM then $\overline{\Delta K}^*_{as,1k}$ is PM.
R_3: If $\bar{\sigma}_1\bar{x}_{a,k}$ is PZ then $\overline{\Delta K}^*_{as,1k}$ is PS.
R_4: If $\bar{\sigma}_1\bar{x}_{a,k}$ is NZ then $\overline{\Delta K}^*_{as,1k}$ is NS.

R_5: If $\bar{\sigma}_1\bar{x}_{a,k}$ is NM then $\overline{\Delta K}^*_{as,1k}$ is NM.
R_6: If $\bar{\sigma}_1\bar{x}_{a,k}$ is NL then $\overline{\Delta K}^*_{as,1k}$ is NL.

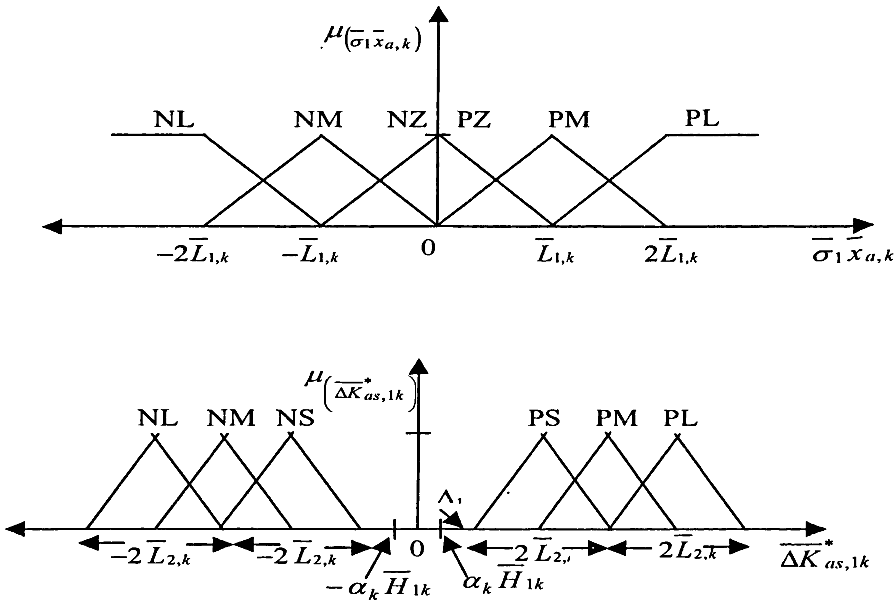

Fig. 1. Membership functions for each input and output.

Defuzzification. The crisp output $\overline{\Delta K}^*_{as,ij}$ is obtained by choosing the center-of-area (centroid) defuzzification method and it is given by

$$\overline{\Delta K}^*_{as,ij} = \frac{\int \mu_{\left(\overline{\Delta K}^*_{as,ij}\right)} \overline{\Delta K}^*_{as,ij}\, d\left(\overline{\Delta K}^*_{as,ij}\right)}{\int \mu_{\left(\overline{\Delta K}^*_{as,ij}\right)} d\left(\overline{\Delta K}^*_{as,ij}\right)}. \tag{42}$$

It has been observed that a large switching gain with proper sign of $\overline{\Delta K}^*_{as,ij}$ will drive the state trajectories to the sliding surface rapidly. Transformed switch gain matrix $\overline{\Delta K}^*_{as,ij}$ is then changed to original switch gain matrix $\overline{\Delta K}_{as}$ by using the proper matrix inverse HB_2 (as discussed in Section 3). Furthermore, when the state trajectories hitting the sliding surface an equivalent control law (35) is then applied to maintain the motion of the states along sliding hyper plane and ensures the trajectory remains on the surface once it gets there.

5. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed controllers in presence of parameter perturbation and external disturbances, a load-frequency control problem of two-area interconnected power system is considered. The nominal system is represented in the state space form by the equation

$$\dot{X}(t) = AX(t) + BU(t) + \Gamma d(t) \tag{43a}$$

$$Y(t) = CX(t) \tag{43b}$$

where $X(t) = \begin{bmatrix} \Delta f_1 & \Delta P_{g1} & \Delta X_{g1} & \Delta P_{tie} & \Delta f_2 & \Delta P_{g2} & \Delta X_{g2} \end{bmatrix}^T$.

Δf_1 and Δf_2 are the deviation in frequencies, ΔP_{tie} is the change in tie-line power, ΔP_{g1} and ΔP_{g2} are the change in turbine-generator outputs, ΔX_{g1} and ΔX_{g2} are the change in outputs of the governors. Furthermore,

$$U = \begin{bmatrix} \Delta P_{c1} & \Delta P_{c2} \end{bmatrix}^T, \quad d = \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T.$$

Area-control error in Area-1 $\text{ACE}_1 = \Delta f_1 + \Delta P_{tie}$ and in Area-2 $\text{ACE}_2 = \Delta f_2 - \Delta P_{tie}$ are the outputs of the composite system. The following are the nominal system matrices:

$$A = \begin{bmatrix} -\frac{1}{T_{P1}} & \frac{K_{P1}}{T_{P1}} & 0 & -\frac{K_{P1}}{T_{P1}} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 \\ T_{12}^* & 0 & 0 & 0 & -T_{12}^* & 0 & 0 \\ 0 & 0 & 0 & \frac{K_{P2}}{T_{P2}} & -\frac{1}{T_{P2}} & \frac{K_{P2}}{T_{P2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{R_2 T_{G2}} & 0 & -\frac{1}{T_{G2}} \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & \frac{1}{T_{G1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{G2}} \end{bmatrix} \quad \Gamma^T = \begin{bmatrix} -\frac{K_{P1}}{T_{P1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{K_{P1}}{T_{P1}} & 0 & 0 \end{bmatrix}.$$

The following nominal parameters are used [5]:

$T_P = T_{P1} = T_{P2} = 20.0$ s; $T_T = T_{T1} = T_{T2} = 0.3$ s; $T_G = T_{G1} = T_{G2} = 0.08$ s;

$K_P = K_{P1} = K_{P2} = 120$ Hz / p.u.MW; $R = R_1 = R_2 = 2.4$ Hz / p.u.MW.

There are always errors present in such models due to linearization, unmodelled dynamics, etc. Moreover, the power system operating conditions change with time leading to changes in system linearized parameters and the following range of system parameter variations are considered:

$$\frac{1}{T_P} \in [\ 0.025 \quad 0.075\ ], \quad \frac{K_P}{T_P} \in [\ 3.0 \quad 9.0\ ], \quad \frac{1}{T_T} \in [\ 2.333 \quad 4.333\ ]$$

$$\frac{1}{RT_G} \in [\ 2.6041 \quad 7.8124\ ], \quad \frac{1}{T_G} \in [\ 8.75 \quad 16.25\ ]$$

The nominal system matrices are as follows:

$$A = \begin{bmatrix} -0.05 & 6.0 & 0.0 & -6.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.33 & 3.33 & 0.0 & 0.0 & 0.0 & 0.0 \\ -5.2083 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 6.0 & -0.05 & 6.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.33 & 3.33 \\ 0.0 & 0.0 & 0.0 & 0.0 & -5.2083 & 0.0 & -12.5 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0.0 & 0.0 & 12.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 \end{bmatrix}$$

$$\Gamma^T = \begin{bmatrix} -6.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -6.0 & 0.0 & 0.0 \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}.$$

Note that our nominal system is not in a regular form of equation (18) and (19). One can use a suitable state transformation to get the desired form. In the present example, the states are rearranged to obtain the system description in regular form and it is given by

$$\overline{X}(t) = [\ \Delta f_1 \quad \Delta P_{g1} \quad \Delta P_{g2} \quad \Delta P_{tie} \quad \Delta f_2 \quad \Delta X_{g1} \quad \Delta X_{g2}\ ]^T.$$

For the present example, we note that the physical interpretation of the states is remaining same after transformation and in general, it is not true. Corresponding transformed nominal system matrices are

$$\overline{A} = \begin{bmatrix} -0.05 & 6.0 & 0.0 & -6.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.33 & 0.0 & 0.0 & 0.0 & 3.33 & 0.0 \\ 0.0 & 0.0 & -3.33 & 0.0 & 0.0 & 0.0 & 3.33 \\ 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 \\ 0.0 & 0.0 & 6.0 & 6.0 & -0.05 & 0.0 & 0.0 \\ -5.2083 & 0.0 & 0.0 & 0.0 & 0.0 & -12.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -5.2083 & 0.0 & -12.5 \end{bmatrix}$$

$$\overline{B}^T = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 \end{bmatrix}$$

$$\overline{\Gamma}^T = \begin{bmatrix} -6.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -6.0 & 0.0 & 0.0 \end{bmatrix}$$

and

$$\overline{C} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}.$$

Define $\overline{X}_1^T = [\ \Delta f_1 \quad \Delta P_{g1} \quad \Delta P_{g2} \quad \Delta P_{tie} \quad \Delta f_2\ ]$ and $\overline{X}_2^T = [\ \Delta X_{g1} \quad \Delta X_{g2}\ ]$.

Case A: It should be pointed out that if the original system parameters are free from perturbation and not excited by external disturbance then it is sufficient to design a P-type sliding surface. When the system state trajectory comes on the sliding surface the closed loop dynamics are described by reduced order model (8). The state feedback control gain S_1 of the reduced order model (8) (switching function) can be found out by minimizing the performance index

$$J = \int_0^t (\overline{X}_1^T Q \overline{X}_1 + \overline{X}_2^T R \overline{X}_2)\,\mathrm{d}t \tag{44}$$

where $Q = 15 \cdot I_{5\times 5}$ and $R = 10 \cdot I_{2\times 2}$.

The resulting value of switching surface gain matrix

$$S = \begin{bmatrix} 1.2053 & 1.6153 & -0.0016 & -0.8141 & -0.0023 & 1 & 0 \\ -0.0023 & -0.0016 & 1.6153 & 0.8141 & 1.2053 & 0 & 1 \end{bmatrix}. \tag{45}$$

The equivalent control law (10) is give by

$$U_{\mathrm{eq}}(t) = -\begin{bmatrix} -0.4568 & 0.1479 & -0.0007 & -0.5797 & 0.0355 & -0.5693 & -0.0004 \\ 0.0355 & -0.0007 & 0.1479 & 0.5797 & -0.4568 & -0.0004 & -0.5693 \end{bmatrix} \overline{X}(t). \tag{46}$$

The range space eigenvalues are located at 0.4 and 0.3 that are unstable and the corresponding fixed gain matrix is given by

$$K_f = \begin{bmatrix} -0.4954 & 0.0962 & -0.0006 & -0.5536 & 0.0356 & -0.6013 & -0.0004 \\ 0.0356 & -0.0006 & 0.1091 & 0.5601 & -0.4857 & -0.0004 & -0.5933 \end{bmatrix}.$$

To satisfy the reaching conditions (28) based on Householder method the elements of transformed switched gain matrix ($\overline{\Delta K}^*_{as}$) are functions of time and they are shown in Figures 5–6.

Computation of $\overline{\Delta K}^*_{as,1k}$ based on fuzzy logic approach (see Figure 1):
Width of input ($\bar{\sigma}_i \bar{x}_{a,k}$) membership function, $2\overline{L}_{1,k} = 2,\ k = 1, 2, \ldots, m$.
Width of output ($\overline{\Delta K}^*_{as,1k}$) membership function, $2\overline{L}_{2,k} = 2,\ k = 1, 2, \ldots, m$.

The computer simulation of the composite system has been performed taking a initial state disturbance of $\overline{X}(0) = [\ 0.5\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ ]^T$. Design of switch gain matrix based on fuzzy logic approach (soft computing) is compared with that of hard switching gain matrix and comparison of system responses using the proposed control strategies are shown in Figures 2–6. It is observed that the switched gain components designed based on fuzzy logic approach are much smooth than the hard switched gain components (see Figures 5–6). As a result, the frequency deviation and area-control error responses based on soft switching seem to be better than the hard switching control actions (see Figure 2).

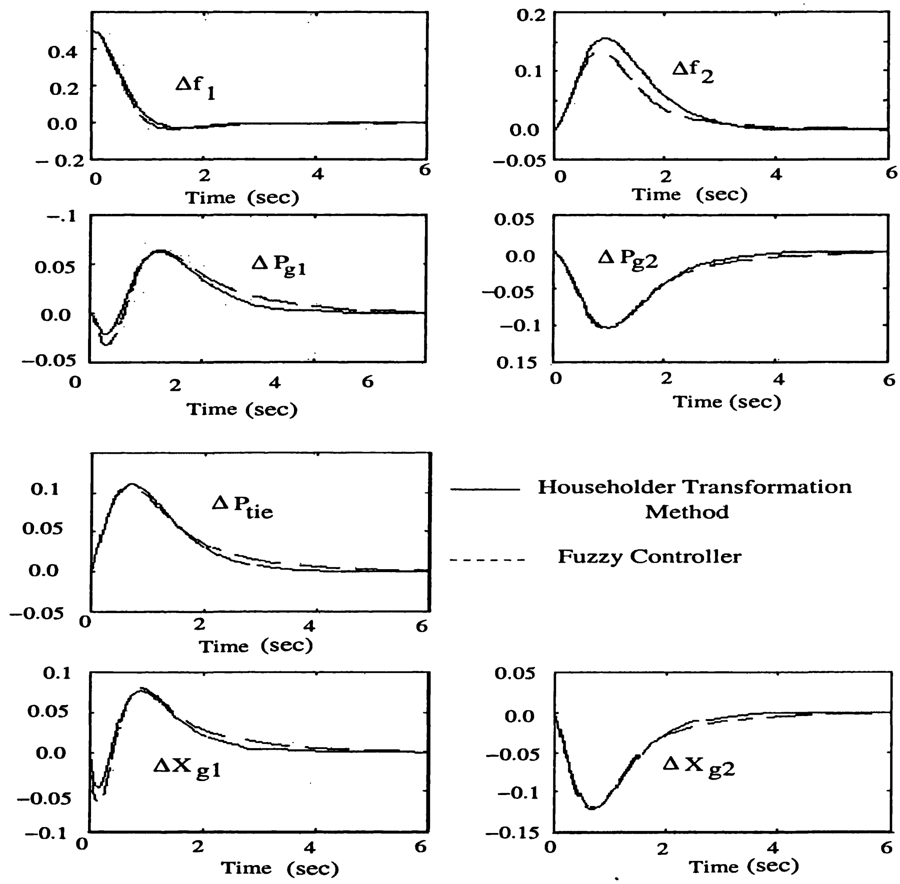

Fig. 2. Comparison of nominal system responses with state disturbance only.

Case B: A PI-type sliding is chosen as in equation (20) when the system response is affected with parameter perturbations and external disturbances. Switching function is then designed by adopting the procedure as discussed in Section 2. Selecting the value of $Q_a = 15I_{7\times 7}$ and $R_a = 10I_{2\times 2}$, the corresponding sliding surface gain matrices are

$$S = \begin{bmatrix} 1.5874 & 1.8662 & 0.0167 & -1.0052 & 0.0266 & 1 & 0 \\ 0.0266 & 0.0167 & 1.8662 & 1.0052 & 1.5874 & 0 & 1 \end{bmatrix}$$

and

$$W = \begin{bmatrix} 1.2247 & 0 \\ 0 & 1.2247 \end{bmatrix}.$$

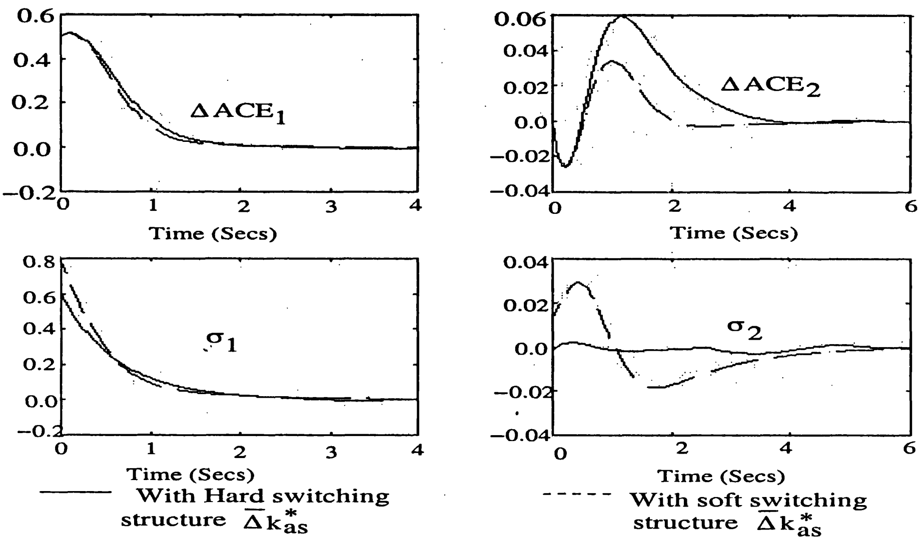

Fig. 3. Area control errors and sliding surface trajectories.

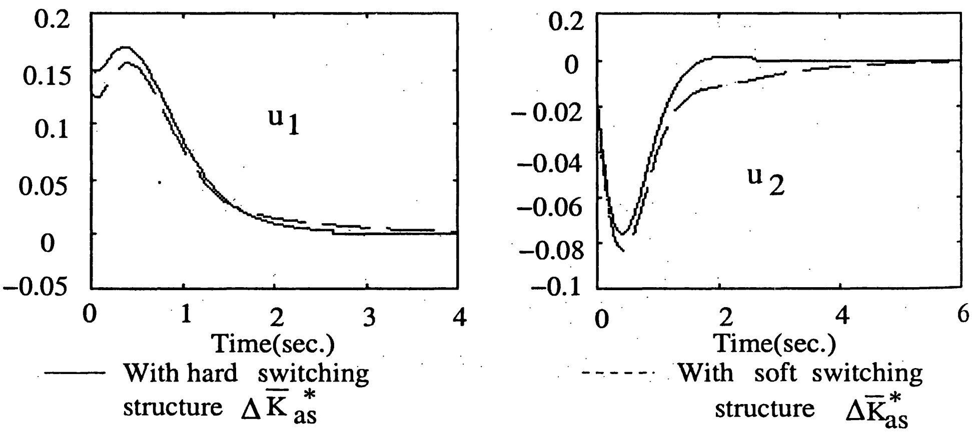

Fig. 4. Comparison of control input structure.

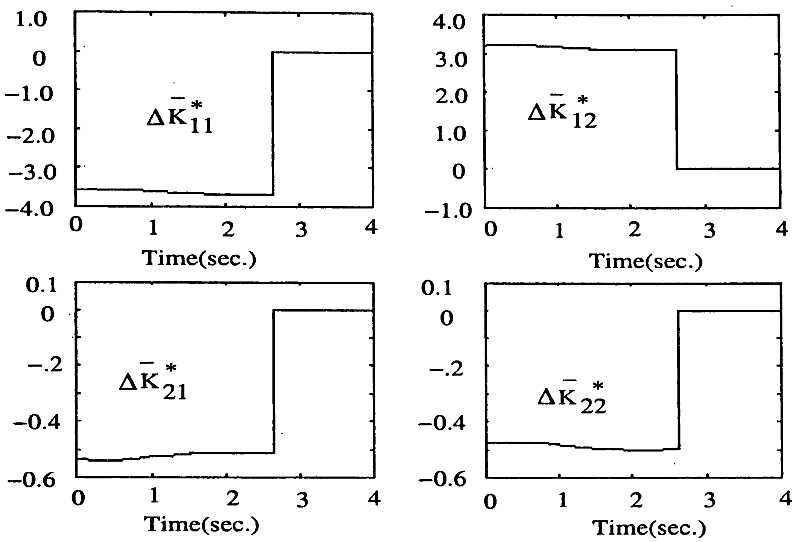

Fig. 5. Hard switching structures (Householder method) $\Delta\overline{K}^*_{as}$.

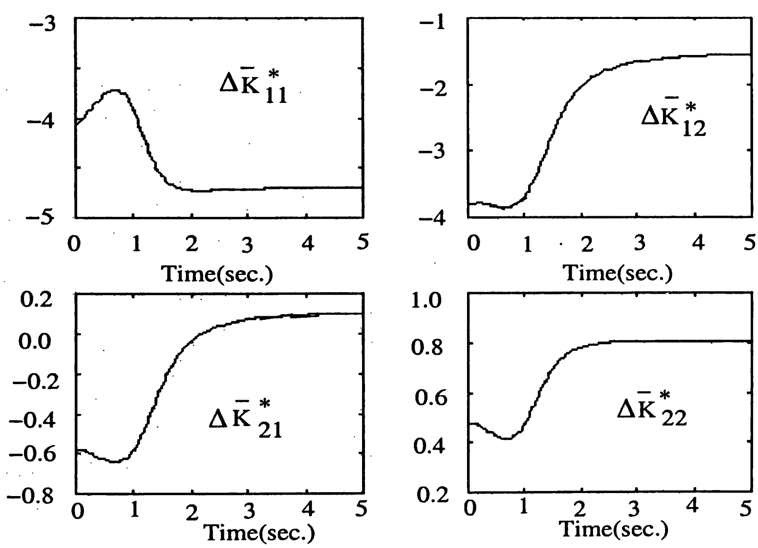

Fig. 6. Soft switching structures (fuzzy logic controller) $\Delta\overline{K}^*_{as}$.

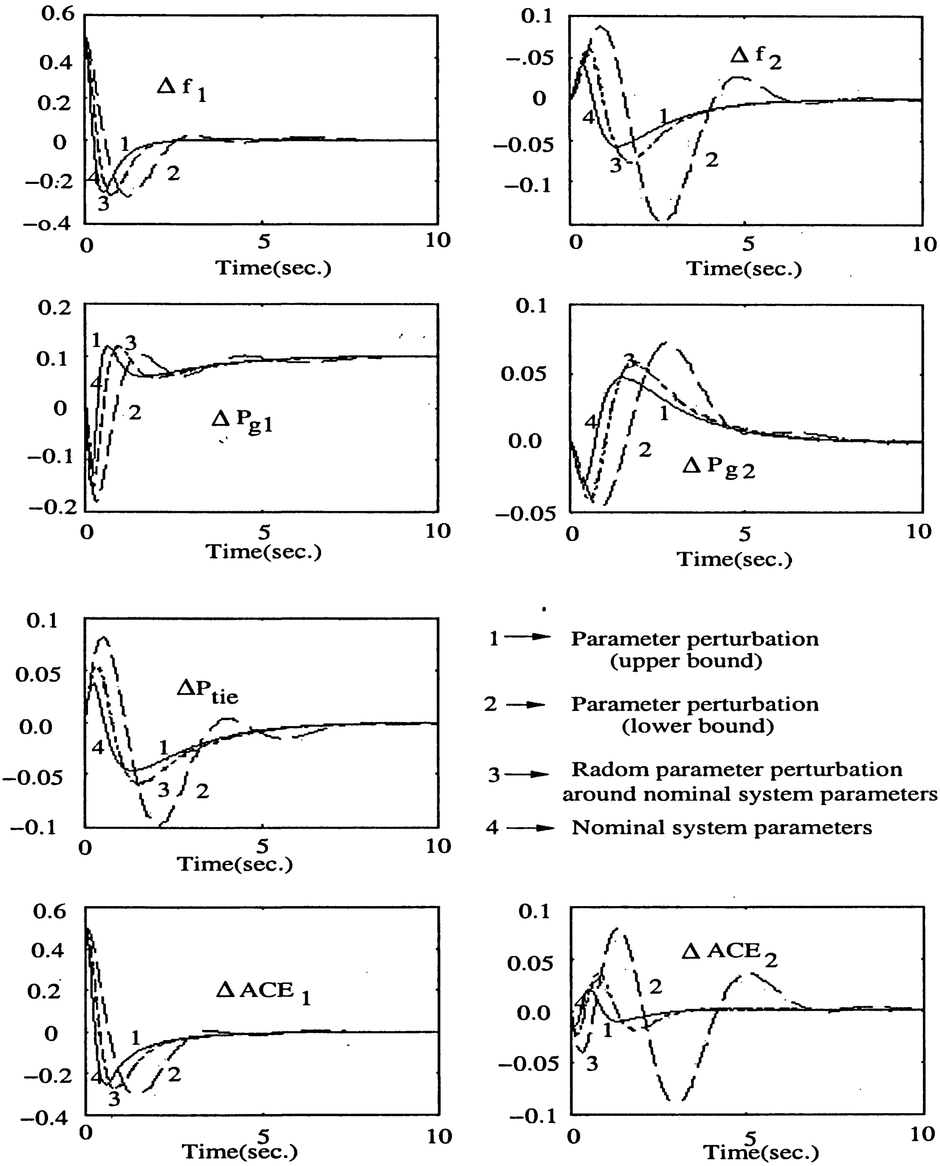

Fig. 7. System responses for state disturbance and 10 % step change in the load demand in area 1 (Householder Method).

The range space eigenvalues are placed at 0.4 and 0.3 and the corresponding fixed gain matrix K_{af} of the augmented system is obtained using the expression $S_{ai}A_{ai} = \lambda_{ri}S_{ai}$, $i = 1, 2, \ldots, m$ (see Section 2, after equation (22))

$$K_{af} = \begin{bmatrix} -0.4195 & 0.2046 & 0.0078 & -0.6190 & 0.0429 & -0.0392 & 0 & -0.5344 & 0.0045 \\ 0.0431 & 0.0079 & 0.2196 & 0.6271 & -0.4068 & 0 & -0.0294 & 0.0045 & -0.5264 \end{bmatrix}.$$

The equivalent control law (U^*_{eq}) (see equation (35)) is employed to the system while the states slide along the sliding surface and does not satisfy the matching condition. To satisfy the reaching conditions (28) based on Householder method, the time varying switched gain matrix is designed and subsequently the reaching conditions are satisfied.

Simulation results are shown with an initial state disturbance of

$$\overline{X}(0) = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

and 10 % step change in load demand in area-1. Performance of the system based on the proposed variable structure control schemes has been studied qualitatively and system responses are shown in Figure 7, which proves the robustness of the designed techniques. An equivalent control law $U^*_{\text{eq}}(t)$ is employed (see equation (35)) while the system trajectory hits the sliding surface to maintain the state trajectory on the sliding surface. The term U_{eq} in equation (35) is given by

$$U_{\text{eq}}(t) = -\begin{bmatrix} -0.3687 & 0.2644 & 0.0083 & -0.6512 & 0.0437 & -0.5824 & 0.0845 \\ 0.0437 & 0.0083 & 0.2644 & 0.6512 & -0.5770 & 0.0045 & -0.8024 \end{bmatrix} \overline{X}(t).$$

6. CONCLUSIONS

A new moving sliding surface based on Householder method is considered for application to higher order variable structure systems. The proposed method requires less number of switching gain vector elements as compared to that of White et al. method [8]. The proposed method does not require to satisfying any additional inequality constraints to meet the reaching condition. Design of transformed switch gain matrix elements based on fuzzy logic approach also provides useful result for smooth control action. It is to be noted that the Householder based VSS with sliding mode controller increases on-line computation burden but works satisfactorily while disturbance-matching condition is not satisfied. The simulation results reveal the system dynamics remain insensitive to the parametric uncertainties and external disturbances only after the system reaches the sliding surface.

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